Zero-phonon branches of the Bose spectrum in the A phase of a system of the He^3 type

P. N. Brusov

Leningrad Polytechnic Institute

V. N. Popov

Leningrad Division of the Steklov Mathematics Institute of the USSR Academy of Sciences
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We have investigated all the zero-phonon branches of the Bose spectrum in the A phase of a model Fermi system with pairing in the p-state. The Bose spectrum of the collective excitations is determined by a functional of the hydrodynamic action obtained by the functional-integration method. In the A phase, all the zero-phonon spectra $E(\mathbf{k})$ are complex even at $\mathbf{k}=0$, and the corresponding collective excitations attenuate moderately. The energies $E_1(0)=\Delta_0(1.96-i0.31)$ and $E_2(0)=\Delta_0(1.17=i0.13)$ are calculated.

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1. INTRODUCTION

We continue here an investigation of a the Bose spectrum of a model previously proposed[1] for He^3. In this model, the Bose spectrum is defined by a "hydrodynamic-action" functional obtained by the method of functional integration, after integrating over the Fermi fields. In a preceding paper[2] we investigated the phonon branches of the spectrum and calculated the corrections to the linear dispersion law, while in Ref. 3 we obtained all the zero-phonon Bose branches in the B-phase of the model.

In this paper we investigated all the zero-phonon Bose branches in the A phase of the model at $T=0$. The main result is that the energy $E(k)$ of all the zero-phonon Bose excitations in the A phase are complex, while the imaginary parts differ from zero even at $\mathbf{k}=0$. This result is physically obvious and is due to the possibility of the decay of a Bose excitation into two fermions. A Bose excitation with nonzero energy and with small momentum can always decay kinematically into two fermions whose momenta are almost opposite and close to the preferred directions along which the gap in the Fermi spectrum vanishes.

Zero-phonon branches of the Bose spectrum in the A phase of He^3 were considered in a number of studies.[4-8] The imaginary parts of the spectrum were calculated only in Ref. 8, but not for all the modes. In this paper we calculate the complex energies of all the zero-phonon Bose branches at zero momentum in the A phase of the model, using the technique developed in Refs. 1, 2, and 3.

The calculation of the zero-phonon Bose spectrum in the A phase is technically more complicated than in the B phase.[3] Even at $\mathbf{k}=0$, the equation $F(E)=0$ for the spectrum has in the left-hand side of $F(E)$ an integral that cannot be expressed in terms of elementary functions, while the roots of the equation turn out to be complex. They were calculated with a computer.

2. He^3 MODEL AND ZERO-PHONON BRANCHES OF THE SPECTRUM

The model system considered here[1] is determined by the hydrodynamic-action functional

$$S_h=g^{-1}\sum_{p,i,a}c_{ia}^{+}(p)c_{ia}(p)+\frac{1}{2}\ln\det\hat{M}(c,c^{+})/\hat{M}(0,0), \tag{2.1}$$

obtained by functional integration after integrating over the Fermi fields. In (2.1), $c_{ia}(p)$ is the Fourier transform of the Bose field $c_{ia}(\mathbf{x},\tau)$ with vector index i and isotopic index a, while M is the operator

$$\begin{pmatrix} Z^{-1}(i\omega-\xi+\mu H\sigma_3)\delta_{p_1p_2}, & (\beta V)^{-1/2}(n_{1i}-n_{2i})\sigma_a c_{ia}(p_1+p_2) \\ -(\beta V)^{-1/2}(n_{1i}-n_{2i})\sigma_a c_{ia}^{+}(p_1+p_2), & Z^{-1}(-i\omega+\xi+\mu H\sigma_3)\delta_{p_1p_2} \end{pmatrix}. \tag{2.2}$$

Here $\xi=c_F(k-k_F)$, $n_i=k_i/k_F$, H is the magnetic field, μ is the magnetic moment of the quasiparticle, σ_a $(a=1,2,3)$ are Pauli matrices, and $\omega=(2n+1)\pi T$ are the Fermi frequencies. The negative constant g is proportional to the amplitude of scattering of two fermions near the Fermi sphere, under the assumption that the amplitude is $g(\mathbf{k}_1-\mathbf{k}_2,\mathbf{k}_3-\mathbf{k}_4)$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the incoming fermions, and $\mathbf{k}_3$ and $\mathbf{k}_4$ are those of the outgoing ones. The method of obtaining the functional S_h is described in greater detail in Ref. 1.

