Quantum magnetosize effects in metals

S. S. Nedorezov

Physicotechnical Institute of Low Temperatures, Ukrainian Academy of Sciences

V. G. Peschanskiĭ

A. M. Gor'kiĭ Kharkov State University

(Submitted 8 July 1980)
Zh. Eksp. Teor. Fiz. **80**, 368–379 (January 1981)

An analysis is carried out of the magnetic size-effect oscillations of the thermodynamic and kinetic characteristics of metallic films. It is shown that the density of electron states in a film in a parallel magnetic field **H** is very sensitive to the topology of the Fermi surface, and upon variation of **H** or of the film thickness L it either changes discontinuously or possesses logarithmic singularities. The latter are due to saddle points on the Fermi surface. In the case of open sections of the Fermi surface, strong modulation arises of the oscillations of the magnetization of the heat capacity, and of the magnetoresistance of the films.

PACS numbers: 75.70.Dp, 75.80. + q

INTRODUCTION

The study of quantum oscillations of thermodynamic and kinetic characteristics of conduction electrons in metallic films in a magnetic field is of significant interest in connection with the study of the electron energy spectrum of metals and the interaction of electrons with the boundaries of the sample. The magnetic size-effect oscillations predicted by Kosevich and Lifshitz[1] were observed in the investigation of the electrical conductivity of filamentary crystals (whiskers) of antimony,[2] and also in the measurement of the magnetization of the whiskers of a number of metals.[3] Quantum oscillations are possible in metallic films[4] in weak magnetic fields. Similar oscillations have been observed in the electrical conductivity of bismuth films.[5]

The effects noted above apply to metals with closed electron orbits in a magnetic field **H** under the assumption of specular reflection of the electrons by the boundaries of the sample. As was shown in Ref. 6, magnetic size-effect oscillations are possible that are due to electrons with open orbits. In the bulk metal, as is well known, electrons on open sections of the Fermi surface (FS) do not participate in the establishment of quantum oscillations. In a thin sample, whose thickness L is less than the free path length l of the carriers, the finiteness of the motion of the electrons along the normal to the boundary of the sample leads to quantization of the energy of the carriers belonging to the open sections of the FS. The contribution of these electrons to the thermodynamic and kinetic characteristics of the thin plates turns out to be quite substantial and causes the appearance of new oscillation effects, the investigation of which allows us to obtain detailed information on the shape of the open electron orbits.

In the present work, we have carried out a detailed analysis of the quantum magnetic size-effect oscillations of the thermodynamic and kinetic electron characteristics of metallic films in a parallel magnetic field. It is shown that the density of electron states ν either changes discontinuously with change in H or else has logarithmic singularities. As $H\to 0$, the noted singularities of ν are identical with the corresponding state-density singularities obtained by one of the authors[4] in the investigation of quantum oscillations in weak magnetic fields. The presence of logarithmic singularities of the density of electron states is connected with the presence of saddle points on the constant-energy surfaces. In the case of open electron orbits, the amplitude of the quantum oscillations of the density of states increases like $|H-H_j|^{-1/2}$ as $H\to H_j$, where

$$H_j = jc\hbar G/eL,\quad j=1,2,3,\ldots \tag{1}$$

is the intensity of the magnetic field at which degeneracy[6] of the electron energy levels relative to the quasi-momentum component p_x takes place. (The magnetic field **H** is directed along the z axis, the normal to the film is directed along the y axis, and G is the period of the reciprocal lattice in the direction of openness of the electron orbits.)

The mentioned properties of the density of electron states arise in the magnetic size-effect oscillations of the thermodynamic and kinetic quantities. The corresponding formulas were obtained for the quantum oscillations of the thermodynamic potential Ω, the heat capacity, the magnetic susceptibility, and the transverse magnetoresistance.

DENSITY OF ELECTRON STATES

Under the assumption of specular reflection of the carriers from the boundary of the sample, the quantum energy levels ε_n of the conduction electrons in a plate in a parallel magnetic field are determined by the quasiclassical quantization condition:[1]

$$S\left(\varepsilon, p_z, p_x; \frac{LeH}{c}\right) = \frac{2\pi\hbar eH}{c}(n+\gamma),\quad n=0,1,2,\ldots, \tag{2}$$

where S is the area of the intersection of the constant energy-surface $\mathscr{E}(\mathbf{p})=\varepsilon$ with the plane $p_z=\text{const}$. This intersection is bounded by the straight lines p_x and $p_x + LeH/c$ (Fig. 1). The quantity γ in formula (2) is less than or equal to unity, the latter corresponding to approximation of the surface potential by an infinitely high potential barrier. In the case of an arbitrary surface potential, γ can be expressed in terms of the change in phase of the wave function of the electron upon reflection from the boundary of the sample.[7] In the present work, we shall set $\gamma=1$ for simplicity. Here e is the

 0038-5646/81/010188-06$02.40

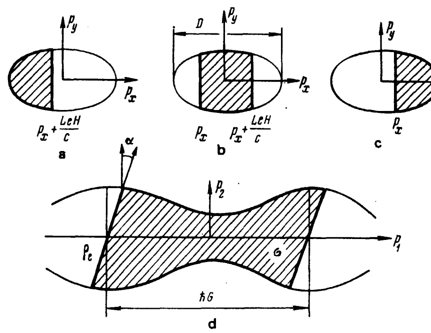

FIG. 1. Trajectories in momentum space (heavy lines) of electrons colliding with film walls in the case of closed (a, b, c) and open (d) sections of the Fermi surface. The magnetic field is parallel to the film surface and in the case (d) it is equal to $H_i = c\hbar G/eL$. At $H \gg H_1$, the electron trajectory spans many cells of momentum space. Case (b) is possible only at $H < H_c = cD/eL$.

charge of the electron, $\hbar$ is Planck's constant, and c is the speed of light.

