[3] Indeed, by changing from the expression for the Green's function (2.5) to the phonon distribution function (see Ref. 2) we can verify by direct summation over the frequencies that the resultant distribution function coincides in the case of integer n with the equilibrium Bose distribution: $N^0_\omega = [\exp(\omega/T) - 1]^{-1}$.

[4] Such a schematic representation of the nonequilibrium phonon Green's function was used in Ref. 7 and turns out to be quite useful in the discussion of many physical problems.

[5] We did not dwell in detail on the proof on this statement. A more detailed exposition of this problem as well as of other aspects touched upon in the paper will be given elsewhere.

[6] This estimate is satisfactory in those cases when the frequency of the emitted phonons is less than the threshold of quasiparticle production in the superconductor.

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Nonlocal electron-interaction effects in the spontaneous-current model

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We consider the influence of hybridization of the electron bands of a semimetal on the symmetry of the order parameter $\Delta(\mathbf{k})$ of the exciton phase. It is shown that an arbitrarily small hybridization suppresses the symmetrical component $\Delta_s(\mathbf{k})$ of the order parameter in a one-dimensional system with weakly screened Coulomb interaction of the electrons and holes. In a three-dimensional system one can indicate for the hybridization a limit below which a first-order phase transition is possible with formation of a symmetrical component $\Delta_s(\mathbf{k}) = \Delta_s(-\mathbf{k})$. One of the possibilities of formation of an inhomogeneous state for an excitonic dielectric is noted. The results of the study have a direct bearing on the spontaneous-current model.

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INTRODUCTION

An indispensable part of the model of spontaneous currents in an excitonic dielectric[1] is the presence of interband dipole transitions in the system. If an electron-hole condensate with an imaginary component of the order parameter appears in a semimetal (semiconductor) system, then a macroscopic electron current can flow in the presence of interband dipole transitions in this system.[1] This, however, still leaves open the question of the influence of the interband dipole transitions of the electrons on the phase transition of the semimetal (semiconductor) into the state of an excitonic dielectric with imaginary order parameter. As will be shown in the present paper this influence is particularly important when the electron-hole interaction is nonlocal.

The most widely used and simplest description of electron bands, which takes into account the interband dipole transitions, is the approximation of Luttinger and Kohn.[2] In this approximation these transitions are represented in the form of band hybridization. The hybridization is expressed in the Hamiltonian in the form of a single-particle operator.

$$\sum_{\substack{\mathbf{k}\\ n\neq n'}} W_{nn'}(\mathbf{k}) a_{n\mathbf{k}}^{+} a_{n'\mathbf{k}} + \mathrm{H.c.}\,, \tag{1}$$

$$W_{nn'}(\mathbf{k}) = \mathbf{P}_{nn'}\mathbf{k}, \quad \mathbf{P}_{nn'} = -\frac{i}{m}\left\langle u_{n\mathbf{k}_0}\left|\frac{\partial}{\partial \mathbf{r}}\right| u_{n'\mathbf{k}_0}\right\rangle,$$

where a_{nk}^{+} is the creation operator for an electron with momentum k in the band n, and $u_{nk_0}(r)$ is the Bloch state wave function at the extremum of the band.

In most problems in the theory of the excitonic dielectric, an electron-hole point interaction is assumed. In this simplest approximation, the order parameter $\Delta(\mathbf{k})$ is independent of the momentum k, and the presence of the hybridization (1) in such a system leads to two effects. First, owing to the angular dependence, the system acquires an anisotropy that lowers the phase-transition temperature in a certain manner.[3] Second, the hybridization fixes the phase of the order parameter Δ, giving preference to a phase that is a multiple of π,[4] i.e., to a state in which there are no spontaneous currents.

