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Contribution to the theory of heat exchange due to a fluctuating electromagnetic field

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The generalized Kirchhoff's law [M. L. Levin and S. M. Rytov, Theory of Equilibrium Thermal Fluctuations in Electrodynamics [in Russian], Nauka, 1967] is used to obtain general expressions for the spectral and total Poynting vector of a fluctuating electromagnetic field in a flat vacuum gap between two arbitrary semi-infinite media of different temperature (Sec. 2; to simplify the derivation, the medium is assumed to be isotropic and spatially local). Some general consequences and particular cases are discussed (Sec. 3), and the case of good conductors is investigated in detail in the impedence approximation both for the normal and for the anomalous skin effect (Sec. 4). The heat-flow formulas are generalized in Sec. 5 (using the concept of the generalized surface impedance) to include anisotropic media with spatial dispersion.

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1. INTRODUCTION

The question of "radiant" heat exchange was posed already in the classical theory of thermal radiation. An example of its solution is the Cristiansen integral formula (with respect to the frequency ω)[1] for the energy flux between "gray" bodies. Interest in this problem was again increased in the 60's, when, in connection with experiments at cryogenic temperatures, attention was called to the fact that consideration of only traveling waves (of radiation) is valid only if the gap thickness l between the bodies is large, i.e., when $l \gg \lambda_W$, where λ_W is the Wien wavelength corresponding to the temperature of the colder body. On the other hand in the case of thin gaps ($l \lesssim \lambda_W$) inhomogeneous waves (the near field) come into play and as a result the coefficient of heat transfer through the gap can depend on the gap width l.[2]

The presence (and even the role) of a near fluctuating field, of which sight was lost completely in the classical (i.e., geometrical-optics) theory of thermal radiation, was pointed out long ago.[3] This field is essential in all cases close enough to the surfaces of the body and by the same token for all sufficiently thin cavities or gaps.[1)] In particular, a general theory of thermal fluctuations of the electromagnetic field, developed by one of us,[3] was used by Lifshitz[5] to calculate the molecular adhesion forces between arbitrary bodies.

The solution of any problem dealing with average bilinear quantities was subsequently greatly facilitated when a simpler and general method was developed[6] for the calculation of correlators of a fluctuating electromagnetic field—the generalized Kirchhoff's law—and made it possible in many other applications to obtain an even shorter solution of the aforementioned adhesion-force problem (Ref. 6, Sec. 18). It is clear that also for heat exchange (average Poynting vector in a gap between two bodies) the near field should also play an essential role at small gap thicknesses l.

The heat flux in a flat gap between a semi-infinite medium and a well conducting nonradiating (cold) mirror was obtained already in Ref. 3 in connection with the question of experimental observation of the near field. There were pointed out, in particular, interference deviations from homogeneity of the field at

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small l, an asymptotic transition to equilibrium (Planck) intensity as $l \to \infty$, and an inversely proportional increase of the energy absorbed by the mirror as $l \to 0$ (Ref. 3, Sec. 7).[2)]

The authors of a number of succeeding papers, evidently unaware of correlation theory,[3] solved the heat-transfer problem by other methods. It was customary to consider (just as in the present article) the simplest geometrical conditions: a flat gap between two semi-infinite media (1 and 2 in Fig. 1). For the case of transparent media, the solution was obtained in Ref. 8 by summing the multiply re-reflected traveling waves and taking into account the seepage (tunneling) of the inhomogeneous waves, under the natural assumption that the primary field inside each of the media is the same as the equilibrium field in this medium at its temperature. The energy flux, expressed in quadratures, was calculated then with a computer under the assumption that there is no frequency dispersion of the refractive indices.

In the opposite case of strongly absorbing media (metals) separated by an insulator, the problem was solved by the same method of multiple reflections as at $l \gg \lambda_W$,[9] as well as for thin gaps.[10] The primary radiation of each metal, however, was described in the cited papers on the basis of the concept of the radiation intensity within the absorbing medium,[11] although it has become clear long ago[3] that in strongly absorbing media the intensity concept connected with traveling natural waves becomes meaningless (extraneous–Langevin–sources are distributed in the medium).

The first investigation of heat transfer between arbitrary media, in which the previously developed[3] correlation theory was employed, was made by Polder and Van Hove.[13] They presented their correct result only for identical media one and two [formula (19) of Ref. 13]. In the subsequent treatment, use was made of numerical calculations, and furthermore not of the flux itself but of its derivative with respect to temperature, i.e., their results[13] pertain only to small temperature differences. Unfortunately, their paper contains no analytic formulas in closed form whatever. The case of metallic surfaces and low temperatures was again considered by Caren[14] with the aid of the same "mode" approach which he used earlier[4], i.e., without allowance for the near field. The general theory[3] was used by him only in a later paper,[15] in which he considered the case of metals with anomalous skin effect, and in which the impedance approximation was used from the very outset. However, certain aspects of Caren's later paper[15] are doubtful. For example, the presence of a factor $1/l^4$ in formulas (26) of his paper contradicts the perfectly general conclusion that the heat flux does not contain terms with powers of $1/l$ higher than the second (see Sec. 3).

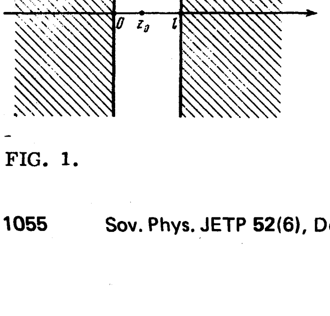

FIG. 1.

Thus, the investigations cited considered only particular cases of media (identical media, transparent insulators without dispersion, well-conducting metals). In addition they made no use of the simplest formulas of the theory of electromagnetic fields of thermal origin—the generalized Kirchhoff's law.[6] It is precisely this form of the theory which makes it possible to obtain in briefest form the final result for the general case of two arbitrary media, including anisotropic ones and those having not only temporal but also spatial dispersion. Initially, in Sec. 2, we confine ourselves (to simplify the exposition) to homogeneous and isotropic media without spatial dispersion.

2. EXPRESSION FOR HEAT FLUX

We are interested in the z-component of the total Poynting vector of the thermal field in a vacuum gap

$$P=\int_0^\infty P(\omega)d\omega,$$

which can be expressed as a sum of opposing fluxes from medium 1 into 2 and from 2 into 1:

$$P=P_{12}+P_{21}.$$

This equality is satisfied, of course, also for the spectral densities

$$P(\omega)=P_{12}(\omega)+P_{21}(\omega). \tag{1}$$

It must be emphasized that $P_{12}(\omega)$ is calculated for the heat flux produced in a gap by medium 1 [permittivity $\varepsilon_1(\omega, T_1)$] in the presence of medium 2 [permittivity $\omega_2(\omega, T_2)$]and conversely. Of course, the determination of this field, which reduces to a solution of a certain boundary-value problem,[6] yields automatically a result that already incorporates multiple re-reflections of traveling waves and diffusion (tunneling) of inhomogeneous waves generated by each of the media. Since the sources of the thermal fluctuations in media 1 and 2 are statistically independent, both fields are incoherent, and it is this which causes the additivity of the fluxes in (1). It is furthermore evident that it suffices only to determine $P_{12}(\omega)$, since $P_{21}(\omega)$ can be immediately obtained by permutation of the indices $1 \rightleftarrows 2$ and by reversal of the sign. We therefore comment here on the case when the thermal field is produced by medium 1, while medium 2 is present passively, as if it were absolutely called, although its permittivity is $\varepsilon_2(\omega, T_2)$.

