Nonlinear plasma instabilities in semiconductors subjected to strong electric fields in the case of inelastic scattering of electrons by optical phonons

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The method of numerical modeling was used to investigate nonlinear plasma instabilities in semiconductors in strong electric fields. It was assumed that the dominant mechanism of the scattering of electrons with energies exceeding that of optical phonons is the spontaneous emission of such phonons. It was found that the application of an electric field produces an instability in two stages, each of which begins with an exponential rise of the energy of the self-consistent electric field. During the first stage, whose duration amounts to two or three cycles of electron acceleration until the optical phonon energy is reached, the electron distribution function becomes strongly modified because of the interaction between electrons and the self-consistent electric field. In the second stage, characterized by a relatively smaller increment, the exponential rise of the field energy results in plasma bunching and formation of one or several solitons. The electric field in a soliton may exceed considerably the external field. Elastic electron collisions are an important stabilizing factor. The attainment of the nonlinear stage of an instability is possible only if the value of τ_0/τ (τ_0 is the time of spontaneous emission of an optical phonon and τ is the relaxation time of the momentum due to the interaction with impurities and acoustic phonons) is sufficiently small.

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1. In some semiconductors (for example, those of the AgBr type) the time of spontaneous emission of an optical phonon τ_0 may be considerably shorter than the momentum relaxation time τ in the case of quasielastic collisions with impurities and acoustic phonons:[1]

$$\tau_0 \ll \tau. \tag{1}$$

If such a semiconductor is subjected to a strong electric field, the electron distribution function is found to be strongly anisotropic. The electron momentum then varies periodically with time. This combination of a plasma nonequilibrium with periodic electron motion should give rise to a number of interesting features of the high-frequency plasma characteristics (see, for example, Refs. 2–6). For example, Gulyaev and Chusov[5] demonstrated that an electron plasma satisfying the condition (1) and subjected to a sufficiently strong electric field may be unstable with respect to buildup of oscillations of the electron density and of the self-consistent electric field. The range of validity of their results[5] is limited to small perturbations of the electric field and small deviations of the electron distribution function from that corresponding to a steady-state distribution in the presence of a strong electric field and scattering by optical phonons. However, it would be interesting to study collective processes in an electron plasma in a semiconductor under these conditions beginning from the moment of application of an electric field when the plasma is still in equilibrium and right up to the time when the growth of the instability makes the energy of the self-consistent electric field considerable and a nonlinear mechanism begins to limit the rise of this field. The present paper reports an investigation of transient nonlinear processes in an electron plasma in a semiconductor subjected to a strong electric field when electrons are scattered inelastically by optical phonons.

2. In analyzing the plasma dynamics in external and self-consistent electric fields we used numerical modeling based on the method of macroparticles similar to that employed in Refs. 7 and 8. If we confine ourselves to one-dimensional motions of a plasma, which is the assumption made below, we find that the distribution functions of electrons $f=f(p,z,t)$ and of the electric field intensity $E=E(z,t)$ are

$$\frac{\partial f}{\partial t}+\frac{p}{m}\frac{\partial f}{\partial z}+eE\frac{\partial f}{\partial p}=I\{f\}, \tag{2}$$

$$\frac{\partial E}{\partial z}=\frac{4\pi e}{\varkappa}\left[\int_{-\infty}^{\infty} f\,dp-N\right]. \tag{3}$$

Here, $\varkappa$ is the lattice permittivity; N is the density of the compensating background (donor concentration), which is assumed to be homogeneous and immobile; $I\{f\}$ is the integral representing collisions of electrons with impurities and phonons. This collision integral is

$$I\{f\}=I_i\{f\}+I_a\{f\}+I_0\{f\},$$

where the terms on the right-hand side are related to the interaction of electrons with impurities, acoustic phonons, and optical phonons, respectively.

We shall assume next that the lattice temperature T_0 is much less than the Debye value:

$$T_0 \ll \hbar\omega_0, \tag{4}$$

where ω_0 is the limiting frequency of optical phonons. Then, the term describing collisions of electrons with optical phonons can be in the form

$$I_0\{f\}=\frac{1}{\tau_0}\left[f((p^2+2m\hbar\omega_0)^{1/2},z,t)-f(p,z,t)\theta\left(\frac{p^2}{2m}-\hbar\omega_0\right)\right], \tag{5}$$

where

$$\theta(x)=\begin{cases}1, & x\geqslant 0\\ 0, & x<0\end{cases}.$$

We shall allow for the quasielastic scattering by impurities and acoustic phonons, ignore a weak inelasticity, and use the following expression which corresponds formally to one-dimensional scattering:

$$I_i\{f\}+I_a\{f\}\approx\tau^{-1}[f(-p,z,t)-f(p,z,t)]. \tag{6}$$

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Strictly speaking, this expression is valid for electrons in quantizing magnetic fields in the ultraquantum limit[8,9] when the elastic scattering of an electron along a magnetic field reverses the electron momentum. The use of more general expressions for collisional terms in the adopted method of numerical modeling presents no fundamental difficulties.