The electron-hole point interaction approximation is to a certain degree an idealization that is justified for a system with a high density of free carriers, and hence with a strongly screened interaction. In real systems, for example in semiconductors, the Coulomb interaction of the electrons is weakly screened and is essentially nonlocal. In particular, it is easy to verify that the amplitudes of the first few harmonics of the Coulomb interaction are practically equal. Therefore the structure of the order parameter $\Delta(\mathbf{k})$ is determined by the competition between different angular harmonics of the dependences of the parameter $\Delta(\mathbf{k})$ on the directions of the vector k.

In a system with the hybridization (1), the nonlocality of the electron-hole interaction becomes particularly important. Indeed, the hybrdization (1) takes the form of a single-particle interband interaction that is antisummetrical in k, and enters explicitly in the self-consistency equation for the order parameter of the exciton phase:

$$\Delta(\mathbf{p}) = T\sum_{n}\int \frac{d^3k}{(2\pi)^3} V(\mathbf{p}-\mathbf{k}) \frac{\Delta(\mathbf{k})+W(\mathbf{k})}{\omega_n^2+\varepsilon_{\mathbf{k}}^2+|\Delta+W|^2}. \tag{2}$$

As already mentioned, in the case of a point potential $V(\mathbf{p}-\mathbf{k}) = g = \mathrm{const}$, the parameter $\Delta(\mathbf{k})$ does not depend on the momentum, and Eq. (2) determines its temperature dependence. For a nonlocal potential $V(\mathbf{p}-\mathbf{k})$ the term linear in the hybridization in Eq. (2) is the source for the parameter $\Delta(\mathbf{k})$ at any temperature, and in this sense the phase transition for $\Delta(\mathbf{k})$ vanishes ($T_c\to\infty$). We note, however, that the angular dependence of that part of $\Delta(\mathbf{k})$ which comes from the source $W(\mathbf{k})$ is antisymmetrical in k, i.e., $\Delta(\mathbf{k}) = -\Delta(-\mathbf{k})$, and makes no contribution to the spontaneous current.[1]

In general form, the solution $\Delta(\mathbf{k})$ of Eq. (2) can have an arbitrary angular dependence. Interest attaches therefore to a phase transition such that a symmetrical component $\Delta_s(\mathbf{k})$ appears in the system with induced antisymmetric parameter $\Delta_a(\mathbf{k})$. It should be recalled that the spherically symmetrical component Δ_s of the imaginary order parameter is of interest because it is just this component which contributes to the macroscopic spontaneous current of the model.[1]

It will be shown in this paper that an arbitrary small hybridization in a quasi-one-dimensional system with a weakly screened Coulomb interaction makes a phase transition impossible for the symmetrical component of the imaginary order parameter. On going to a three dimensional system, the integration with respect to angle changes the statistical weight with which each component of the parameter $\Delta(\mathbf{k})$ enters in the free energy functional, and electron-hole pairing with a symmetrical order parameter corresponds to the largest statistical weight, in addition, in the three-dimensional case, unlike the one-dimensional one, the symmetrical pairing of Δ_s is least suppressed by the hybridization in solid angles directed perpendicular to the vector $\mathbf{P}_{12}$. Therefore a first-order phase transition becomes possible for the component Δ_s at a sufficiently low value of the hybridization W.

In the second part of the paper we consider qualitatively one more effect that takes place only in models with nonlocal electron-hole interaction. It is shown, for an arbitrarily small noncongruence of the Fermi surfaces of the electrons and holes, that the expansion of the free-energy functional in the components of the complex order parameter $\Delta(\mathbf{k}, \mathbf{R})$ contains a term linear in the spatial derivative $\Delta\partial\Delta/\partial\mathbf{R}$, made up of a symmetrical $\Delta_s(\mathbf{R})$ and antisymmetrical $\Delta_a(\mathbf{R})$ component of the parameter $\Delta(\mathbf{k}, \mathbf{R})$. The presence in the functional of this term, called the Lifshitz invariant,[5] makes possible inhomogeneous states of an excitonic dielectric.

