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Distinguishing features of generation of the second harmonic of an electromagnetic wave in a magnetoactive plasma

N. S. Erokhin, S. S. Moiseev, and A. P. Shuklin

Kharkov State University
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Second-harmonic generation upon incidence of an electromagnetic wave on a weakly inhomogeneous plasma is investigated under conditions when the plasma density gradient is perpendicular to the external magnetic field. The efficiency of energy conversion into the second harmonic is calculated and the dependence of the effect on the polarization of the wave incident on the plasma is indicated.

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INTRODUCTION

It was shown earlier[1] that when a p-polarized electromagnetic wave is incident on a weakly inhomogeneous cold isotropic plasma, a second harmonic is observed in the reflected signal. This effect occurs only in an inhomogeneous plasma because of the following circumstances. First, in an inhomogeneous isotropic plasma the nonlinear current induced at double the frequency by the incident wave is not purely longitudinal, but has a transverse component proportional to the gradient of the plasma density. Second, in an inhomogeneous isotropic plasma there exists near the critical-density surface a unique high-Q resonator in which the amplitude of the first field harmonic reaches anomalously high values. Owing to the small thickness of this resonator (compared with the wavelength of the second harmonic), the spectrum of the spatial harmonics of the nonlinear current broadens and occupies the region of the second-harmonic phase velocities, and it is this which leads to effective generation of the double-frequency wave from the region of the critical plasma density.

Second-harmonic generation has by now been investigated by many workers (see, e.g., Refs. 2–5), the main premises of the theory have been confirmed, and the effect is being used in plasma diagnostics. Since the plasma contains frequently quasistationary magnetic field (either produced by extraneous sources or spontaneously generated in the plasma as a result of the development of various instabilities[6-10]), a need arises for the study of the effect of the magnetic field on the generation of the second harmonic of the electromagnetic wave. This is important both for a correct interpretation of the experimental data and to determine the conditions under which the effect can be used, as well as the distinguishing features of the second-harmonic generation in a magnetoactive plasma.

The features of second-harmonic generation in a magnetoactive plasma are connected mainly with the change of the dispersion of the electromagnetic waves. Thus, in a magnetoactive plasma it is necessary to take into account the possibility of synchronism (coincidence of the refractive indices) between the harmonics, even if the thermal corrections to the wave dispersion are negligibly small. An important role is played by the presence of points of intersection of the electromagnetic-oscillation modes, as well as the agreement between the location of the singularities of the first harmonic and of its refractive index. It will be shown below that in a magnetoactive plasma a singularity of the second-harmonic field can be located in the region of propaga-

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tion of the first harmonic. Whereas in a cold isotropic plasma the second harmonic is radiated from the region of a singularity of the first-harmonic field, in a magnetoactive plasma the principal radiation can come from the synchronism region of the harmonics, where their refractive indices are equal, while the radiation from the regions of the singularity of the first-harmonic field is negligible because of interference between radiation coming from separate layers. It is of interest to investigate the influence of the magnetic field on the polarization of the second harmonic. It must be noted here that in an isotropic plasma the second harmonic is p-polarized and is present only in the reflected signal.

The present paper deals with the foregoing questions. Although second-harmonic generation in magnetoactive plasma has already been investigated before,[11] only a particular limiting case was considered there, that of a weak external magnetic field and of electromagnetic waves propagating strictly perpendicularly to this field, when the coupling coefficient of the harmonics is small in terms of the parameter ω_{He}/ω and the problem does not deal with a number of important aspects, including the interaction between differently polarized waves in an inhomogeneous plasma. In addition, the analysis of the present paper covers the situations of greatest interest from the experimental point of view. It will be shown, in particular, that the effect increases substantially with increasing length of the wave that probes the plasma, a factor of particular importance for plasma diagnostics in large installations. Estimates made, for example, in the centimeter band yield for the energy conversion into the second harmonic an efficiency on the order of one per cent at an intensity of several kilowatts per square centimeter of the wave incident on the plasma.

1. BASIC EQUATIONS

We consider the propagation of an electromagnetic wave in a cold magnetoactive plasma. Let a homogeneous external magnetic field $\mathbf{H}_0$ be directed along the z axis, let the plasma density gradient $n_0(x)$ be perpendicular to it and directed along the x axis, and let the wave vector $\mathbf{k}$ lie in the (xz) plane. For high-frequency oscillations, in which the principal part is assumed by electrons, we use the following system of equations:

$$\frac{\partial \mathbf{H}}{\partial t}+c\,\mathrm{rot}\,\mathbf{E}=0,\quad \frac{\partial \mathbf{E}}{\partial t}=c\,\mathrm{rot}\,\mathbf{H}+4\pi e n \mathbf{v},$$
$$\frac{\partial n}{\partial t}+\mathrm{div}\, n\mathbf{v}=0,\quad \frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}+\frac{e}{m}\mathbf{E}+\frac{e}{mc}[\mathbf{v}\times\mathbf{H}]=0. \qquad (1.1)$$

We solve the problem by successive approximations relative to the first-harmonic amplitude. The hydrodynamic electron velocity v and the electron density n are represented in the form

$$\mathbf{v}=\mathrm{Re}\sum_s \mathbf{v}_s(x)\exp(is\omega t-isk_z z),$$

$$n=n_0(x)+\mathrm{Re}\sum_s n_s(x)\exp(is\omega t-isk_z z).$$

In first-order perturbation theory we obtain from (1.1) equations for the first-harmonic fields in an inhomogeneous magnetoactive plasma

$$\frac{d}{dx}\varepsilon_\parallel^{-1}(\omega)\frac{dH_{y1}}{dx}+k_H^2(\omega)H_{y1}=i\alpha(\omega)E_{y1},$$
$$\frac{d^2E_{y1}}{dx^2}+k_E^2(\omega)E_{y1}=-i\alpha(\omega)H_{y1}, \qquad (1.2)$$

where we have introduced the notation

$$k_H^2(\omega)=\frac{\omega^2}{c^2}\left[1-\frac{N_z^2}{\varepsilon_\perp(\omega)}\right],\quad k_E^2(\omega)=\frac{\omega^2}{c^2}\left[\varepsilon_\perp(\omega)-\frac{g^2(\omega)}{\varepsilon_\perp(\omega)}-N_z^2\right].$$

Here $\varepsilon_\parallel$, $\varepsilon_\perp$, and g are components of the dielectric tensor of the plasma:

$$\varepsilon_\parallel(\omega)=1-\frac{\omega_{pe}^2}{\omega^2},\quad \varepsilon_\perp(\omega)=1-\frac{\omega_{pe}^2}{\omega^2-\omega_{He}^2},\quad g(\omega)=\frac{\omega_{He}\omega_{pe}^2}{\omega(\omega^2-\omega_{He}^2)};$$

$\alpha(\omega)=(\omega/c)^2N_z g(\omega)/\varepsilon_\perp(\omega)$ is the coupling coefficient of the ordinary and extraordinary waves, and N_z is the longitudinal refractive index, which is less than unity for a field propagating in a vacuum.