In the case of open electron orbits (see Fig. 1d), the area S in (2) is a periodic function of p_x with period $\hbar G$. At values of the magnetic field $H = H_j$ [see (1)] the area S is independent of p_x (Ref. 6) and is equal to

$$S(\varepsilon, p_z, p_x; LeH_j/c) = j\sigma(\varepsilon, p_z), \tag{3}$$

where σ is the area of the open section of the constant-energy surface within the limits of the reciprocal lattice cell. It then follows from (2) that the quantized energy levels, in the principal approximation in the quasiclassical small parameter $(1/n \ll 1)$, do not depend on the quasimomentum component p_x.[1)] In fields $H \neq H_j$, the degeneracy of the levels is removed, which leads to characteristic singularities in the density of states of electrons with open orbits.

The density of the electron states $\nu(\varepsilon, H)$ is determined by the expression:

$$\nu(\varepsilon, H) = \frac{1}{(2\pi\hbar)^2 L} \sum_{i=1}^{2} \sum_{n=0}^{\infty} \int \delta(\varepsilon + (-1)^i \mu H - \varepsilon_n(\mathbf{p}_\parallel, H))\, d^2\mathbf{p}_\parallel, \tag{4}$$

where $\mu = e\hbar/2m_0c$, and $\mathbf{p}_\parallel$ is the component of the quasimomentum in the plane of the plate. Summing (4) by Poisson's formula, and using the quantization condition (2), we obtain the following for the oscillating part ν_{osc} of the density of states ($\nu = \tilde{\nu} + \nu_{osc}$, where $\tilde{\nu}$ is some smooth function of ε and H):

$$\nu_{osc}(\varepsilon, H) = \frac{2c}{(2\pi\hbar)^3 LeH} \sum_{i=1}^{2} \sum_{k=1}^{\infty} \int \frac{\partial S}{\partial \varepsilon} \cdot \times \cos\left[\frac{kc}{\hbar eH} S\left(\varepsilon + (-1)^i \mu H, \mathbf{p}_\parallel; \frac{LeH}{c}\right)\right] d^2\mathbf{p}_\parallel. \tag{5}$$

Hence, calculating the integral and summing over k in (5), we have

$$\nu_{osc}(\varepsilon, H) = \frac{m^*}{\hbar^2 L} J^{-1/2} \sum_{i=1}^{2} \left(\frac{1}{2} - \left\{\frac{cS_e}{2\pi\hbar eH} + (-1)^i \frac{m^*}{m_0}\right\}\right), \quad J > 0,$$

$$\nu_{osc}(\varepsilon, H) = -\frac{m^*}{\pi\hbar^2 L} |J|^{-1/2} \sum_{i=1}^{2} \ln\left|2 \sin\left(\frac{cS_e}{2\hbar eH} + (-1)^i \pi \frac{m^*}{m_0}\right)\right|, \quad J < 0, \tag{6}$$

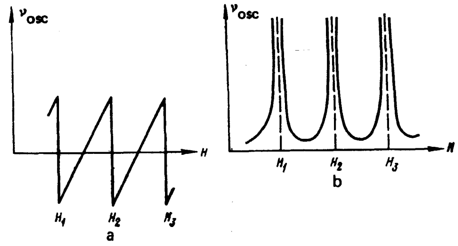

FIG. 2. Oscillations of the density of electron states with the magnetic field: a) $J > 0$, b) $J < 0$.

where

$$m^* = \frac{1}{2\pi} \frac{\partial S_e}{\partial \varepsilon}, \quad S_e \equiv S\left(\varepsilon, p_{ze}, p_{xe}; \frac{LeH}{c}\right)$$

is the extremal value of the area S with respect to the variables p_x and p_z $(\partial S/\partial p_{xe} = \partial S/\partial p_{ze})$, $\{x\}$ is the fractional part of x,

$$J = \frac{\partial^2 S}{\partial p_{xe}^2} \frac{\partial^2 S}{\partial p_{ze}^2} - \left(\frac{\partial^2 S}{\partial p_{xe} \partial p_{ze}}\right)^2. \tag{7}$$

With change in ε or H, when the energy ε is identical with the quantum level $\varepsilon_n(\mathbf{p}_\parallel, H) \pm \mu H$, the density of states $\nu(\varepsilon, H)$ changes either discontinuously ($J > 0$) or has a logarithmic singularity ($J < 0$), depending on the sign of J (Fig. 2). As $H \to 0$ the formulas (6) transform into the corresponding expressions obtained for ν_{osc} in Ref. 4, where an analysis is given constant-energy surface topology that gives rise to the presence of the logarithmic singularities in the density of states.