1. HYBRIDIZATION AND NONLOCALITY OF THE POTENTIAL V (p - k)

1. We consider an exactly solvable example of electron-hole pairing in a one-dimensional semimetal with the Fermi level at $\mathbf{k} = \mathbf{k}_0$, $\varepsilon(\mathbf{k}_0) = 0$ at zero temperature. Equation (2) integrated over the frequencies takes the form

$$\Delta(\mathbf{p}) = \int \frac{dk}{2\pi} V(\mathbf{p}-\mathbf{k}) \frac{\Delta(\mathbf{k})+W(\mathbf{k})}{[\varepsilon^2(\mathbf{k})+|\Delta+W|^2]^{1/2}} \tag{3}$$

in the case of weak screening the one-dimensional Coulomb potential $V(\mathbf{p}-\mathbf{k})$ can be assumed to be given by $V(\mathbf{p}-\mathbf{k}) = V_0\delta(\mathbf{p}-\mathbf{k})$. Equation (3) is then easily integrated and yields

$$\Delta(\mathbf{k}_0) = V_0\,\mathrm{sign}\,[\Delta(\mathbf{k}_0)+\mathbf{P}_{12}\mathbf{k}_0]. \tag{4}$$

It is easily seen that the solution of (4) is of the form (see Fig. 1)

$$\Delta(\mathbf{k}_0) = \begin{cases} V_0, & -V_0<\mathbf{P}_{12}\mathbf{k}_0<\infty \\ -V_0, & -\infty<\mathbf{P}_{12}k_0<V_0. \end{cases}$$

In the region defined by the inequality $|\mathbf{P}_{12}\cdot\mathbf{k}_0| < V_0$, where the hybridization is small, there exist two solutions for $\Delta(\mathbf{k}_0)$: odd, $\Delta(\mathbf{k}_0) = -\Delta(-\mathbf{k}_0) = \pm V_0$, and even $\Delta(\mathbf{k}_0) = \Delta(-\mathbf{k}_0) = \pm V_0$. Outside this region there exists at $|\mathbf{P}_{12}\cdot\mathbf{k}_0| > V_0$ only the odd solution $\Delta(\mathbf{k}_0) = -\Delta(-k_0) = \pm V_0$.

Of the two solutions in the ambiguity region, it is

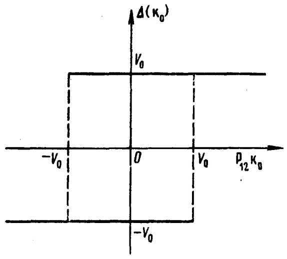

FIG. 1.

necessary to choose the more convenient one on the basis of minimization of the energy. If it is recognized that the energy difference between the normal ($\Delta_s = 0$) and bound states is given by the expression

$$\delta E = \sum_{\mathbf{k}} [\,|\varepsilon(\mathbf{k})| - (\varepsilon^2(\mathbf{k}) + |\Delta(\mathbf{k}) + \mathbf{P}_{12}\mathbf{k}|^2)^{1/2}], \tag{5}$$

then the energy difference between the odd and even solution will take the form

$$\Delta E = \sum_{V_0 > |\mathbf{P}_{12}\mathbf{k}_0| > 0} [(\varepsilon^2(\mathbf{k}) + [V_0 + \mathbf{P}_{12}\mathbf{k}]^2)^{1/2} - (\varepsilon^2(\mathbf{k}) + [V_0 - \mathbf{P}_{12}\mathbf{k}]^2)^{1/2}].$$

This quantity is always positive, and consequently the odd solution is always favored. By the same token we have shown that in the considered case there is no phase transition with formation of a symmetrical component Δ_s of the order parameter.