The spectral density $P_{12}(\omega)$ at a frequency $\omega \geqslant 0$ is

$$P_{12}(\omega)=\frac{c}{4\pi}\{\langle E_x(\omega,z_0)H_y^*(\omega,z_0)\rangle-\langle E_y(\omega,z_0)H_x^*(\omega,z_0)\rangle+\text{c.c.}\}, \tag{2}$$

where the asterisk denotes complex conjugation, and the angle brackets denote statistical averaging. The correlators of the spectral components of the thermal field at the point $\mathbf{r}_0=\{0,0,z_0\}$ (Fig. 1) are expressed, in accord with the generalized Kirchhoff's law, in terms of the Joule losses of certain auxiliary ("diffraction") fields in the radiating medium 1.[6] The latter constitute Green's functions with sources chosen in a definite manner (dipoles placed at the point $\mathbf{r}_0$). For planar-layered media this boundary-value problem was solved long ago (see, e.g., Ref. 16), the diffraction fields are known and can be expressed in the form of spatial integrals containing both traveling and inhomogeneous waves.

Finding the correlators in (2), we arrive at the following expression for the spectral density of the energy flux of the thermal field produced in the system by medium 1:

$$P_{12}(\omega)=\frac{1}{\pi^2}\Pi_1 M. \tag{3}$$

Here

$$\Pi_1=\hbar\omega(\exp(\hbar\omega/k_BT_1)-1)^{-1}$$

is the Planck function,[3)]

$$M=-\int_0^\infty\left\{\frac{1}{|\Delta_\varepsilon|^2}\left(\frac{p_1}{\varepsilon_1}-\frac{p_1^*}{\varepsilon_1^*}\right)\left(\frac{p_2}{\varepsilon_2}-\frac{p_2^*}{\varepsilon_2^*}\right)+\frac{1}{|\Delta_\mu|^2}\left(\frac{p_1}{\mu_1}-\frac{p_1^*}{\mu_1^*}\right)\left(\frac{p_2}{\mu_2}-\frac{p_2^*}{\mu_2^*}\right)\right\}|p|^2\varkappa\,d\varkappa=M_\varepsilon+M_\mu, \tag{4}$$

$$p=(\varkappa^2-k^2)^{1/2},\quad p_\nu=(\varkappa^2-k^2\varepsilon_\nu\mu_\nu)^{1/2},\quad k=\omega/c\quad(\nu=1,2)$$

$$\Delta_\varepsilon=\left(\frac{p_1}{\varepsilon_1}+p\right)\left(\frac{p_2}{\varepsilon_2}+p\right)e^{pl}-\left(\frac{p_1}{\varepsilon_1}-p\right)\left(\frac{p_2}{\varepsilon_2}-p\right)e^{-pl}, \tag{5}$$

and for Δ_μ we have the same expression as for Δ_ε, but with the interchange $\varepsilon_\nu \rightleftarrows \mu_\nu$. For p_ν we choose the root with $\mathrm{Re}\,p_\nu>0$, and formula (4) is independent of the choice of the sign of p. The integral M is written out here for media that have, in addition to ε_1 and ε_2, also complex permeabilities μ_1 and μ_2.[4)]

We note that the obvious requirement that $P_{12}(\omega)$ be independent of the position of the point of observation in the gap (of the distance z_0 to the boundary of the medium 1) is automatically satisfied: z_0 drops out of the integrand of (4) if it is recognized that p is either real or pure imaginary, and only the width of the gap l remains.

The integral M is symmetrical with respect to permutation of the indices 1 and 2, so that the flux from medium 2 is equal to $P_{21}(\omega)=-\Pi_2 M/\pi^2$, and the resultant flux is

$$P=\frac{1}{\pi^2}\int_0^\infty(\Pi_1-\Pi_2)M\,d\omega. \tag{6}$$

The integral (4) contains contributions from traveling waves ($0\leq\varkappa\leq k$, p imaginary) as well as from inhomogeneous waves ($\varkappa>k$, p real), i.e., from the near or quasistationary field.

3. CERTAIN CONSEQUENCES AND PARTICULAR CASES

It is clear that when the width l of the gap is increased the role of the near field becomes smaller and smaller (at real p, the quantities Δ_ε and Δ_μ increase like e^{pl}), and in the limit only the traveling-wave field is left, when, in essence, the heat-exchange can be meaningfully called radiant. To calculate the integral with respect to $\varkappa$ from zero to k as $l\to\infty$ we proceed in the following manner. We introduce in place of $\varkappa$ the variable $h=(k^2-\varkappa^2)^{1/2}$ $(p=ih)$. The term $M\varkappa$ in (4) then takes the form

$$M_\varepsilon=-\int_0^k\frac{(\gamma_1-\gamma_1^*)(\gamma_2-\gamma_2^*)}{|\Delta_\varepsilon|^2}h^3\,dh;$$

$$\gamma_\nu=p_\nu/\varepsilon_\nu\quad(\nu=1,2),$$

$$\Delta_\varepsilon=2i(\alpha\sin hl+\beta\cos hl),\quad \alpha=\gamma_1\gamma_2-h^2,\quad \beta=(\gamma_1+\gamma_2)h. \tag{7}$$

In expanded form

$$M_\varepsilon=-\frac{1}{2}\int_0^k\frac{(\gamma_1-\gamma_1^*)(\gamma_2-\gamma_2^*)}{a+b\cos 2hl+c\sin 2hl}h^3dh;$$

$$a=|\alpha|^2+|\beta|^2,\quad b=|\alpha|^2-|\beta|^2,\quad c=\alpha\beta^*+\alpha^*\beta.$$

With increasing l, the period of the oscillations of the denominator, namely $\Delta h=\Pi/l$, decreases and it is therefore expedient to calculate the integral in two stages: the interval $(0,k)$ is broken up into periods Δh (their number is $\approx 2l/\lambda$), and the integration is carried out over each of the intervals Δh, assuming that within the limits of this interval all the quantities except $\sin 2hl$ and $\cos 2hl$ of constant values corresponding to $p=p_j=j\Delta h$, and then take the sum over all the intervals Δh:

$$M_\varepsilon\approx-\frac{1}{2}\sum_{j=0}^{2l/\lambda}[(\gamma_1-\gamma_1^*)(\gamma_2-\gamma_2^*)h^3]_{p=p_j}\int_{p_j}^{p_j+\Delta h}\frac{dh}{a_j+b_j\cos 2hl+c_j\sin 2hl}.$$

We denote the integral that enters in this expression by I_j and change its integration variable to $x=2hl$:

$$I_j=\frac{1}{2l}\int_0^{2\pi}\frac{dx}{a_j+b_j\cos x+c_j\sin x}.$$

Since the condition

$$a_j^2-(b_j^2+c_j^2)=-(\alpha_j\beta_j^*-\alpha_j^*\beta_j)^2\geqslant 0$$

is satisfied, the integral is

$$I_j=\frac{\Delta h}{(a_j^2-(b_j^2+c_j^2))^{1/2}}=\frac{\Delta h}{|\alpha_j\beta_j^*-\alpha_j^*\beta_j|}.$$

When this result is substituted in M_ε, it is natural to change, as $l\to\infty$, from a sum to an integral. This yields, after substituting expressions (7) for α and β,

$$M_\varepsilon=-\frac{1}{2}\int_0^k\frac{(\gamma_1-\gamma_1^*)(\gamma_2-\gamma_2^*)}{|(|\gamma_1|^2+h^2)(\gamma_2-\gamma_2^*)+(|\gamma_2|^2+h^2)(\gamma_1-\gamma_1^*)|}h^2\,dh. \tag{8}$$

To obtain M_μ we must make the interchange $\varepsilon\rightleftarrows\mu$, i.e., put $\gamma_\nu=p_\nu/\mu_\nu$.

