Spin glasses with random anisotropic exchange

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It is shown that in the limit as $n \to \infty$, at space dimensionalities $2 < d < 4$, a phase transition into spin glass takes place in a magnet with random anisotropic exchange. This transition corresponds to Bose condensation into the first localized state of the random exchange-interaction matrix. The local frozen magnetization in the low-temperature phase is determined by the wave function of this state. The order parameter, the field conjugate to it, and two generalized susceptibilities, which become infinite in the low-temperature phase, are determined. This fact attests to the presence of a soft mode in the system. The behavior of various thermodynamic quantities in a constant magnetic field and in a random one is investigated. The correlator of the local frozen-in magnetization is calculated.

1. INTRODUCTION

Many recent papers deal with phase transitions in strongly disordered magnetics of the spin-glass type (see, e.g., the reviews[1,2]). In almost all the papers, the theory is passed on the Edwards–Anderson order parameter.[3] There is at present, however, no sufficient understanding of the stability of this order parameter to fluctuations (see, e.g., Refs. 4–6).

Hertz, Fleishman, and Anderson (HFA)[7] have developed in this connection an alternate approach to spin-glass theory. In this approach the paramagnet-spin glass phase transition is associated with the mobility edge, and the frozen local magnetization in the spin-glass phase corresponds to the first delocalized eigenstate of the random exchange-interaction matrix. What takes place here is, in essence, a transition of the type of Bose condensation in this state.

However, the model with random Heisenberg exchange interaction, investigated by HFA, is too complicated, and it was therefore impossible to obtain a consistent solution of the problem. HFA could advance only intuitive ideas concerning the character of the solution. In particular, it has turned out that for magnets with random exchange with $n \geq 2$ (n is the number of the spin components) there is no phase transition into spin glass at any dimensionality of space.

We consider in this paper an exactly solvable spin-glass model, namely a problem with random anisotropic exchange in the limit as $n \to \infty$. It turns out that a transition into the spin-glass phase takes place in such a system. This transition is analogous in many respects to the phase transition in an ordered magnet as $n \to \infty$, which is known to be equivalent to a phase transition of the Bose-condensation type in an ideal Bose gas in terms of the variables N and T. Therefore in our case this transition turns out to be equivalent to Bose condensation into the first delocalized state of a disordered system, while the frozen magnetization is determined by the wave function of the first delocalized state of a random exchange-interaction matrix.

Thus, the situation investigated by use corresponds fully to the HFA approach, except that the solution is carried through to conclusion. The frozen magnetiza-

tion obtained by us, at space dimensionalities $2 < d < 4$ has all the properties typical of spin glass. In particular, the correlator of this local magnetization is calculated. The order parameter is the quantity M, with the Edwards-Anderson parameter $q = M^2$ playing the role of the Bose-condensate density (just as in ordinary Bose condensation the order parameter is $\langle \psi \rangle$ and not $\rho_s = |\langle \psi \rangle|^2$). We investigate the behavior of such a system in a constant magnetic field, in a random magnetic field, and in the field conjugate to the order parameter M. We calculate the usual susceptibility, the susceptibility connected with the parameter q, and two susceptibilities connected with the order parameter M. The last two become infinite below the Curie point, thus attesting to the presence of a soft mode in the system. We note that since the initial Hamiltonian is not invariant to the $O(n)$ rotation group, this soft mode is not connected with rotation of the local magnetization, but is apparently connected with the usual invariance of the order parameter with respect to the group $U(1)$ in Bose condensation. This means that the system contains a certain hidden symmetry.

It is important to note that in our model there are no localized states of the random-exchange matrix. This makes the model greatly different from real systems. It seems to us, however, that the study of such a simplified model is nevertheless of definite interest. Of fundamental interest, in our opinion, is the possibility of correctly defining in the spin-glass problem the order parameter, the conjugate magnetic field, and the susceptibility corresponding to this order parameter, inasmuch as these questions are the most important from the point of view of the theory.

We note, however, that all our results pertain only to the case $n \to \infty$. At finite n it is perfectly possible for the situation to change completely already in first order in $1/n$. It is possible, in particular, that the lowest critical dimensionality for the spin-glass phase, which in our case is two, will become equal to four.

We indicate now two possible realizations of our model. We note first that a particular case of our problem is a magnet with randomly rotating anisotropy (this case is obtained by putting $J_{ik}^{\alpha\beta} = \delta_{ik} J_{\alpha\beta}$ in Eq. (1) and bearing in mind that in our case the fluctuations of $J_{\alpha\beta}$

are Gaussian, which differs somewhat from the usual random anisotropy). Random anisotropy is realized in many amorphous magnets. It is interesting to note that the expression obtained in the last section for the correlation radius coincides with the estimate of Imry and Ma[8] (although they have considered random fields, it is easy to show that their estimate is valid also for random anisotropy).