The system (1.2) describes the propagation of two types of waves in an inhomogeneous magnetoactive plasma. On the plasma-vacuum interface the field coupling coefficient $\alpha(\omega)$ vanishes. In the vacuum, the p-polarized wave has field components H_y, E_x, and E_z, and in an s-polarized wave, the components E_y, H_x, and H_z.

In the next approximation we obtain from the system (1.1) the equations for the fields of the second harmonic excited by the nonlinear current induced by the first harmonic. They take the form

$$\frac{d}{dx}\varepsilon_\parallel^{-1}(2\omega)\frac{dH_{y2}}{dx}+k_H^2(2\omega)H_{y2}=i\alpha(2\omega)E_{y2}+\frac{4\omega^2}{c^2}f_H,$$
$$\frac{d^2}{dx^2}E_{y2}+k_E^2(2\omega)E_{y2}=-i\alpha(2\omega)H_{y2}+\frac{4\omega^2}{c^2}f_E, \qquad (1.3)$$

where f_H and f_e are nonlinear sources.

We express the perturbation of the electron density and velocity, as well as the components $H_{x,z}$ and $E_{x,z}$ of the first-harmonic in terms of H_{y1} and E_{y1}, and introduce in analogy with Ref. 12 the dimensionless plasma density $v=n_0(x)/n_c$ ($n_c\equiv m_e\omega^2/4\pi e^2$ is the critical density) and the dimensionless parameter $u\doteq(\omega_{He}/\omega)^2$. The nonlinear sources are then defined by the formulas

$$f_H=\frac{evN_zU_1/4m\omega^2}{(1-v)(1-u-v)(4-u-v)}+\frac{evN_zU_3/4m\omega^2}{(1-u-v)(4-u-v)}\frac{d}{dx}\left(\frac{U_3}{1-u-v}\right)$$
$$-\frac{(eN_zU_2/4m\omega^2)(4-u)}{(1-u-v)(4-u-v)}\frac{d}{dx}\left(\frac{vU_2}{1-u-v}\right)$$
$$+\frac{e}{4m\omega^2}\frac{d}{dx}\left[\frac{3vN_z}{(v-4)(1-v)^2}\left(\frac{c}{\omega}\frac{dH_{y1}}{dx}\right)^2\right.$$
$$\left.+\frac{2c^2/\omega^2}{(4-v)(1-v)}\frac{dH_{y1}}{dx}\frac{d}{dx}\left(\frac{vU_2}{1-u-v}\right)+\frac{vN_zU_4}{(4-v)(1-u-v)^2}\right],$$
$$f_E=\frac{evU_5/8m\omega^2}{(1-v)(1-u-v)(4-u-v)}+\frac{evU_2/8m\omega^2}{(1-u-v)(4-u-v)} \qquad (1.4)$$
$$\times\left[v\frac{d}{dx}\left(\frac{U_6}{1-u-v}\right)+\frac{d}{dx}\left(\frac{U_7}{1-u-v}\right)\right]$$
$$+\frac{eU_8/8m\omega^2}{(1-u-v)(4-u-v)}\left[\frac{d}{dx}\left(\frac{vU_2}{1-u-v}\right)-\frac{vN_z}{1-v}\frac{dH_{y1}}{dx}\right].$$

The purpose of expressing the nonlinear sources in the form (1.4) is to separate the resonant denominators $(1-u-v)$ and $(4-u-v)$ and to gather the bounded functions into blocks U_s, so that the nonlinear sources can be represented in a more compact form. It follows from formulas (1.4) that the nonlinear effects are strongest in the vicinity of the hybrid-resonance points in at the first and second harmonics, $v=1-u$ and $v=4-u$, respectively. The blocks U_s are expressed in terms of the first-harmonic fields in the following manner:

$$U_1=6iN_zu^{1/2}E_{y1}\frac{dH_{y1}}{dx}+3iN_zu^{1/2}(1-v)H_{y1}\frac{dE_{y1}}{dx}+2(u+v-1+3N_z^2)H_{y1}\frac{dH_{y1}}{dx}-(1-v)(2+u-2v)E_{y1}\frac{dE_{y1}}{dx},\quad U_2=N_zH_{y1}+iu^{1/2}E_{y1},$$

$$U_3=iu^{1/2}(v-3)E_{y1}-N_z(2+u)H_{y1},\quad U_6=(1-v)E_{y1}-iu^{1/2}N_zH_{y1},$$

$$U_4=N_z^2(1-u)H_{y1}^2+2iN_zu^{1/2}vH_{y1}E_{y1}+[(1-v)^2-u]E_{y1}^2,$$

$$U_7=6iN_zu^{1/2}H_{y1}-2(2+u-2v)E_{y1},$$

$$U_8=iu^{1/2}N_z(8-2u-v)H_{y1}-[uv+2(1-v)(4-u-v)]E_{y1},$$

$$U_9=iu^{1/2}[2(1-u-v)+N_z^2(v-6)]H_{y1}\frac{dH_{y1}}{dx}+3iu^{1/2}(1-v)(2-v)E_{y1}\frac{dE_{y1}}{dx}+uN_z(6-v)E_{y1}\frac{dH_{y1}}{dx}+N_z(1-v)(4-v+2u)H_{y1}\frac{dE_{y1}}{dx}. \tag{1.5}$$

Equations (1.3) allow us to calculate the second-harmonic fields from the given first-harmonic field. We call attention to the fact that the nonlinear sources do not contain resonant factors of the type $(\omega^2-\omega_{He}^2)^{-1}$ or $(4\omega^2-\omega_{He}^2)^{-1}$. Since we are considering second-harmonic generation when an electromagnetic wave is incident from vacuum on an inhomogeneous plasma, we must establish the relative arrangement of the regions of harmonic propagation, of the hybrid-resonance points, and of the intersection points of the oscillation modes as well as of the points of the synchronism $k_x(2\omega)=2k_x(\omega)$ in an inhomogeneous magnetoactive plasma.