2. We turn now to the three-dimensional situation. For a qualitative analysis of the problem we make certain assumptions. We consider an order parameter $\Delta(\mathbf{k})$ in the form of two components:

$$\Delta(\mathbf{k}) = \Delta_s + \Delta_a(\mathbf{k}), \quad \Delta_a(\mathbf{k}) = \Delta_a \mathbf{n}_P \mathbf{n}_{\mathbf{k}},$$

where $\mathbf{n}_P$ is the unit vector in a direction parallel to the hybridization vector $\mathbf{P}_{12}$ [Eq. (1)] and $\mathbf{n}_k = \mathbf{k}/|\mathbf{k}|$. In the Coulomb interaction $V(\mathbf{k}-\mathbf{k}')$ we also confine ourselves to the first two harmonics:

$$V(\mathbf{k}-\mathbf{k}') = \frac{4\pi e^2}{\varepsilon(|\mathbf{k}-\mathbf{k}'|^2 + \varkappa_D^2)} \approx g_s + 3g_a \mathbf{n}_{\mathbf{k}} \mathbf{n}_{\mathbf{k}'}. \tag{6}$$

The constants g_s and g_a can be calculated as the mean values of $V(\mathbf{k}-\mathbf{k}')$ on the Fermi surface $|\mathbf{k}| = |\mathbf{k}'| = k_0$:

$$g_s = \frac{2\pi e^2}{\varepsilon k_0^2} \ln \frac{2k_0}{\varkappa_D}, \quad g_a = \frac{2\pi e^2}{\varepsilon k_0^2} \left[\ln \frac{2k_0}{\varkappa_D} - 1\right]. \tag{7}$$

From which it is seen that in the weak-screening limit $k_0 \gg \varkappa_D$ the constants g_s and g_a are practically equal. Therefore the temperatures $T_{c0}^{s,a}$ for each of the components Δ_s and Δ_a, defined by the relation

$$\frac{1}{g_{s,a}} = T_{c0}^{s,a} \ln \frac{2\tilde{\omega}}{\pi T_{c0}^{s,a}}, \tag{7a}$$

where $\tilde{\omega}$ is the cutoff frequency, can be regarded as close to each other.

It must be noted that the temperatures $T_{c0}^{s,a}$ are not the true critical temperatures of the corresponding phase transitions, since when one of the components of the paraneter $\Delta(\mathbf{k})$ is produced and increases if the temperature is lowered, the next low-temperature phase transition is shifted in temperature or is completely suppressed.

In our problem, the antisymmetrical imaginary component $\Delta_a(\mathbf{k})$ exists at all temperatures [on account of the source $W(\mathbf{k})$] and means many-electron renormalization of the single-particle hybridization $W(\mathbf{k})$. The temperature T_{c0}^{a} (7a) for this parameter means formally the temperature near which Δ_a increases abruptly. To solve the problem of the phase transition Δ_s, it is not sufficient to find the temperature at which the corresponding solution of Eq. (2) appears. It is necessary to compare the free energy corresponding to these solutions with the free energy of the normal state.

We shall solve Eq. (2) near the temperature T_c of the proposed phase transition for Δ_s. We assume that the hybridization W is small enough, so that Δ_a is also small. Then Eq. (2) can be expanded in the vicinity of T_c up to terms cubic in $\Delta(\mathbf{k})$ and integrated over the angles. After separating the symmetrical and antisymmetrical parts of Eq. (92) we obtain the following nonlinear system of equations:

$$\begin{aligned} &\Gamma^3 - {}^5/_3(\Delta_0^2(T) - 3\Delta_s^2)\Gamma - w = 0, \\ &\Delta_s^2 = \Delta_0^2(T) - \Gamma^2, \end{aligned} \tag{8}$$

where

$$\begin{aligned} &\Gamma = \Delta_a + \tilde{W}, \quad \tilde{W} = \mathbf{P}_{12}\mathbf{k}_0/g_a, \quad w = {}^5/_3 \tilde{T}^2 \tilde{W}, \\ &\tilde{T}^2 = 8\pi^2 T^2/7\zeta(3), \quad \Delta_0^2(T) = -\tilde{T}^2 \ln(T/T_{c0}). \end{aligned} \tag{8a}$$

The numerical coefficients in the system (8) are determined by integration over the angles and are typical of only the three-dimensional case.