The second possible realization likewise involves amorphous magnets. It sometimes happens that in such magnets a random rotating anisotropy acts directly on magnetic atoms that effect an indirect exchange between other atoms. It is easily shown that random anisotropic exchange takes place in this case.
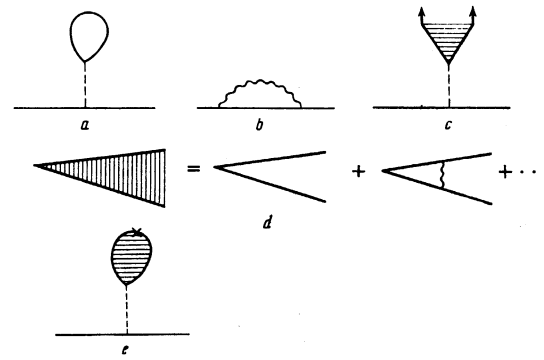
2. DERIVATION OF BASIC EQUATIONS

We choose the Hamiltonian in the form

$$\frac{H}{T} = -\frac{1}{T}\sum_{ik} v_{ik} \mathbf{m}_i \mathbf{m}_k - \frac{1}{T}\sum_{ik\alpha\beta} J_{ik}^{\alpha\beta} m_i^\alpha m_k^\beta$$

$$+ \frac{1}{2a}\sum_{i\alpha}(m_i^\alpha)^2 + \frac{\lambda}{8n}\sum_{i\alpha\beta}(m_i^\alpha)^2(m_i^\beta)^2 - \frac{h}{T}\sum_{i\alpha} m_i^\alpha. \tag{1}$$

Here m_i^α are the spin variables (i and α number the sites and the components, respectively), v_{ik} is the exchange ferromagnetic integral, $J_{ik}^{\alpha\beta}$ is the random anisotropic exchange integral, T is the temperature, λ is the interaction constant, h is the external magnetic field, and a is a constant. We assume that the random matrix $J_{ik}^{\alpha\beta}$ has a Gaussian distribution and

$$\langle J_{ik}^{\alpha\beta} J_{lm}^{\gamma\delta}\rangle = \frac{1}{n} I(\mathbf{r}_i - \mathbf{r}_k)(\delta_{il}\delta_{km}\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{im}\delta_{kl}\delta_{\alpha\delta}\delta_{\beta\gamma}). \tag{2}$$

We are considering thus a system with a nonrandom ferromagnetic interaction v_{ik} and with a random anisotropic exchange $J_{ik}^{\alpha\beta}$. We shall assume hereafter that $n \to \infty$ and confine ourselves to the zeroth approximation in $1/n$. For convenience in the calculations we assume the magnetic field to be diagonally directed, with $|h| = hn^{1/2}$.

We note that a Hamiltonian of the form (1) with the usual random exchange is usually investigated (see, e.g., Ref. 7). We, however, investigate a system with random anisotropic exchange, and this problem can be solved exactly as $n \to \infty$. The Green's function is in this case

$$G_{\alpha\beta}(\mathbf{r}_i - \mathbf{r}_k) = \langle m_i^\alpha m_k^\beta\rangle = \delta_{\alpha\beta} G(\mathbf{r}_i - \mathbf{r}_k),$$

$$G(\mathbf{r}) = d^3 \int \frac{d\mathbf{k}}{(2\pi)^3} G(\mathbf{k}) e^{i\mathbf{k}\mathbf{r}}, \tag{3}$$

$$[G(\mathbf{k})]^{-1} = 1/a - 2v(\mathbf{k})/T - \Sigma.$$

Here d^3 is the volume of the unit cell and Σ is the self-energy part. As $n \to \infty$, Σ is determined by three diagrams (see a and c in Fig. 1). The block shown in Fig. c is interpreted in Fig. d. A straight line corresponds to a complete Green's function, a dashed one to the interaction λ, a wavy line to the correlator $I(\mathbf{r})$, and the arrows represent the external magnetic field.

FIG. 1.

Calculating these diagrams, we obtain the following expression for Σ.

$$\Sigma = 4I_0 G_0/T^2 - \lambda P_0/2,$$

$$G_0 = d^3 \int \frac{d\mathbf{k}}{(2\pi)^3} G(\mathbf{k}), \qquad P_0 = G_0 + \frac{h^2}{T^2}\frac{G^2(\mathbf{k}=0)}{1 - 4I_0\Pi_0/T^2}, \tag{4}$$

$$\Pi_0 = d^3 \int \frac{d\mathbf{k}}{(2\pi)^3} G^2(\mathbf{k}), \qquad I_0 = I(\mathbf{k}=0) = \sum_{\mathbf{r}} I(\mathbf{r}).$$