If the media are identical, i.e., despite the difference between their temperatures we can assume that $\varepsilon_2=\varepsilon_1=\varepsilon$ and $\mu_2=\mu_1=\mu$, then

$$\gamma_2=\gamma_1=\frac{1}{\varepsilon}(k^2-k^2\varepsilon\mu-h^2)^{1/2}\equiv\gamma$$

and (8) takes the form

$$M_\varepsilon=\frac{1}{4}\int_0^k\frac{|\gamma-\gamma^*|}{|\gamma|^2+h^2}h^2\,dh.$$

In particular, for transparent identical media (ε and μ real) we obtain

$$M_\varepsilon=\frac{\varepsilon}{2}\int_0^k\frac{(h^2+q^2)^{1/2}}{(1+\varepsilon^2)h^2+q^2}h^2\,dh\quad(q^2=k^2(n^2-1),\ n^2=\varepsilon\mu). \tag{9}$$

This case (in addition to the more general case of two different transparent media) was investigated by a numerical method in Ref. 8.

The integral in (9) is easily evaluated:

$$M_\varepsilon=\frac{k^3\varepsilon}{8(\varepsilon^2+1)}\left\{2n+\frac{n^2-1}{\varepsilon^2+1}\left[(\varepsilon^2-1)\ln\frac{n+1}{n-1}-4\varepsilon\operatorname{arctg}\frac{\varepsilon}{n}\right]\right\}.$$

Replacement of ε by μ yields M_μ. In particular, for identical nonmagnetic transparent media ($\mu_1=\mu_2=1$, $\varepsilon_1=\varepsilon_2=n^2$) we have at $l=\infty$

$$M(\infty)=M_\varepsilon+M_\mu=\frac{k^3}{8}\left\{\frac{n^2(n^2+1)^2}{n^4+1}+\frac{n^2(n^2-1)}{(n^4+1)^2}\left[(n^4-1)\ln\frac{n+1}{n-1}\right.\right.$$
$$\left.\left.-4n^2\operatorname{arctg}n\right]-(n^2-1)\operatorname{arctg}\frac{1}{n}\right\}. \tag{10}$$

In the case $l=0$ (media in direct contact)

$$M_\varepsilon=-\frac{1}{4}\int_0^\infty\frac{(\gamma_1-\gamma_1^*)(\gamma_2-\gamma_2^*)}{|\gamma_1+\gamma_2|^2}\varkappa\,d\varkappa\quad\left(\gamma_\nu=\frac{p_\nu}{\varepsilon_\nu}\right). \tag{11}$$

If both media absorb (ε_ν and (or) μ_ν complex), then the integral (11) diverges at the upper limit, as should be the case in the absence of spatial dispersion, i.e., for spatial locality of the material equations.

It is of interest to establish in the general case the order of the divergence as $l\to0$. Replacement of the integration variable in (4) by $x=\varkappa l$ yields

$$M_\varepsilon=-\frac{1}{l^2}\int_0^\infty\frac{1}{|D_\varepsilon|^2}\left(\frac{\beta_1}{\varepsilon_1}-\frac{\beta_1^*}{\varepsilon_1^*}\right)\left(\frac{\beta_2}{\varepsilon_2}-\frac{\beta_2^*}{\varepsilon_2^*}\right)|\beta|^2x\,dx, \tag{12}$$

where

$$\beta=(x^2-(kl)^2)^{1/2},\quad\beta_\nu=(x^2-(kl)^2\varepsilon_\nu\mu_\nu)^{1/2},$$
$$D_\varepsilon=\left(\frac{\beta_1}{\varepsilon_1}+\beta\right)\left(\frac{\beta_2}{\varepsilon_2}+\beta\right)e^\beta-\left(\frac{\beta_1}{\varepsilon_1}-\beta\right)\left(\frac{\beta_2}{\varepsilon_2}-\beta\right)e^{-\beta}.$$

The integrals that remain in (12) converge already at all l, including $l=0$, so that the expansion of M in powers of l at small l is of the form

$$M=\frac{C_{-2}}{l^2}+\frac{C_{-1}}{l}+C_0+\ldots, \tag{13}$$

with ($\varepsilon_\nu=\varepsilon_\nu'-i\varepsilon_\nu''$)

$$C_{-2}=\varepsilon_1''\varepsilon_2''\int_0^\infty\frac{x\,dx}{|(1+\varepsilon_1\varepsilon_2)\operatorname{sh}x+(\varepsilon_1+\varepsilon_2)\operatorname{ch}x|^2}+(\varepsilon\rightleftharpoons\mu).$$

Thus, the first term of the expansion is in the general case of the order of $1/l^2$, and the presence of higher powers of $1/l$ in Caren's paper[15] is an error. With increasing losses, the term of order $1/l^2$ remains, but the region of l in which it plays a role becomes ever narrower ($C_{-2}\to0$) and can reach, in the absence of spatial dispersion with macroscopic scale, microscopic scales at which the phenomenological theory no longer holds at all. It is furthermore clear that the term of order $1/l^2$ vanishes if at least one of the media is transparent.

Assume for the sake of argument that medium 2 is transparent, then

$$\frac{p_2}{\varepsilon_2}-\frac{p_2^*}{\varepsilon_2^*}=\frac{p_2-p_2^*}{\varepsilon_2}=\begin{cases}2i(k^2n_2^2-\varkappa^2)^{1/2}, & \varkappa<kn_2\\ 0, & \varkappa>kn_2\end{cases},$$

where $n_2=(\varepsilon_2\mu_2)^{1/2}$. Thus, the integration with respect to $\varkappa$ is cut off at $\varkappa=kn_2$, and there is no divergence at any l, including $l=0$. This last case of contact between an absorbing medium in the half-space $z<0$ with a transparent medium, was considered in detail in Refs. 5 and 6. We note that the convergence of the integral (11) in the case of one transparent medium means that not only C_{-2} but also C_{-1} vanishes in the expansion (13).

If both media are transparent, then the upper limit in (4) is determined by the medium with the smaller refractive index, i.e., it is equal to kn_2 at $n_1>n_2$. Formula (11) takes therefore the form

$$M_\varepsilon=\frac{1}{\varepsilon_1\varepsilon_2}\int_0^{kn_2}[(k^2n_1^2-\varkappa^2)(k^2n_2^2-\varkappa^2)]^{1/2}$$
$$\times\left[\frac{1}{\varepsilon_1}(k^2n_1^2-\varkappa^2)^{1/2}+\frac{1}{\varepsilon_2}(k^2n_2^2-\varkappa^2)^{1/2}\right]^{-2}\varkappa d\varkappa\quad(n_1>n_2).$$

The integral can be evaluated in close form, but the resultant expression is rather unwieldy and will not be presented here. On the other hand, for the case of identical refractive indices ($n_2=n_1=n$) we obtain

$$M_\varepsilon=k^3n^3\varepsilon_1\varepsilon_2/2(\varepsilon_1+\varepsilon_2)^2,$$

and for perfectly identical media ($\varepsilon_1=\varepsilon_2$, $\mu_1=\mu_2$)

$$M_\varepsilon=M_\mu={}^1/_8k^3n^3,$$

so that at $l=0$ we have

$$M(0)=M_\varepsilon+M_\mu={}^1/_4k^3n^3. \tag{14}$$

If, as in Ref. 8, we neglect the dispersion (n independent of ω), then it is easy to integrate with respect to ω in (6), and we obtain the total energy flux[5] at $l=0$. In the absence of dispersion, the integral with respect to ω for (14) is the same as for M at $l=\infty$. In other words, the ratio $P(0)/P(\infty)$ of the total fluxes is equal to ratio $M(0)/M(\infty)$. According to (10) and (14)

$$P(0)/P(\infty)=1/F(n),$$

where

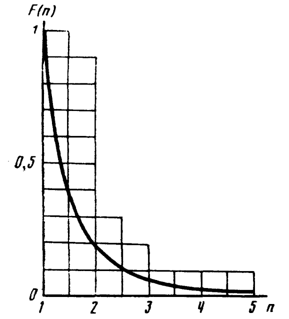

FIG. 2.

$$F(n)=\frac{1}{2}\left\{\frac{(n^2+1)^2}{n(n^4+1)}+\frac{n^2-1}{(n^4+1)^2}\left[(n^4-1)\ln\frac{n+1}{n-1}-4n^2\operatorname{arctg} n\right]-\frac{n^2-1}{n^2}\operatorname{arctg}\frac{1}{n}\right\}. \quad (15)$$

At small values of $n-1$, as follows from (15),

$$F(n)\approx 1-(1+\pi/2)(n-1).$$

Figure 2 shows a plot of the function $F(n)$. We note that the values of the fluxes $P(0)$ and $P(\infty)$, calculated in accordance with our formulas for the same values of the temperatures and refractive indices for which a computer calculation was performed in Ref. 8, agrees with the results obtained in that reference.

