Heating of electrons and holes in bismuth and their energy-relaxation times

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(Submitted 25 May 1980)
Zh. Eksp. Teor. Fiz. **79**, 2282–2290 (December 1980)

The heating of electrons and holes in bismuth by an electric field is investigated experimentally as a function of the time after turning on the current through the sample. The measurements were made in a quantizing magnetic field. The carrier temperature was determined from the amplitude of the Shubnikov–de Haas oscillations. The energy relaxation time of the holes was determined from the time dependence of the temperature. In the purest and most perfect crystals, its value is 10^{-8} sec. The temperature of the electrons on levels with small quantum numbers changed monotonically following the application of the heating current. An increase of the effective temperature was observed, with a characteristic time shorter than 10^{-9} sec, followed by an increase with a time constant 3×10^{-9} sec.

PACS numbers: 72.15.Gd, 72.15.Lh

An electric field not only causes an electron system to drift as a result of the electron-current transport, but also alters the energy of the disoredere thermal motion. If the time in which the electron loses the excess energy acquired from the field is longer than the momentum relaxation time, then the change of the random-motion energy can exceed the drift energy substantially. The main result of the field action is then the heating of the electron system. Effects due to electron heating can be observed in semiconductors without noticeably disturbing the phonon system (see, e.g., Ref. 1). One of the methods of experimentally determining the electron temperature is to measure the amplitude of the Shubnikov-de Haas oscillations.[2-6] The dependence of the amplitude of the quantum oscillations of the resistance on the time elapsed from the instant when the heating current is switched on was used by Bauer and Kahlert[5] to measure directly the energy-relaxation time in n-InAs.

A deviation from Ohm's law at helium temperatures in bismuth, due to heating effects, was first observed by Borovik.[7] His results, however, did not make it clear whether the dependence of the resistance on the current is due to heating of only the electron system or to heating of the entire metal as a whole. Krylov and Sharvin[6] studied hot electrons which they injected into bismuth through a point contact. These experiments have shown that the temperatures of the electron and phonon systems can differ appreciably, at any rate when the volume in which the electron system is heated is small compared with the sample volume, and made possible a rough estimate of the carrier-energy relaxation time.

In the present study we investigated the heating of bismuth holes and electrons in a quantizing magnetic field. Measurements in a strong magnetic field made it possible to observe heating at low currents, owing to the increase of the sample resistance in the magnetic field, and permitted the use of the amplitude of the Shubnikov–de Haas oscillations to monitor the carrier temperature. A method similar to that of Bauer and Kahlert[5] was used to determine the energy-relaxation times.

PROCEDURE

The experiments were performed on bismuth single crystals whose crystallographic-axis orientations and geometrical dimensions are indicated in Table I. The samples were placed directly in liquid helium. The electric current flowed along the sample axis, and the constant magnetic field was oriented perpendicular to the axis. The internal resistance of the current source was large compared with the sample resistance. In the magnetic-field interval used for the measurements, the sample resistance was of the order of 10 Ω, so that a voltage proportional to the sample resistance could be picked off directly from the current contacts.

The contacts were narrow strips of a conducting adhesive deposited on the upper face of the sample perpendicular to its axis. Since the longitudinal magnetoresistance of the bismuth is much less than the transverse one, it follows that, in a magnetic field perpendicular to the upper face of the samples, planes passing through the conducting strips and normal to the sample axis are equipotential surfaces and the current distribution in the space between the contacts is uniform.

Measurements of two types were performed. In the experiments of the first type, a study was made of the dependence of the amplitude of the Shubnikov–de Haas oscillations on the direct current through the sample. To this end, plots were obtained of $R(H)$ or $\partial R(H)/\partial H$, where R is the sample resistance at various currents and temperatures. $\partial R/\partial H$ was measured by a standard procedure, with modulation of the magnetic field, narrow-band amplification, and synchronous detection.