We note first the following. As seen from Figs. a and c,

$$P_0 = \langle(m_i^\alpha)^2\rangle. \tag{5}$$

If we take this fact into account, then Eq. (4) can be easily obtained by splitting the fourth-power term of the Hamiltonian (1):

$$\sum_{i\alpha\beta}(m_i^\alpha)^2(m_i^\beta)^2 \to 2n\langle(m_i^\beta)^2\rangle\sum_{i\alpha}(m_i^\alpha)^2 = 2nP_0\sum_{i\alpha}(m_i^\alpha)^2. \tag{6}$$

This makes the Hamiltonian (1) quadratic, and we obtain from it directly our needed expressions, particularly the one for ϵ, and use next (5) as the self-consistency condition. This is the usual method of taking the fourth-power term into account as $n \to \infty$, and we shall use it hereafter.

From (3) and (4) we get

$$[G(\mathbf{k})]^{-1} = 1/a + \lambda P_0/2 - 4I_0 G_0/T^2 - 2v(\mathbf{k})/T. \tag{7}$$

This formula contains two parameters this must be determined, P_0 and G_0. To take the term with G_0 into account, we consider the problem of the spectrum of the random-exchange matrix $J_{ik}^{\alpha\beta} + v_{ik}\delta_{\alpha\beta}$. To this end we write down the Schrödinger equation

$$\sum_{k\beta}(J_{ik}^{\alpha\beta} + v_{ik}\delta_{\alpha\beta})a_k^\beta = \omega a_i^\alpha \tag{8}$$

and obtain the correlator $R(\omega, \mathbf{k})$ of the operators a and a^+ and the state density $\rho(\omega)$. In the limit as $n \to \infty$ these functions are calculated exactly from the simple equation (corresponding to diagram b).

$$[R(\omega, \mathbf{k})]^{-1} = \omega - v(\mathbf{k}) - I_0 R_0,$$

$$R_0 = d^3 \int \frac{d\mathbf{k}}{(2\pi)^3} R(\omega, \mathbf{k}), \tag{9}$$

$$\rho(\omega) = -\mathrm{Im}\, R_0(\omega)/\pi.$$

Treating P_0 in (7) as a parameter, it is easy to connect the Green's functions of our problem with the Green's functions

$$G(\mathbf{k}) = \frac{T}{2} R\left\{\frac{T}{2}\left(\frac{1}{a} + \frac{\lambda P_0}{2}\right), \mathbf{k}\right\},$$

$$G_0 = \frac{T}{2} R_0\left\{\frac{T}{2}\left(\frac{1}{a} + \frac{\lambda P_0}{2}\right)\right\} = \int \frac{\rho(\omega)\,d\omega}{1/a + \lambda P_0/2 - 2\omega/T}. \qquad (10)$$

We have thus replaced the problem of determining the parameter G_0 in (7) by a solution of the problem of the spectrum and the Green's function of the random-exchange matrix. To close the system of equations we must write down a self-consistency equation for P_0. To this end we introduce the quantity

$$\Phi(\omega) = \Pi_1(\omega)/[1 - I_0 \Pi_1(\omega)],$$

$$\Pi_1(\omega) = d^3 \int \frac{d\mathbf{k}}{(2\pi)^3} R^2(\mathbf{k}, \omega), \qquad (11)$$

$$\Phi(\omega) = -\frac{\partial R_0}{\partial \omega} = \int \frac{\rho(\omega')\,d\omega'}{(\omega - \omega' + i\delta)^2}.$$

The last equation is easily obtained from (9), and is valid only in the forbidden band, but this is precisely the case of interest to us. From (4) and (9)–(11) we obtain the following equation for P_0:

$$P_0 = \int \frac{\rho(\omega)\,d\omega}{1/a + \lambda P_0/2 - 2\omega/T} + \frac{Ah^2}{T^2} \int \frac{\rho(\omega)\,d\omega}{(1/a + \lambda P_0/2 - 2\omega/T)^2},$$

$$A = \frac{G^2(\mathbf{k}=0)}{\Pi_0} = \frac{R^2[{}^1\!/_2 T(1/a + \lambda P_0/2), \mathbf{k}=0]}{\Pi_1[{}^1\!/_2 T(1/a + \lambda P_0/2)]}. \qquad (12)$$