The Bose spectrum is determined in first-order approximation by the quadratic part of S_h, which is different for the different superfluid phases. The quadratic part of S_h for the A phase of the model is a sum of three quadratic forms, the first of which depends on the variables c_{i1}, the second on c_{i2}, and the third on c_{i3}. The second and third form are transformed into the first by the substitutions $c_{i2}\to c_{i1}$ and $c_{i3}\to ic_{i2}$. The quadratic form of the variables c_{i1} is[2]

$$\sum_p\Big\{c_{i1}^{+}(p)c_{j1}(p)\Big[\frac{\delta_{ij}}{g}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}n_{1i}n_{1j}G_1G_2(\xi_1+i\omega_1)(\xi_2+i\omega_2)\Big]$$
$$+(c_{i1}^{+}(p)c_{j1}^{+}(-p)+c_{i1}(p)c_{j1}(-p))\frac{2\Delta_0^2Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1\pm in_2)^2n_{1i}n_{1j}G_1G_2\Big\}, \tag{2.3}$$

where

$$G(p)=(\omega^2+\xi^2+\Delta^2)^{-1},\qquad \Delta^2=\Delta_0^2(n_1^2+n_2^2)=\Delta_0^2\sin^2\theta. \tag{2.4}$$

Here Δ_0 is the maximum value of the energy gap of the Fermi spectrum. In the term $(n_1\pm in_2)^2$ of (2.3) the upper and lower signs are taken when the multiplication is by $c_{i1}^{+}c_{j1}^{+}$ and by $c_{i1}c_{j1}$, respectively.

We investigate now all the Bose-spectrum branches

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defined by (2.3) at zero momentum k. At $\mathbf{k}=0$ the form of the variables $c_{f1}(\omega, \mathbf{k}=0)$ and $c_{f1}^{+}(\omega, \mathbf{k}=0)$ is a sum of a form of c_{f1}, c_{11}^{+}, c_{21}, c_{21}^{+} and a form of c_{31}, c_{31}^{+}. The functions that are the coefficients of $c_{f1}^{+}c_{j1}$, $c_{f1}^{+}c_{j1}^{+}$, and $c_{f1}c_{j1}$ $(i,j=1,2)$ can be expressed in the form

$$\frac{\delta_{ij}}{g}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p} n_{1i}n_{1j}(\xi_1+i\omega_1)(\xi_2+i\omega_2)G_1G_2 = \frac{2\delta_{ij}Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1^2+n_2^2)[(\xi_1+i\omega_1)(\xi_2+i\omega_2)G_1G_2-G_1], \tag{2.5}$$

$$\frac{2\Delta_0^2Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1\pm in_2)^2 n_{1i}n_{1j}G_1G_2 = b_{ij}\frac{Z^2\Delta_0^2}{2\beta V}\sum_{p_1+p_2=p}(n_1^2+n_2^2)G_1G_2.$$

Here b_{ij} $(i,j=1,2)$ are the elements of the matrix

$$\begin{pmatrix} 1 & \pm i \\ \pm i & -1 \end{pmatrix} \tag{2.6}$$

in which the minus sign corresponds to the variables $c_{f1}c_{j1}$, and the plus sign to $c_{i1}^{+}c_{j1}^{+}$.

On going from the left to the right sides of the formulas in (2.5) we used the possibility of averaging (at $\mathbf{k}=0$) over the azimuthal angle, on which the functions G_1, G_2, ξ_1, and ξ_2 are independent. We have used also the inequality

$$\frac{\delta_{ij}}{g}+\frac{Z^2}{\beta V}\sum_{p_1} n_{1i}n_{1j}G_1=0, \tag{2.7}$$

which determines the value of the gap that enters in $G_1 = (\omega_1^2+\xi_1^2+\Delta_0^2\sin^2\theta_1)^{-1}$.