4. CASE OF WELL-CONDUCTING METALS

By a well-conducting metal we mean one whose permittivity $\varepsilon(\omega)=-4\pi i\sigma/\omega$ (σ is the conductivity) has an absolute value much larger than unity. Then, in first-order approximation, at any structure of the field outside the metal, the Leontovich boundary condition $\mathbf{E}=\zeta\mathbf{H}\times\mathbf{n}$ is satisfied on the surface of the metal; here $\zeta=(\mu/\varepsilon)^{1/2}=(i\omega\mu/4\pi\sigma)^{1/2}$ is the surface impedance, which is obviously a small quantity ($|\zeta(\omega)|\ll 1$). The entire subsequent calculation of the heat exchange is accurate to first order in ζ.

In this approximation, the values of p_ν are

$$p_\nu=(\varkappa^2-k^2\varepsilon_\nu\mu_\nu)^{1/2}\approx ik(\varepsilon_\nu\mu_\nu)^{1/2}\ (\nu=1,2),$$

so that the ratios

$$p_\nu/\varepsilon_\nu\approx ik\zeta_\nu,\ p_\nu/\mu_\nu\approx ik/\zeta_\nu=ik\zeta_\nu^*/|\zeta_\nu|^2$$

are likewise independent of $\varkappa$. As a result we obtain for the terms M_ε and M_μ in (4), in the considered impedance approximation

$$M_\varepsilon=k^2\zeta_1'\zeta_2'\int_0^\infty\frac{\varkappa d\varkappa}{|p\operatorname{sh}pl+ik\zeta\operatorname{ch}pl|^2}=k^2\zeta_1'\zeta_2'I_\varepsilon,$$

$$M_\mu=\zeta_1'\zeta_2'\int_0^\infty\frac{|p|^2\varkappa d\varkappa}{|k\operatorname{sh}pl-ip\zeta\operatorname{ch}pl|^2}=k^2\zeta_1'\zeta_2'I_\mu, \quad (16)$$

where we have introduced the notation

$$\zeta=\zeta'+i\zeta''=\zeta_1+\zeta_2=(\zeta_1'+\zeta_2')+i(\zeta_1''+\zeta_2'').$$

The factor $\zeta_1'\zeta_2'$ is already present in front of the integrals in (16), and we wish to take into account only first-order effects in the impedances. It is clear therefore that in the integrals I_ε and I_μ we are interested only in the singularities—the terms of order $1/\zeta$. We obtain them separately for the near field ($\varkappa>k$) and for the wave field ($0<\varkappa<k$).

Near field. Changing over to an integration variable $p=(\varkappa^2-k^2)^{1/2}$, we have

$$I_\varepsilon^{\text{near}}=\int_0^\infty\frac{pdp}{|p\operatorname{sh}pl+ik\zeta\operatorname{ch}pl|^2},\ k^2I_\mu^{\text{near}}=\int_0^\infty\frac{p^3dp}{|k\operatorname{sh}pl-ik\zeta\operatorname{ch}pl|^2}.$$

As $p\to 0$ ($\varkappa\to k$), there is no singularity in I_μ^{near}, and for $I_\varepsilon^{\text{near}}$, inasmuch as $\sinh pl\approx pl$ and $\cosh pl\approx 1$, we obtain

$$I_\varepsilon^{\text{near}}\approx\int_0\frac{pdp}{|p^2l+ik\zeta|^2}=\frac{1}{2kl\zeta'}\left(\frac{\pi}{2}+\operatorname{arctg}\frac{\zeta''}{\zeta'}\right). \quad (17)$$

Wave field. Putting $p=ih$, we obtain

$$I_\varepsilon^{\text{wave}}=\int_0^k\frac{h\,dh}{|h\sin hl-ik\zeta\cos hl|^2},\quad k^2I_\mu^{w}=\int_0^k\frac{h^3\,dh}{|k\sin hl+ih\zeta\cos hl|^2}.$$

As $h\to 0$, the integral I_μ^{wave} has no singularities, and we obtain for I^{wave}

$$I_\varepsilon^{\text{wave}}\approx\int_0\frac{hdh}{|h^2l-ik\zeta|^2}=\frac{1}{2kl\zeta'}\left(\frac{\pi}{2}-\operatorname{arctg}\frac{\zeta''}{\zeta'}\right). \quad (18)$$

In addition, I^{wave} has singularities at the other zeros of $\sin hl$, i.e., near the values $h_n=n\pi/l<k$ (n is an integer). In the vicinity of h_n, putting $h=h_n+h'$, we have $\sin hl\approx(-1)^nh'l$ and $\cos hl\approx(-1)^n$, and the contribution made by the integral over the vicinity of h_n is

$$\int\frac{h_n\,dh'}{|h_nlh'-ik\zeta|^2}=\frac{\pi}{kl\zeta'},$$

i.e., it is independent of n. The number m of such contributions is obviously equal to the integer part of the quantity $y=kl/\pi$ ($m=[y]$), so that all h_n (with the exception of $h=0$) make a summary contribution

$$m\pi/kl\zeta'=m/\zeta'y. \quad (19)$$

It follows from (17)–(19) that

$$I_\varepsilon=\frac{1}{\zeta'}\left(\frac{1}{2y}+\frac{m}{y}\right).$$

In the integral I_μ^{wave}, the contribution from the vicinity of the point h_n is

$$\frac{h_n^3}{k^2}\int\frac{dh'}{|klh'+ih_n\zeta|^2}=\frac{\pi h_n^2}{k^3l\zeta'}=\frac{n^2}{\zeta'y^3}$$

and consequently

$$I_\mu^{\text{wave}}=\frac{1}{\zeta'y^3}\sum_1^m n^2=\frac{1}{6\zeta'y^3}m(m+1)(2m+1).$$

Since I_μ^{near} has no singularity, we obtain in accord with (14)

$$M=M_\varepsilon+M_\mu=k^2\frac{\zeta_1'\zeta_2'}{\zeta_1'+\zeta_2'}f\left(\frac{kl}{\pi}\right); \quad (20)$$

$$f(y)=\frac{1}{2y}+\frac{m}{y}+\frac{m(m+1)(2m+1)}{6y^3},\quad y=\frac{kl}{\pi},\quad m=[y]. \tag{21}$$

At $y<1$ we have $f(y)=1/2y$, and at $y \gg 1$, when we can assume $y \approx m$, we obtain $f(y)\approx 4/3$. At each passage of y through an integer value, the function $f(y)$ undergoes a jump due to the introduction of a new mode[6]: $f(m+0)-f(m-0)=2/m$.