The expression for the expansion of the free energy in the parameters Δ_s and Δ_a can be obtained by direct summation of the perturbation theory diagrams, or reconstructed from Eqs. (5). Accurate to a constant that stems from the change of notation $\Gamma = \Delta_a + \tilde{W}$, we obtain

$$\tilde{T}^2 \delta F = -\frac{1}{2}\Delta_0^2(T)\Delta_s^2 + \frac{1}{4}\Delta_s^4 + \frac{1}{2}\Delta_s^2\Gamma^2 - \frac{1}{6}\Delta_0^2(T)\Gamma^2 + \frac{1}{20}\Gamma^4 - \frac{1}{5}w\Gamma. \tag{9}$$

For comparison, it is useful to write down the expression for δF in the one-dimensional case

$$\tilde{T}^2 \delta F = -\frac{1}{2}\Delta_0^2(T)\Delta_s^2 + \frac{1}{4}\Delta_s^4 + \frac{3}{2}\Delta_s^2\Gamma^2 - \frac{1}{2}\Delta_0^2(T)\Gamma^2 + \frac{1}{4}\Gamma^4 - \tilde{W}\Gamma. \tag{9a}$$

It is seen that in both cases the hybridization $W(\tilde{W})$ causes the parameter Γ to make a linear contribution to the free energy. This is precisely why in the one-dimensional case, when the statistical weight (the coefficient of the second and fourth powers of the parameter) of both parameters Δ_s and Γ is the same, and pairing with the antisymmetry parameter $\Delta_a(\Gamma)$ is always more convenient. In the three-dimensional case described by (9) the statistical weight for the parameter Γ is smaller than for Δ_s because of the angular dependence of the former. A possibility is thus uncovered of the formation of a state with $\Delta_s \neq 0$.

To find the simultaneous solutions of the system (8) we substitute the second equation in the first, thus eliminating the variable Δ_s. The resultant cubic equation for the parameter Γ

$$\Gamma^3 - \frac{5}{6}\Delta_0^2(T)\Gamma = -\frac{1}{4}w \tag{10}$$

and the corresponding functional

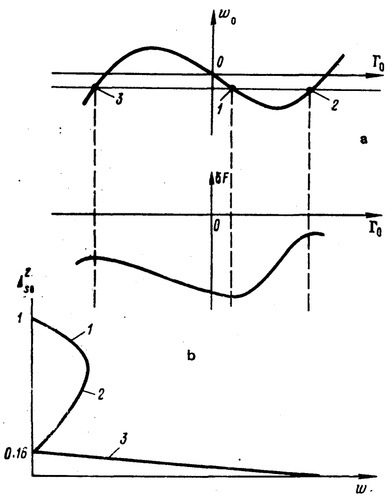

FIG. 2. a) Graphic solution of Eq. (10). $\Gamma_0=\Gamma/\Delta_0(T,\ \Delta_{s0}=\Delta_s/\Delta_0(T),\ w_0=w/\Delta_0^3(T)$. Solution 1 is stable, while 2 and 3 are unstable. b) Dependence of the order parameter Δ_{s0} on w_0: $\Delta^2_{s0}\ (w_0)=1-\Gamma_0^2(w_0)$.

$$T^2\delta F_s=-\frac{1}{4}\Delta_0^4(T)$$

$$+\frac{1}{3}\Delta_0^2(T)\Gamma^2-\frac{1}{5}\Gamma^4-\frac{1}{5}w\Gamma$$

describe the behavior of the complete hybridization $\Gamma=\Delta_a+\tilde{W}$ for coexisting Δ_s and Γ. The region in which Eq. (10) is valid is determined by the condition $\Delta_3^2\geqslant 0$, corresponding to $\Gamma^2\leqslant\Delta_0^2(T)$.