We denote the coefficient δ_{ij} in (1.5) by $f(\omega)$, and the coefficient of b_{ij} by $g(\omega)$. We put also

$$u_1=\mathrm{Re}\,c_{11},\quad v_1=\mathrm{Im}\,c_{11},\quad u_2=\mathrm{Re}\,c_{21},\quad v_2=\mathrm{Im}\,c_{21}. \tag{2.8}$$

The quadratic form of the variables u_1, u_2, v_1, v_2 $(\mathbf{k}=0)$ can then be expressed as a sum of two forms:

$$[(f(\omega)+g(\omega))(u_1^2+v_2^2)-2g(\omega)u_1v_2] + [(f(\omega)-g(\omega))(v_1^2+u_2^2)-2g(\omega)v_1u_2]. \tag{2.9}$$

These forms correspond to the matrices

$$\begin{pmatrix} f(\omega)+g(\omega) & -g(\omega) \\ -g(\omega) & f(\omega)+g(\omega) \end{pmatrix},\quad \begin{pmatrix} f(\omega)-g(\omega) & -g(\omega) \\ -g(\omega) & f(\omega)-g(\omega) \end{pmatrix} \tag{2.10}$$

Equating to zero the determinants of the matrices (2.10), we obtain the equations

$$f(\omega)(f(\omega)+2g(\omega))=0,\quad f(\omega)(f(\omega)-2g(\omega))=0$$

or

$$f(\omega)=0,\quad f(\omega)+2g(\omega)=0,\quad f(\omega)-2g(\omega)=0. \tag{2.11}$$

We add to (2.11) the equation obtained from an examination of the terms with c_{31} and c_{31}^{+}:

$$h(\omega)=g^{-1}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p} n_3^2(\xi_1+i\omega_1)(\xi_2+i\omega_2)G_1G_2 = \frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}\{2n_3^2(\xi_1+i\omega_1)(\xi_2+i\omega_2)G_1G_2-(n_1^2+n_2^2)G_1\}=0. \tag{2.12}$$

The three equations of (2.11) can be combined into one:

$$\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}(n_1^2+n_2^2)[((\xi_1+i\omega_1)(\xi_2+i\omega_2)\pm(1,0)\Delta^2)G_1G_2-G_1]=0, \tag{2.13}$$

in which $\pm(1,0)\Delta^2$ denotes either Δ^2 or $-\Delta^2$ or 0.

Changing over (at $T=0$) in (2.12) and (2.13) from the sums to integrals and substituting the expressions for G_1 and G_2, we rewrite (2.12) and (2.13) in the form

$$\frac{2Z^2k_F^2}{(2\pi)^4c_F}\int d\Omega\, d\omega_1\, d\xi_1\left[\frac{2\cos^2\theta(\xi_1+i\omega_1)(\xi_2+i\omega_2)}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}-\frac{\sin^2\theta}{\omega_1^2+\xi_1^2+\Delta^2}\right]=0 \tag{2.14}$$

$$\frac{2Z^2k_F^2}{(2\pi)^4c_F}\int\sin^2\theta\, d\Omega\, d\omega_1\, d\xi_1\left[\frac{(\xi_1+i\omega_1)(\xi_2+i\omega_2)\pm(1,0)\Delta^2}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}-\frac{1}{\omega_1^2+\xi_1^2+\Delta^2}\right]=0.$$

Taking the integrals with respect to ω_1 and ξ_1 with the aid of the Feynman technique (see Ref. 2), we get

$$\frac{Z^2k_F^2}{4\pi^3c_F}\int_0^1 d\alpha\int\cos^2\theta\, d\Omega\left[\ln\frac{\Delta^2}{\Delta^2+\alpha(1-\alpha)\omega^2}-\frac{\alpha(1-\alpha)\omega^2}{\Delta^2+\alpha(1-\alpha)\omega^2}\right]=0, \tag{2.15}$$

$$\frac{Z^2k_F^2}{4\pi^3c_F}\int_0^1 d\alpha\int\sin^2\theta\, d\Omega\left[\ln\frac{\Delta^2}{\Delta^2+\alpha(1-\alpha)\omega^2}-\frac{2\alpha(1-\alpha)\omega^2+\Delta^2\mp(1,0)\Delta^2}{\Delta^2+\alpha(1-\alpha)\omega^2}\right]=0.$$

Calculating the integrals with respect to α, substituting $\omega\to\Delta_0\omega$, and putting $\cos\theta=x$, we arrive at the equations

$$\int_0^1 dx(1-x^2)\frac{\omega^2+4(1-x^2)}{\omega[\omega^2+4(1-x^2)]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}=0,$$

$$\int_0^1 dx(1-x^2)\frac{\omega^2+2(1-x^2)}{\omega[\omega^2+4(1-x^2)]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}=0, \tag{2.16}$$

$$\int_0^1 dx(1-x^2)\frac{\omega}{[\omega^2+4(1-x^2)]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}=0,$$

$$\int_0^1 dx\,x^2\left[\frac{\omega^2+2(1-x^2)}{\omega[\omega^2+4(1-x^2)]^{1/2}}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega}-1\right]=0.$$