Formula (20) was derived from the general expression (4), which was obtained in turn assuming absence of spatial dispersion. But (20) contains only surface impedances that describe the ratio of the tangential components of the electric and magnetic fields on the boundary of the body. The result (20) therefore remains in force also in the presence of spatial dispersion, provided only that the surface impedance of the medium is small enough. We would have arrived at (20) also if we were to solve from the very beginning the auxiliary boundary-value problem of diffraction field with impedance conditions on the boundaries of the vacuum gap (see Sec. 5).

Thus, in the impedance approximation, the spectral density of the heat flux is described by the simple analytic formulas (20) and (21). These formulas make it possible also to calculate the total flux, if the impedance approximation is valid in the entire region of frequencies ω in which the Planck function differs noticeably from zero, i.e., the conditions $|\zeta_1(\omega)|\ll 1$ and $|\zeta_2(\omega)|\ll 1$ are satisfied in a sufficiently large vicinity of the Wien frequency $\omega_W=2\pi c/\lambda_W$, where $\lambda_W T=0.29$ (λ_W is in centimeters and T is in degrees Kelvin). The total flux, according to (6) and (20), then takes the form

$$P=P_{12}+P_{21}=\frac{1}{\pi^2c^2}\int_0^\infty[\Pi_1(\omega)-\Pi_2(\omega)]Z(\omega)f\left(\frac{\omega l}{\pi c}\right)\omega^2\,d\omega, \tag{22}$$

where

$$Z(\omega)=\zeta_1'\zeta_2'/(\zeta_1'+\zeta_2'). \tag{23}$$

We note that the case (frequently encountered in experiment) when the impedance of the cold medium is much smaller than the impedance of the hot medium ($\zeta_2'\ll\zeta_1'$ and $T_1>T_2$), we have $Z(\omega)\approx\zeta_2'$, and the flux P depends mainly on the temperature of the hot medium and on the impedance of the cold one.

At not too low temperatures, the impedances of metals are given by the formula for the normal skin effect

$$\zeta_\nu'=(\omega/8\pi\sigma_\nu)^{1/2}\quad(\nu=1,\ 2) \tag{24}$$

and consequently $Z(\omega)\propto\omega^{1/2}$ if, of course, there is no dispersion of the conductivities σ_ν as yet in the Wien region of frequencies. At room temperatures, as already indicated by Rubens and Hagen (see, e.g., Ref. 17), we can use the static value of σ at $\lambda>1.2\times10^{-3}$ cm. The Wien wavelength is $\lambda_W=10^{-3}$ cm at $T=290$ K. It can therefore be assumed that in the cryogenic temperature region the dispersion of σ is negligibly small, as will in fact be assumed hereafter. Furthermore, in this region the Wien frequencies are lower, and the conductivities higher than at room temperature, making the accuracy of the impedance approximation higher.

At very low temperatures T_2 of the cold metal, the anomalous skin-effect regime may set in, and spatial dispersion can be produced because the electron mean free path becomes comparable with the thickness of the normal skin layer, and subsequently also much larger. In the latter case[18] $\zeta_2'\propto\omega^{2/3}$ and, if $\zeta_1'\gg\zeta_2'$ (or if the skin effect is anomalous for both metals, i.e., if also $\zeta_1'\propto\omega^{2/3}$), then $Z(\omega)\propto\omega^{2/3}$.

We confine ourselves below only to these two forms of the $Z(\omega)$ frequency dependence, and express this quantity in the form

$$Z(\omega)=Z_\alpha(\omega_1)(\omega/\omega_1)^\alpha=Z_\alpha(\omega_2)(\omega/\omega_2)^\alpha, \tag{25}$$

where $\alpha=1/2$ for the normal skin effect and $\alpha=2/3$ for the anomalous one. Here $\omega_\nu=k_BT_\nu/\hbar\approx\omega_{W\nu}/5$, so that

$$\Pi_1(\omega)=\hbar\omega_1\frac{\omega/\omega_1}{e^{\omega/\omega_1}-1},\quad \Pi_2(\omega)=\hbar\omega_2\frac{\omega/\omega_2}{e^{\omega/\omega_2}-1}.$$

In the calculation of P_{12}, introducing the characteristic length $\lambda_1=2\pi c/\omega_1=2\pi c\hbar/k_BT_1\approx5\lambda_{W1}$, and the dimensionless parameters

$$t=\omega/\omega_1,\quad x=2l/\lambda_1\quad(l/\lambda_W=2.483x) \tag{26}$$

(the argument of the function f in (22) is in this case $y=\omega l/\pi c=xt$), we reduce the total flux P_{12} to the form

$$P_{12}=\frac{\hbar\omega_1^4Z_\alpha(\omega_1)}{\pi^2c^2}W_\alpha(x),\quad W_\alpha(x)=\int_0^\infty\frac{t^{3+\alpha}f(xt)}{e^t-1}dt. \tag{27}$$

For a thin gap ($l\ll\lambda_1$, $x\ll1$), the only region of importance is $xt<1$, in which $f(xt)\approx1/2xt$, so that

$$W_\alpha(x)\approx L_{\alpha-1}/2x; \tag{28}$$

$$L_\alpha=\int_0^\infty\frac{t^{3+\alpha}}{e^t-1}dt=\Gamma(4+\alpha)\zeta(4+\alpha) \tag{29}$$

(Γ is the gamma function and ζ is the Riemann zeta function). Thus, in the case of thin gaps, the total heat flux increases with decreasing l like $1/l$.

For a wide gap ($l\gg\lambda_1$, $x\gg1$), the main contribution to the integral (22) is made by the region $t\sim1$, in which $xt\gg1$ and $f(xt)\approx4/3$. Therefore at $x\gg1$, we have

$$W_\alpha(x)\approx{}^4/_3L_\alpha. \tag{30}$$

This formula corresponds to an asymptotic flux value independent of l (the approximation of the classical theory of thermal radiation).

Replacing the frequency ω_1 by its expression in terms of the temperature ($\omega_1=k_BT_1/\hbar$) and denoting by S_α the quantity

$$S_\alpha=\frac{4}{3}L_\alpha\frac{k_B^4}{\pi^2c^2\hbar^3}=\frac{80}{\pi^4}\sigma_{SB}L_\alpha,\qquad S_{1/2}=10.076\sigma_{SB},\quad S_{2/3}=12.663\sigma_{SB}, \tag{31}$$

we can rewrite (27) in the form

$$P_{12} = S_\alpha T_1^4 Z_\alpha(\omega_1) w_\alpha(x), \quad w_\alpha(x) = W_\alpha(x)/W_\alpha(\infty). \tag{32}$$

The universal (independent of the parameters of the metal) function $w_\alpha(x)$ takes at $x \ll 1$ (thin gap), according to (28) and (29), the form

$$w_\alpha(x) \approx 3L_{\alpha-1}/8L_\alpha x \equiv A_\alpha/x, \tag{33}$$

with $A_{1/2} = 0.1450$ and $A_{2/3} = 0.1082$. It can be shown that in the opposite case at $x \gg 1$ (wide gap)

$$w_\alpha(x) \approx 1 - \frac{B_\alpha}{x^{3+\alpha}}, \tag{34}$$

$$B_\alpha = \frac{3\zeta(3+\alpha)(1+\alpha)}{(2\pi)^{3+\alpha}\zeta(4+\alpha)\alpha(2+\alpha)(3+\alpha)} \sin\frac{\pi\alpha}{2},$$

so that $B_{1/2} = 0.00125$ and $B_{2/3} = 0.00083$. It is clear from (34) that $w_\alpha(x)$ approaches its unity asymptotic value from below, i.e., $w_\alpha(x)$ has a minimum.