It is easy to verify that Eq. (10) has three solutions (see Fig. 2), two of which correspond to local maxima of the free-energy functional δF_s, and one to a minimum and is therefore the only stable one.

It is convenient to solve Eq. (10) in the dimensionless variables $\Gamma_0=\Gamma/\Delta_0(T)$ and $w_0=w/\Delta_0^3(T)$. A stable solution of (10) is given by the approximate equality $\Gamma_0^*\approx 3w_0/10$ and its energy is equal to

$$\delta E_s^*=-\frac{1}{4}-\frac{3}{100}w_0^2. \tag{11}$$

We note that the temperature dependence of the energy δE^*_s is incorporated in the scale that make the quantity w dimensionless.

The energy (11) for $\Delta(k)$, which contains the symmetrical component Δ_s, must be compared with the energy of the normal state, which is characterized by a zero symmetrical component $\Delta_s\equiv 0$. To determine the temperature dependence of the total hybridization in the normal phase one can use the first equation of (8) with Δ_s^2 term omitted:

$$\Gamma_N^3-{}^5/_3\Delta_0^2(T)\Gamma_N=w. \tag{12}$$

The stable and most convenient solution of this equation takes in the dimensionless variables the approximate form

$$\Gamma_N^*=({}^5/_3)^{1/2}+{}^3/_{10}w_0$$

and has an energy

$$\delta E_N^*=-\frac{5}{36}-\frac{21}{600}w_0^2-\frac{1}{3}\left(\frac{3}{5}\right)^{1/2}w_0. \tag{13}$$

It remains to take the difference between the energies (11) and (13)

$$\Delta E=\delta E_s^*-\delta E_N^*=-\frac{1}{9}+15^{-1/2}w_0+\frac{1}{200}w_0^2.$$

It is seen that with decreasing w_0 (this corresponds exactly to a lowering of the temperature or to a decrease of the hybridization), the energy ΔE becomes negative at $w_0\leqslant 15^{1/2}/9$. This determines the temperature below which a first-order phase transition leads to a state with a symmetrical component Δ_s of the order parameter $\Delta(\mathbf{k})=\Delta_s+\Delta_a(\mathbf{k})$.

2. PHASE TRANSITION INTO AN INHOMOGENEOUS STATE

We consider now the question of one feasibility of inhomogeneous states of an excitonic dielectric. As already noted, departure from the framework of the approximation of a pointlike electron hole interaction provides the system with additional degrees of freedom, with angular harmonics appearing in the complex order parameter $\Delta(\mathbf{k})$. Above the phase transition point, the interaction of fluctuations of different angular harmonics of the parameter $\Delta(\mathbf{k})$ determines to a considerable degree the character of the phase transition.

In the general case the order parameter $\Delta(\mathbf{r},\mathbf{R})$ is described by two coordinates, $\mathbf{r}$ and $\mathbf{R}$. The coordinate $\mathbf{r}$ corresponds to the relative motion of the electron and hole in the exciton pair and is already conjugate to the employed momentum $\mathbf{k}$. The coordinate $\mathbf{R}$ describes the position of the mass center of the exciton, while the $\Delta(\mathbf{R})$ dependence means an inhomogeneous state of the electron-hole condensate.

An analysis of the expansion of the free-energy functional δF in terms of the components of the complex order parameter $\Delta^\alpha(\mathbf{k})$ shows that if the system has no inversion center then the expansion of δF can contain terms that are linear in the spatial derivative $\Delta^\alpha\partial\Delta^\beta/\partial\mathbf{R}$. The structure of these terms should have the following invariant form[5]:

$$\mathbf{d}\left(\Delta^\alpha\frac{\partial\Delta^\beta}{\partial\mathbf{R}}-\Delta^\beta\frac{\partial\Delta^\alpha}{\partial\mathbf{R}}\right), \tag{14}$$

where α and β are indices that denote the corresponding angular harmonic. The coefficients d can be determined from the microscopic theory of the excitonic dielectric. It turns out that the invariant (14) can be made up of the component pairs Δ^s_{Im}, Δ^a_{Re} and Δ^a_{Im}, Δ^s_{Re}.