2. DISPERSION CHARACTERISTICS OF FIRST AND SECOND HARMONICS IN AN INHOMOGENEOUS MAGNETOACTIVE PLASMA

We investigate now the dependence of the transverse component k_x of the wave vector on the plasma density, assuming the longitudinal refractive index N_z to be less than unity. According to (1.2) we have for the first harmonic $(k_x^2-k_E^2)(k_x^2-k_H^2\varepsilon_\parallel)=\alpha^2\varepsilon_\parallel$. From this we obtain the transverse refractive index of the first harmonic as a function of the plasma density v:

$$N_x^2(\omega,v)=\{2(1-u)(1-N_z^2)+2v(1-v)+v(1-N_z^2)(u-2)\pm v[u^2(1-N_z^2)^2+4u(1-v)N_z^2]^{1/2}\}/2(1-u-v). \tag{2.1}$$

We shall adhere to the terminology used by Ginzburg.[12] Then the plus and minus signs in (2.1) correspond respectively to the ordinary and extraordinary waves.

It is seen from (2.1) that the intersection of the oscillation modes takes place at a plasma density

$$v_t(\omega)=1+u(1-N_z^2)^2/4N_z^2.$$

At the intersection point, the transverse refractive index of the ordinary and extraordinary wave is

$$N_x^2(\omega,v_t)=\frac{N_z^2(1-N_z^2)}{(1+N_z^2)}\left[1-\frac{u(1-N_z^2)^2}{4N_z^4}\right]\equiv N_t^2(\omega).$$

The point of intersection of the oscillation modes is located in the wave propagation region if $N_z^2>u^{1/2}/(2+u^{1/2})$. In this case, in the vicinity of the intersection point, the dispersion of the ordinary wave is anomalous, $\partial(c^2k_x^2)/d(\omega^2)<0$, i.e., the phase and group velocities of the ordinary wave along the density gradient are anti-parallel.

1. Let the electron cyclotron frequency ω_{He} be less than the wave frequency ω $(u<1)$. The field has then three reflection points:

$$v_1(\omega)=(1-N_z^2)(1-u^{1/2}),\quad v_2(\omega)=1,\quad v_3(\omega)=(1-N_z^2)(1+u^{1/2})$$

and the hybrid-resonance point for the extraordinary wave is $v_r(\omega)=1-u$. If the angles of wave incidence on the plasma satisfy the condition $N_z^2<u^{1/2}/(1+u^{1/2})$, then the relative arrangement of the reflection points is $v_1(\omega)<v_2(\omega)<v_3(\omega)$. In the incidence-angle region $u^{1/2}/(1+u^{1/2})<N_z^2<u^{1/2}$ we have $v_r(\omega)<v_3(\omega)<v_2(\omega)$. Finally, for $u^{1/2}<N_z^2<1$ we obtain $v_1<v_3<v_r<v_2$. In a weakly inhomogeneous plasma the hybrid-resonance region $v\approx 1-u$ is accessible to a wave incident from vacuum if the longitudinal refractive index N_z is close to

$$N_0(\omega)=[u^{1/2}/(1+u^{1/2})]^{1/2}.$$

In this case the ordinary wave, propagating from the plasma boundary, reaches the intersection points

$$v_t(\omega)=1+u^{1/2}/4(1+u^{1/2}),$$

where it is transformed into an extraordinary wave that returns to the hybrid-resonance layer $v\approx 1-u$. When an extraordinary wave propagates from the plasma boundary, the hybrid-resonance point is obscured by an opacity region $v_1(\omega)<v<v_r(\omega)$ and is therefore inaccessible.

We note in addition that for $N_z>N_0(\omega)$ the extraordinary point has only one reflection point, $v_1(\omega)$. By making the substitutions $v\to v/4$ and $u\to u/4$ we obtain from (2.1) an expression for the transverse refractive index of the second harmonic:

$$N_x^2(2\omega,v)=\{8(1-N_z^2)(4-u)+2v(v-4)+v(1-N_z^2)(u-8)\pm v[u^2(1-N_z^2)^2+4u(4-v)N_z^2]^{1/2}\}/8(4-u-v). \tag{2.2}$$

According to (2.2), the intersection of the oscillation modes at the second harmonic takes place at the point

$$v_t(2\omega)=4+[u(1-N_z^2)^2/4N_z^2].$$

For the case $u<1$ the hybrid-resonance point of the extraordinary wave at the second harmonic point and the reflection points are given by

$$v_1(2\omega)=(1-N_z^2)(4-2u^{1/2}),\quad v_2(2\omega)=4,\quad v_r(2\omega)=4-u,\quad v_3(2\omega)=(1-N_z^2)(4+2u^{1/2}), \tag{2.3}$$

with $v_3(\omega)<v_1(2\omega)$.

For the choice $N_z=N_0(\omega)$, when $v_2(\omega)=v_3(\omega)=1$ and the hybrid-resonance region $v\approx v_r(\omega)$ is accessible to the wave incident on the plasma, the first harmonic has in a plasma layer

$$1<v<1+u^{1/2}[4(1+u^{1/2})]^{-1}$$

points of synchronism $v_s(\omega,l)$ with all the higher harmonics. With increasing number l of the harmonic that is synchronized with the wave incident on the plasma, the synchronism point $v_s(\omega,l)$ approaches the reflection point $v=1$ of the first harmonic. A typical plot of the squared refractive indices N_x^2 of the harmonics is shown in Fig. 1(a) for $N_z<N_0(\omega)$.

Figure 2(a) shows plots of the wave vectors k_x of the

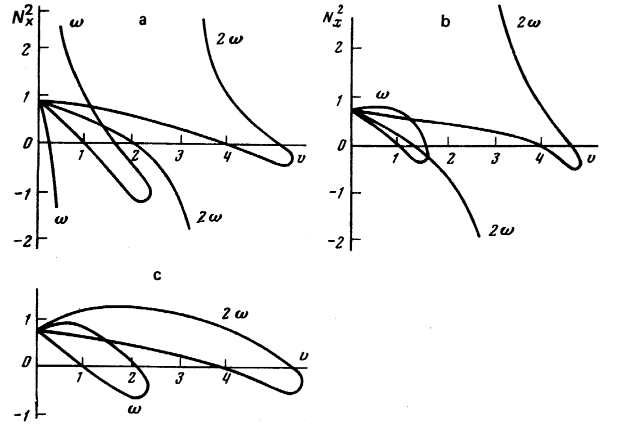

FIG. 1. Dependence of the squares of the transverse-wave refractive indices at the frequencies ω and 2ω on the plasma density $v=n_0(x)/n_c$: a) $\omega_{He}<\omega$, $N_z<N_0$; b) $\omega_{Hz}/2<\omega_{He}<\omega$, $N_z<N_0(2\omega)$; c) $\omega<\omega_{He}/2$, $N_z<N_0(2\omega)$. For each harmonic, the point of oscillation modes is located in the transparency region.

first harmonic for $N_z=N_0(\omega)$, the arrows indicating the directions of the group velocities $\partial\omega/\partial k_x$ along the plasma-density gradient. It is seen from the plots in Fig. 2(a) that the wave incident on the plasma, on passing past the point $v=1$, is synchronized with harmonics that propagate from the synchronism points towards the edge of the plasma. Consequently, generation of harmonics in the synchronism region leads in this case to the appearance of waves of higher frequencies in the reflected signal.