The solid line 1 of Fig. 3 is the computer-calculated plot of $w_{1/2}(x)$. The dashed plots are the asymptotic dependences (33) and (34) for $\alpha = 1/2$. The curve for the anomalous skin effect ($\alpha = 2/3$) is similar in form. The values of $w_\alpha(x)$ in the vicinity of the minimum are given in the table for $\alpha = 1/2$ and $\alpha = 2/3$. The minimum of $w_\alpha(x)$ at $\alpha = 1/2$ occurs at $x = 0.180$ and is equal to 0.8960, while at $\alpha = 2/3$ it occurs at $x = 0.1680$ and amounts to 0.8840.

Thus, according to (32), the total heat flux is given by

$$P = S_\alpha\{Z_\alpha(\omega_1) w_\alpha(x_1) T_1^4 - Z_\alpha(\omega_2) w_\alpha(x_2) T_2^4\}. \tag{35}$$

In the cases considered by us, the frequency dependence of $Z_\alpha(\omega)$ was taken in the form (25), i.e.,

$$Z_\alpha(\omega) = G_\alpha \omega^\alpha,$$

where G_α is independent of the frequency ω. Recognizing that $\omega_\nu = k_B T_\nu/h$, we can rewrite (35) in the form

$$P = S_\alpha\left(\frac{k_B}{\hbar}\right)^\alpha G_\alpha\left\{w_\alpha(x_1) T_1^{4+\alpha} - w_\alpha(x_2) T_2^{4+\alpha}\right\}. \tag{36}$$

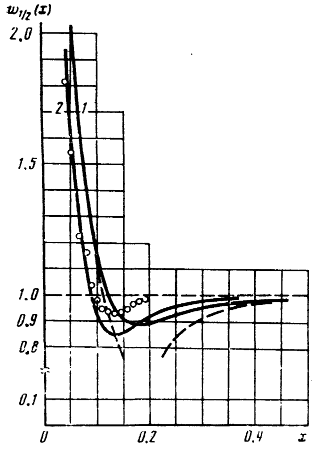

FIG. 3.

TABLE I. Values of the functions $w_\alpha(x)$.

x	$w_{1/2}(x)$	$w_{2/3}(x)$	x	$w_{1/2}(x)$	$w_{2/3}(x)$
0,10 *	1.1655	1.1064	0.25	0,9289	0.9315
0,11	1.0780	1,0271	0,26	0,9344	0.9375
0.12	1.0135	0.9704	0.27	0.9396	0.9431
0.13	0.9674	0.9315	0.28	0,9444	0.9482
0,14	0.9358	0.9064	0,29	0.9489	0.9528
0.15	0,9153	0.8919	0,30	0.9530	0.9570
0.16	0.9033	0,8851	0.31	0.9568	0.9607
0.17	0.8974	0.8837	0.32	0,9602	0.9642
0.18	0.8959	0.8860	0.33	0.9633	0,9672
0.19	0.8975	0.8908	0.34	0.9662	0.9700
0.20	0.9011	0.8969	0,35	0,9688	0.9725
0.21	0.9060	0.9039	0,36	0.9712	0.9748
0.22	0,9115	0,9110	0.37	0,9734	0.9768
0.23	0.9173	0.9182	0,38	0.9753	0.9787
0.24	0,9232	0,9250	0.39 **	0.9771	0.9803

*The asymptotic formula (33) yields at $x = 0.10$ the values $w_{1/2} \approx 1.1450$ and $w_{2/3} \approx 1.028$.
**The asymptotic formula (34) at $x = 0.39$ yields $w_{1/2} \approx 0.9663$ and $w_{2/3} \approx 0.9737$.

For a wide gap, when $w_\alpha(x_1) = w_\alpha(x_2) = 1$, we have

$$P_{\text{wide}} = S_\alpha\left(\frac{k_B}{\hbar}\right)^\alpha G_\alpha\{T_1^{4+\alpha} - T_2^{4+\alpha}\}, \tag{37}$$

and for a thin gap, when expression (33) is valid, i.e., $w_\alpha(x) = A_\alpha/x = \pi\,\hbar c A_\alpha/k_B T l$, we obtain

$$P_{\text{thin}} = S_\alpha A_\alpha \frac{\pi c}{l}\left(\frac{k_B}{\hbar}\right)^{\alpha-1} G_\alpha\{T_1^{3+\alpha} - T_2^{3+\alpha}\}. \tag{38}$$

Thus, the exponents of T in the curly brackets are smaller by unity in the case of a thin gap than in the case of a wide one. In the anomalous skin effect, $G_{2/3}$ does not depend on temperature,[18] so that the entire temperature dependence of the total flux is given by the curly brackets in (36)–(38). In the case of the normal skin effect on the other hand, according to (23) and (24),

$$G_{1/2} = [(8\pi\sigma_1(T_1))^{1/2} + (8\pi\sigma_2(T_2))^{1/2}]^{-1}$$

and it is necessary generally speaking to take into account the temperature dependences of the conductivities.

For comparison with experiment it is more convenient to use the normalized flux—the ratio P/P_{wide}, which according to (36) and (37) is given by

$$\frac{P}{P_{\text{wide}}} = \frac{w_\alpha(x_1) T_1^{4+\alpha} - w_\alpha(x_2) T_2^{4+\alpha}}{T_1^{4+\alpha} - T_2^{4+\alpha}}. \tag{39}$$

It is seen therefore that if T_1 is as little as double T_2, the ratio P/P_{wide} is practically equal to the universal function $w_\alpha(x_1)$.

At small temperature differences, formula (36) changes into a differential equation

$$\frac{P}{\Delta T} = (4+\alpha) S_\alpha\left(\frac{k_B}{\hbar}\right)^\alpha G_\alpha T^{3+\alpha} u_\alpha(x), \quad u_\alpha(x) \equiv w_\alpha(x) + \frac{x}{4+\alpha}\frac{dw_\alpha(x)}{dx}.$$

The universal function $u_{1/2}(x)$ is represented by curve 2 of Fig. 3. The circles show the mean values of Hargreaves' experimental results.[19]

Both the asymptotic behavior of curve 2 with decreasing l and the position of the maximum agree well with experiment. The difference in the depth of the minimum can be due either to the fact that the experiment was carried out with thin (10^{-5} cm) chromium films or to the possible influence of the dispersion of σ at room temperatures ($T_1 = 323$ K, $T_2 = 306$ K). We note that in the experiments of Refs. 12 and 20 the growth of the heat flux $P(l)$ begins at gap widths larger by one order of magnitude than expected from the theory. We have therefore doubts concerning the results of these experiments.

5. CASE OF ARBITRARY MEDIA

We turn to the solution of the problem of heat exchange under more general assumptions with respect to the media 1 and 2 than in Sec. 2 above, where our purpose was to demonstrate the usefulness and convenience of using the generalized Kirchhoff's law.

As already mentioned, to calculate the heat flux in the gap it is necessary to calculate the Joule losses produced in each of the media by certain auxiliary (diffraction) fields. These fields (Green's functions) had to be known only in the gap, where the point sources of these fields (dipoles) were also placed. In this situation, it was convenient to use the concept of generalized surface impedance (see, e.g., Ref. 21). The boundary value of interest to us then splits into two:

a) Determination of the dipole field in the gap between the media, which are described by their surface impedances. By the same token, the final expression for the energy flux of the thermal field contains these impedances.

b) Calculation of the general impedances of the two media themselves, which is a separate problem.