The coefficient d is the same for each invariant and is given by

$$\mathbf{d}=\mathbf{n}_\Delta\frac{v_F^2}{6T}\int_0^\infty\frac{d\varepsilon}{\varepsilon}\left(\mathrm{ch}^{-2}\frac{\mu+\varepsilon}{2T}-\mathrm{ch}^{-2}\frac{\mu-\varepsilon}{2T}\right). \tag{15}$$

Here $\mathbf{n}_\Delta$ is a unit vector that defines the axis of the antisymmetrical component $\Delta_a(\mathbf{k})$, the direction of which will be discussed below. In the general case the coefficient d is different from zero to the extent that the Fermi surfaces of the electrons and holes are not congruent, and only the simplest noncongruence that results from the degree of doping μ is incorporated in Eq. (15).

The presence of the linear invariants (14) in the expansion of the functional of the free energy raises the question of the feasibility of inhomogeneous states. For examples, invariants of this type are actively used to construct helicoidal structures in spin magnets. We write down part of the free-energy expansion for two components of the order parameter $\Delta(\mathbf{k})$, which are connected with the invariant (14). For the sake of argument we take the parameters Δ^s_{Im} and Δ^a_{Re} and assume them to be small, so that we retain only the linear and quadratic terms of the expansion:

$$\delta F=\int d\tau\left\{\alpha_s|\Delta_{\mathrm{Im}}{}^s(\mathbf{R})|^2+\gamma_1\left|\frac{\partial\Delta_{\mathrm{Im}}{}^s(\mathbf{R})}{\partial\mathbf{R}}\right|^2+\alpha_a|\Delta_{\mathrm{Re}}{}^a(\mathbf{R})|^2+\gamma_2\left|\frac{\partial\Delta_{\mathrm{Re}}{}^a(\mathbf{R})}{\partial\mathbf{R}}\right|^2+2\mathbf{d}\left[\Delta_{\mathrm{Im}}{}^s\frac{\partial\Delta_{\mathrm{Re}}{}^a}{\partial\mathbf{R}}-\Delta_{\mathrm{Re}}{}^a\frac{\partial\Delta_{\mathrm{Im}}{}^s}{\partial\mathbf{R}}\right]\right\}, \tag{16}$$

where

$$\alpha_{a,s}=(T-T_{c0}^{a,s})/T_{c0}^{a,s},\quad \gamma_i>0,$$

and the temperatures $T^{a;s}_{c0}$ are connected with the structure of the potential of the electron-hole interaction by the relation (7a).

In this form, the functional (16) makes it possible to determine the mutual influence of the fluctuations of the parameters Δ^s_{Im} and Δ^a_{Re} above the phase transition point. The minimum of the functional (16) corresponds to the system of equations

$$\begin{aligned}&\left(\alpha_s-\gamma_1\frac{\partial^2}{\partial\mathbf{R}^2}\right)\Delta_{\mathrm{Im}}{}^s(\mathbf{R})+\mathbf{d}\frac{\partial\Delta_{\mathrm{Re}}{}^a}{\partial\mathbf{R}}=0,\\&\left(\alpha_a-\gamma_2\frac{\partial^2}{\partial\mathbf{R}^2}\right)\Delta_{\mathrm{Re}}{}^a(\mathbf{R})-\mathbf{d}\frac{\partial\Delta_{\mathrm{Im}}{}^s}{\partial\mathbf{R}}=0.\end{aligned} \tag{17}$$