2. We consider now the case $1<u<4$, when $\omega<\omega_{He}<2\omega$. Now there is no hybrid-resonance point for the first harmonic. For $N_z<N_0(\omega)$ the ordinary wave has a reflection point $v_2(\omega)=1$, and the extraordinary wave has $v=v_3(\omega)>1$. At $H_z>N_0(\omega)$ the reflection points $v_{2,3}(\omega)$ pertain to the ordinary wave, the extraordinary wave has no reflection points, and the branches coalesce again at the intersection point

$$v_t=1+u(1-N_z^2)^2/4N_z^2.$$

For the second harmonic, the resonance point and the reflection points are calculated from formulas (2.3). In the region of the electromagnetic-wave incidence angles corresponding to $N_z^2<u^{1/2}(2+u^{1/2})^{-1}$, the first harmonic penetrates into the plasma to a depth $v\leq v_3(\omega)$ and reaches the hybrid resonance point of the second harmonic $v=4-u$ under the condition

$$N_z^2<(u+u^{1/2}-3)(1+u^{1/2})^{-1},$$

which is possible at $N_z^2<\frac{1}{2}$ and $u<4$. For $N_z^2<u^{1/2}(2+u^{1/2})^{-1}$ the depth of penetration of the first harmonic into the plasma is $v_t(\omega)$. The hybrid-resonance point of the second harmonic is accessible to the first harmonic if $u>12N_z^2(1+N_z^2)^{-2}$, which takes place at $u<4$ and $N_z>N_m$, where N_m is a number determined from the equation $N_m^3+3^{1/2}N_m^2+N_m=3^{1/2}$, i.e., $N_m\approx 0.666$. In the case $1<u<4$ the second-harmonic reflection point $v_1(2\omega)$ is closer to the plasma edge, $v_1(2\omega)<v_3(\omega)$, than the reflection point of the first harmonic $v_3(\omega)$.

In the region $1<u<2.25$ it is possible to satisfy the condition $N_z^2<(3-2u^{1/2})(4-2u^{1/2})^{-1}$. Then $1<v_1(2\omega)<v_3(\omega)$ and the first harmonic incident on the plasma and having the polarization of the extraordinary wave is synchronized in the region $1<v<v_3(\omega)$ with the ordinary wave at the second harmonic, and the second harmonic generated in the synchronization region will be radiated "forward," in the direction of the increase of the plasma density, towards the reflection point $v=4$. In the case $N_z^2>(3-2u^{1/2})(4-2u^{1/2})^{-1}$ the reflection point arrangement is $v_1(2\omega)<v_3(\omega)<1$. Now the point of synchronism of the first harmonic with the second harmonic traveling in the direction of increasing plasma density is located in the region $1<v<v_t(\omega)$. To illustrate the foregoing, Fig. 1(b) shows a typical plot of the refractive indices N_x^2 of the harmonics at $1<u<4$.

3. We consider now the case $u>4(\omega<\omega_{He}/2)$, when there is no hybrid resonance at either harmonic, and the regions of propagation of the first and second harmonics is determined by the location of the reflection points $v_{2,3}(\omega)$ and $v_{2,3}(2\omega)$. When the angles of incidence of the first harmonic on the plasma are set by the condition $N_z<N_0(2\omega)$ we have

$$v_2(\omega)<v_3(\omega)<v_2(2\omega)<v_3(2\omega).$$

If, however, $N_0(2\omega)<N_z<N_0(\omega)$ we obtain

$$v_2(\omega)<v_3(\omega)<v_3(2\omega)<v_2(2\omega).$$

The synchronization of the harmonics in the region of their propagation is possible only under the condition $v_3(2\omega)>1$, and this determines the maximum incidence angle θ of the first harmonic on the plasma:

$$\max N_z^2=(3+2u^{1/2})(4+2u^{1/2})^{-1}\equiv N_c^2(\omega).$$

It is easily seen that the relation $N_0(2\omega)<N_0(\omega)<N_c(\omega)$ is satisfied, with $(7/8)^{1/2}<N_c(\omega)<1$ at $u>4$. In the case $u>4$ an extraordinary first-harmonic wave which propagates in the direction of increasing plasma density is synchronized with an ordinary second-harmonic wave that travels in the same direction and is reflected back from the plasma layer with $v=4$.

In the special case of first-harmonic incidence on the plasma when $N_z=N_0(2\omega)$, the ordinary second-harmonic wave is partially reflected from the $v=4$ layer, and the

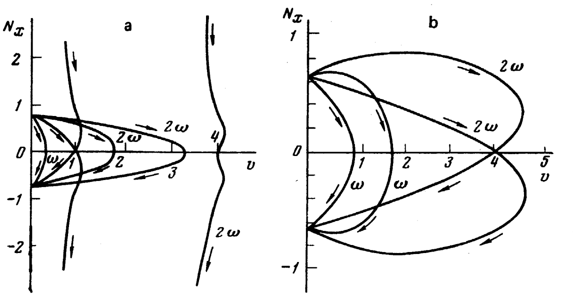

FIG. 2. Dependence of the transverse refractive indices of the harmonics w and $2w$ on the plasma density. The arrows near the curves show the direction of the group velocities of the waves: a) $\omega_{He}<\omega$, $N_z=N_0(\omega)$—special case of penetration of wave incident from vacuum up to the hybrid resonance layer; b) $\omega<\omega_{He}/2$, $N_z=N_0(2\omega)$—special case when the plasma region between the vacuum and the point of mode intersection is transparent at the second harmonic to the waves of both polarizations.

remainder penetrates farther into the interior of the plasma and is fully transformed near the layer $v=4+u^{1/2}(2+u^{1/2})^{-1}$ into an extraordinary wave traveling in the opposite direction, towards the plasma boundary [see Fig. 2(b)]. We note in addition that at $N_z< N_0(2\omega)$ the point of synchronism of the harmonics is located in the plasma-density region $1< v< v_3(\omega)$, while at $N_0(2\omega)< N_z< N_c(\omega)$ it is located in the region $1< v< v_t(\omega)$. A typical plot of the refractive indices of the harmonics for the case $u>4$ is shown in Fig. 1(c).