The density of electron states in a plate in a parallel magnetic field is determined by formulas (6) both in the case of closed and the case of open sections of constant-energy surfaces. In the latter case, near the degeneracy of the quantum levels as $H \to H_j$, we get from (6)

$$\nu_{osc}(\varepsilon, H) \cong \mp \frac{(j \cos\alpha)^{1/2}}{(\pi\hbar)^2 L^{1/2}} \left(\frac{c}{e}\right)^{1/2} \frac{\partial \sigma_e}{\partial \varepsilon} \left|\frac{\partial^2 \sigma}{\partial p_{ze}^2} \sum_{i=1}^{2} \frac{1}{R_i}\right|^{-1/2} |H - H_j|^{-1/2} \cdot \times \sum_{k=1}^{\infty} \frac{1}{k} \cos\left(\frac{kj}{2m_0} \frac{\partial \sigma_e}{\partial \varepsilon}\right) \sin\left[k\left(\frac{L\sigma_e}{\hbar^2 G} + \frac{eL^2}{jc\hbar^2}\left(\rho_e - \frac{\sigma_e}{\hbar G}\right)(H - H_j)\right)\right], \quad H \to H_j, \tag{8}$$

where the upper sign is chosen in the case $\partial^2\sigma/\partial p_{ze}^2 > 0$ and the lower in the case $\partial^2\sigma/\partial p_{ze}^2 < 0$; ρ_e is the extremal chord of the constant-energy surface $\mathscr{E}(\mathbf{p}) = \varepsilon$ in the direction of the normal $\mathbf{N}$ to the plate, α is the angle between $\mathbf{N}$ and the normal to the constant-energy surface at the points of its intersection with the extremal chord, and R_i are the radii of curvature at these points (see Fig. 1d), $\sigma_e \equiv \sigma(\varepsilon, p_{ze})$.

Formula (8) is valid in fields

$$a/L \ll |H - H_j|/H_1 \ll 1, \tag{9}$$

where a is the lattice constant. At $H = H_j$, we have the following expression for ν_{osc}:

$$\nu_{osc}(\varepsilon, H_j) \equiv \nu_j(\varepsilon) = \frac{(G \cos\alpha)^{1/2}}{2^{1/2} \pi^{5/2} \hbar^2 L^{1/2}} \frac{\partial \sigma_e}{\partial \varepsilon} \left|\frac{\partial^2 \sigma}{\partial p_{ze}^2}\right|^{-1/2} \cdot \times \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} \cos\left(\frac{kj}{2m_0} \frac{\partial \sigma_e}{\partial \varepsilon}\right) \cos\left(k \frac{L\sigma_e}{\hbar^2 G} \pm \frac{\pi}{4}\right). \tag{10}$$

The amplitude of the quantum oscillations of the density of states reaches its maximum value in fields $H=H_j$, and significantly exceeds the amplitude of the oscillations at intermediate field values $H\neq H_j$:

$$|\nu_{osc}/\nu_j|\sim(a/L)^{1/2}\ll 1. \qquad (11)$$

QUANTUM OSCILLATIONS OF THERMODYNAMIC QUANTITIES

As a result of standard calculations (see, for example, Refs. 1 and 4) we obtain the following for the oscillating part Ω_{osc} of the thermodynamic potential Ω:

$$\Omega_{osc}=\frac{V}{\pi^2 L}\left(\frac{eH}{c}\right)^2\left(\frac{\partial S_e}{\partial\zeta}\right)^{-1}|J|^{-1/2}\sum_{k=1}^{\infty}\frac{1}{k^3}\Psi\left(\frac{\pi kcT}{\hbar eH}\frac{\partial S_e}{\partial\zeta}\right)$$
$$\times\cos\left(\frac{k}{2m_0}\frac{\partial S_e}{\partial\zeta}\right)\cos\left(\frac{kcS_e}{\hbar eH}\pm\frac{\pi}{4}\pm\frac{\pi}{4}\right). \qquad (12)$$

Here T is the temperature, V is the volume of the sample, ζ is the chemical potential, and $\Psi(x)=x/\sinh x$. In the argument of the cosine, identical signs were chosen in the case $J>0$; the sign is plus in the case $\partial^2 S/\partial p_{xe}^2>0$ and minus in the case $\partial^2 S/\partial p_{xe}^2<0$; the signs are different in the case $J<0$.