The first of this equation is the equation $f-2g=0$, the second is $f=0$, the third is $f+2g=0$, and the fourth is $h=0$. It is they which determine the Bose spectrum at $\mathbf{k}=0$ following the analytic continuation $i\omega\to E$. The spectrum branches corresponding to the second and fourth equations are doubly degenerate. To take into account the forms of the variables c_{f2} and c_{f3} which lead to similar equations for the spectrum, it is necessary to multiply by 3 the multiplicity of each branch in the considered model.

The third and fourth equations in (2.16) have roots $\omega=0$ and correspond to the phonon branches. From the first and second equation we can obtain the complex energies of the zero-phonon branches $E_1(\mathbf{k}=0)$ and $E_2(\mathbf{k}=0)$.

3. CALCULATION OF $E_1(0)$ AND $E_2(0)$

In an actual calculation of the nontrivial roots of the equations in (2.16) we encounter the problem of the analytic continuation $i\omega\to E$ of the function

$$[\omega^2+4(1-x^2)]^{-1/2}\ln\frac{[\omega^2+4(1-x^2)]^{1/2}+\omega}{[\omega^2+4(1-x^2)]^{1/2}-\omega} = [4(1-x^2)-E^2]^{-1/2}\ln\frac{[4(1-x^2)-E^2]^{1/2}-iE}{[4(1-x^2)-E^2]^{1/2}+iE} = [4(1-x^2)-E^2]^{-1/2}\ln\frac{E+i[4(1-x^2)-E^2]^{1/2}}{E-i[4(1-x^2)-E^2]^{1/2}}-\frac{i\pi}{[4(1-x^2)-E^2]^{1/2}}. \tag{3.1}$$

in the integrands. The right-hand side of (3.1) is convenient for a continuation first from the positive imaginary axis $E=i\omega$ $(\omega>0)$ to the upper (physical) half-plane, and then to the lower one through the interval $[0,2]$ of the real axis. The first term in the right-hand side of (3.1) is analytic in the vicinity of the interval

[0, 2]. The contribution made to the integrals of (2.16) by the second term can be calculated in terms of elementary functions that can be easily continued to the unphysical sheet through [0, 2].

Using also the formula

$$\ln\frac{E+i[4(1-x^2)-E^2]^{1/2}}{E-i[4(1-x^2)-E^2]^{1/2}}=-2\ln\frac{E-i[4(1-x^2)-E^2]^{1/2}}{2(1-x^2)^{1/2}}, \qquad (3.2)$$

we can write down the first and second equations of (2.16) in the form

$$F_1(E)=-2\int_0^1 dx(1-x^2)[4(1-x^2)-E^2]^{1/2}\ln\frac{E-i[4(1-x^2)-E^2]^{1/2}}{2(1-x^2)^{1/2}}$$
$$-\frac{\pi}{128}[(E^2-4)(E^2+12)\ln((E+2)/(E-2))+4E(12-E^2)]=0,$$
$$F_2(E)=-2\int_0^1 dx\frac{(1-x^2)[2(1-x^2)-E^2]}{[4(1-x^2)-E^2]^{1/2}}\ln\frac{E-i[4(1-x^2)-E^2]^{1/2}}{2(1-x^2)^{1/2}} \qquad (3.3)$$
$$-\frac{\pi}{256}[(5E^4+24E^2-48)\ln((E+2)/(E-2))+4E(12-5E^2)]=0.$$

The logarithm outside the integral sign in (3.3) takes, upon analytic continuation into the lower half-plane through the segment [0, 2], the form

$$\ln((E+2)/(E-2))=-i\pi+\ln(2+E)-\ln(2-E). \qquad (3.4)$$

Formulas (3.3) and (3.4) were used to calculate the roots $E_1(0)$ and $E_2(0)$ with a computer. The result takes the form

$$E_1(0)=\Delta_0(1.96-i0.31),\quad E_2(0)=\Delta_0(1.17-i0.13), \qquad (3.5)$$

the second of the branches being doubly degenerate.