We shall assume that the initial conditions are

$$f(p, z, t=0)=f_0(p), \quad E(z, t=0)=E_0, \tag{7}$$

where $f_0(p)$ is the distribution function of the initial state which may be an equilibrium state; E_0 is an external static electric field. The boundary conditions are taken in the form

$$E(z=0, t)=E(z=L, t), \quad \int\limits_0^L E(z,t)\,dz=E_0L, \tag{8}$$

where L is the thickness of the sample in the direction of the electric field. The conditions (8) should be supplemented by the boundary conditions for electrons and these are in the form of the periodicity condition.

3. The system (2), (3) with allowance for electron scattering described by the collisional term (5) and (6) was solved by the method of macroparticles implemented in the form of a uniform code.[7,9] Both types of scattering were modeled by Monte Carlo algorithms (see Ref. 11). The algorithm employed with sequences of random numbers was similar to that used by others[12,13] who modeled pair collisions in a gaseous plasma using the macroparticle method.

The initial electron distributions were assumed to be Maxwellian with the temperature T_0, which varied within the limits of the inequality (4). The numerical values of the parameters τ_0 and τ

$$\tau_E=(2m\hbar\omega_0)^{1/2}/eE_0, \quad \omega_p=(4\pi e^2N/\varkappa m)^{1/2}, \quad l_E=(2m\hbar\omega_0)^{1/2}\tau_E/2m$$

and of L were selected in these numerical experiments from the conditions

$$\tau_0<\tau_E\ll\tau, \quad l_E\ll L. \tag{9}$$

The first of these conditions means that the process of electron acceleration by a field is interrupted abruptly by the emission of an optical phonon and elastic collisions are rare. The second condition corresponds to the fact that during the transit time across a sample an electron undergoes many cycles of acceleration to the optical phonon energy. One of the consequencies is a reduction in the role of the boundary effects.

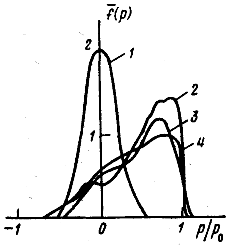

FIG. 2. Evolution of the electron distribution calculated for $T_0/\hbar\omega_0 = 0.2$, $\tau\omega_p = 80$, $\tau_E\omega_p = 20$, and the following values of $t\,\omega_p$: 1) 0; 2) 20; 3) 30; 4) 60.

The following sets of parameters were used in the calculations: $\tau_0\omega_p = 0.5$; $\tau_E\omega_p = 2,\ 20$; $\tau\omega_p = 20, 26, 80, \infty$; $L/l_E = 12,\ 18.25$; $T_0/\hbar\omega_0 = 0.1, 0.2, 0.3$, and 0.4. The number of macroparticles (electrons) was 20×1024, 30×1024, and 35×1024.

4. The main results of the numerical experiments are presented graphically. Figures 1 and 2 show the evolution of the thickness-average distribution function of electrons after the application of an electric field:

$$\overline{f(p,t)}=L^{-1}\int\limits_0^L f(p,z,t)\,dz.$$

For comparison, Fig. 1 includes (dashed curves) the steady-state electron distributions found by solving Eq. (2) for a homogeneous steady-state system.

We can see from the figures that distributions close to the steady-state form are established after two or three electron acceleration cycles. It is characteristic that the occurrence of elastic collisions results in significant symmetrization of the distribution function relative to the point $p = 0$.

Figure 3 shows, in a logarithmic scale, the time dependence of the energy of the self-consistent electric field. We can see that there are two regions of exponential rise of the energy of the field with time. The first corresponds to the time interval $0<\tau_E$, at the end of which the peak of the distribution function approaches the point $p = p_0$. When the major fraction of electrons returns to the passive region ($|p|<p_0$), the field energy

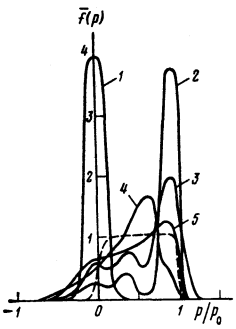

FIG. 1. Evolution of the electron distribution calculated for $T_0/\hbar\omega_0 = 0.1$, $\tau\omega_p = \infty$, $\tau_E\omega_p = 40$, and the following values of $t\,\omega_p$: 1) 0; 2) 18; 3) 20; 4) 26; 5) 80.