Since the functions $R(\omega, \mathbf{k})$, $R_0(\omega)$, and $\Pi_1(\omega)$ can be regarded as given, Eqs. (10) and (12) comprise a closed system of equations for the Green's function. We shall investigate them in detail below. At present, however, we note the main consequences of these equations. It is seen from (12) that at $h=0$ this equation is equivalent to the Bose-condensation problem in terms of the variables N and T, in a system with a specified state density $\rho(\omega)$. The phase transition will take place at a temperature T_c determined from the equation

$$1/a + \lambda P_0(T_c)/2 = 2\omega_0/T_c, \qquad (13)$$

where ω_0 is the edge of the band in the problem of the spectrum of the random exchange matrix (it is known[9] that in this problem there is no region of localized states as $n \to \infty$, therefore ω_0 separates the forbidden and conduction bands). Bose condensation takes place in a state C_{0i}^α corresponding to the energy ω_0, and this results, as will be shown below, in local quenched moments M_i^α whose statistical properties are determined by the correlator $K(\mathbf{r})$. These quantities are equal to

$$M_i^\alpha = \langle m_i^\alpha \rangle_T = (nN)^{\frac{1}{2}} M C_{0i}^\alpha,$$

$$\langle M_i^\alpha M_k^\beta \rangle_{\text{conf}} = M^2 \delta_{\alpha\beta} K(\mathbf{r}_i - \mathbf{r}_k), \qquad (14)$$

where $\langle \cdots \rangle_T$ and $\langle \cdots \rangle_{\text{conf}}$ denote averaging over the temperature and over the configurations, N is the number of sites in the crystals, and M is the order parameter. We shall show below that $K(\mathbf{r})$ decreases exponentially at large distances, i.e., there is a finite correlation radius in which M_i^α is completely turned, and obviously $\langle M_i^\alpha \rangle_{\text{conf}} = 0$. This means that the Bose condensation took place into the spin-glass state.

We note also the following fact. If the system contains in place of a constant magnetic field a random magnetic field with a Gaussian distribution and with a

correlator

$$\langle h_i^\alpha h_k^\beta \rangle = \delta_{\alpha\beta} \delta_{ik} h^2,$$

then we get in place of the diagram c the diagram e, where the cross denotes the correlator of the random fields. Carrying the same calculations as before, we obtain again Eq. (12), but with $A = 1$. It is of interest to note that in a constant field at $v(\mathbf{k}) = 0$ we also have $A = 1$. On the other hand it turns out that A is finite near the Curie temperature, and therefore we can regard it in (12) as an inessential constant (of course, only in weak fields). Physically this means that the constant field acts on our system in the same way as the random field. This is easily understood. Actually, for random frozen magnetization the only thing that matters is whether the magnetic field is directed along $\mathbf{M}_i$ or not. Therefore both types of field act on our system in the same way. The situation is different if the external magnetic field is correlated with $\mathbf{M}_i$. In this case it will act entirely differently, since such a field is conjugate to our order parameter. The equation for P_0 in such a field is the equation of state for our system. Let us derive this equation. The term corresponding to this field in the Hamiltonian is of the form

$$\frac{\Delta H}{T} = -\frac{1}{T} \sum_{i\alpha} h_i^\alpha m_i^\alpha, \qquad (15)$$

$$h_i^\alpha = h_0 M_i^\alpha / M = h_0 (nN)^{\frac{1}{2}} C_{0i}^\alpha.$$

Substituting (15) in (1) in place of the last term, uncoupling the fourth-order term in accordance with (6), and diagonalizing the Hamiltonian, we obtain

$$\frac{H}{T} = -\frac{1}{T} \sum_\lambda \omega_\lambda m_\lambda^2 + \frac{1}{2}\left(\frac{1}{a} + \frac{\lambda P_0}{2}\right) \sum_\lambda m_\lambda^2 - \frac{h_0}{T} m_0 (nN)^{\frac{1}{2}},$$

$$m_\lambda = \sum_{i\beta} m_i^\beta C_{\lambda i}^\beta, \qquad P_0 = \frac{1}{nN} \sum_\lambda \langle m_\lambda^2 \rangle, \qquad (16)$$

where $C_{\lambda i}^\beta$ and ω_λ are the eigenfunction and eigenenergies corresponding to the states λ of the random-exchange matrix. From (16) we easily obtain

$$\langle m_\lambda \rangle = \frac{h_0}{T} \frac{(nN)^{\frac{1}{2}}}{1/a + \lambda P_0/2 - 2\omega_0/T} \delta_{\lambda,0},$$

$$\langle m_\lambda^2 \rangle - \langle m_\lambda \rangle^2 = (1/a + \lambda P_0/2 - 2\omega_\lambda/T)^{-1}. \qquad (17)$$

