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Propagation of an electromagnetic field in noninertially moving optically active media

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The propagation of electromagnetic waves in an optically active medium at rest in a rotating coordinate frame is investigated on the basis of the method of local Lorentz transformations. The results are used to determine the splitting of the natural frequencies of a ring resonator rotating together with the medium. The theoretical analysis data agree well with the results of the performed experiment.

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1. To determine the character of propagation of electromagnetic waves in an arbitrary medium that is at rest in a noninertial reference frame, it is necessary to solve Maxwell's equations, which in covariant form are given by¹

$$\nabla_{\alpha} H^{\mu\alpha} = 0; \quad \nabla_{\alpha} F_{\mu\nu} = 0, \quad (1)$$

where $H^{\mu\alpha}$ and $F_{\mu\alpha}$ are bivectors of the electromagnetic field, ∇_{α} is the covariant derivative, and a correspondence is established between the components of the tensors $H^{\mu\alpha}$, $F_{\mu\alpha}$ and the three-dimensional vectors in accordance with the rule

$$H^{\mu\nu} = \{H^{0i}; H^{mn}\} = \{(-g)^{1/2} \mathbf{D}; (-g)^{1/2} \mathbf{H}\}, \quad (2)$$

$$F_{\mu\nu} = \{F_{0i}; F_{mn}\} = \{\mathbf{E}; \mathbf{B}\},$$

which transforms (1) to the usual three-dimensional form.¹

The system (1) must be supplemented by the equations that connect the bivectors $H^{\mu\alpha}$ and $F_{\mu\alpha}$ —the material equations. To establish the covariant form of this connection in noninertially moving media one used customarily generalizations of various schemes for the construction of the material equations of inertially moving media.² However, even in the first few theoretical papers the use of different versions of generalizations of the electrodynamics of moving isotropic media, within

the framework of special relativity theory, to include the case of general relativity theory, has led to discrepancies in the main results.³⁻⁵ It has been shown⁶⁻⁷ that these discrepancies are due to the erroneous assumptions⁵ made in the construction of the material tensor $\epsilon^{\mu\nu\alpha\beta}$ that determines the coupling equations. This was confirmed in an experimental determination⁸ of the influence of the refractive index n of an isotropic medium on the splitting $\Delta\nu$ of two opposing waves of a rotating laser with a ring resonator (RR). Volkov and Kiselev⁹, within the framework of a method developed by them,⁵ determined the splitting $\Delta\nu$ for the case when, in the proper system of the rotating resonator, the permittivity and the permeability of the medium are specified by gyrotensors $\hat{\epsilon}$ and $\hat{\mu}$. The result in Ref. 9 differs from an analogous result obtained in their own paper¹⁰ with qualitative account taken of the gyrotropy of the medium.

Thus, the available theoretical data do not make it possible to determine rigorously the character of the propagation of the electromagnetic field in rotating optically active media, and the solution of this problem calls for a detailed theoretical and experimental study.

In the present paper we construct the material equations and solve subsequently the propagation problem on the basis of the method of local Lorentz transforma-

tions,¹¹ which makes it possible to determine the components of the tensor $\varepsilon^{\mu\nu\alpha\beta}$ for an arbitrary medium and, if necessary, to analyze a large class of electrodynamic problems in non-inertial reference frames in gravitational fields. The theoretical results of the present paper are in good agreement with experiments aimed at determining the splitting of opposing waves in a rotating ring laser containing an optically active medium.

2. We expand the bivectors of the electromagnetic field in terms of the vectors of a locally inertial reference frame

$$H^{\nu\alpha} = h_{(\alpha)}^{\mu} h_{(\beta)}^{\nu} H^{(\alpha)(\beta)}; \quad (3)$$

$$F_{\mu\nu} = h_{\mu}^{(\alpha)} h_{\nu}^{(\beta)} F_{(\alpha)(\beta)},$$

where $h_{(\alpha)}^{\mu}$ is a tetrad,

$$h_{(\alpha)}^{\mu} h_{(\nu)}^{\alpha} = \delta_{\nu}^{\mu}.$$

Equation (3) can be expressed in terms of its components, with account taken of the skew symmetry of the field tensors, by unifying the pair of indices (μ, ν) into a single collective index A in accordance with the rule $(1, 0) \rightarrow 1; (2, 0) \rightarrow 2; (3, 0) \rightarrow 3; (2, 3) \rightarrow 4; (3, 1) \rightarrow 5$, and $(1, 2) \rightarrow 6$. This corresponds to a transition to a six-dimensional bivector space.¹² The expansion (3) in this space takes the form