In the experiments of the second type we studied the time dependence of the Shubnikov–de Haas oscillation amplitude A following a short rise time the current through the sample. To this end, a rectangular pulse from a G5-48 generator was applied to a load consisting of the sample and a series-connected resistor $R_0=47$ Ω. The pulse rise time was 2 nsec. The voltage from the sample was fed to the input of a stroboscopic oscilloscope, from the output of which we could pick off a voltage proportional to the voltage on the sample at specified instants of time relative to the

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TABLE I.

Sample	Length, cm	Section area, 10^{-3} cm²	Distance between contacts, cm	Crystallographic axis along sample axis	Magnetic-field direction
Bi3	1.2	2	0.2	C_2	$H\|C_1$
Bi7	1.3	3	0.67	C_2	$H\|C_1$
Bi8	1.6	4	0.75	C_2	$H\|C_3$
Bi8	1.6	4	0.45	C_2	$H\|C_1$

leading front of the pulse. It was thus possible to obtain the $R(H)$ dependence at a given instant of time and, determine the variation of the Shubnikov–de Haas oscillation amplitude with time with a set of $R(H,t)$ curves. The use of a modulation technique yielded the time dependence of the amplitude of the quantum oscillations of the derivative $\partial R/\partial H$.

In measurements of time dependences it is important that a uniform current distribution over the sample be established rapidly enough. The time of establishment of the uniform current distribution can be estimated from the sample conductivity. For our samples it was of the order of 10^{-10} sec in the field interval 10–20 kOe.

2. EXPERIMENTAL RESULTS

The oscillation picture obtained for the samples used in the present study agree fully with the one known from earlier studies.[8-10] A typical $R(H)$ plot at two values of the measuring current is shown in Fig. 1. Short-period hole oscillations can be seen in the magnetic-field range 15–20 kOe. Contributions to the long-period oscillations is made by all three electron ellipsoids–e_1, e_2, and e_3. At the orientation corresponding to Fig. 1, the central intersections of the ellipsoids e_2 and e_3 with a planes perpendicular to the magnetic field are equal and are approximately double the intersection area of the ellipsoid e_1. The minimum of the transverse resistance in a magnetic field ~24 kOe is due to the alignment of the Landau levels 0^+ and 1^- of the ellipsoids e_2 and e_3 with the Fermi level. The electrons of the ellipsoid e_1 are then on the 0^- level (the number here is that of the level, and the + and – signs correspond to the different spin orientations). The preceding minimum is due to the alignment of the bottoms of the levels 1^+ and 2^- of ellopsoids e_2 and e_3 and of the bottoms of the levels 1^- and 0^+ of ellipsoid e_1 with the Fermi level, etc. It is seen from Fig. 1 that the depth of the electron minma and the amplitude of the hole oscillations decreases with increasing measuring current, whereas the monotonic part of the resistance is practically independent of the measuring current.

For a quantitative description, we used the effective temperature T^* determined experimentally in the following manner. Calibration measurements were made of the dependence of weak-current oscillation amplitudes on the sample temperature. To determine T^* we compared the calibration curve with the amplitude of the oscillations obtained by flow of the heating current through the sample. The amplitude of the hole oscillations on the $R(H)$ curves was small, and the plots of $\partial R/\partial H$ were therefore used in this case to measure T^*.

At large oscillation quantum numbers n, the heating changes little with change of the oscillation phase. At small n, when the state density on the Fermi level is greatly altered, the depth of the minimum is determined by the electron heating in that magnetic field in which the corresponding Landau level is aligned with the Fermi level.