A computer search was made also of nontrivial roots of the third and fourth equations of (2.17) (which have trivial roots $\omega=0$). No nontrivial roots were found for the third equation. For the fourth equations we found the nontrivial root

$$E_4(0)=\Delta_0(0.7-i0.5). \qquad (3.6)$$

4. CONCLUSIONS

The obtained energies of the collective modes can be compared with the results of a number of studies. We recall first that in the A phase of the model the number of phonon modes (9) is larger than in real He^3-A (5), and all the modes are triply degenerate. The degeneracy is due to the use of the weak-coupling approximation in the model. Allowance for close-coupling effects (see the Appendix) decreases the number of phonon modes from 9 to 5. Application of a magnetic field decreases the number of phonon modes from 9 to 6 in the model considered here and from 5 to 4 when close-coupling is taken into account.

Despite the additional degeneracy inherent in the model, the modes calculated here agree well with those obtained by essentially different methods. The zero-phonon spectrum of the considered model consists of three E_1 branches and six E_2 branches. The real part Re $E_1(0)$ agrees within 2% with the branch $E=2\Delta_0$ obtained in Ref. 7 by the kinetic-equation method, while Re$E_2(0)$ agrees within 4% with the energy of the $E=1.22\Delta_0$ of the clapping (spin) mode. The difference is apparently due to the neglect of the weak coupling, since the error in the numerical calculation does not exceed 1%. The flapping (orbital) mode $E=1.56\Delta_0$ (Refs. 4 and 5) ($E=1.58\Delta_0$, Refs. 7 and 8) does not appear in the considered model of the A phase of He^3 (see the Appendix). The width $\Gamma_n\cong\pi\omega_n$ of this mode[7] is so large (larger by an order of magnitude than for the clapping mode), that the flapping mode is poorly defined and is therefore not as interesting from the experimental point of view as the well defined clapping mode.

The results reported here lead to the conclusion that the use of the weak-coupling approximation has little effect on the calculated frequencies of the collective modes. We note in this connection that in the B-phase the agreement between the frequencies in the weak and close-coupling approximations was exact.[3]

The identification of the modes E_1 and E_2 with the modes obtained in Refs. 4–8 is based on the values of the real parts of the mode energies. A more detailed classification of the spectrum with respect to the degrees of freedom of the order parameter (see the Appendix) shows that the number of the $2\Delta_0$ modes (3) and of the clapping modes (6) in the present paper and in that of Wolfle[3] are the same. The only difference is that the four phonon modes of the orbital waves go over in our model into the flapping modes of Ref. 5 when the close-coupling effects are taken into account.

As for the imaginary parts of the zero-phonon mode energies, they were not calculated in Refs. 4–7. The damping of the clapping mode $0.4\Delta_0$ obtained in Ref. 8 is larger than our value $0.13\Delta_0$. The branches $E=2\Delta_0$ were not obtained in Ref. 8 at all.

Our calculated imaginary parts of the energies $E_1(0)$ and $E_2(0)$ are of the order of 15% of the real ones, so that the damping can be regarded as moderate, and the excitations themselves can be regarded as resonances. We note that the damping increases with increasing real part of the energy (from 13 to 17%).

The solution (3.6) has an imaginary part of the order of the real one, and it cannot be interpreted as a resonance. This is not surprising, since the variable corresponding to the solution (3.6) is of the phonon type, and with it is already associated a weakly damped phonon mode.

We (as well as the authors of Refs. 4, 6, and 7) did not consider the case of nonzero k. It is clear that at small k the energy $E(\mathbf{k})$ remains complex.

Wolfle[5] and Tewordt *et al.*[8] considered the case of small $\mathbf{k}\parallel\mathbf{l}$. The zero-sound absorption at which zero-phonon modes can be excited[9] become noticeable only when the orientations of k and l are different.[5] Just as in the B-phase, experiments aimed at observing zero-phonon modes can yield detailed information on the temperature dependence of the gap $\Delta_0=\Delta_0(T)$.

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APPENDIX

INFLUENCE OF CLOSE COUPLING AND OF A MAGNETIC FIELD ON THE STRUCTURE OF THE BOSE SPECTRUM

We shall show that allowance for the close coupling effects decreases the number of phonon modes from 9 to 5, and that turning-on a magnetic field decreases the number of phonon modes from 9 to 6 for weak coupling and from 5 to 4 when close-coupling effects are taken into account.