Recognizing that the sum over λ goes over into an integral with respect to ω with weight $\rho(\omega)$, we obtain from (17) the following equation of state:

$$P_0 = M^2 + \int \frac{\rho(\omega)\,d\omega}{1/a + \lambda P_0/2 - 2\omega/T}, \qquad (18)$$

$$M = \frac{h_0}{T}(1/a + \lambda P_0/2 - 2\omega_0/T)^{-1}.$$

Equation (18) is completely analogous to the equation of state of an n-vector model in an ordered system as $n \to \infty$ (Ref. 10), and is equivalent to the problem of Bose condensation in the variables N and T.[11] The quantity M is the order parameter and analogous to magnetization in an ordered system. It follows from (18) that in our problem the role of the transverse susceptibility $\chi_\perp$ and of the longitudinal susceptibility $\chi_\parallel$

are assumed by the quantities

$$\chi_\perp = \frac{M}{h_0} = \frac{1}{T}(1/a+\lambda P_0/2-2\omega_0/T)^{-1},\tag{19}$$

$$\chi_{\shortparallel} = \frac{\partial M}{\partial h_0}.$$

It follows from (19) that at $T < T_c$ the susceptibility is $\chi_\perp \sim h_0^{-1}$ as $h_0 \to 0$. As will be seen below, $\chi_{\shortparallel} \sim 1/h_0^{1/2}$, i.e., $\chi_{\shortparallel}$ also diverges at $T < T_c$, just as in the ordered case. It follows therefore that a soft mode is indeed present in the problem.

3. CALCULATION OF THE CORRELATOR $R(\omega, k)$ AND OF THE STATE DENSITY

As seen from the result of the preceding section, to solve the problem we must know the correlator $R(\omega, k)$ and the state density $\rho(\omega)$. These quantities are defined in Eq. (9). In the general case this equation cannot be solved analytically. We shall therefore solve it in two particular cases: first, at $v(\mathbf{k}) = 0$, and second in the case of weak disorder, i.e., at $v(0) \gg I_0^{1/2}$ near the edge of the band, and confine ourselves in the latter case to space dimensionalities $2 < d < 4$. In the first case we have from (9) (Ref. 9)

$$R(\omega, k) = [\omega - (\omega^2 - 4I_0)^{1/2}]/2I_0,$$

$$\rho(\omega) = (4I_0 - \omega^2)^{1/2}\vartheta(4I_0 - \omega^2)/2\pi I_0.\tag{20}$$

We see that in this case $R(\omega, k)$ does not depend on $\mathbf{k}$.

We consider now the much more interesting second case. In the case of weak disorder, i.e., at $I_0 \ll v^2(0)$, the main singular contribution is made by small k. Separating in (9) the contribution of the small k, we obtain for ω_1 from (9) the following equation, which is simply related to R_0:

$$\omega_1 - \xi\omega_1^{1/2} - (\omega - I_0R_0 {}_{\text{reg}} - v(0)) = 0,$$

$$\omega_1 = \omega - I_0R_0 - v(0),$$

$$\xi = \frac{I_0}{\beta^{3/2}}d^3\int\frac{d\mathbf{x}}{(2\pi)^3}\frac{1}{x^2(x^2+1)} = \frac{I_0d^3}{4\pi\beta^{3/2}},$$

$$R_0 {}_{\text{reg}} = d^3\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{v(0)-v(\mathbf{k})},\tag{21}$$

$$v(\mathbf{k}) = v(0) - \beta k^2, \quad kd \ll 1,$$

$$\rho(\omega) = \text{Im}\,\omega_1/\pi I_0,$$

$$[R(\omega, k)]^{-1} = \omega_1 - v(\mathbf{k}) + v(0).$$

As seen from (21), the state density becomes different from zero where $\omega_1(\omega)$ has a branch point. The energy ω_0 at which this takes place is the edge of the band. At this energy, however, the equality $\omega_1(\omega) = 0$ is not at all obligatory, and then $R(\omega_0, \mathbf{k} = 0)$ does not become infinite. It is seen, however, from (11) that the two-particle Green's function becomes infinite at this energy if $\rho(\omega) \sim \omega^\alpha$ with $\alpha < 1$. This means that at $\omega = \omega_0$ there is produced also conductivity, i.e., there is no region of localized states in this model.

Solving the equation for ω_1 in (21), we obtain near ω_0

$$\omega_1 = \omega - \omega_0 + \xi(\omega - \omega_0)^{1/2} + \xi^2/4,$$

$$\omega_0 = v(0) + I_0R_0 {}_{\text{reg}} - \xi^2/4, \quad \omega_1(\omega_0) = \xi^2/4,\tag{22}$$

$$\rho(\omega) = \xi(\omega_0 - \omega)^{1/2}\vartheta(\omega_0 - \omega)/\pi I_0.$$

It is seen from (22) that actually $\omega_1(\omega_0)$ does not vanish. Near the edge of the band

$$R(\omega_0, k) = 1/\beta(k^2 + \varkappa^2), \quad \varkappa = \xi/2\beta^{1/2},\tag{23}$$

which leads to a finite correlation radius of the magnetization correlator below T_c.