Plots of the effective electron and hole temperatures as functions of the power released in the sample are shown in Fig. 2. As seen from the figure, at a fixed power the electron and hole temperatures can differ noticeably even when the heating is with direct current. What remained uncertain, however, was whether the minimum temperature (in Fig. 2– the hole temperature) differs from the crystal-lattice temperature. To clarify this point we measured the amplitudes of the quantum oscillations in a pulsed regime. The current-pulse duration was 0.1 μsec and their repetition frequency was 0.5 MHz. The sample voltage was measured at those instants of time when all the processes that establish the carrier temperature and which will be discussed below had terminated. An estimate based on the energy released per unit sample volume and on the known[11] heat capacity of bismuth shows that the lattice temperature cannot change noticeably during the time of the pulse, and the average power fed to the sample decreases by more than one order of magnitude compared with the dc measurements. Comparing the hole-oscillation amplitude measured in direct current and in the pulsed regime, we could determine the change of lattice temperature for the case of dc heating. For the sample Bi7, for example, at a direct current 40 mA, the change of the lattice temperature was 0.3 K, whereas the hole temperature increased by 0.7 K, and the maximum lattice heating during the time of one pulse

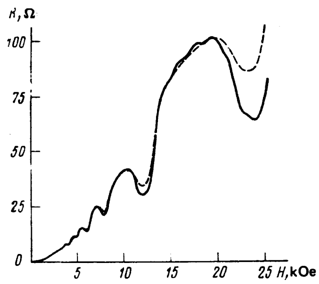

FIG. 1. Resistance of sample Bi7 vs. magnetic field at two values of the current. Solid curve) $J = 4$ mA, dashed) $J = 40$ mA, H Cl, $T = 1.65$ K.

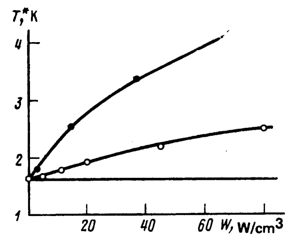

FIG. 2. Dependence of the effective temperatures T^* of the electrons of ellipsoids e_2 and e_3 on the Landau levels 0+ and 1- (H 24k Oe) and of the holes (H 18 kOE) on the power released in sample Bi7. H|| C_1, $T = 1.65$ K.

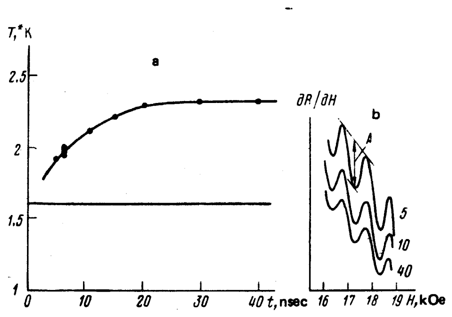

FIG. 3. a) Time dependence of effective hole temperature sample Bi3, H|| C_1, T = 1.6 K. b) Experimental plots of hole oscillations; the numbers at the curves are the times in nanoseconds.

in the pulsed regime was 0.04 K.

The initial sections of the curves on Fig. 2 are linear. Inasmuch as in our samples the momentum relaxation is due mainly to scattering by impurities and lattice defects, and consequently depends little on temperature, the presence of linear sections of the $T^*(W)$ curves serves as an experimental justification for introducing an energy relaxation time that is temperature-independent within the limits of the linear section.

The dependence of the effective hole temperature on the time elapsed from the start of the current pulse is shown in Fig. 3a. (Naturally, such a dependence was observed only at a sufficiently high current. No noticeable change of the oscillation amplitude with time was observed with a weak pulse current.) The time in Fig. 3 is reckoned from the instant at which the current through the sample reaches half its maximum.

The hole energy-relaxation time τ_e^h determined from Fig. 3a is 10 ± 2 nsec. The same energy relaxation time was obtained in sample Bi7 for holes having the same orientation. The hole energy relaxation time for sample Bi3, measured in a weaker magnetic field 9–12 kOe is of the same order, 9 ± 2 nsec. In sample Bi8 ($H\approx 18$ kOe) the hole energy relaxation time was shorter, $\tau_e^h = 4\pm 2$ nsec. The value of τ_e^h of the same sample measured in the case H ∥ C_3(H ≈ 5.5 kOe) was 7 ± 2 nsec.