We consider, in the Ginzburg–Landau region $|T-T_c| \ll T_c$, that part Π of the action which is independent of the gradients. In the weak coupling model we have[1]

$$\Pi=-\operatorname{tr} AA^{+}+\nu \operatorname{tr} A^{+}AP+(\operatorname{tr} AA^{+})^{2}+\operatorname{tr} AA^{+}AA^{+} +\operatorname{tr} AA^{+}A^{*}A^{T}-\operatorname{tr} AA^{T}A^{*}A^{+}-{}^{1}/_{2}\operatorname{tr} AA^{T}\operatorname{tr} A^{+}A^{*}, \quad (A.1)$$

where A (the order parameter) is a complex matrix with elements A_{ik}. The A-phase in the weak coupling is described by the order parameter

$$\frac{1}{2}\begin{pmatrix}1&0&0\\ i&0&0\\ 0&0&0\end{pmatrix}, \quad (A.2)$$

and the phonon variables are

$$u_{21}-v_{11},\ u_{12}+v_{22},\ u_{13}+v_{23},\ u_{31},\ v_{31},\ u_{32},\ v_{32},\ u_{33},\ v_{33}, \quad (A.3)$$

where $u_{ik}=\operatorname{Re}A_{ik}$ and $v_{ik}=\operatorname{Im}A_{ik}$. These variables correspond to the branches of the spectrum not only in the Ginzburg–Landau region, but also at all $T<T_c$. In the limit as $T\to 0$, the first three of the variables in (A.3) correspond to sound waves with $c_F k/\sqrt{3}$, and the six remaining to orbital waves $c_F k_{\parallel}$.[1] The phonon spectrum is thus degenerate in the spin index.

To take into account the close-coupling effects, we consider Π with arbitrary coefficients of the fourth-order terms:

$$\Pi=-\operatorname{tr} A^{+}A+\nu \operatorname{tr} A^{+}AP+a(\operatorname{tr} A^{+}A)^{2}+b\operatorname{tr} AA^{+}AA^{+} +c\operatorname{tr} AA^{+}A^{*}A^{T}+d\operatorname{tr} AA^{T}A^{*}A^{+}+e\operatorname{tr} AA^{T}\operatorname{tr} A^{+}A^{*}. \quad (A.4)$$

The condition $\delta\Pi=0$ yields in the A-phase an order parameter in the form

$$\frac{1}{2}(a+b+d)^{-1/2}\begin{pmatrix}1&0&0\\ i&0&0\\ 0&0&0\end{pmatrix}. \quad (A.5)$$

To find the phonon variables, we calculate the second variation δ^2

$$\delta^{2}\Pi=-\operatorname{tr} AA^{+}+\nu \operatorname{tr} A^{+}AP+a\operatorname{tr}[(A^{+}C)^{2}+(C^{+}A)^{2} +2A^{+}AC^{+}C+2A^{+}CC^{+}A]+b\operatorname{tr}[2AA^{+}CC^{+}+2A^{+}AC^{+}C +AC^{+}AC^{+}+A^{+}CA^{+}C]+c\operatorname{tr}[AA^{+}C^{*}C^{T}+A^{+}A^{*}C^{T}C +A^{*}A^{T}CC^{+}+A^{T}AC^{+}C^{*}+AC^{+}A^{*}C^{T}+A^{T}CA^{+}C^{*}] +d\operatorname{tr}[AA^{T}C^{*}C^{+}+A^{T}A^{*}C^{+}C+A^{*}A^{+}CC^{+}+A^{+}AC^{T}C^{*}]+4e|\operatorname{tr} C^{T}A|^{2}, \quad (A.6)$$

where C is the matrix (A.5), and A is a variable matrix. Substituting the values of C, C^{+}, C^{*}, and C^{T} we get