Differentiating Ω_{osc} with respect to the field, we obtain the oscillating part of the magnetic moment $M_{osc}=-\partial\Omega_{osc}/\partial H$. We consider in more detail the manifestations of the singularities in the density of states (see the previous section) in quantum oscillations of the magnetic susceptibility $\chi_{osc}=-\partial^2\Omega_{osc}/\partial H^2$:

$$\chi_{osc}=\frac{V}{(\pi\hbar)^2 L}\left(\frac{\partial S_e}{\partial\zeta}\right)^{-1}|J|^{-1/2}\left(\frac{\partial S_e}{\partial H}-\frac{S_e}{H}\right)^2\sum_{k=1}^{\infty}\frac{1}{k}\Psi\left(2\pi^2 k\frac{T}{\Delta\varepsilon}\right)$$
$$\times\cos\left(\frac{k}{2m_0}\frac{\partial S_e}{\partial\zeta}\right)\cos\left(\frac{kcS_e}{\hbar eH}\pm\frac{\pi}{4}\pm\frac{\pi}{4}\right), \qquad (13)$$

and of the heat capacity $C_{osc}=-T\partial^2\Omega_{osc}/\partial T^2$:

$$C_{osc}=-\frac{VT}{\hbar^2 L}\frac{\partial S_e}{\partial\zeta}|J|^{-1/2}\sum_{k=1}^{\infty}\frac{1}{k}\Psi''\left(2\pi^2 k\frac{T}{\Delta\varepsilon}\right)$$
$$\times\cos\left(\frac{k}{2m_0}\frac{\partial S_e}{\partial\zeta}\right)\cos\left(\frac{kcS_e}{\hbar eH}\pm\frac{\pi}{4}\pm\frac{\pi}{4}\right), \qquad (14)$$

where $\Delta\varepsilon=2\pi\hbar eH/(c\partial S_e/\partial\zeta)$ is the distance between the quantum energy levels.

In the region of low temperatures, $T\ll\Delta\varepsilon$, we get from (13) at $J<0$:

$$\chi_{osc}=\begin{cases}-\dfrac{V}{2\pi^2\hbar^2 L}\left(\dfrac{\partial S_e}{\partial\zeta}\right)^{-1}|J|^{-1/2}\left(\dfrac{\partial S_e}{\partial H}-\dfrac{S_e}{H}\right)^2\\ \times\displaystyle\sum_{i=1}^{2}\ln\left|2\sin\left(\frac{cS_e}{2\hbar eH}+\frac{(-1)^i}{4m_0}\frac{\partial S_e}{\partial\zeta}\right)\right|, & H\neq H_{ni},\\ -\dfrac{V}{2\pi^2\hbar^2 L}\left(\dfrac{\partial S_e}{\partial\zeta}\right)^{-1}|J|^{-1/2}\left(\dfrac{\partial S_e}{\partial H}-\dfrac{S_e}{H}\right)^2\ln\left(\dfrac{T}{\Delta\varepsilon}\right), & H=H_{ni},\end{cases} \qquad (15)$$

where the values of the field H_{ni} are determined by the condition

$$\frac{c}{\hbar eH_{ni}}S_e\left(\zeta,p_{\parallel e};\frac{LeH_{ni}}{c}\right)+\frac{(-1)^i}{2m_0}\frac{\partial S_e}{\partial\zeta}=2\pi n,$$
$$n=1,2,3,\ldots;\quad i=1,2. \qquad (15')$$

Summing over k in formula (13) at $T=0$, and comparing the result obtained with (6), we can express the oscillations χ_{osc} directly in terms of the oscillations $\nu_{osc}(\zeta,H)$ of the density of states at any sign of J:

$$\chi_{osc}(H)=V\left(\frac{\partial S_e}{\partial\zeta}\right)^{-2}\left(\frac{\partial S_e}{\partial H}-\frac{S_e}{H}\right)^2\nu_{osc}(\zeta,H). \qquad (16)$$

At the points H_{ni}, the magnetic susceptibility either changes discontinuously or has logarithmic singularities, depending on the sign of J. There are similar singularities in the heat capacity C_{osc}/T as $T\to 0$:

$$C_{osc}(H,T)=V\frac{\pi}{3}\nu_{osc}(\zeta,H)T. \qquad (17)$$

At finite but sufficiently low temperatures, $T\ll\Delta$, we have from (14) at $H=H_{ni}$ and $J<0$,

$$C_{osc}=-V\frac{T}{6\pi\hbar^2 L}\frac{\partial S_e}{\partial\zeta}|J|^{-1/2}\ln\left(\frac{T}{\Delta\varepsilon}\right), \qquad (17')$$

while at $H\neq H_{ni}$ the oscillations of the heat capacity are determined by formulas (17) and (16) [similar to the expressions (15) for oscillations of the magnetic susceptibility].

In the region of weak magnetic fields

$$H\ll H_c, \qquad (18)$$

where $H_c=cD/eL$ is the field at which the extremal orbit is cut off by the dimensions of the plate (D is the size of the extremal cross section of the FS along p_x), the obtained formulas (13)–(17) transform into the corresponding expressions given in Ref. 4. The singularities in the oscillations of the thermodynamic quantities, pointed out above, occur in fields $H<H_c$. At $H>H_c$, in the case of closed sections of the FS, the effect of the boundaries of the plate can be neglected and the quantum oscillations of the thermodynamic characteristics of the conduction electrons are described by the theory of Ref. 8. The transition field region $H\sim H_c$ was investigated in Ref. 9. In the case of diffuse scattering of the electrons by the boundaries of the sample, the oscillations that correspond to the specified extremal orbit drop out[10] and in fields $H<H_c$ quantum oscillations take place on nonextremal sections of the FS.[11]