The electron energy relaxation was investigated only at small numbers n (Fig. 4). The minimum B (see Fig. 4b) corresponded to alignment of the bottom of the Landay 0^+ subbands of ellipsoids e_2 and e_3 with the Fermi level, and the minimum C to the same for the 1^- levels of the same ellipsoids. The effective temperature was determined using the quantities

$$\Delta R=R(A)-R(C) \quad \text{or} \quad \Delta R=R(A)-R(B).$$

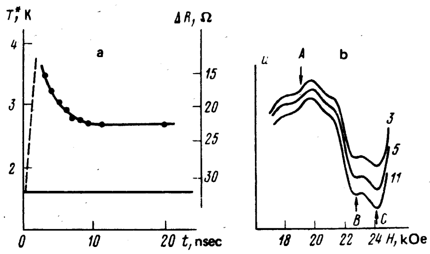

FIG. 4. a) Time dependence of the effective temperature of the electrons of ellipsoids e_2 and e_3 on the levels 0^+ and 1^- ($H\approx 24$ kOe), sample Bi7, H|| C_1, T = 1.6 K. b) Experimental plots, the numbers at the curves are the times in nanoseconds.

As seen from Fig. 4, the electron temperature varies nonmonotonically with time. The temperature rises within times not obtainable with our installation, after which it decreases with a time constant $\tau_e^e\approx 3\pm 0.5$ nsec. We note that since the sample resistance at the point A is practically independent of temperature, these measurements show the time variation of the electron temperatures at the bottom of the corresponding Landau subband.

Similar measurements were made in a weaker magnetic field with the Fermi level aligned with the bottoms of the Landau subbands 2^+ and 1^- of ellipsoids e_2 and e_3. In this case, too, a nonmonotonic time variation of T^* was observed, and the temperature decreased with a constant $\tau_e^e\approx 3\pm 0.5$ nsec.

DISCUSSION

In a quantizing magnetic field the electron energy depends on three quantities: the Landau-level number n, the spin projection s on the direction of the magnetic field, and the momentum component p_z along the field. The same quantities determine the equilibrium distribution function. It is therefore possible to distinguish in a heated electron system between the distribution of the electrons with respect to the quantum levels and their distribution in momentum on a fixed level, i.e., given s and n.

In a semimetal, if the intervalley scattering is weak, different carrier groups can be differently heated, and in each of the valleys the perturbed distribution functions do not have a Fermi distribution and cannot be described by any temperature. If the probability of scattering with change of the quantum number n is low, then the energy distributions on different Landau levels will also differ. Nonetheless,[12] even in the case of weak electron-electron scattering it is possible to introduce an electron temperature different from the lattice temperature T for electrons from any of the valleys, subject to the conditions

$$kT/s\gg(m\varepsilon_F)^{1/2},\ (m\hbar\Omega)^{1/2},$$

where s is the speed of sound, ε_F is the Fermi energy, m is the mass, and Ω is the cyclotron frequency of the electrons of the valley under consideration.

In the case when the mass in the magnetic-field direction is $m_z\gg m_c$ (m_c is the cyclotron mass), and the scattering by impurities and lattice defects with change of n is small, the electron distribution on the filled Landau level closest to the Fermi level is thermal under the conditions

$$p_z(n_0),\ (m_c\hbar\Omega)^{1/2},\ (m_c\varepsilon_F)^{1/2}\ll kT/s\ll(m_z\hbar\Omega)^{1/2}. \tag{1}$$

Here $p_z(n_0)$ is the characteristic value of p_z on the level n_0. The effective electron temperature on this Landau level is determined by the value of the electric field.

The electron distribution over the levels need not necessarily correspond to this temperature.

The conditions (1) can be regarded, stretching the point somewhat, as satisfied only for electrons in bismuth at small n and at $H \parallel C_1$ or $H \parallel C_2$. Therefore in the discussion that follows we shall take special notice of those conclusions that depend essentially on the assumption that the carriers have a thermal distribution.