$$H^A = T_{(B)}^A H^{(B)}; \quad F_A = T_A^{(B)} F_{(B)}, \quad (4)$$

where $T_{(B)}^A$, in accordance with our preceding paper,⁷ in first order in the speed of rotation about the z axis, is represented in the following form:

$$T_{(B)}^A = \begin{pmatrix} h_{(0)}^0 h_{(1)}^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_{(0)}^0 h_{(2)}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{(0)}^0 h_{(3)}^3 & 0 & 0 & 0 \\ 0 & h_{(2)}^0 h_{(3)}^0 & -h_{(2)}^0 h_{(3)}^0 & h_{(2)}^0 h_{(3)}^0 & 0 & 0 \\ -h_{(1)}^1 h_{(3)}^0 & 0 & h_{(1)}^1 h_{(3)}^0 & 0 & h_{(1)}^1 h_{(3)}^0 & 0 \\ h_{(1)}^1 h_{(2)}^0 & -h_{(1)}^1 h_{(2)}^0 & 0 & 0 & 0 & h_{(1)}^1 h_{(2)}^0 \end{pmatrix}, \quad (5)$$

where $T_{(B)}^A T_{(C)}^B = \delta_C^A$.

In a locally inertial reference frame, the coupling equations of the induction and intensity vectors in three-dimensional form are known.¹³ The tensor $\varepsilon^{(A)(B)}$ of the medium is represented in this case in the form of a block matrix whose diagonal blocks correspond to the dielectric tensor $\hat{\varepsilon}$ and to the inverse permeability tensor $\hat{\mu}^{-1}$. Projecting the tensor ε^{AB} of the medium on the locally inertial reference frame, we obtain the equations for the components of ε^{AB} under the conditions that $\varepsilon^{(A)(B)}$ is known:

$$\varepsilon^{AB} = T_{(C)}^A T_{(D)}^B \varepsilon^{(C)(D)}, \quad (6)$$

meaning that the material relations are also known:

$$H^A = T_{(C)}^A T_{(D)}^B \varepsilon^{(C)(D)} F_B; \quad F_A = T_A^{(C)} T_B^{(D)} \varepsilon_{(C)(D)} H^B. \quad (7)$$

Retaining only the terms of first order in Ω , we obtain the tensor of the medium in the form

$$\varepsilon^{AB} = \begin{pmatrix} \hat{\varepsilon} & \hat{K}^* \\ \hat{K} & \hat{\mu}^{-1} \end{pmatrix}, \quad (8)$$

$$\hat{K} = \begin{pmatrix} i\bar{m}_z g_{03} + i\bar{m}_y g_{02} & -i\bar{m}_y g_{01} - \bar{\mu}_1 g_{03} & -i\bar{m}_z g_{01} + \bar{\mu}_1 g_{02} \\ \bar{\mu}_2 g_{03} - i\bar{m}_x g_{02} & i\bar{m}_z g_{03} + i\bar{m}_x g_{01} & -i\bar{m}_z g_{02} - \bar{\mu}_2 g_{01} \\ -i\bar{m}_x g_{03} - \bar{\mu}_3 g_{02} & -i\bar{m}_y g_{03} + \bar{\mu}_3 g_{01} & i\bar{m}_y g_{02} + i\bar{m}_x g_{01} \end{pmatrix},$$

where \hat{K}^* is the transposed and complex-conjugate matrix of \hat{K} ,

$$\varepsilon_{mn} = \varepsilon_{mnh} g^h, \quad \mu_{mn}^{-1} = \varepsilon_{mnh} \bar{m}^h \quad (m \neq n),$$

and ε_{mnh} is the Levi-Civita symbol. Hence the sought three-dimensional form of the coupling equations in an optically active medium that moves together with the rotating observer is represented in the form

$$D = \hat{\varepsilon} E - \hat{K}^* B; \quad H = \hat{\mu}^{-1} B + \hat{K} E. \quad (9)$$