$$\delta^{2}\Pi=\nu(a+b+d)(u_{13}^{2}+v_{13}^{2}+u_{23}^{2}+v_{23}^{2}+u_{33}^{2}+v_{33}^{2}) +4a(u_{11}+v_{21})^{2}+2b[2(u_{11}+v_{21})^{2}-(u_{13}-v_{23})^{2} -(u_{12}-v_{22})^{2}-(u_{22}+v_{12})^{2}-(u_{23}+v_{13})^{2}-2(u_{32}^{2}+v_{32}^{2} +u_{33}^{2}+v_{33}^{2})]+2c[2(u_{11}-v_{21})^{2}+2(u_{21}+v_{11})^{2} +(u_{21}-v_{22})^{2}+(u_{22}+v_{12})^{2}+(u_{13}-v_{23})^{2}+(u_{23}+v_{13})^{2}] +2d[2(u_{11}+v_{21})^{2}-(u_{12}-v_{22})^{2}-(u_{13}-v_{23})^{2}-(u_{22}+v_{12})^{2} -2(u_{22}-v_{12})^{2}-2(u_{23}-v_{13})^{2}-(u_{23}+v_{13})^{2}-2(u_{32}^{2}+v_{32}^{2} +u_{33}^{2}+v_{33}^{2})]+4e[(u_{11}-v_{21})^{2}+(u_{21}+v_{11})^{2}]. \quad (A.7)$$

We consider first the system in a zero magnetic field ($\nu=0$). Then (A.7) is the sum of five quadratic forms multiplied by the independent coefficients a, b, c, d, and e. The variables

$$u_{12}+v_{22},\ u_{13}+v_{23},\ u_{21}-v_{11},\ u_{31},\ v_{31} \quad (A.8)$$

do not enter in any of these forms, and corresponding to them are therefore phonon modes. Thus, allowance for the close-coupling effects decreases the number of phonon branches from 9 to 5. The modes u_{32}, v_{32}, u_{33}, and v_{33}, which correspond in the weak-coupling approximation to orbital waves, become zero-phonon modes when the close-coupling effects are taken into account.

Expression (A.7) at $\nu\neq 0$ describes the system in a magnetic field. In the weak binding approximation the number of phonon modes decreases from 9 to 6, and the variables $u_{13}+v_{23}$, u_{33}, and v_{33} become zero-phonon because of the appearance of the gap $\sim\mu H$ in the spectrum. In a system with close-coupling the mode that becomes zero-phonon upon application of a magnetic field is $u_{13}+v_{22}$ (the modes u_{33} and v_{33} in the case of close-coupling zero-phonon also at $\nu=0$), and the number of phonon modes decreases from 5 to 4.

To gain an idea of the total Bose spectrum (including the zero-phonon branches) when close-coupling effects are taken into account, we rewrite (A.7) at $H=0$ ($\nu=0$) in the form

$$\delta^{2}\Pi=4(a+b+d)(u_{11}+v_{21})^{2}+4(c+e)[(u_{11}-v_{21})^{2} +(u_{21}+v_{11})^{2}]+2(c-b-d)[(u_{13}-v_{23})^{2}+(u_{12}-v_{22})^{2} +(u_{22}+v_{12})^{2}+(u_{23}+v_{13})^{2}]-4d[(u_{23}-v_{13})^{2} +(u_{22}-v_{12})^{2}]-4(b+d)(u_{32}^{2}+u_{33}^{2}+v_{32}^{2}+v_{33}^{2}). \quad (A.9)$$

For comparison, we write down $\delta^2\Pi$ in the weak coupling approximation, putting in (A.9) $a=b=c=-d=-2e=1$:

$$\delta^{2}\Pi=4[(u_{11}+v_{21})^{2}+(u_{23}-v_{13})^{2}+(u_{22}-v_{12})^{2}] +2[(u_{11}-v_{21})^{2}+(u_{21}+v_{11})^{2}+(u_{13}-v_{23})^{2}+(u_{13}-v_{22})^{2} (u_{22}+v_{12})^{2}+(u_{23}+v_{13})^{2}]+0\cdot[u_{32}^{2}+u_{33}^{2}+v_{32}^{2}+v_{33}^{2}]. \quad (A.10)$$

The form (A.10) has three eigenvalues 4, corresponding to the variables $u_{11}+v_{21}$, $u_{22}-v_{12}$, and $u_{23}-v_{13}$. It is just to these variables that the branches E_1 correspond as $T\to 0$. The other nonzero eigenvalue 2 correspond to six variables: $u_{21}+v_{11}$, $u_{12}-v_{22}$, $u_{13}-v_{23}$, $u_{11}-v_{21}$, $u_{23}+v_{13}$, and six E_2 branches as $T\to 0$.