We have a quite different situation in the case of open sections of the FS, when the existence of quantum oscillations is determined entirely by the effect of the boundary of the plate.[6] As follows from formulas (8), (10), (15) and (16), the amplitudes of the oscillations χ_{osc} and C_{osc} near the degeneracy of the quantum energy levels increases in the case $H\to H_j$ like $|H-H_j|^{-1/2}$, reaching maximal values at $H=H_j$. The period of the oscillations near H_j is equal to

$$\Delta H=2\pi j\frac{c\hbar^2 G}{eL^2}\left|\rho_e-\frac{\sigma}{\hbar G}\right|^{-1}. \qquad (19)$$

In intermediate fields $H\neq H_j$, the amplitude of the oscillations falls off and is of the order of $(a/L)^{1/2}$ relative to the maximum amplitude. The period of the oscillations in this region of fields has a complicated field dependence

$$\Delta H=\frac{2\pi\hbar eH}{c}\left|\frac{\partial S_e}{\partial H}-\frac{S_e}{H}\right|^{-1}, \qquad (20)$$

which is determined by the extremal area of the FS section.[1]

QUANTUM OSCILLATIONS OF THE TRANSVERSE MAGNETORESISTANCE

The quantum oscillations of the kinetic characteristics are due to the singularities of the density of states, and also to the singularities of the scattering amplitude of the carriers. The latter manifests itself only in very pure metals and at low temperatures. Thus, in a bulk sample, the singularities of the amplitude of electron scattering from impurities in a magnetic field have a significant effect on the oscillatory dependence of the magnetoresistance only under the conditions:[12,13]

$$\max\{T, T_D\} \ll (\hbar\omega_H)^2/\zeta, \tag{21}$$

where T_D is the Dingle temperature and ω_H is the cyclotron frequency.

In metallic films in a parallel magnetic field, the singularities of the scattering amplitude turn out to be much weaker than in the bulk metal, and the region where account of them is significant is much narrower, to wit,

$$\max\{T, T_D\} < \zeta \exp(-\zeta/\hbar\omega_H). \tag{22}$$

Therefore, in a very wide range of temperatures and magnetic fields, the oscillations of the kinetic characteristics are connected with the singularities of the density of states of the conduction electrons and are proportional to $\partial M_{osc}/\partial H$ (see Ref. 14). We limit ourselves below to the case and analyze the quantum oscillations of the transverse magnetoresistance, i.e., we shall assume the magnetic field H to be located in the plane of the plate orthogonal to the electric current density j. In this case, in strong magnetic fields, when the radius of the classical trajectory of the electron r_H is much less than the free path length l, we can use the Kubo method[15] for calculation of the electrical conductivity and take into account the scattering amplitude of the charge carriers in the Born approximation.[2)]

The symmetric part of the electrical conductivity tensor can easily be expressed in terms of the shift of the center of the electron orbit, brought about by the action of the film potential on the electron and to scattering from impurities. The film potential has no effect on the drift of the carriers along the normal to the surface of the film (the y axis). Therefore, the shift of the center of the orbit in this direction, brought about by the action of the scattering potential of the impurities, and determining the quantity σ_{yy}, can be calculated by analogy with the bulk samples.[16] The drift of the carriers along the surface of the film is essentially due to the action of the film potential. Therefore, σ_{xx} depends essentially on the form of the film potential. Under conditions of the static skin effect,[17] the contribution to σ_{xx} from electrons glancing along the surface film is decisive. The calculation of σ_{xx} is obviously more convenient to carry out by using the quantum kinetic equation.[14,18] However, the oscillating part of the resistance is determined in most cases, as will be seen below, by the oscillations of the component σ_{yy}, which we calculate within the framework of the Kubo theory,[15,16] taking into account the specifics of the electron states in the film.

As a result, we get for σ_{yy}

$$\sigma_{yy} = -\frac{\pi\hbar c^2}{H^2}\sum_{s=1}^{2}\sum_{n,n'=1}^{\infty}\iint f_F'(\varepsilon_n(\mathbf{p}_\parallel, H) + (-1)^s\mu H)\cdot \times\delta(\varepsilon_n(\mathbf{p}_\parallel, H) - \varepsilon_{n'}(\mathbf{p}_\parallel', H))\left|\left\langle n\mathbf{p}_\parallel\left|\frac{\partial U}{\partial x}\right|n'\mathbf{p}_\parallel'\right\rangle\right|^2 d^2\mathbf{p}_\parallel d^2\mathbf{p}_\parallel', \tag{23}$$

where $f_F(\varepsilon)$ is the fermi function,

$$U(\mathbf{r}) = \sum_j u(\mathbf{r} - \mathbf{R}_j), \tag{23'}$$

$\mathbf{R}_j$ is the radius vector of the scattering center and u is the potential energy of the electron in the field of such a center. It is assumed that $u(r)$ is independent of the electron spin.