The effective temperature of the holes was measured for large quantum numbers of the Landau levels, so that the quasiclassical approach is applicable. For isotropic scattering, the power absorbed by the carriers per unit volume is

$$W=R_F^2\nu(\varepsilon_F)e^2E^2\tau^{-1}/3, \qquad (2)$$

where R_F is the elliptic-orbit semi-axis in the electric-field direction, $\nu(\varepsilon_F)$ is the state density on the Fermi level, and τ is the momentum relaxation time. Under stationary conditions it is equal to the power lost per unit time:

$$W=\frac{\pi^2}{3}\nu(\varepsilon_F)kT\frac{k(T^*-T)}{\tau_e}, \quad \frac{T^*-T}{T}\ll 1. \qquad (3)$$

Equating (1) and (3), we obtain for the change of the carrier temperature

$$T^*-T=\frac{1}{\pi^2}\frac{(eER_F)^2}{k^2T}\frac{\tau_e}{\tau}=\frac{1}{\pi^2}\frac{\tau_e}{\tau}\left(\frac{pcE}{H}\right)^2k^{-2}T^{-1}. \qquad (4)$$

In the last relation, which is valid for large quantum numbers n, p is the projection of the momentum on the direction perpendicular to H and E.

Equation (4) can be used to determine the momentum relaxation time τ, since all the quantities that enter in this equation, with the exception of τ, are either well known or can be measured in experiment. Although the momentum-relaxation times obtained in this manner depend on the extent to which the true distribution of the heated carriers is close to thermal, they can be used to to compare the quality of different samples. To exclude the lattice heating, T^* was measured by a pulse technique. The hole momentum relaxation time determined from (4) was 10^{-9} sec for sample Bi7. (τ was determined from the plots of the oscillations in the $H \parallel C_1$ orientation at $H\approx 18$ kOe.)

The holes can lose energy as a result of intravalley scattering with emission of a phonon, or else as a result of intervalley scattering. The last process is possible at helium temperatures only on account of scattering from an impurity or from a structural defect, and is therefore elastic. However, if the electron and hole temperatures are not equal, then elastic intervalley scattering leads to equalization of the temperature and can cool the holes if $T^*_e<T^*_h$. According to Lopez,[13] the relaxation time for intervalley scattering in the most perfect bismuth samples is of the order of 10^{-8} sec and agrees with the hole energy relaxation time obtained in our experiments. The momentum relaxation time of the sample Bi8, determined in accord with (4), turned out to be lower by one order of magnitude than that of the other samples, $\tau\sim 10^{-10}$ sec. Therefore the shorter relaxation times of the holes of this sample can be interpreted as a result of an increased intervalley scattering probability.

The values of the energy relaxation times depend little on the method used to reduce the experimental data. At any rate, the relaxation time determined from the time dependence of the quantum oscillations agrees with the time obtained after calculating the oscillation amplitude and the effective temperature.

What remains unclear is the origin of the nonmonotonic time dependence of the effective electron temperature. In principle, this behavior is not surprising in a system of several levels with weak exchange energy between them.

We consider by way of example the system of two Landau levels, shown in Fig. 5, and assume that conditions (1) are satisfied. Transitions with change of the quantum number of n are possible in elastic scattering by an impurity and upon emission or absorption of phonons with momentum $\sim(2m_z\hbar\Omega)^{1/2}$. Scattering by phonons seems to predominate. The basis for this assumption is the strong decrease of the amplitude of the quantum oscillations of the longitudinal magnetoresistance of bismuth with decrease of temperature in the interval from 4.2 to 1.3 K.[9,10] (Experiments on the temperature dependence of the oscillations of the longitudinal magnetoresistance were performed on our samples and yielded an analogous result.)