We note also a curious situation that arises when a plasma of frequency ω lower than the electron cyclotron frequency ω_{He} is incident on the plasma at an angle θ equal to $\sin^{-1}N_0(\omega)$, i.e., when $N_z=N_0(\omega)$ and the plasma region $v< v_t(\omega)$ ahead of the point of intersection of the oscillation modes is transparent to both types of wave. In this case, when the wave incident on the plasma has a polarization corresponding to the limiting polarization of the ordinary wave for low plasma densities, the second harmonic is radiated from the synchronism region in the direction of increasing plasma density. On the other hand if the wave incident on the plasma has the limiting polarization of the ordinary wave, then the second harmonic is radiated from the region of the synchronization of the harmonics in the direction of decreasing plasma density.

It should be noted here that for a probing first-harmonic wave beam that is bounded in the transverse direction it is necessary to take into account the k_y component of the wave vector. Equations (2.1) and (2.2) then describe the total transverse refractive index $N_\perp^2=N_x^2+N_y^2$. Inasmuch as the component k_y remains constant when the plasma density varies, by virtue of the homogeneity of the system along the y axis, the value of N_x^2 calculated from formulas (2.1) and (2.2) is now decreased by N_y^2, i.e., the $N_x^2(\omega,v)$ and $N_x^2(2\omega,v)$ curves on Fig. 1 need nearly be shifted downward by an amount N_y^2 along the ordinate axis.

3. SECOND HARMONIC GENERATION IN A HYBRID-RESONANCE LAYER

We proceed now to calculate the second-harmonic radiation field. Let $H_k(2\omega,x)$ be four linearly independent solutions of the system of equations (1.3) for the magnetic field H_{y2} when there are no nonlinear sources, and $E_k(2\omega,x)$ the corresponding solutions for the electric-field component E_{y2}. We express the solution of the system (1.3) with nonlinear sources f_H and f_E in the form

$$H_{y2}=\sum_k A_k(x)H_k(2\omega,x),\quad E_{y2}=\sum_k A_k(x)E_k(2\omega,x).$$

The functions $A_k(x)$ then satisfy the equations

$$\begin{gathered}\sum_k H_k(2\omega,x)\frac{dA_k}{dx}=0,\quad \sum_k E_k(2\omega,x)\frac{dA_k}{dx}=0,\\ \sum_k\frac{dH_k(2\omega,x)}{dx}\frac{dA_k}{dx}=\frac{4\omega^2}{c^2}\varepsilon_\parallel(2\omega)f_H,\\ \sum_k\frac{dE_k(2\omega,x)}{dx}\frac{dA_k}{dx}=\frac{4\omega^2}{c^2}f_E.\end{gathered}\tag{3.1}$$

Solving the system of linear equations (3.1) for dA_k/dx, we obtain the unknown functions $A_k(x)$:

$$A_k(x)=\int_{x_k}^{x}dx'\frac{\Delta_k(2\omega,x')}{\Delta(2\omega,x')},\tag{3.2}$$

where $\Delta_k(2\omega,x)$ and $\Delta(2\omega,x)$ are the corresponding fourth-order determinants of the system (3.1), and the choice of the end points x_k of the integration interval is connected with the boundary conditions of the second-harmonic radiation.

1. Let us investigate second-harmonic generation in the region $v\approx 1-u$ ($u<1$) of the first-harmonic resonance, where the field of the extraordinary wave has a singularity $E_{x1}\sim(1-u-v)^{-1}$. In a weakly inhomogeneous plasma it is necessary to ensure passage of the extraordinary wave towards the hybrid resonance point. According to the results of the preceding section, this is achieved by incidence of the first harmonic on the plasma at an angle close to critical, when $N_z\approx N_0(\omega)$. We put $v=1-u+x/L$ (L is the plasma-density inhomogeneity length) and assume, to simplify the formulas, that N_z is small.

In the first-harmonic hybrid-resonance region, Eqs. (1.2)–(1.5) take the form

$$\begin{gathered}\frac{d^2}{dx^2}E_{y2}+k_x^2(2\omega)E_{y2}=-\frac{ieu^{1/2}}{mc^2}(1-u)\frac{d}{dx}\left(\frac{E_{y1}}{1-u-v}\right)^2,\\ \frac{d^2}{dx^2}E_{y1}+\frac{\omega^2u(u-1)}{c^2(1-u-v)}(1+N_z^2)E_{y1}=0.\end{gathered}\tag{3.3}$$

At the chosen $v(x)$ dependence, the solution of Eq. (3.3) for the first-harmonic field is expressed in terms of a Hankel function:

$$E_{y1}=\left(\frac{8\pi^2S_1}{\omega Luv_0}\right)^{1/2}\xi^{1/2}H_1^{(1)}(2\xi^{1/2}),\quad \xi=\frac{\omega^2Lx}{c^2}uv_0(1+N_z^2).\tag{3.4}$$

Here S_1 is the flux of the first-harmonic energy along the plasma-inhomogeneity direction, $v_0=1-u$, and $0\leqslant\arg\xi\leqslant\pi$.

The solution (3.4) describes the propagation of an extraordinary wave in the direction of the decrease of the plasma density towards the hybrid resonance point $\xi=0$, in the vicinity of which the wave is completely absorbed at arbitrarily small dissipation. We write out the solution for the second-harmonic field. From (3.3) it follows that

$$E_{y2}=\frac{ev_0u^{1/2}}{2mc^2k_2}\int_{-\infty}^{+\infty}dx'\exp(-ik_2|x-x'|)\frac{d}{dx'}\left[\frac{L^2}{x'^2}E_{y1}^2\right],\tag{3.5}$$

where the singular point $x'=0$ is circled from above, and $k_2\equiv k_x(2\omega,v_0)$ is the wave vector of the second harmonic in the region of the resonance. An analysis of (3.5) shows that the second harmonic is radiated forward—in the propagation direction of the first harmonic.