For experiments with a ring laser, an important role is played only by the electric gyrotropy of the medium.¹⁴ In this case the matrix K simplifies considerably and we have for the coupling equations

$$D = \hat{\varepsilon} E + \frac{1}{\mu} \left[\left[r \times \frac{\Omega}{c} \right] \times B \right]; \quad H = \frac{1}{\mu} B + \frac{1}{\mu} \left[\left[r \times \frac{\Omega}{c} \right] \times E \right]. \quad (10)$$

The latter differ from the coupling equations that follow from the work of Volkov and Kiselev.⁹

3. Maxwell's equations (1) jointly with the material relations (10) for a monochromatic field lead to the wave equation

$$\text{rot rot } E = k_0^2 \mu \hat{\varepsilon} E - ik_0 \left[\left[r \times \frac{\Omega}{c} \right] \times \text{rot } E \right] - ik_0 \text{rot} \left[\left[r \times \frac{\Omega}{c} \right] \times E \right], \quad (11)$$

where $k_0 = 2\pi\nu_0/c$ and ν_0 is the oscillation frequency of the nonrotating resonator. We consider the electromagnetic field of a ring-laser resonator in the form of a superposition of two plane waves propagating in opposite directions, i.e.,

$$E = E_0 \exp(-in \cdot kr),$$

where k is the wave vector and n^* is the effective refractive index, the latter defined both by the properties of the medium and by the character of their motion.

In the coordinate frame in which the angular velocity is directed along the z axis, the wave vector $k = (0, k, 0)$, and the gyration vector $G = (0, g, 0)$,¹³ Eq. (11) is satisfied under the condition

$$E_x = \pm i E_y; \quad (12a)$$

$$n^* = n_0 + \frac{1}{k} \left(k \times \left[r \times \frac{\Omega}{c} \right] \right) \pm \frac{g}{2n_0}, \quad (12b)$$

where the \pm sign corresponds to waves of opposite directions. The frequency difference $\Delta\nu$ of opposing circularly polarized modes (12a), due to the rotation of the RR and to the presence of a gyrotropic medium in it, can be determined from the condition that the phase advances of these modes be equal after a single circuit over the resonator filled with a medium having a refractive index (12b):

$$\frac{2\pi\nu_1}{c} \oint n_1 \cdot dl = \frac{2\pi\nu_2}{c} \oint n_2 \cdot dl, \quad (13)$$

from which we obtain for (12)

$$\Delta\nu' = \frac{4\nu_0}{L+d(n_0-1)} \left(\frac{\Omega S}{c} + \frac{R\lambda H l}{4\pi} \right), \quad (14)$$

where $g = R\lambda H n_0 / \pi$, R is the Verdet constant, and H is the magnetic field intensity, and $n_0 = (\varepsilon\mu)^{1/2}$.

Expression (14) differs from the one that follows from the paper of Volkov and Kiselev⁹, which takes under similar conditions the form

$$\Delta\nu'' = \frac{4\nu_0}{L+d(n_0-1)} \left(\frac{\Omega S_2}{c} + \frac{(1+n_0^2)\Omega S_1}{2c} + \frac{R\lambda H l}{4\pi} \right). \quad (15)$$

Despite the use of the Volkov and Kiselev material relations,⁹ Bychkov, Luk'yanov, and Bakalyar¹⁴ obtained in their book¹⁴ for $\Delta\nu$ an expression that does not contain the term $(1+n_0^2)\Omega\cdot S_1/2c$. The reason is that they investigated the propagation of an electromagnetic wave along the angular-velocity vector ($k\parallel\Omega$), which, however, does not correspond to the case expressed by formulas (14) and (15), and realized in the rotating RR.

According to (14) and (15), the beat frequency of the opposing waves of a ring laser can be expressed in the form

$$\Delta\nu = K_1/T + \Delta\nu_0 + K_{-1}/\Omega, \quad (16)$$

where K_1 is a scale factor^{7,8}; T is the period of the rotation; $\Delta\nu_0$ is the splitting due to the gyrotropic medium and to other nonreciprocal effects included in (16) in the form of additional terms⁷; K_{-1} is the coupling characteristic of the opposing waves and leads to effects that are smaller by five or six orders of magnitude than the two preceding terms of (16).⁸ It must be noted that relation (16) generalizes the result obtained in Ref. 8 for isotropic media to include the case of rotation of optically active media. According to (14) and (15), the discrepancies between the premises of the present paper and of Ref. 9, just as in the case of an isotropic medium, are reflected only in the magnitude of the scale factor K_1 , whose values for (14) and for (15) are respectively