At $d \neq 3$ dimensions, Eq. (21) takes the form

$$\omega_1 - \xi\omega_1^{(d-2)/2} - (\omega - I_0R_0 {}_{\text{reg}} - v(0)) = 0.\tag{24}$$

The integral that determines ξ in (21) then diverges at $d = 2$ and $d = 4$. It is clear therefore that these are the borderline dimensionalities. Obviously, $d = 2$ is the lower critical dimensionality, and at $d = 4$ the character of the solution for weak interaction simply changes. In the interval $2 < d < 4$ the solution (24) retains as before the features of the solution (22), i.e., $\omega_1(\omega_0)$ is finite and the state density $\rho(\omega) \sim (\omega - \omega_0)^{1/2}$ near the band edge. As seen from (20), in the case of strong disorder the picture is the same, and it is therefore clear that at $I_0 \sim v^2(0)$ the qualitative character of the solution is preserved. Since only the form of the state density near the band edge is of importance for the study of the phase transition, it can be assumed that at $2 < d < 4$ the square-root state density exists at any degree of disorder. Inasmuch as in this dimensionality range only the case $d = 3$ is of interest, we shall consider hereafter only the three-dimensional case. The main results connected with the square-root singularity of $\rho(\omega)$ will remain in force in the interval $2 < d < 4$.

We parametrize $\rho(\omega)$ near the edge of the band in the following manner:

$$\rho(\omega) = \varepsilon_2^{-1/2}(\omega_0 - \omega)^{1/2}\vartheta(\omega_0 - \omega),\tag{25}$$

where ω_0 and ε_2 are certain parameters that can be calculated in the particular cases (20) and (22).

4. SOLUTION OF THE EQUATION OF STATE

We proceed now to solve the equation of state (18). From (18) we have the following equation for the quantity ε, which is simply related to P_0 and to the generalized magnetization M:

$$\frac{4}{\lambda T}(\varepsilon - \tau) = \frac{h_0^2}{4\varepsilon^2} - \frac{T\varepsilon}{2}\int\frac{\rho(\omega)d\omega}{(\omega_0 - \omega)(\varepsilon + \omega_0 - \omega)},$$

$$\varepsilon = \frac{T}{2}\left(\frac{1}{a} + \frac{\lambda P_0}{2} - \frac{2\omega_0}{T}\right) = \frac{h_0}{2M} = \frac{1}{2\chi_\perp},\tag{26}$$

$$\tau = \frac{T}{2a} - \omega_0 + \frac{\lambda T^2}{8}\int\frac{\rho(\omega)d\omega}{\omega_0 - \omega}.$$

In the derivation of (26), a subtraction was made from (18), therefore the integral in (26) is determined at small ε by small $\omega_0 - \omega$ and we can use $\rho(\omega)$ from (25). We then obtain

$$\varepsilon + \mu\varepsilon^{1/2} - \alpha h_0^2/\varepsilon^2 = \tau,\tag{27}$$

$$\alpha = \lambda T/16, \quad \mu = \pi\lambda T^2/8\varepsilon_2^{1/2}.$$

In the zeroth order in the parameter $h_0|\tau|^{-5/2}$ the solution of (27) is

$$\varepsilon = \frac{\mu^2}{2}\left[1 + \frac{2\tau}{\mu^2} - \left(1 + \frac{4\tau}{\mu^2}\right)^{1/2}\right], \quad \tau > 0,$$

$$\varepsilon = 0, \quad \tau < 0,\tag{28}$$

$$\varepsilon = \tau^2/\mu^2, \quad 0 < \tau \ll \mu^2.$$

In the derivation of (28) we cannot simply put $h_0 = 0$ in (27), but must assume h_0 to be a sufficiently small but finite quantity. Since (27) has been derived at $\varepsilon \ll \omega_0 \sim T_c$, Eq. (28) is also valid under these very same conditions, which hold true at all $\tau \sim \mu^2$ if $\lambda \ll 1$, and only at $\tau \ll \mu^2$ if $\lambda \sim 1$. In the latter case only the last asymptotic formula for $\varepsilon(\tau)$ is valid.

It is seen from (28) that ε has a kink at $\tau = 0$. This means that the phase transition takes place at $\varepsilon = \tau = 0$. The condition $\varepsilon = 0$ coincides with Eq. (13). The condition $\tau = 0$ yields an explicit expression for T_c:

$$T_c = \frac{\Omega}{4a}\left[\left(1 + \frac{16a^2\omega_0}{\Omega}\right)^{1/2} - 1\right],$$
$$\Omega^{-1} = \frac{\lambda}{8}\int\frac{\rho(\omega)d\omega}{\omega_0 - \omega}. \tag{29}$$

If $\lambda \to 0$, then $\Omega \gg \omega_0 a^2$ and $T_c = 2a\omega_0$, corresponding to perturbation theory.

