where j_l is a spherical Bessel function.

Let $\varkappa_0 = (U_0)^{1/2}$ be the critical value at which the level with the given angular momentum appears. The condition for the appearance of the level is

$$j_{l-1}(\varkappa_0 r_0) = 0. \tag{8}$$

We set $U = U_0 + \lambda$ and assume $\lambda \ll U_0$. Then

$$\varkappa \approx \varkappa_0 + (\lambda - \alpha^2)/2\varkappa_0. \tag{9}$$

We form the expression

$$F = \left[\frac{1}{r^{l+1}R_l}\frac{d}{dr}(r^{l+1}R_l)\right]_{r=r_0}. \tag{10}$$

Taking into account (8) and (9) and using the well-known properties of the Bessel functions, we obtain in the first approximation

$$F = (\alpha^2 - \lambda) r_0/2. \tag{11}$$

In the region outside the well, $r \geq r_0$, the wave function is

$$R_l = Bh_l^{(1)}(i\alpha r), \tag{12}$$

where $h_l^{(1)}$ is a spherical Hankel function. The value of F when (12) is substituted in (10) under the condition $\alpha r_0 \ll 1$ is

$$F = -\alpha^2 r_0/(2l-1). \tag{13}$$

Equating (11) and (13), we find the connection between λ and α:

$$\lambda = \alpha^2(2l+1)/(2l-1). \tag{14}$$

Note that the expression (13) is valid only for $l \neq 0$. In the case $l = 0$ we have $F = -\alpha$, and therefore the connection between λ and α in the first approximation in αr_0 has the form $\alpha = \lambda r_0/2$. We now consider the scattering problem in which we are interested.

Within the well for $r \leq r_0$ the wave function has, as before, the form (7), but in the expression (9) we have k^2 instead of $-\alpha^2$. Accordingly, instead of (11) we obtain $F = -(k^2 + \lambda) r_0/2$. Substituting the expression (14) for λ, we find

$$F = -\frac{r_0}{2}\left[k^2 + \alpha^2\frac{2l+1}{2l-1}\right]. \tag{15}$$

Outside the well, the wave function can be represented in the form

$$R_l = B[j_l(kr)\operatorname{ctg}\delta_l - n_l(kr)] \tag{16}$$

(n_l is a spherical Neumann function).

Calculating F with the function (16) and using the condition $kr \ll 1$, we obtain

$$F = \frac{k^{2l+1} r_0^{2l}\operatorname{ctg}\delta_l}{[(2l-1)!!]^2} + \frac{k^2 r_0}{2l-1}. \tag{17}$$

Equating (15) and (17), we find

$$k^{2l+1}\operatorname{ctg}\delta_l = -\frac{2l+1}{2l-1}[(2l-1)!!]^2\frac{k^2+\alpha^2}{2r_0^{2l-1}}. \tag{18}$$

I am grateful to Yu. N. Demkov for discussions and valuable comments.

[1]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka (1974); English translation: Quantum Mechanics, Pergamon Press, Oxford (1974).
[2]Yu. N. Demkov and G. F. Drukarev, Zh. Eksp. Teor. Fiz. 49, 691 (1965) [Sov. Phys. JETP 22, 479 (1966)].

Translated by Julian B. Barbour

Motion of atoms and molecules in a resonant light field

A. N. Kazantsev, G. I. Surdutovich, and V. P. Yakovlev

Institute of Semiconductor Physics, Siberian Division, USSR Academy of Sciences
(Submitted 7 July 1980)
Zh. Eksp. Teor. Fiz. **80**, 541–550 (February 1981)

The effect of resonant light pressure on the motion of atoms and molecules is investigated. A kinetic equation is obtained that describes the action exerted on the particles by both the average light-pressure forces and by their fluctuations. The forces due to spontaneous and induced transitions are considered, as well as those due to some combination of both. The kinetic equation is used to describe atom scattering under conditions close to those in experiment [A. Arimonodo, H. Lew, and T. Oka, Phys. Rev. Lett. **43**, 753 (1979)]. Also obtained is the temperature of atoms cooled in a standing wave by a light-pressure force of mixed type.

PACS numbers: 41.70. + t, 51.10. + y

§1. INTRODUCTION

Resonant particles are acted upon in the field of laser emission by rather appreciable light-pressure forces.[1-3] These forces depend on the intensity of the external field, on the proximity of its frequency to resonance, and on its spatial structure. The light pressure on an individual particle is determined in final analysis by the rate of scattering of the external-field photons.

In a traveling light wave, only spontaneous transitions can contribute to the pressure. In non-uniform fields (e.g., in a standing wave), a stimulated light-pressure force (gradient force) is also produced. Inasmuch as the absorption and emission of external-field quanta are correlated processes in stimulated transitions, the

 0038-5646/81/020272-06$02.40

gradient force is a regular quantity. Spontaneous emission upsets the coherence of the interaction with the field and influences the motion of the atom in two ways. First, a certain average force is produced, which can cause acceleration or deceleration, depending on the detuning.[4] Second, the atoms begin to diffuse in velocity space. Depending on the field parameters and on the interaction time, under real conditions the particles can either drift or diffuse. To determine these motions, the kinetic equation must be used.

This paper is devoted to a derivation of a kinetic equation for the atoms and molecules in the quasiclassical limit. The motion of slow quasiclassical atoms is governed by an equation of the Fokker-Planck type (§3). We obtain the average force and the diffusion coefficient with allowance for the spontaneous and induced transitions. By way of example of the use of this equation, we consider the scattering of an atom beam (§ 4) under conditions close to those of the experiment of Oka *et al.*[5] The average scattering angle is calculated as a function of the detuning and of the field intensity. In §5 is obtained the exact distribution function of atoms cooled by a standing light wave. If it is approximated by a Boltzmann distribution, then the particle temperature can be determined as a function of the field parameters.