This system is linear in the parameters $\Delta^{a,s}$ and therefore does not make it possible to find the structure of the nucleating phase, but determines the temperature of the phase transition. We change over in (17) to the momentum representation, and then the phase transition temperature is determined from the condition that the determinant of the system

$$\det\equiv(\alpha_s+\gamma_1\mathbf{q}^2)(\alpha_a+\gamma_2\mathbf{q}^2)-(\mathbf{dq})^2 \tag{18}$$

vanish. If we put $\alpha_a=\alpha_s=\alpha$ and $\gamma_1=\gamma_2=\gamma$, then it is seen directly from (18) that with increase in temperature the phase transition will take place at the point $\alpha=(d^2/4\gamma)>0$, $\mathbf{q}_0=\mathbf{d}/2\gamma$. This means that above the temperature $T_{c0}(\alpha=0)$ for the homogeneous phase transition ($\mathbf{q}_0=0$) there appears an inhomogeneous state ($\mathbf{q}_0\neq 0$).

In the general case, the critical temperatures are different for the parameters Δ^s_{Im} and Δ^a_{Re}. Let for the sake of argument $\alpha_a>\alpha_s$ ($T^a_{c0}<T^s_{c0}$). The condition for the appearance of inhomogeneous can be regarded to be the condition when the temperature at which the determinant (18) vanishes is higher than the critical temperatures T^s_{c0} and T^a_{c0}, at which the onset of an inhomogeneous state is still possible, is determined from Eq. (18) at $\alpha_s=0$:

$$\alpha_0=\frac{T_{c0}{}^s-T_{c0}{}^a}{T_{c0}{}^a}\leqslant\frac{d^2(\mu)}{\gamma(\mu)}. \tag{19}$$

It should be noted that the difference between the temperatures T^s_{c0} and T^a_{c0} determines in accord with (7a) the degree of screening of the potential of the electron-electron interaction. An increase of α_0 corresponds to a larger localization of this interaction, $g_s\gg g_a$. The coefficients $d(\mu)$ and $\gamma(\mu)$ depend on the degree of doping of the sample, with $d(\mu)$ increasing with increasing μ, while the coefficient $\gamma(\mu)$, on the contrary, decreases with increasing μ.[6] Consequently, in accord with the inequality (19), the permissible difference $T^s_{c0}-T^a_{c0}$ [or the degree of screening of the potential $V(\mathbf{k}-\mathbf{k}')$ (6)] will increase with increase in the doping μ.

As seen from (15), the vector $\mathbf{q}_0$ will be parallel to the vector $\mathbf{n}_\Delta$, which determines the antisymmetric component of the parameter Δ^a_{Re}. In the isotropic model of a semimetal the direction of $\mathbf{n}_\Delta$ is not fixed. The direction of $\mathbf{n}_\Delta$ and hence of $\mathbf{q}_0$ becomes fixed when account is taken of the anisotropy of the electron spectrum, and in particular when account is taken of the hybridization $W(\mathbf{k})$. The anisotropy that stems from the hybridization (1) is characterized by the fact that in directions parallel to the vector $\mathbf{P}_{12}$ there is a single-particle gap in the spectrum of the electrons and holes. which decreases to zero in directions perpendicular to $\mathbf{P}_{12}$. This is precisely why the most effective electron-hole pairing is one in which the vector $\mathbf{n}_\Delta$ is perpendicular to $\mathbf{P}_{12}$, for in this direction the single-particle forbidden gap is minimal. The bare vector inhomogeneity $\mathbf{q}_0$, which is parallel to $\mathbf{n}_\Delta$, is also perpendicular to the vector $\mathbf{P}_{12}$. It is just in this case, as already noted,[7] that a spontaneous current is produced in a system with an inhomogeneous imaginary component $\Delta^s_{\mathrm{Im}}(\mathbf{q}_0)$ (at $\mathbf{q}_0\perp\mathbf{P}_{12}$).

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