As the basis functions in (23), we use the eigenfunctions of the electron in a plate in a parallel magnetic field:

$$\psi_{np_\parallel}(\mathbf{r}) = \frac{1}{2\pi\hbar}\exp\left(\frac{i\mathbf{p}_\parallel\mathbf{r}_\parallel}{\hbar}\right)\left\langle y - \frac{cp_x}{eH}\right|n, p_z\right\rangle, \tag{24}$$

where the function $\langle y - cp_x/eH|n, p_z\rangle$ satisfies the equation

$$\mathscr{E}_{\rm sym}\left(p_x - \frac{eHy}{c}, p_y, p_z\right)\left\langle y - \frac{cp_x}{eH}\right| n, p_z\right\rangle = \varepsilon_n(\mathbf{p}_\parallel, H)\left\langle y - \frac{cp_x}{eH}\right| n, p_z\right\rangle \tag{25}$$

with the zero boundary conditions

$$\left\langle -\frac{cp_x}{eH}\right| n, p_z\right\rangle = \left\langle L - \frac{cp_x}{eH}\right| n, p_z\right\rangle = 0. \tag{25'}$$

Here $\mathscr{E}_{\rm sym}$ is the function $\mathscr{E}(p_x, p_y, p_z)$, completely symmetrized with respect to p_x and p_y. This function determines the dispersion law of the considered conduction electrons, $\hat{p}_y = -i\hbar\partial/\partial y$.

The effects considered are important at low temperatures, so that one must take into account only the scattering of the electrons by the impurities. Averaging over the impurities, and calculating the sums in (23) according to the Poisson formula, we get the following for the oscillating part of the conductivity:

$$\sigma_{yy}^{\rm osc} = -\sum_{s=1}^{2}\sum_{k=1}^{\infty}\int d\varepsilon\, f_F'(\varepsilon + (-1)^s\mu H)\int d^2\mathbf{p}_\parallel W(\varepsilon, \mathbf{p}_\parallel; H) \times\cos\left[\frac{kc}{\hbar eH}S\left(\varepsilon, \mathbf{p}_\parallel; \frac{LeH}{c}\right)\right], \tag{26}$$

$$W(\varepsilon, \mathbf{p}_\parallel; H) = \frac{\varkappa c}{2\pi^2\hbar LH^2}\frac{\partial n(\varepsilon, \mathbf{p}_\parallel)}{\partial\varepsilon}\int d^2\mathbf{q}\, q_x^2|u(\mathbf{q})|^2\frac{\partial n(\varepsilon, \mathbf{p}_\parallel - \hbar\mathbf{q}_\parallel)}{\partial\varepsilon}\cdot \times\left|\int_{-cp_x/eH}^{L-cp_x/eH} dy\langle n(\varepsilon, \mathbf{p}_\parallel), p_z|y\rangle\left\langle y + \frac{c\hbar q_x}{eH}\right| n(\varepsilon, \mathbf{p}_\parallel - \hbar\mathbf{q}_\parallel), p_z - \hbar q_z\right\rangle\right|^2, \tag{27}$$

where $\varkappa$ is the impurity density, $u(\mathbf{q})$ is the coefficient of expansion of the potential energy $u(\mathbf{r})$ in a Fourier integral,

$$n(\varepsilon, \mathbf{p}_\parallel) = \frac{c}{2\pi\hbar eH}S\left(\varepsilon, \mathbf{p}_\parallel; \frac{LeH}{c}\right). \tag{28}$$

Calculation of the integrals in (26) leads to the following expression for the conductivity oscillations:

$$\sigma_{yy}^{\rm osc} = 4\pi\frac{\hbar eH}{c}W(\zeta, \mathbf{p}_{\parallel e}; H)|J|^{-1/2}\sum_{k=1}^{\infty}\frac{1}{k}\Psi\left(\frac{\pi kcT}{\hbar eH}\frac{\partial S_e}{\partial\zeta}\right) \times\cos\left(\frac{k}{2m_0}\frac{\partial S_e}{\partial\zeta}\right)\cos\left[\frac{kc}{\hbar eH}S\left(\zeta, \mathbf{p}_{\parallel e}; \frac{LeH}{c}\right) \pm \frac{\pi}{4} \pm \frac{\pi}{4}\right]. \tag{29}$$

Comparing (29) with the formulas of the preceding section, we see that, as in the case of the bulk metal (see Ref. 14), the quantum oscillations of the conductivity in the plate are proportional to the quantum oscillations of the differential magnetic susceptibility χ_{osc}. At low temperatures, $T \ll \Delta\varepsilon$, the oscillations of σ_{yy}^{osc} are expressed in terms of the quantum oscillations $\nu_{osc}(\zeta, H)$ of the density of electron states:

$$\sigma_{yy}^{osc} = (2\pi\hbar)^3 \frac{LeH}{c} \left(\frac{\partial S_e}{\partial \zeta}\right)^{-1} W(\zeta, p_{\parallel e} H) \nu_{osc}(\zeta, H). \qquad (30)$$

It follows from (6) and (30) that the conductivity σ_{yy}^{osc} either changes discontinuously upon variation of the magnetic field at the points H_{ni} when $J > 0$, or has a logarithmic singularities ($J < 0$). In the latter case $|\sigma_{yy}^{osc}|$ reaches maximal values at $H = H_{ni}$:

$$\sigma_{yy}^{osc} = -2\pi \frac{\hbar e H_{ni}}{c} W(\zeta, p_{\parallel e}; H_{ni}) |J|^{-1/2} \ln\left(\frac{T}{\Delta\varepsilon}\right), \qquad (31)$$

where the distance between the quantum levels[3)]

$$\Delta\varepsilon = 2\pi\hbar e H_{ni}/c \frac{\partial S_e}{\partial \zeta}.$$

The indicated conductivity singularities take place in fields that are less than the cutoff field H_c. In fields $H > H_c$, the effect of the boundary of the plate (in the case of closed sections of the FS) disappears and the oscillations are identical with the corresponding oscillations of the conductivity of the bulk sample.