Next, calculating the amplitude of the second harmonic, which is proportional to the integral

$$C(\mu)=\int_{-\infty}^{+\infty}\frac{d\xi}{\xi}[H_1^{(1)}(2\xi^{1/2})]^2e^{i\mu\xi},\quad \mu=\frac{ck_2}{\omega}\frac{c/\omega L}{uv_0(1+N_z^2)},\tag{3.6}$$

we note that the singular point $\xi=0$ makes no contribution, and the value of the integral is determined by the contribution of the synchronism point $\xi_s=(4/\mu^2)\gg 1$ of the harmonics. It can be verified that passing through the synchronism point is the stationary phase line

$$\rho(\varphi)=\frac{4}{\mu^2}\left(\cos\frac{\varphi}{2}-\sin\frac{\varphi}{2}\right)^{-2}, \quad \xi=\rho e^{i\varphi}, \quad -\frac{\pi}{2}\leq\varphi\leq\frac{3\pi}{2}, \tag{3.7}$$

which goes around the singularity in the complex plane at a large distance, where the asymptotic expression for the Hankel function $H_1^{(1)}\sim\exp(2i\xi^{1/2})$ is valid. On the stationary phase line defined by the equation $\mathrm{Re}(4\xi^{1/2}-\mu\xi)=4/\mu$, the integrand in (3.6) has an absolute maximum at the saddle point ξ_s, and decreases exponentially with increasing distance from the point ξ_s. This result is directly connected with the smallness of the parameter μ, which is equal to the ratio of the characteristic wavelength of the first harmonic to the wavelength of the second harmonic, and as a result the nonlinear source turns out to be rapidly varying at distances of the order of the wavelength of the second harmonic radiated from this point.

Thus, in contrast to the case of an isotropic plasma,[1] despite the presence of a singularity in the nonlinear source, the radiation of the second harmonic from the region of the hybrid resonance of the first harmonic in a weakly inhomogeneous plasma (when the points of reflection, synchronism, and hybrid resonances are separated by distances greater than the characteristic wavelengths) is negligible as a consequence of the interference of the radiations from the individual layers in the vicinity of the hybrid-resonance point. In the case of an isotropic plasma[1] the amplitude of the second harmonic is also proportional to an integral of the type (3.6), but now with a large value of the parameter μ. It is easily seen from (3.6) that at $\mu\gg1$ the asymptotic value of the function $C(\mu)$ is the residue of the integrand at the point $\xi=0$. In a weakly inhomogeneous magnetoactive plasma, a similar situation arises in the high-frequency range, $\omega\gg\omega_{He}$, at almost normal incidence, $N_z\ll1$, of the first harmonic on the plasma, when the reflection points of the first harmonic are in the immediate vicinity of the hybrid-resonance point. In this limited case, which is considered at $N_z=0$ in Ref. 11, the second-harmonic generation effect, in analogy with the situation in a cold isotropic plasma,[1] is due to the singularity of the first-harmonic fields. Consequently, if $N_z=0$, the contribution of the singularity must be taken into account in the frequency region $\omega\gtrsim\omega_{He}(L/\lambda)^{2/3}$, where L is the plasma-density inhomogeneity length and λ is the vacuum wavelength of the first harmonic.

2. Since the nonlinear source f_H and f_E have a singularity at the point of the hybrid resonance of the second harmonic $v_r(2\omega)\approx4-u$ $(u<4)$, we shall investigate second-harmonic generation in the vicinity of a layer of plasma of density $v\approx4-u$. The accessibility of this layer for a first harmonic incident from vacuum imposes on the first harmonic the limitation $\frac{1}{2}\omega_{He}<\omega<\omega_{He}$, as well as the restriction indicated in the preceding section on the incidence angle. To clarify the qualitative aspect of the question, it suffices to consider the case of normal incidence of the first harmonic on the plasma. In the vicinity of the hybrid-resonance point of the second harmonic we obtain from (1.3)–(1.5) the equation

$$\frac{d^2E_{y2}}{dx^2}-\frac{\omega^2u(4-u)}{c^2(4-u-v)}E_{y2}\approx\frac{ieu^{1/2}(u-1)(u-6)}{6mc^2(4-u-v)}E_{y1}\frac{dE_{y1}}{dx}. \tag{3.8}$$

Assuming a linear, $v=(4-u)(1+x/L)$, variation of the density near the point $v=4-u$ and neglecting the variation of the first-harmonic wave vector $k_x(\omega)\equiv k_1$, we transform (3.8) into

$$\tilde{\xi}\frac{d^2E_{y2}}{d\tilde{\xi}^2}+E_{y2}=\beta\exp(-i\tilde{\mu}\tilde{\xi}), \tag{3.9}$$

where $\tilde{\xi}=\omega^2uLXx/c^2$, β is a certain constant, and $\tilde{\mu}=(2ck_1/\omega)(c/\omega Lu)\ll1$. The solution of (3.9) is expressed by the integral

$$E_{y2}=i\beta\exp\left(\frac{i}{\tilde{\mu}}\right)\int_0^{\tilde{\mu}}\frac{ds}{s^2}\exp\left(-i\tilde{\xi}s-\frac{i}{s}\right). \tag{3.10}$$

Since the parameter $\tilde{\mu}$ is small, we can investigate (3.10) by asymptotic methods. The contribution of the upper limit $s=\tilde{\mu}$ of the integral describes the induced part of the field, while the contribution of the saddle point $s=\tilde{\xi}^{-1/2}$ describes the second-harmonic radiation proper. It turns out that the second-harmonic radiation appears after the first harmonic passes through the synchronism point $\tilde{\xi}_s=1/\tilde{\mu}^2\gg1$ and propagates in the same direction as the first harmonic.

We thus arrive again at the conclusion that in a weakly inhomogeneous magnetoactive plasma the singularity of the nonlinear sources, which is connected with the presence of a second-harmonic hybrid-resonance layer in the plasma, makes no contribution to the emission of the second-harmonic wave.

4. SECOND-HARMONIC GENERATION IN THE SYNCHRONISM REGION

We consider now second-harmonic generation in the vicinity of an isolated synchronism point v_s of the harmonics in a weakly inhomogeneous plasma. The term "isolated synchronism point" means that the region where the second-harmonic radiation is produced contains no reflection points, intersection points, or hybrid-resonance points.