We recall the definition of the generalized surface impedance, bearing in mind the case of flat boundary of interest to us. Assume that we are dealing with medium 1, which fills the half space $z<0$, and locate on its boundary $z=0$ the axes $x_1 \equiv x$ and $x_2 \equiv y$. The possibility of introducing a general impedance is based on the fact that the solution of the boundary-value electrodynamic problem inside a linear medium, when the tangential components of only the electric field (or only the magnetic field) are specified on its boundary, is unique and determines the linear connection between the values of the components of both fields on the boundary of the medium. Denoting by $\mathbf{E}(t,\mathbf{x})$ and $\mathbf{H}(t,\mathbf{x})$ the tangential components of the fields on the boundary ($\mathbf{x} = \{x_1, x_2\}$ is the two-dimensional radius-vector in the plane $z=0$), we write down the indicated connection in the form

$$E_\alpha(t,\mathbf{x}) = \int d\tau \int d^2\rho\, \zeta_{\alpha\beta}(\tau,\rho)\,[H(t-\tau,\mathbf{x}-\rho),\mathbf{n}]_\beta, \qquad (40)$$

where $\mathbf{n}$ is the unit inward normal vector to the boundary of the medium $z=0$, the spatial integration is over the entire plane $z=0$, the indices α and β take on values 1 and 2, and summation is carried out as usual over the repeated index β. The matrix $\zeta_{\alpha\beta}(\tau,\rho)$ is in fact called the generalized impedance tensor of the medium.

A connection of the form (40) between $\mathbf{E}$ and $\mathbf{H}$ covers the case when the medium is anisotropic and is spatially nonlocal, but is homogeneous in the planes $z=\text{const}$; this, of course, does not exclude a dependence of its properties on z.

Changing over in (40) to the Fourier transforms

$$\mathbf{E}(\omega,\varkappa) = \frac{1}{(2\pi)^3}\int dt \int d^2x \exp(-i\omega t + i\varkappa\mathbf{x})\,\mathbf{E}(t,x),$$

and analogously for $\mathbf{H}$, we obtain

$$E_\alpha(\omega,\varkappa) = \zeta_{\alpha\beta}(\omega,\varkappa)\,[\mathbf{H}(\omega,\varkappa)\times\mathbf{n}]_\beta, \qquad (41)$$

where

$$\zeta_{\alpha\beta}(\omega,\varkappa) = \int d\tau \int d^2\rho\, \zeta_{\alpha\beta}(\tau,\rho)\exp(-i\omega\tau + i\varkappa\rho) \qquad (42)$$

is the impedance tensor in the $\omega\,\varkappa$ representation.

Of course, the concrete expression for the impedance (42) depends essentially on the structure of the medium. If, for example, the medium is isotropic in the planes $z=\text{const}$ (we call this a z-uniaxial medium), then $\zeta_{\alpha\beta}(\omega,\varkappa)$ takes the form

$$\zeta_{\alpha\beta}(\omega,\varkappa) = \left(\delta_{\alpha\beta} - \frac{\varkappa_\alpha\varkappa_\beta}{\varkappa^2}\right)\zeta_t(\omega,\varkappa) + \frac{\varkappa_\alpha\varkappa_\beta}{\varkappa^2}\zeta_l(\omega,\varkappa), \qquad (43)$$

where ζ_t and ζ_l are the transverse and longitudinal (relative to $\varkappa$) impedances and depend only on the modulus of $\varkappa$, i.e., on $\varkappa = (\varkappa_1^2 + \varkappa_2^2)^{1/2}$. We note that the inverse tensor $\xi_{\alpha\beta} \equiv \zeta_{\alpha\beta}^{-1}$ is similar in form, with $\xi_t = 1/\zeta_t$ and $\xi_l = 1/\zeta_l$. In the particular case of a completely isotropic and homogeneous semi-infinite medium with material constants $\varepsilon_1(\omega)$ and $\mu_1(\omega)$, the internal boundary-value problem (problem b) is very simple and yields for ζ_t and ζ_l the expressions

$$\zeta_t = ik\mu_1/p_1, \quad \zeta_l = p_1/ik\varepsilon_1, \quad p_1 = (\varkappa^2 - k^2\varepsilon_1\mu_1)^{1/2}. \qquad (44)$$

The sign of the root should be chosen such that $\mathrm{Re}\,p_1 > 0$ (the waves attenuate in the interior of the medium), and in the case of a transparent medium, when p_1 is either real or imaginary, it is necessary to choose for imaginary $p_1 = ih_1$ the value of the root with $h_1 > 0$ (waves traveling away from the boundary). Putting $\varepsilon_1 = 1$ and $\mu_1 = 1$, we obtain from (44) the vacuum impedance $\zeta_{\alpha\beta}^{(0)}$, for which

$$\zeta_t = ik/p, \quad \zeta_l = p/ik, \quad p = (\varkappa^2 - k^2)^{1/2}.$$

If we can neglect $\varkappa$ in (44) compared with $k|\varepsilon_1\mu_1|$, then

$$p_1 \approx ik(\varepsilon_1\mu_1)^{1/2}, \quad \zeta_t \approx \zeta_l \equiv \zeta = (\mu_1/\varepsilon_1)^{1/2},$$

so that the tensor $\zeta_{\alpha\beta}(\omega,\varkappa) \approx \zeta(\omega)\delta_{\alpha\beta}$ does not depend on $\varkappa$ and formula (41) takes the form

$$E_\alpha(\omega,\varkappa) = \zeta(\omega)\,[\mathbf{H}(\omega,\varkappa)\mathbf{n}]_\alpha. \qquad (45)$$

This is precisely the case dealt with in Sec. 4, i.e., the case of well-conducting metals with $|\zeta(\omega)| \ll 1$.

We emphasize that in the present section we make no assumptions whatever concerning the smallness of $\zeta_{\alpha\beta}(\omega, \varkappa)$, so that the results pertain to any spatially nonlocal and anisotropic medium that is inhomogeneous in z.

Since the correlators of the fluctuation field are determined by the Joule losses of the diffraction fields, we present an expression for these losses in the medium in terms of its generalized impedance and the tangential components of the electric field at the boundary. The losses are expressed by the flux of the Poynting vector through the boundary $z=0$:

$$q(t)=\frac{c}{4\pi}d^2x(\mathbf{n}[\mathbf{E}\times\mathbf{H}])_{z=0}=\frac{c}{4\pi}\int d^2x(\mathbf{E}[\mathbf{H}\times\mathbf{n}])_{z=0}. \tag{46}$$

for fields that vary harmonically with time

$$\mathbf{E}(t, \mathbf{x})=\mathrm{Re}\{\mathbf{E}(\omega, \mathbf{x})e^{i\omega t}\}, \quad \mathbf{H}(t, \mathbf{x})=\mathrm{Re}\{\mathbf{H}(\omega, \mathbf{x})e^{i\omega t}\},$$

the averaging of (46) over the period $2\pi/\omega$ and the use of (41) yields for the averaged flux $Q=\langle q(t)\rangle$ the expression

$$Q=\frac{\pi c}{4}\int d^2\varkappa\{\xi_{\alpha\beta}(\omega, \varkappa)+\xi_{\beta\alpha}^*(\omega, \varkappa)\}E_\alpha^*(\omega, \varkappa)E_\beta(\omega, \varkappa),$$

where we have used the surface-admittance tensor $\xi_{\alpha\beta}(\omega, \varkappa)=\zeta^{-1}_{\alpha\beta}(\omega, \varkappa)$ and introduced the Fourier transforms of the components $\mathbf{E}(\omega, \varkappa)$.

We can now turn to the principal problem of heat exchange between two media separated by a vacuum gap of width l, assuming that the surface-impedance tensors of both media ζ_1 and ζ_2 are known [problem a]. Finding the mixed heat losses of the diffraction fields, produced by suitable oriented dipoles with moments $\mathbf{p}$ and $\mathbf{m}$, located in the gap (we omit here the straightforward but rather cumbersome derivations), we arrive at the following final expression for the resultant heat flux:

$$P=\frac{1}{\pi^2}\int_0^\infty d\omega(\Pi_1-\Pi_2)\int d^2\varkappa N(\omega, \varkappa); \tag{47}$$

$$N(\omega, \varkappa)=\frac{1}{2\pi}\mathrm{Sp}\{\xi_0^+R^+(\zeta_1+\zeta_1^+)R\xi_0(\zeta_2+\zeta_2^+)\}, \tag{48}$$

$$R^{-1}=(I+\xi_0\zeta_2)(I+\xi_0\zeta_1)e^{pl}-(I-\xi_0\zeta_2)(I-\xi_0\zeta_1)e^{-pl}, \tag{49}$$

where ξ_0 denotes the matrix inverse to the vacuum impedance, I is a unit matrix, and the cross marks the Hermitian-conjugation operation.