In the case of open sections of the FS, the quantum oscillations of the conductivity, which are determined by the formulas (29) and (30), increase essentially as $|H - H_j|^{-1/2}$ in the case $H \to H_j$; this follows directly from the expression (8) for the density of the electron states. The maximum values (apart from the sign) of the conductivity σ_{yy}^{osc} occur at the points $H = H_j$:

$$\sigma_{yy}^{osc} = (2\pi)^{1/2} \hbar \frac{G^{3/2}}{L^{1/2}} \overline{W}(\zeta, p_{ze}; H_j) |\partial^2\sigma/\partial p_{ze}^2|^{-1/2} (\cos\alpha)^{1/2}$$

$$\times \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} \Psi\left(\frac{\pi kTL}{\hbar^2 G} \frac{\partial\sigma_e}{\partial\zeta}\right) \cos\left(\frac{kj}{2m_0} \frac{\partial\sigma_e}{\partial\zeta}\right) \cos\left(\frac{kL\sigma_e}{\hbar^2 G} \pm \frac{\pi}{4}\right), \qquad (32)$$

$$\overline{W}(\zeta, p_z; H) = \frac{1}{\hbar G} \int_0^{\hbar G} W(\zeta, p_z, p_x; H)\, dp_x. \qquad (33)$$

The period of the oscillations ΔH near H_j is determined by the formula (19). In intermediate fields $H \neq H_j$, the amplitude of the oscillations falls off to values of the order of $(a/L)^{1/2}$ relative to the maximum value of the

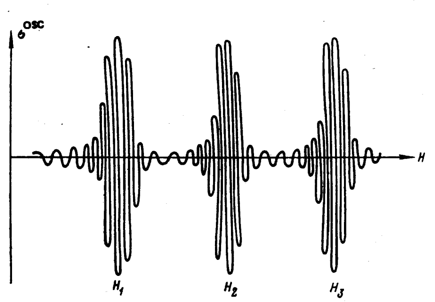

FIG. 3. Modulation of quantum oscilations, due to the electrons on open sections of the Fermi surface, of the transverse electric conductivity of metallic films.

conductivity at the points H_j (Fig. 3). The period of the oscillations ΔH has a complicated dependence on H and L, determined by formula (20).

The difference of the potentials is usually measured at a specified value of the electric current flowing through the current contacts, and it is necessary for us to find the matrix that is inverse to the matrix of the electrical conductivity σ_{ik}. If the current contacts are located at the ends of the plate and the distance between them appreciably exceeds l, then the electric field along the x axis can be assumed to be uniform with sufficient accuracy, while the inhomogeneous field $E_y(y)$ must be found from the condition of electrical neutrality of the metal.[17] Specular reflection of the carriers ensures that the current cannot flow through the surface of the plate, while the law of charge conservation div $\mathbf{j} = 0$ ensures the vanishing of j_y at any depth y in the conductor.

It is not difficult to verify that the inhomogenity of the field E_y is most appreciable at distances of the order r_H at the surface of the film; however, its account does not lead to a qualitative change in the quantum oscillations of the resistance. Therefore, in the calculation of the transverse resistivity $\rho = \sigma_\perp^{-1}$, we limit ourselves to the approximation of a homogeneous electric field.

In strong magnetic fields, when $r_H \ll L$, the asymptotic behavior of the electrical conductivity of the plate depends essentially on the topology of the FS. If the FS has open sections at given orientation of the magnetic field and the electrons belonging to these sections drift into the interior of the metal at some angle α to the normal, then the expression for $\sigma_\perp$ has the following form:

$$\sigma_\perp = \sigma_{xx} - \frac{\sigma_{xy}\sigma_{yx}}{\sigma_{yy}} \approx \sigma_0 \frac{r_H}{L} + \sigma_{xx}^{osc} + \sigma_{yy}^{osc}\left(\delta\, \mathrm{tg}^2\alpha - \frac{r_H^2 l^2}{L^4}\right)$$
$$+ \frac{r_H l}{L^2}(\sigma_{xy}^{osc} - \sigma_{yx}^{osc}) - (\sigma_{xy}^{osc} + \sigma_{yx}^{osc})\,\delta\, \mathrm{tg}\,\alpha, \qquad (34)$$

where σ_0 is the electrical conductivity of the bulk sample at $H = 0$; δ is the relative fraction of electron belonging to the open sections of the FS. Numerical factors of the order of unity are omitted in the formula (34). Also omitted are small quantum corrections, which vary smoothly with the magnetic field, such that $\sigma_\perp^{sm}$ is identical with the classical expression.[19] Even at small δ, but large r_H^2/lL, the basic contribution to the electrical conductivity of the plate is made by electrons belonging to closed sections of the FS which, reflected specularly from the surface of the plate, carry out unbounded motion along the direction of the electric current [the first term in formula (34)], while the interior of the plate hardly participate in the charge transport (the static skin effect).