$$K_1' = \frac{4\nu_0 S}{c[L+d(n_0-1)]}, \quad (17)$$

$$K_1'' = \frac{4\nu_0[S_2+(n_0^2+1)S_1/2]}{c[L+d(n_0-1)]}. \quad (18)$$

On the contrary, the results of Ref. 10 lead, first, to violation of (16) and, second, to the conclusion that the scale factor depends on the gyrotropy of the medium. In addition, in the expression for $\Delta\nu$ in Ref. 9 between the oppositely directed modes of two equivalent rotating RR, only one of which contains an optically active medium, there is contained, in contrast to (12)–(14), a term that includes simultaneously the property of optical activity of the medium and its rotation:

$$\frac{RHn_0\Omega S}{\pi[L+d(n_0-1)]}. \quad (19)$$

The latter means that the considered case should, according to Ref. 9, also lead to violation of (16).

4. To check on the results of the foregoing analysis and to compare them with the results of Volkov and Kiselev,⁹ we have investigated experimentally a rotating ring He-Ne laser ($\lambda = 0.63 \mu\text{m}$) with a resonator in the form of a triangle with sides $l_1 = 0.22 \text{ m}$, $l_2 = 0.27 \text{ m}$, and $l_3 = 0.31 \text{ m}$. In the arm l_3 was located a two-anode laser tube that produced an amplification of the order of 2%. In arm l_2 was located a non-reciprocal Faraday element constructed in accordance with a differential scheme¹⁴ and made up of two plane-parallel plates of K-8 glass ($n_{01} = 1.5163$), whose total length was $3 \times 10^{-3} \text{ m}$. The plates, as well as the two quartz windows ($n_{02} = 1.457$) of the laser tube (each $2 \times 10^{-3} \text{ m}$ thick) were secured at the Brewster angle. A longitudinal magnetic field of intensity 516 Oe was produced by a bulky permanent magnet. Stability during the time of the experiment was ensured by assembling the resonator on a quartz plate,

TABLE I.

| Rotation direction | $\Delta\nu_0$, Hz | K_1 | Rotation direction | $\Delta\nu_0$, Hz | K_1 |
|--|---|---|--|--|---|
| $\left\{ \begin{array}{l} H=0 \\ H=516 \text{ Oe} \end{array} \right.$ | $\left\{ \begin{array}{l} 607 \pm 2 \\ 121410 \pm 50 \end{array} \right.$ | $\left\{ \begin{array}{l} 1434535 \pm 11 \\ 1434390 \pm 80 \end{array} \right.$ | $\left. \begin{array}{l} H=0 \\ H=516 \text{ Oe} \end{array} \right\}$ | $\left\{ \begin{array}{l} -624 \pm 11 \\ -121464 \pm 25 \end{array} \right.$ | $\left\{ \begin{array}{l} 1434408 \pm 25 \\ 1434291 \pm 45 \end{array} \right.$ |

by feeding the laser tube from a source with relative current instability $\sim 10^{-4}$, by using an air suspension rotated by a jet thrust.⁸ The splitting of the frequencies of the opposing wave on account of the rotation could be varied in the range from 200 kHz to 1 MHz. The relative error in the measurement of the beat frequency was in this case $\sim 10^{-5}$. Markers corresponding to the angle 2π from revolution to revolution were produced by a beam at the exit from the ring laser, focused on the slip of a photoreceiver that was stationary relative to the ground. The laser revolution period (T) and the number of beat periods of the opposing modes were measured with digital frequency meter and were recorded with digital printing units.

The description of the experimental apparatus explains readily the experimental procedure and the reduction of the experimental results. By varying the frequency splitting of the opposing waves either as a result of rotation only (with the non-reciprocal element turned off), or by varying the gyration vector (with the laser at rest) it is possible to determine whether the splitting is equal to the sum of the corresponding splittings due to the rotation and gyrotropy (14)–(16). It is then possible to compare the experimental value of the scale factor K_1 with K_1' and K_1'' . Thus, the experiment consisted of finding the value of the scale factor and determining its dependence on the magnitude of the gyration vector.