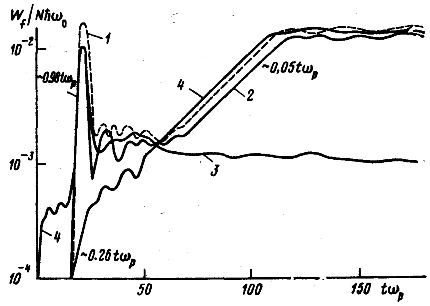

FIG. 3. Time dependences of the self-consistent electric field energy calculated for $\tau_0\omega_p = 0.5$, $\tau_E\omega_p = 20$ and different values of the parameters $T_0/\hbar\omega_0$ and $\tau\omega_p$: 1) $T_0/\hbar\omega_0 = 0.1$, $\tau\omega_p = \infty$; 2) 0.2, ∞; 3) 0.2, 80; 4) 0.4, ∞.

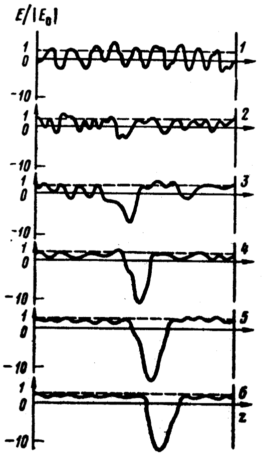

FIG. 4. Time dependences of the distribution of the self-consistent electric field $E(z)$ for $L/l_B = 12$ (formation of one soliton) plotted for various values of $t\omega_p$: 1) 20; 2) 50; 3) 80; 4) 110; 5) 140; 6) 170.

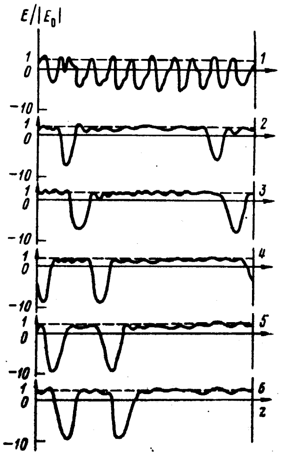

FIG. 5. Time dependences of the distribution of the self-consistent electric field $E(z)$ for $L/l_B = 18.25$ (formation of two solitons) plotted for various values of $t\omega_p$: 1) 20; 2) 50; 3) 80; 4) 110; 5) 140; 6) 170.

decreases in a certain time interval. The observed features of the behavior of the field energy indicate that plasma oscillations initiated by inelastic electron scattering are excited and grow rapidly. The growth increment of these oscillations in the time interval $0<t<\tau_B$ is found to depend on the relative width $T_0/\hbar\omega_0$ of the initial electron distribution. A similar dependence on the initial temperature is exhibited also by the maximum obtained value of the self-consistent field energy.

The second stage of practically exponential rise of the energy of the self-consistent electric field ($t \gtrsim 3\tau_B$) is characterized by an increment $\gamma \sim \tau_B^{-1}$, which is independent of the width of the initial electron distribution. This corresponds to the linear growth of an instability predicted by Gulyaev and Chusov.[5] The difference between the instability increments in the first and second stages is due to a considerable difference in the average distribution function after the first acceleration cycle and after a large number of such cycles. The fact that the values of the growth increments obtained for the second stage ($\gamma \sim \tau_B^{-1}$) are somewhat greater than the values predicted in Ref. 5 is due to the fact that in numerical experiments it was assumed that $\tau_B\omega_p \gg 1$, whereas in analytic calculations[5] it was postulated that $\tau_B\omega_p \ll 1$. However, a comparison of the analytic and numerical calculations for the same values of the parameter $\tau_B\omega_p$ is complicated by the circumstance that when $\tau_B\omega_p \gg 1$ it is not possible to obtain analytic expressions for the increment, whereas in the $\tau_B\omega_p \ll 1$ case the computer time needed for calculations increases strongly.

In our opinion, the greatest interest lies in the behavior of an electron plasma at a later stage of the instability growth ($t \gtrsim 5\tau_B$) when the energy of the self-consistent field exhibits nonlinear saturation of its rise.