It is easy to note that in a sufficiently broad range of magnetic fields satisfying the condition $r_H > L^2/l$ the resistance ρ_{xx}^{osc} is determined by the quantity σ_{yy}^{osc}, i.e.,

$$\frac{\rho_{xx}^{osc}}{\rho^{sm}} = \frac{\tilde{\sigma}_{yy}^{osc}}{\sigma_0} \frac{r_H l^2}{\delta^2 L^3}, \quad \delta \gg \frac{r_H^2}{lL}. \qquad (35)$$

Here $\tilde{\sigma}_{zz}^{osc}$ describes the contribution of the volume electrons, those not touching the boundaries of the sample, and the contribution of electrons on open orbits of the FS, which are described by formulas (30) and (32).

Formula (35) remains valid so long as the maximal diameter of the electron orbit $2r_H$ is less than the thickness of the plate. At $2r_H > L$, the distribution of the electric current over the cross section of the sample is practically uniform, but the specific conductivity of the plate agrees with σ_0 in order of magnitude. In this case σ_{yy}^{osc} is determined by electrons colliding with both surfaces of the plate, while the specifics of the open cross sections of the FS do not enter into the smooth and oscillating portions of the resistance.

1) The dependence of the energy levels on p_x is preserved in the next terms in the quasiclassical expansion of the energy in $1/n$; this is not significant for the effects considered below.

2) Following Refs, 12 and 13, we can express the formulas for σ_{osc} and ρ_{osc} obtained below in terms of the exact amplitude of electron scattering from point impurities, which would allow us to consider the range of fields and temperatures satisfying the condition (22).

3) According to what was said above, at too low temperatures (22), the formula (31) for σ_{yy}^{osc} is not applicable. In the case we have considered $|\sigma_{yy}^{osc}| \ll \tilde{\sigma}_{yy}$m, where σ_{yy} is the smooth part of the conductivity.

[1]A. M. Kosevich and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 29, 743 (1955) [Sov. Phys. JETP 2, 646 (1956)].

[2]Yu. P. Gaĭdukov and E. M. Golyamina, Zh. Eksp. Teor. Fiz. 74, 1936 (1978) [Sov. Phys. JETP 47, 1008 (1978)].

[3]V. M. Pudalov and S. G. Semenchinskiĭ, Pis'ma Zh. Tekh. Fiz. 4, 38 (1978) [Sov. Tech. Phys. Lett. 4, 15 (1978)].

[4]S. S. Nedorezov, Zh. Eksp. Teor. Fiz. 56, 299 (1969) [Sov. Phys. JETP 29, 164 (1969)].

[5]V. V. Andreiskiĭ and Yu. F. Komnik, Fiz. Tverd. Tela (Lenigrad) 12, 1582 (1970) [Sov. Phys. Solid State 12, 1254 (1979)].

[6]S. S. Nedorezov and V. G. Peschanskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. 31, 577 (1980) [JETP Lett. 31, 542 (1980)].

[7]S. S. Nedorezov, Fiz. Nizk. Temp. 6, 924 (1980) [Sov. J. Low Temp. Phys. 6, 449 (1980)].

[8]I. M. Lifshitz and A. M. Kosevich. Zh. Eksp. Teor. Fiz. 29, 730 (1955) [Sov. Phys. JETP 2, 636 (1956)].

[9]S. S. Nedorezov, Zh. Eksp. Teor. Fiz. 67, 1544 (1974) [Sov. Phys. JETP 40, 769 (1975)].

[10]M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 34, 754 (1958) [Sov. Phys. JETP 7, 518 (1958)].

[11]V. G. Peschansky, M. S. Awad-Alla, and V. V. Sinolitsky, Phys. Stat. Sol. 60, K37 (1973).

[12]V. G. Skobov, Zh. Eksp. Teor. Fiz. 38, 1204 (1960) [Sov. Phys. JETP 11, 1941 (1960)].

[13]Yu. A. Bychkov, Zh. Eksp. Teor. Fiz. 39, 689 (1960) [Sov. Phys. JETP 12, 483 (1961)].

[14]I. M. Lifshitz, M. Ya. Azbel' and M. I. Kaganov, Elektronnaya teoriya metallov (Electron Theory of Metals), Nauka, 1974.

[15]R. Kubo, J. Phys. Soc. Japan 12, 570 (1957).

[16]R. Kubo, H. Hasegawa, and N. Hashitsume, J. Phys. Soc. Japan 14, 56 (1959).

[17]M. Ya. Azbel', V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 49, 572 (1965); 52, 1003 (1967); 55, 1980 (1968) [Sov. Phys. JETP 22, 399 (1966); 25, 665 (1967); 28, 1045 (1969)].

[18]A. M. Kosevich and V. V. Andreev, Zh. Eksp. Teor. Fiz. 38, 882 (1960) [Sov. Phys. JETP 11, 637 (1960)].

[19]O. V. Kirichenko, V. G. Peschanskiĭ, and S. N. Savel'eva, Zh. Eksp. Teor. Fiz. 77, 2045 (1979) [Sov. Phys. JETP 50, 976 (1979)].

Translated by R. T. Beyer