Since $(m_z\hbar\Omega)^{1/2}>kT/s$ and the elastic scattering with change of n is weak, the temperatures of electrons of various levels can differ from one another and from the lattice temperature T. The higher the temperature of each of the levels, the higher the transition probability between levels with emission of a phonon, since both the number of electrons that can be scattered and the number of free places in which they can land increase with rising temperature. It is important that in scattering with change of n there is emitted a phonon with energy higher than thermal, and this scattering leads to cooling of each of the considered levels. The power given up by each level to the lattice can be represented in the form $Q=\alpha(T^*_0+T^*_1-2T)$, where T^*_0 and T^*_1 are the temperatures of the electrons on the levels $n=0$ and $n=1$.

Two variants are possible in this case. In the first, the electrons of the lower level acquire from the electric field less energy than they give up to the photon

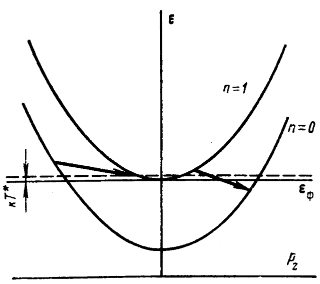

FIG. 5. Scheme of transitions between different Landau levels with phonon emission.

system via transitions with change of n. In this case the result of applying an electric field will be cooling of the level $n=0$ below the lattice temperature, and heating of the level $n=1$. In the second case the power acquired by the lower level from the electric field exceeds Q. The steady-state value of the temperature on each of the levels is higher than that lattice temperature, but the process of establishment of the temperaon the level $n=1$ is nonmonotonic. The decreasing section of the $T^*_1(t)$ plot is due to slow heating of the lower-level electrons and to the associated increase of heat transfer from the upper-level electrons.

The considered mechanism demonstrates the possibility, in principle, of a nonmonotonic electron-temperature variation in a two level system. However, as shown by the numerical estimates, it can not yield results that are in agreement. The correction that must be introduced in the upper-level temperature to account for the heating of $n=0$ level is smaller by two orders of magnitude than that observed in experiment.

CONCLUSION

Direct measurement of the energy relaxation time by a method similar to that used in the present study is possible in any compensated metal in the presence of a sufficiently strong magnetic field. As seen with bismuth as the example, the energy relaxation process can be quite complicated, and different carrier groups can relax with different characteristic times, and even a nonmonotonic time variation of the carriers from some group is possible. Experiments aimed at determining the energy relaxation times in metals can define the frequency range in which nonlinear effects connected with carrier heating manifest themselves significantly.

The authors are sincerely grateful to V. F. Gantmakher for helpful discussion and N. V. Yashinkova for preparing the samples used in this study.

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Translated by J. G. Adashko

Resonant recombination of photoexcited light holes in germanium in a magnetic field

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(Submitted 23 May 1980)
Zh. Eksp. Teor. Fiz. **79**, 2291–2301 (December 1980)

Resonant capture of light holes on the ground and excited acceptor levels, with emission of optical phonons, was observed in the study of the photoconductivity of p-Ge in a magnetic field. An analysis of the experimental spectra yielded additional data on the energy dependence of the effective masses and g-factors of the light holes. It is shown that recombination takes place not from the very bottom of the magnetic subband, but from a discrete level separated from this subbband by the Coulomb field of the ionized center. It is shown that the cross sections for capture onto the ground and excited levels of the acceptor are of the same order in this process.

PACS numbers: 72.20.Jv, 72.20.My, 72.40. + w

1. INTRODUCTION

The study of nonradiative carrier recombination in semiconductors is made difficult by the fact that it is usually impossible to observe the elementary recombination acts themselves, since the recombination must be assessed by analyzing such macroscopic characteristics as, e.g., the temperature dependence of the electric resistivity. One of the recombination processes, however, namely carrier capture by an ionized center with emission of on optical phonon $\hbar\omega_0$, can proceed in a magnetic field in resonant fashion.[11] The resonance sets in because of singularities that appear in the state density in the magnetic subband when the condition

$$E_n - E^{(i)} = \hbar\omega_0 \quad (1)$$

is satisfied, where E_n is the energy of the bottom of the n-th magnetic subband for the free carrier and $E^{(i)}$ is the energy of one of the carrier bound states on the impurity center, reckoned from one level. The

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