The main feature of the nonlinear regime of the investigated instability is the "bunching" of an electron plasma, namely the appearance of solitary traveling waves (solitons or domains) of the electron density and of the associated waves of the self-consistent electric field (Figs. 4 and 5). The resultant nonlinear waves travel in the direction of the electron drift at a constant phase velocity $v_{ph} \approx 0.3p_0/m$. Experiments indicate that the number of solitons created in the nonlinear stage is governed by the ratio L/l_B. If $L/l_B = 12$, all the variants predict formation of one soliton, whereas for $L/l_B = 18.25$ the number of solitons is two. The amplitude of the electric field E_s in a wave is many times greater than the electric field intensity E_0. A typical value of the ratio of the fields is $E_s/E_0 \sim 10$.

Formation of large-amplitude solitons accounts for the instability saturation mechanism. It is clear from Figs. 4 and 5 that everywhere, except in the region of a soliton, the self-consistent electric field is directed opposite to the external field. Thus, the flux of electrons to the active region ($|p|>p_0$) of the phase space, which is the source of energy for the growing waves, decreases strongly.

Our numerical experiments dealt also with the influence of elastic scattering on the growth of the investigated instability. The results indicated that the role of such scattering could be considerable. For example, it is clear from Fig. 3, showing the time dependence of the field energy in the case when $\tau_B/\tau = \frac{1}{4}$ and $\tau_0/\tau_B = \frac{1}{40}$, that there is no region of rise of the field energy corresponding to the instability. However, in the variant $\tau_B/\tau = \frac{1}{10}$, $\tau_0/\tau_B = \frac{1}{4}$ the elastic collisions reduce the growth increment of the field energy but do not suppress the instability completely. Hence, it follows that the attainment of the nonlinear stage of the instability and generally its effective manifestation in the physical experiments requires that the condition (1) be satisfied with a sufficient margin. In this sense, semiconductors of the AgBr type, for which it is estimated that $\tau/\tau_0 \sim 500$, are very promising.

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Self-focusing of light in nematic liquid crystals as a method of investigation of the orienting effect of a free surface

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The huge optical nonlinearity of the mesophase of a nematic liquid crystal (NLC), recently predicted and observed by the self-focusing of light, and caused by reorientation of the NLC director under the influence of light fields, is discussed. A calculation is carried out of the nonlinear advance of phase and of the optical power of the nonlinear lens for a layer of NLC oriented by means of one or two surfaces. Proposed experiments would enable one to obtain quantitative information about the orienting action of a free surface. Methods of increasing the accuracy of the experiment are discussed. Expressions are also obtained for the power of the nonlinear lens in a number of specific problems on external self-focusing of light in NLC.

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1. INTRODUCTION

A huge optical nonlinearity of the oriented mesophase of a nematic liquid crystal (NLC) was recently predicted theoretically and observed experimentally.[1] This nonlinearity is caused by reorientation of the NLC director by the electric field of the light wave. In the experiment,[1] the original uniform planar orientation of the NLC was preserved because of the rigid orientation of the director on the rubbed surface of the cell walls. A suitable departure

$$\delta\mathbf{n}(\mathbf{r})=\mathbf{n}(\mathbf{r})-\mathbf{n}_0$$

of the director from the unperturbed direction lowers the energy of interaction with the light wave but leads to the appearance of a positive energy of nonuniform deformation

$$F[\text{erg/cm}^3]\sim K(\nabla\delta\mathbf{n})^2,$$

where K is a Frank constant (see below). Minimization of the sum of these energies leads to a local equation for $\delta\mathbf{n}$, whose solution was carried out[1] with allowance for the rigid pinning of the director at the boundaries and gave a completely satisfactory agreement with experiment.

Papers of Mada[2,3] discuss theoretically a possible mechanism of the orienting effect of a free NLC surface (that is, for example, the boundary between the NLC and air). The point is that there is a preferred orientation of the NLC director with respect to such a free surface, and this orientation may be different for different specific shapes of the NLC (see Refs. 4 and 5). The degree of rigidity of the orientation along such a preferred direction can be characterized[2] by the orientation-dependent part of the surface energy density,

$$\Lambda[\text{erg/cm}^2]\sim\sigma_a(\delta\mathbf{n})^2$$

(see below for a more exact definition). From the constants σ_a and K we can form a quantity of dimensions length, $l=K/\sigma_a$. If the total thickness L of the cell is much larger than l, i.e., if $L\gg K/\sigma_a$, then the effect of the surface may be considered to be practically a rigid pinning of the director. If, on the contrary, $L\ll K/\sigma_a$ (or equivalently, if $\sigma_a\to 0$), the free surface exerts no influence at all on the orientation of the director. Mada[2,3] notes that so far no methods are known for experimental measurement of the value of the orientation-

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