Let $k_+(\omega,x)$ and $k_-(\omega,x)$ be the components, calculated with the aid of (2.1), of the wave vector $k_x(\omega,v)$ of the ordinary and extraordinary waves, respectively, in an inhomogeneous plasma. In the propagation region, the quasiclassical solutions of Eqs. (1.2) are of the form

$$\begin{gathered}H_{1,2}(\omega,x)=\Pi_H^{(+)}(\omega,x)\exp\left[\mp i\int_{x_{1,2}}^{x}k_+(\omega,x')dx'\right],\\ H_{3,4}(\omega,x)=\Pi_H^{(-)}(\omega,x)\exp\left[\mp i\int_{x_{3,4}}^{x}k_-(\omega,x')dx'\right],\\ \Pi_H(\omega,x)=\left\{\frac{\varepsilon_\parallel(\omega)\left[k_x^2(\omega,x)-k_E^2(\omega,x)\right]}{k_x(\omega,x)\left[k_+^2(\omega,x)-k_-^2(\omega,x)\right]}\right\}^{1/2},\\ E_k(\omega,x)=i\frac{\alpha(\omega)}{\left[k_x^2(\omega,x)-k_E^2(\omega,x)\right]}H_k(\omega,x).\end{gathered} \tag{4.1}$$

Taking into account (4.1) and the ratio of the field components, we obtain the polarization coefficients of the ordinary (+) and extraordinary (−) waves at the first harmonic:

$$K(\omega)=\frac{i}{N(\omega)}\left[\sigma\pm(\sigma^2+1-v)^{1/2}\right]\equiv-\frac{H_{y1}}{N(\omega)E_{y1}}, \tag{4.2}$$

where $N(\omega,x)$ is the total refractive index, and $\sigma=u^{1/2}(1-N_z^2)/2N_z$. It is seen from (4.2) that the waves are elliptically polarized. The limiting polarization of the waves at the plasma boundary ($v \to 0$) is equal to $K_\infty(\omega) = i[\sigma \pm (\sigma^2+1)^{1/2}]$.

For the second harmonic, the quasiclassical solutions $H_k(2\omega, x)$ and $E_k(2\omega, x)$ are obtained by simply replacing ω by 2ω in (4.1), and the wave amplitudes A_k are expressed with the aid of the integrals (3.2) in terms of the determinants $\Delta(2\omega, x)$ and $\Delta_k(2\omega, x)$ of the system (3.1). Using (3.1) and (4.1), we obtain

$$\Delta_{1,2}(2\omega, x) = \frac{\pm 8\omega^2 \varepsilon_\parallel(2\omega)\alpha(2\omega)H_{2,1}}{c^2[k_+^2(2\omega) - k_E^2(2\omega)]}\left[f_E - \frac{if_H\alpha(2\omega)\varepsilon_\parallel(2\omega)}{k_-^2(\omega) - k_E^2(2\omega)}\right], \quad (4.3)$$

as well as the determinant $\Delta(2\omega, x) = 4\varepsilon_\parallel(2\omega)$. The determinants $\Delta_{4,3}(2\omega, x)$ are obtained from (4.3) by replacing $H_{2,1}(2\omega)$ by $H_{3,4}(2\omega)$ and by interchanging $k_+(2\omega)$ and $k_-(2\omega)$. The nonlinear sources f_E and f_H are given by (1.4) and (1.5), in which the differentiation operator acts, for the isolated synchronism point $x_s(\omega)$, only on the exponentials in the functions H_k and E_k, and yields a factor $-ikx(\omega)$ or $-2ikx(\omega)$. Next, the first-harmonic magnetic and electric fields H_{y1} and E_{y1} must be taken in the form (4.1) with a normalization factor $(8\pi\omega S_1/c^2)^{1/2}$, where S_1 is the first-harmonic energy flux along the density gradient.

To calculate the second-harmonic amplitude, we separate in the functions $\Delta_k(2\omega, x)$ the rapidly oscillating factors

$$\Delta_k(2\omega, x) = F_k(x)\exp\left[i\int_{x_s}\{2k_x(\omega, x') - k_x(2\omega, x')\}dx'\right]. \quad (4.4)$$

Here $F_k(x)$ is a slowly varying pre-exponential factor. In the vicinity of the synchronism point we assume a linear variation of the density:

$$v = v_s[1 + (x - x_s)/L_s],$$

where L_s is the inhomogeneity length. We than have

$$\int_{x_s}^{x}[k_x(2\omega, x') - 2k_x(\omega, x')]dx' = \frac{\omega q_s(\omega)}{cL_s}(x - x_s)^2 \operatorname{sign} k_x(\omega, v_s),$$

$$q_s(\omega) = v_s\frac{\partial}{\partial v_s}[N_x(2\omega, v_s) - N_x(\omega, v_s)]. \quad (4.5)$$

After substituting (4.4) and (4.5) in the integral (3.2) we obtain the amplitudes of the second harmonic far from the synchronism point, where the radiation field at the doubled field has already been formed:

$$A_k = \left(\frac{c\pi L_s}{\omega q_s}\right)^{1/2}\frac{F_k(x_s)}{\Delta(2\omega, x_s)}\exp\left[-i\frac{\pi}{4}\operatorname{sign} k_x(\omega, v_s)\right]. \quad (4.6)$$

The second-harmonic energy flux along the plasma-density gradient is $S_2 = c^2|A_k|^2/16\pi\omega$.

Formulas (4.1)–(4.6) in conjunction with the expressions for the nonlinear sources (1.4) and (1.5) make it possible, given the magnetic field H_0, the wave frequency ω, the wave incidence angle, and the energy flux S_1 at the first harmonic, to calculate the coefficient Q_{12} of conversion of the energy of the wave incident plasma into the second harmonic, $Q_{12} = c_2|A_k|^2/16\omega S_1$ (see the Appendix). In order of magnitude, $Q_{12} = \varkappa(\omega L_s/c)S_1/S_*$, where $\varkappa$ is a certain number, $\omega L_s/c$ is the quasiclassicism parameter, and S_* is the normalization energy flux, equal to $m_e^2\omega^2c^3/e^2$. It is convenient to express the normalization flux S_* in terms of the vacuum first-harmonic wavelength $\lambda_1 = 2\pi c/\omega$:

$$S_* = (\lambda_1[\text{cm}])^{-2}\cdot 3.4\cdot 10^{11}\ [\text{W/cm}^2].$$

Taking $S_1 = 105$ W/cm^2 and $\omega L_s/c = 10^2$, we obtain $Q_{12} = 2.9\times 10^5\varkappa(\lambda_1[\text{cm}])^2$ for the order of magnitude of the coefficient of nonlinear conversion into the second harmonic in terms of the energy flux.