We calculate now ε, M, $\chi_\perp$, and $\chi_\parallel$ below T_c in first order in $h_0|\tau|^{-5/2}$. From (27) we obtain readily

$$\varepsilon = h_0\left(\frac{\alpha}{|\tau|}\right)^{1/2}\left(1 - \frac{\mu h_0^{1/2}\alpha^{1/4}}{|\tau|^{5/4}}\right),$$
$$M = \frac{h_0}{2\varepsilon} = \frac{1}{2}\left(\frac{|\tau|}{\alpha}\right)^{1/2}\left(1 + \frac{\mu h_0^{1/2}\alpha^{1/4}}{|\tau|^{5/4}}\right), \tag{30}$$
$$\chi_\parallel = \frac{\partial M}{\partial h_0} = \mu/4\alpha^{1/4}h_0^{1/2}|\tau|^{1/4},$$
$$\chi_\perp = 1/2\varepsilon = (|\tau|/\alpha)^{1/2}/h_0.$$

It is seen from (30) that the susceptibilities below T_c are $\chi_\perp \sim 1/h_0$ and $\chi_\parallel \sim 1/h_0^{1/2}$, just as for an ordered system.

At $\tau > 0$, naturally, the two susceptibilities are equal

$$\chi_\parallel = \chi_\perp = 1/2\varepsilon = \mu^2/\tau^2. \tag{31}$$

From (30) and (31) we determine the critical exponents

$$\beta = 1/2, \quad \gamma = 2, \quad \delta = 5, \tag{32}$$

corresponding to a phase transition in an ideal Bose gas in terms of the variables N and T.

5. SUSCEPTIBILITY AND THE EDWARDS-ANDERSON PARAMETER

The usual physical susceptibility χ and the Edwards-Anderson parameter q are obviously given by

$$\chi = G(k = 0) = \frac{T}{2}R\left\{\frac{T}{2}\left(\frac{1}{a} + \frac{\lambda P_0}{2}\right), \quad k = 0\right\}, \tag{33}$$
$$q = P_0 - G_0.$$

We now connect q^2 with M. We calculate below the function $K(\mathbf{r})$ that enters in (14), and see that $K(\mathbf{r} = 0) = 1$. It follows then from (14) and from the definition of q that

$$q = \langle(M_i^\alpha)^2\rangle_{\text{conf}} = M^2. \tag{34}$$

To calculate χ and q at finite h we consider the Eq. (12). Since it can be readily seen that A is finite at the transition point, it follows that in the calculation of the principal singularity at small h we must regard A as a constant. We then obtain from (12) an equation similar to (27):

$$\varepsilon + \mu\varepsilon^{1/2} - \nu h^2/\varepsilon^{1/2} = \tau,$$
$$\nu = \pi\lambda TA/32\varepsilon_2^{3/2}. \tag{35}$$

We solve this equation near T_c. In this region, the first term is small, and we have

$$\varepsilon^{1/2} = [\tau + (\tau^2 + 4\mu\nu h^2)^{1/2}]/2\mu. \tag{36}$$

As $h \to 0$ we obtain the asymptotic form of (28) at small τ. We calculate now the susceptibility. From the definition of ε in (26), it is seen from (21), (22), and (33) that near T_c

$$\chi = \frac{2T_c}{\xi^2}\left\{1 - \frac{2}{\mu\xi}[\tau + (\tau^2 + 4\mu\nu h^2)^{1/2}]\right\}. \tag{37}$$

It follows from (37) that at $h = 0$ the susceptibility has a kink that becomes smoothed out in a magnetic field. Thus, a constant magnetic field eliminates the phase transition. We note that this fact, generally speaking, is not obvious, since the constant magnetic field is not the field conjugate to the order parameter. A random magnetic field acts in exactly the same manner (we recall that in this case we need merely put $A = 1$). We have next from (12), (33), and (36)

$$q = \frac{4\nu h^2}{\lambda T\varepsilon^{1/2}} = 2[(\tau^2 + 4\mu\nu h^2)^{1/2} - \tau]/\lambda T,$$
$$q \sim \begin{cases} h^2/\tau, & \tau > 0 \\ |\tau|, & \tau < 0 \end{cases}, \quad h^2 \ll |\tau|. \tag{38}$$

The second formula in (38) agrees with (30) and (34). It is seen from (38) that the indices corresponding to the parameter q coincide with their classical values:

$$\beta_q = 1, \quad \gamma_q = 1. \tag{39}$$

6. CALCULATION OF THE CORRELATOR OF A LOCAL FROZEN MAGNETIZATION

We calculate now the function $K(\mathbf{r})$ that enters in expression (14) for the correlator of the magnetizations:

$$K(\mathbf{r}_i - \mathbf{r}_j) = nN\langle C_{0i}^\alpha C_{0k}^\beta\rangle_{\text{conf}}. \tag{40}$$