The calculation of the Bose spectrum in Ref. 6 yields 6 spin modes and three $2\Delta_0$ modes, i.e., as many as in the weak binding case considered here. Formula (A.9) shows that in the general case allowance for the close-coupling effects leads to splitting. The spin modes break up into two groups—two branches corresponds to the eigenvalue $4(c+e)$ and four correspond to the number $2(c-b-d)$. The three $2\Delta_0$ branches also break up into one branch with eigenvalue $4(a+b+d)$ and two branches with eigenvalue $-4d$. We note that no conclusion can be drawn from the data of Ref. 5 concerning the splitting of the branches.

The branches u_{32}, u_{33}, v_{32}, and v_{33}, which in the weak-binding approximation are orbital waves, go over into the normal flapping mode and the super-flapping mode when account is taken of the close-coupling effects, as shown by comparison with the data of Ref. 5.

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Dynamics of laser damage in KDP crystals

V. N. Genkin, A. M. Miller, and L. V. Soustov

Institute of Applied Physics, USSR Academy of Sciences
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Photoresponse investigation has revealed the dynamics of laser damage in KDP crystals. It is established that in sufficiently pure crystals the damage develops in the volume at the speed of sound and terminates in formation of cracks characterized by a substantial surface-charge density. Characteristics of the damaged region, such as pressure, temperature, absorbed-energy density, and absorption coefficient are determined. The role played by the pressure wave in the process of damage to dielectrics by nanosecond laser pulses is discussed.

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1. INTRODUCTION

The mechanisms whereby materials are optically damaged have been intensively studied for more than a decade, and it has been established by now that the development of sizeable (10^{-3}–10^{-2} cm) damage in pure substances is not the direct consequence of heating of the absorbing inhomogeneities, but is due to the absorption by the medium itself under the influence of the laser radiation. [A substance is regarded is here as pure if the absorbing inclusions are small enough and their heating cannot cause directly any noticeable damage in the medium, the distance between the inclusion being $l_0 \gg (\tau_p \chi)^{1/2}$, where τ_p is the duration of the laser pulse and χ is the thermal diffusivity of the medium.] The nature of this absorption and the mechanisms of its onset in real crystals are still the subjects of debates. Many papers have dealt with absorption of laser by free carriers, whose appearance is attributed, for example, to a thermal absorption wave[1] or to ultraviolet preionization of the matrix.[2] These mechanisms differ substantially both in the values of the parameters (e.g., the matrix temperature) needed for their realizations, and in the rate of propagation of the absorption produced by them. One should therefore expect an analysis of the dynamics of damage development to cast light on the roles of the different mechanisms in the breakdown.

Most frequently, attempts to track the dynamics of the damage are made by studying the emission that accompanies the damage, and the scattering of the light from the damaged region (see, e.g., Refs. 3 and 4). Experiments have shown, however, that both the light and the scattering appear during later stages of the damage, frequently after the termination of the laser pulse.[1)] It seems promising therefore to investigate the damage dynamics by using the photoelectric response. In fact, the large amount of information provided by this method has been well demonstrated with photoelectric spectroscopy of semiconductors as the example, but the methods developed for semiconductors cannot be directly applied to dielectrics. Owing to the low volume conductivity, the surface and contact phenomena in dielectrics are relatively strong and impede seriously the interpretation of the results, especially in the study of damage. This may be the reason why hardly any investigations of photoconductivity in transparent have been made until now, while Belikova *et al.*[6] attributed the damage in corundum, whose strength is 10^{10} W/cm^2, to the formation of an electron avalanche, an assumption that does not seem to be reliable enough.[2)]

We report below the results of an investigation of the dynamics of the photoresponse in damage of KDP crystals, which were used as an example to develop a technique[8] that eliminated the influence of contact and surface phenomena. It was established with the aid of this technique that the generation of free carriers and nonstationary heating of the crystal lattice produce photoresponses of opposite polarity, J_σ and J_ε, respectively. The current

$$J_\sigma \sim \int_{V_0} \sigma(t,r)\,dV$$

(σ is the light-induced conductivity in the volume V_0) corresponds to an increase of the sample conductivity, while the thermoelectric current

$$J_\varepsilon \sim P_{abs}\, d\varepsilon/dT$$

(P_{abs} is the radiation power absorbed in the sample, ε is the low-frequency permittivity of the crystal, and T is the lattice temperature) decreases the conductivity.

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