It is important to note that the second-harmonic generation efficiency, measured by the quantity Q_{12}, increases in proportion to the square of the first-harmonic vacuum wavelength. It is of interest to know the order of magnitude of the coefficient $\varkappa$. If we take $u = 3$ and $N_z \approx 0.681$, then it turns out unexpectedly that $\varkappa \approx 10^5$. Now at $\lambda_1 = 1$ cm the efficiency Q_{12} of conversion into the second harmonic reaches one per cent at first-harmonic energy flux $S_1 \approx 1.8$ kW/cm^2. In this case the magnetic field H_0 in the plasma should be of the order of 17 kOe, and $L_s \approx 16$ cm.

It can also be noted that the effect increases in proportion to the additional factor $(\omega L_s/c)^{1/2}$ if the synchronism point v_s coincides with the inflection point on the density profile $v(x)$.

CONCLUSION

We summarize now the principal results on second-harmonic generation by an electromagnetic wave incident on a magnetoactive plasma in the vicinity of an isolated synchronism point of the harmonics.

1. The frequency ω of the incident wave is higher than the electron cyclotron frequency ω_{He}. Second-harmonic wave generation takes place at a polarization $K(\omega) = i[\sigma + (\sigma^2+1)^{1/2}]$ of the first harmonic in vacuum and at incidence angles close to the critical $\theta_c = \sin^{-1}[\omega_{He}/(\omega + \omega_{He})]^{1/2}$. The second harmonic is radiated in the direction of decreasing plasma density. The first harmonic is synchronized (at different points) with both the ordinary and extraordinary second-harmonic waves.

2. The frequency of the incident wave lies in the range $\omega_{He}/2 < \omega < \omega_{He}$. Second-harmonic generation takes when the wave incident on the plasma has a polarization $K(\omega) = i[\sigma - (\sigma^2+1)^{1/2}]$. There are no restrictions on the incidence angle θ. The first-harmonic extraordinary wave is synchronized with the second-harmonic ordinary wave. The second harmonic is radiated forward towards the reflection point $n_0 = 4n_c$.

3. The frequency of the wave incident on the plasma is lower than $\omega_{He}/2$. The extraordinary first harmonic wave synchronizes with the second-harmonic ordinary wave at incidence angles smaller than $\sin^{-1}[(3\omega + 2\omega_{He})/(4\omega + 2\omega_{He})]^{1/2}$. The second harmonic is radiated forward into the interior of the plasma towards the layer $n_0 = 4n_c$.

4. The special case $N_z^2 = \omega_{He}/(\omega + \omega_{He})$ and $\omega < \omega_{He}$. When the first harmonic with limiting polarization of the ordinary wave is incident on the plasma, the second harmonic is radiated in the direction of the lower plasma density and has the polarization of the ordinary wave. If, however, the first harmonic has the limiting polarization of the extraordinary wave, then the second

harmonic is radiated in the direction of the higher plasma density.

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APPENDIX

We present now an algorithm for the calculation of the coefficient Q_{12} of conversion of the energy of the wave incident on the plasma into the second harmonic. First, specifying the parameters u and N_z, we obtain the position of the synchronism point of the harmonics, i.e., v_s, the refractive index $N_s \equiv N_x(\omega, v_s)$, and the rate of divergence q_s of the modes in the vicinity of the synchronism point. We then calculate the functions G_k:

$$G_1=\mu_1^2(1-v_s)[(1-v_s)^2+u(v_s-4)]+\mu_1 N_z u^{1/2}[(1-v_s)(2+v_s)+u(4-v_s)]-(1-u-v_s)^2-3uN_z^2,$$

$$G_2=N_z(1-3u-v_s)+2\mu_1 u^{1/2}(1-v_s), \quad \mu_1=[\sigma+g_1(\sigma^2+1-v_s)^{1/2}]^{-1},$$

$$G_3=\mu_1^2[u-(1-v_s)^2]-2\mu_1 v_s N_z u^{1/2}+(1-u)N_z^2,$$

$$G_4=3\mu_1^2(1-v_s)(1+u-v_s)+\mu_1 N_z u^{1/2}[(1-v_s)(v_s-10)-u(2+v_s)]+(1-u-v_s)^2+u(7-u-v_s)N_z^2$$

and the nonlinear source f:

$$f=\frac{2N_z v_s G_1}{(1-u-v_s)^2(4-u-v_s)}+\frac{2N_s^2 v_s G_2}{(1-v_s)(4-v_s)(1-u-v_s)}+\frac{2N_z v_s(1-v_s)G_3}{(4-v_s)(1-u-v_s)^2}+\frac{2u^{1/2}v_s G_4}{(1-u-v_s)^2(4-u-v_s)}[\sigma-g_2(\sigma^2+4-v_s)^{1/2}],$$

where g_1 and g_2 characterize the polarization of the synchronized harmonics: $g=+1$ for the ordinary waves and $g=-1$ for the extraordinary ones. Given the first-harmonic energy flux S_1 in the synchronism region we now determine the nonlinear transformation coefficient

$$Q_{12}=\frac{(\omega L_s/c)}{q_s}\frac{S_1}{S_*}\frac{\pi^2 f^2}{64N_s}\left(1+\frac{g_1\sigma}{(\sigma^2+1-v_s)^{1/2}}\right)^2\left(1-\frac{g_2\sigma}{(\sigma^2+4-v_s)^{1/2}}\right).$$

The polarization coefficients of the harmonics in the synchronism region are respectively

$$K_1=\frac{i}{N_1}[\sigma+g_1(\sigma^2+1-v_s)^{1/2}], \quad K_2=-\frac{i}{2N_2}[\sigma+g_2(\sigma^2+4-v_s)^{1/2}].$$

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Is renormalization necessary in the quasilinear theory of Langmuir oscillations?

A. A. Galeev, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko

Institute of Cosmic Studies, Academy of Sciences of the USSR
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We elucidate the conditions for the applicability of the quasilinear approximation for the description of resonance interactions between waves and particles. We show that when the condition for fast phase mixing and collectivization of resonance particles (overlap of neighboring resonances) is satisfied, the nonlinear corrections to the growth rate and to the diffusion coeeficient are negligibly small.

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1. The formalism for the quasi-linear theory for the description of resonance interactions between waves and particles was developed about two decades ago.[1,2] The theory was based upon the assumption that there exists in the plasma a rather broad packet of oscillations in which rapid phase mixing takes place. The equation for the distribution function of the resonance particles has then the form of the Fokker-Planck diffusion equation, and for the evaluation of the appropriate collision integral it is sufficient to restrict oneself, as was assumed earlier,[1,2] to the contribution from the main terms, quadratic in the field amplitude.

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