Using the spectral representation for the correlator $R(\omega, \mathbf{r})$:

$$\delta_{\alpha\beta}R(\omega, \mathbf{r}_i - \mathbf{r}_k) = \left\langle\sum_\lambda\frac{C_{\lambda i}^\alpha C_{\lambda k}^\beta}{\omega - \omega_\lambda + i\delta}\right\rangle \tag{41}$$

and assuming that the eigenfunction corresponding to ω_0 is nondegenerate, since $\rho(\omega) \to 0$ as $\omega \to \omega_0$, we obtain

$$K(\mathbf{r}) = -\lim_{\omega\to\omega_0}\frac{\text{Im}\,R(\omega, \mathbf{r})}{\pi\rho(\omega)}. \tag{42}$$

If $v(\mathbf{k}) = 0$, then, as seen from (20), $\text{Im}\,R(\omega, \mathbf{r}) = -\pi\rho(\omega)\delta_{\mathbf{r}0}$ and we have

$$K(\mathbf{r}) = \delta_{\mathbf{r}0}. \tag{43}$$

Thus, in the case of strong disorder, M_i^α does not correlate even at neighboring sites. In the case of weak disorder we obtain from (21)–(23)

$$K(\mathbf{r}) = e^{-\varkappa r}, \tag{44}$$

where $\varkappa$ is defined in (23).

If $I_0 \to 0$, then also $\varkappa \to 0$ and we obtain in place of spin glass an ordinary ferromagnet.

For an arbitrary space dimensionality we easily obtain from (24), in lieu of (23), the following expression for $\varkappa$:

$$\varkappa = \beta^{-1/2}(\xi(d-2)/2)^{1/(4-d)}. \tag{45}$$

Since $\xi \sim I_0$, it follows that $\varkappa \sim I_0^{1/(4-d)}$, which coincides with the estimate of Imry and Ma[8] for the correlation radius.

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Investigation of the spatial distribution of acoustic radiation resulting from emergence of a dislocation pile-up on a surface

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Measurements are reported of the spatial distribution of the various components of the stress tensor in the field of transition acoustic radiation generated on emergence of a pile-up of dislocations on the surface of a crystal or on formation of such a pile-up near the surface and its subsequent penetration into the crystal. Theoretical relationships are obtained for the emission of transition sound as a result of emergence of a planar dislocation pile-up and of a Peierls dislocation on the surface. Allowance for the finite width of the dislocation core makes it possible to remove consistently the divergence of the radiation fields and to relate the characteristics of the leading edge of an acoustic radiation pulse to the core width. The components of the stress tensor and of the vector describing the velocity of elements of the medium in the leading edge of an acoustic pulse are inversely proportional to the square root of the width of a dislocation core. A comparison of the experimental and theoretical results made allowing for the influence of the crystal anisotropy demonstrates validity of the theory of transition radiation in describing the spatial distribution of acoustic emission when a pile-up of dislocations emerges on the surface of a crystal.

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1. INTRODUCTION

Natsik[1] used the physical analogy between two field theories—electrodynamics and theory of elasticity—to consider theoretically the transition emission of sound by a dislocation emerging on the surface of a crystal, in the same way as the transition emission of electromagnetic waves has been considered earlier.[2] The first experimental investigation of the transition emission of sound was reported in Ref. 3. Among the latter investigations it is worth noting Ref. 4 reporting the first experimental identification of the transition acoustic radiation in its pure form. Investigation of the transition emission of sound is of general physical interest because it demonstrates the existence of transition radiation for waves of different origin.[5] Moreover, such investigation provides a physical basis for one of the promising nondestructive testing methods, which is the method of acoustic emission.

Comparison of various mechanisms of the emission of sound by moving dislocations, discussed by Natsik

et al. in developing a theory of acoustic emission (for details see the review in Ref. 6), shows that under the conditions usually encountered in plastic deformation the greatest contribution to the acoustic emission is made not by the accelerated motion of dislocations but by the process associated with the appearance or disappearance of dislocations. Formation of dislocations near an interface followed by penetration into the medium is also accompanied by transition radiation. The appearance of dislocations inside a crystal, which—in accordance with the law of conservation of the Burgers vector—is possible only in the form of pairs of dislocations of opposite signs, is accompanied by annihilation radiation. The disappearance of dislocations by emergence on a surface or by annihilation inside a crystal is also accompanied by transition or annihilation radiation, respectively. In an earlier paper[7] we reported an experimental investigation of the annihilation radiation, whereas in the present paper (which is a direct continuation of the investigations reported in Refs. 4 and 7) we shall give the experimental results obtained in a study of the transition radiation.

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