The scale factor is determined by least-squares extrapolation to a zero period of revolution of the RR.⁸ The results for opposite directions of rotation (1 and 2) are given in the table, from which it is seen that the scale factor of a ring laser does not depend on the gyrotropy of the medium, and that the average value of K_1 at $T = 2.5 \text{ sec}$ corresponds to $\Delta\nu = 695.2 \text{ kHz}$.

A theoretical estimate of the scale factor (17) and (18) for the presented geometrical dimensions of the RR, for the substances used in it, and for the radiation wavelength yields the following values of P_1' and k_1'' :

$$K_1' = 1.43408 \cdot 10^6; \quad K_1'' = 1.44200 \cdot 10^6.$$

At a revolution period $T = 2.5 \text{ sec}$, this leads to a splitting frequency

$$\Delta\nu_1' = 573.6 \text{ kHz}; \quad \Delta\nu_1'' = 576.8 \text{ kHz},$$

where $\Delta\nu_1 = K_1\Omega$.

The frequency increment $\Delta\nu_0$ due to the gyrotropy of the medium

$$\frac{1}{2\pi} c l R H [L + d_1(n_{01}-1) + d_2(n_{02}-1)]^{-1},$$

where d_1 and d_2 are respectively the length of the non-reciprocal element and of the quartz plates of the laser tube, is equal to 121.4 kHz, i.e., this calculation leads

to a splitting $\Delta\nu' = 695.0$ kHz, as against $\Delta\nu'' = 698.2$ kHz.² This allows us to state that the experimentally observed value of $\Delta\nu$ agrees with that calculated by us, since the difference between the splittings $\Delta\nu'$ and $\Delta\nu''$ lies within the limits of the measurement accuracy. Unfortunately, the splitting (19) turns out to be much less than the value that can be registered at present in experiment with two different rotating RR even if the Verdet constant is increased by two orders of magnitude compared with R of K-8 glass.

Thus, besides the good quantitative agreement of the theoretical analysis, proposed in the present paper, for the propagation of an electromagnetic field in rotating optically active media, with the experimental data, our experiment has confirmed the validity of relation (16), which is a statement of the additivity of the effect of non-reciprocity when account is taken of the rotation of a gyrotropic medium.

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Polarization of resonance fluorescence of an atom with degenerate levels in a strong field

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The problem of resonance scattering of a monochromatic wave by an atom with allowance for saturation effects is solved in the radiation-field coherent-state representation. A general calculation scheme for scattering by a multilevel atom is presented. The exact solution for a two-level atom is identical with the results obtained by other authors. The method is applied to the problem of scattering of a linearly polarized light wave by two degenerate levels of the $S_{1/2}$ - $P_{1/2}$ type in alkali metal atoms. The polarization characteristics of the process are discussed.

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1. INTRODUCTION

A number of papers are devoted to the question of the fluorescence spectrum of a two-level atom in the field of an intense monochromatic wave for times which are much larger than the lifetime of the free atom.¹⁻¹⁵

There is a well-known expression for the probability of spontaneous emission of a photon with frequency ω per unit frequency interval,³ which in the case of high intensity of the incident field ($\Omega \gg \gamma$, $|\epsilon|$) takes the form

$$I_0(\omega) = \frac{\gamma/4}{(\omega - \omega_k)^2 + \gamma^2/4} + \frac{3\gamma/16}{(\omega - \omega_k - \Omega)^2 + 9\gamma^2/16} + \frac{3\gamma/16}{(\omega - \omega_k + \Omega)^2 + 9\gamma^2/16} \quad (1)$$

Here ω_k is the frequency of the incident wave, ω_0 is the separation between levels, γ is the spontaneous width, $\Omega = |E \cdot d|/\hbar$ is the Rabi frequency, and $\epsilon = \omega_0 - \omega_k$

is the resonance detuning parameter.

We should point out that the results of work on resonance fluorescence substantially depend on the mathematical formulation and the physical set-up of the problem, and very often agree only when the exciting radiation is of high intensity. An analysis of the reasons for such a divergence of the results is presented in papers by Swain¹⁰ and Raman.¹² One frequently used method for solving the problem is expansion of the wave function of the field in states with a fixed number of photons, and an approximate uncoupling of the infinite system of equations for the density matrix elements formed in this case.^{4-6,9-12} We must approach the evaluation of the results obtained by this means very cautiously, since they depend substantially on the uncoupling