We note certain properties of the quantity $N(\omega, \varkappa)$. Although expression (48) does not reveal explicitly the symmetry of $N(\omega, \varkappa)$ with respect to permutation of the media $1 \rightleftarrows 2$, it can be verified that $N(\omega, \varkappa)$ does possess this symmetry. Further, if we replace in (48) all the matrices by their inverses ($\xi_0\to\zeta_0, \zeta_1\to\xi_1, \zeta_2\to\xi_2$), then $N(\omega, \varkappa)$ remains unchanged. Nor does $N(\omega, \varkappa)$ change in the case when ζ_1 and ζ_2 are replaced by the matrices $\tilde{\zeta}_1$ and $\tilde{\zeta}_2$ obtained with the aid of the transformation

$$\tilde{\zeta}_1=-\varepsilon\xi_1\varepsilon, \quad \tilde{\zeta}_2=-\varepsilon\xi_2\varepsilon,$$
$$\varepsilon=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

In this case there is no need to transform the vacuum impedance, since it is invariant to this transformation (it is easy to verify that $\xi_0=-\varepsilon\zeta_0\varepsilon$). It follows from the foregoing that the heat exchange between media with impedance matrices ζ_1 and ζ_2 is the same as between media with impedances $\tilde{\zeta}_1$ and $\tilde{\zeta}_2$. This property reflects the principle of duality of the fields $\mathbf{E}$ and $\mathbf{H}$.

We consider now the value of $N(\omega, \varkappa)$ for z-uniaxial media. In this case the surface-impedance tensors are given by (43) and the trace in (48) can be easily calculated by directing the x_1 axis along the vector $\varkappa$ (in this case the matrices ζ_1, ζ_2, and ξ_0 become diagonal). As a result we have

$$N(\omega, \varkappa)=\frac{1}{2\pi}\left\{\frac{(\zeta_{1l}+\zeta_{1l}^*)(\zeta_{2l}+\zeta_{2l}^*)}{|\zeta_{0l}|^2|\Delta_l|^2}+\frac{(\zeta_{1t}+\zeta_{1t}^*)(\zeta_{2t}+\zeta_{2t}^*)}{|\zeta_{0t}|^2|\Delta_t|^2}\right\}, \tag{50}$$

$$\begin{gathered}\Delta_l=\left(1+\frac{\zeta_{1l}}{\zeta_{0l}}\right)\left(1+\frac{\zeta_{2l}}{\zeta_{0l}}\right)e^{pl}-\left(1-\frac{\zeta_{1l}}{\zeta_{0l}}\right)\left(1-\frac{\zeta_{2l}}{\zeta_{0l}}\right)e^{-pl},\\ \Delta_t=\left(1+\frac{\zeta_{1t}}{\zeta_{0t}}\right)\left(1+\frac{\zeta_{2t}}{\zeta_{0t}}\right)e^{pl}-\left(1-\frac{\zeta_{1t}}{\zeta_{0t}}\right)\left(1-\frac{\zeta_{2t}}{\zeta_{0t}}\right)e^{-pl}.\end{gathered} \tag{51}$$

Since all the quantities in (50) depend only on $\varkappa=(\varkappa_1^2+\varkappa_2^2)^{1/2}$, we can integrate in (47) over the polar angle. If we substitute in (50) and (51) the impedances (44) corresponding to fully isotropic media, then we arrive at formulas (4) and (5).

The results presented are applicable only to nongyrotropic media, but can be readily extended also to the case when gyrotropy is present. As shown earlier,[6] in this case it is necessary to calculate the heat losses of the diffraction fields with the external magnetizing field $\mathbf{B}_0$ inverted. Thus, the formulas obtained will apply also for gyrotropic media, if we substitute in (48) and (49) surface impedances calculated with the field $\mathbf{B}_0$ inverted. It should be noted that the homogeneity assumed above for the media in the planes $z=\mathrm{const}$ means that the magnetizing field can be oriented arbitrarily but must be uniform.

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1) This circumstance is ignored in Caren's paper,[4] in which the calculation of the energy density in an equilibrium rectangular resonator and of the energy flux in its walls, performed with account taken of only discrete wave modes, was extended also to those cases when all or some of the dimensions of the resonator decrease without limit.

2) Much later, but from the same point of view of observing the near field, a purely radiotechnical (spectrally selective) method of its variation was proposed,[7] but at the present time the realization of this method entails apparently great difficulties.

3) The generalized Kirchhoff's law contains the average energy of the oscillator $\Theta(\omega, t)$, but the zero-point oscillations make under no condition any contribution to the energy flux and are therefore discarded here: $\Pi_1=\Theta(\omega, T_1)-\hbar\omega/2$.

4) As stated earlier, the generalization of (4) and (5) to anisotropic media and to the presence of spatial dispersion is given in Sec. 5.

5) Integration with respect to ω yields $P=\sigma_{SB}(T_1^4-T_2^4)n^2$, where σ_{SB} is the Stefan–Boltzmann constant.

6) Jumps, of the same origin, in the average spectral characteristics of the thermal field are encountered also in other waveguide problems (see, e.g., Ref. 6, §9).

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Redistribution of the vibrational energy in the course of laser excitation of high vibrational levels of polyatomic molecules

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An analysis is made of a method for calculating the rate coefficients of transitions in a laser field between vicinities of resonances in the band structure of the vibrational quasicontinuum of polyatomic molecules. It is shown that the dependences of the rate coefficients on the molecular energy and on the laser frequency are governed by the type of the strongest anharmonic interaction which locks the excited mode and by the nature of the redistribution of the vibrational energy of the molecules between the degrees of freedom. A study is made of the possibility of using the model of complete stochastization of vibrations in the description of the excitation of a molecule by infrared laser radiation. A model for the increase in the vibrational energy of the directly excited mode is proposed and this model gives the best agreement with the experimental frequency and energy characteristics of the SF_6 and SiF_4 molecules. The available spectroscopic data on the SF_6 molecule are used to find the dependence of the threshold energy of stochastization on the laser interaction frequency.

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The process of isotopically selective collisionless infrared dissociation of molecules in a laser field is attracting attention[1-3] and this applies particularly to the nature of the excitation of high vibrational states of polyatomic molecules.[4-9] This includes determination of the characteristics governing the excitation dynamics and of the frequency and energy dependences of the efficiency of the laser interaction, studies of the methods and mechanisms of the redistribution of the vibrational energy of the molecules between the degrees of freedom, etc. In contrast to early studies of the interaction of laser radiation with the vibrational degrees of freedom of molecules,[10,11] recent work on the excitation of polyatomic molecules has led the majority of authors to the conclusion of the need to separate the problem into two parts: spectroscopic and kinetic. The spectroscopic problem involves investigation of the internal interactions and formation of the spectra, whereas the kinetic problem involves the dynamics of excitation of such spectroscopically complex systems.

An approach to the solution of the problem of the excitation of a polyatomic molecule in two stages is suggested in Ref. 6. The first stage is a consideration of the model problem of excitation of a complex multilevel quantum system in a laser field and determination of those characteristics of the spectrum and operator of the interaction with the field which govern the dynamics of the populations. The next stage should be determination of the characteristics of Ref. 6 for real molecules

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