§2. INITIAL EQUATIONS

We consider a model of two-level atoms with frequency ω_0 and transition dipole moment d. The lower level 1 is the ground level, and the upper level 2 has a width γ. The resonant field

$$E(r)e^{-i(\omega_0+\Delta)t}+\text{c.c.}$$

has a small detuning Δ. The Hamiltonian of an atom of mass M can be written in the resonance approximation in the form

$$H(r)=-\frac{1}{2M}(\hbar\nabla)^2-\frac{1}{2}\sigma_3\hbar\Delta-d(\sigma_+E(r)+\sigma_-E^*(r)), \tag{1}$$

where $\sigma_{3,\pm}$ are spin matrices.

To take consistent account of the recoil in spontaneous and stimulated transitions, we must start from the quantum equation for the atom density matrix $\rho(r_1r_2t)$[6,7]:

$$\frac{\partial\rho(1,2)}{\partial t}+\hat{\gamma}\rho(1,2)=\frac{1}{i\hbar}[H(1)\rho(1,2)-\rho(1,2)H(2)]. \tag{2}$$

The relaxation operator $\hat{\gamma}$ is of the form

$$\hat{\gamma}\rho_{22}=\gamma\rho_{22},\quad \hat{\gamma}\rho_{21}=\gamma\rho_{21}/2,$$

$$\hat{\gamma}\rho_{11}=-\gamma\rho_{22}\int d\mathbf{n}\,e^{ik(r_1-r_2)}W(\mathbf{n}).$$

Here $W(\mathbf{n})$ is the probability of spontaneous photon emission in the direction $\mathbf{n}$, and $k=\omega_0/c$.

Under ordinary conditions the momenta of the atoms greatly exceed the momentum of the resonant photon

$$v_0/v\ll 1,\quad v_0=\hbar k/M. \tag{3}$$

In this case Eq. (2) can be expanded, in the Wigner representation, in the quasiclassical parameter v_0/v. We then have for the atom distribution function

$$f(rvt)=\rho_{22}(rvt)+\rho_{11}(rvt),$$

the population difference

$$q(rvt)=\rho_{22}(rvt)-\rho_{11}(rvt),$$

and the dipole moment $p(rvt)=\rho_{12}(rvt)$ (in units of d) the following system of equations[8]:

$$\frac{dp}{dt}+\nu p=-i\Omega(r)q-\frac{\hbar}{2M}(\nabla\Omega(r))\frac{\partial f}{\partial\mathbf{v}}, \tag{4}$$

$$dq/dt+\gamma(q+f)=-2i(\Omega^*(r)p-\text{c.c.}), \tag{5}$$

$$\frac{df}{dt}-\hat{\Gamma}(f+q)=-\frac{\hbar}{M}\left(\nabla\Omega^*\frac{\partial p}{\partial\mathbf{v}}+\text{c.c.}\right). \tag{6}$$

Here $d/dt=\partial/\partial t+\mathbf{v}\cdot\nabla$ is the total derivative with respect to time

$$\Omega(r)=dE(r)/\hbar,\quad \nu=-i\Delta+\gamma/2.$$

Equations (4) and (5) correspond to the known system of Bloch equations for two-level atoms. The difference lies only in the second term in the right-hand side of (4), which takes into account the recoil in stimulated transitions.

Equation (6) for the distribution function describes the change of the motion of the center of inertia of the atom both on account of the stimulated transitions (right-hand side) and on account of spontaneous ones. Equation (6) contains the spontaneous-relaxation operator for an atomic dipole linearly polarized along the z axis, namely

$$\hat{\Gamma}=\frac{1}{10}\gamma v_0^2\left(\frac{\partial^2}{\partial v_x^2}+\frac{\partial^2}{\partial v_y^2}+\frac{1}{2}\frac{\partial^2}{\partial v_z^2}\right).$$

Substituting the solution of Bloch's equations (4) and (5) in Eq. (6), we obtain the kinetic equation for the distribution function. For the case of slow atoms (§ 3) it takes the Fokker-Planck form.

§3. THE KINETIC EQUATION

A. Traveling wave

We consider first the simpler case of a traveling wave

$$E(r)=E_0e^{ikx},$$

which admits of a spatially homogeneous solution of the Bloch equations. In the quasistationary limit ($dq/dt \ll \gamma q$, $\partial p/\partial t\ll \nu p$) Eqs. (4) and (5) reduce to algebraic equations for p and q. As a result we obtain for f the kinetic equation

$$\frac{df}{dt}+\frac{\partial}{\partial v_x}\left[\left(\frac{F_s}{M}+5\frac{\partial D_0}{\partial v_x}\right)f\right]=\left(7\frac{\partial^2}{\partial v_x^2}+2\frac{\partial^2}{\partial v_y^2}+\frac{\partial^2}{\partial v_z^2}\right)D_0f, \tag{7}$$

where $F_s=\gamma\hbar k\omega$ is the spontaneous light pressure force, $D_0=\gamma v_0^2w/10$ is the reduced diffusion coefficient, and

$$w=\left|\frac{dE_0}{\hbar}\right|^2\left[(\Delta-kv_x)^2+\frac{\gamma^2}{4}+2\left|\frac{dE_0}{\hbar}\right|^2\right]^{-1}$$

is the probability of populating the upper level.

The difference between the transverse coefficients of diffusion along the y and z axes is connected with the orientation of the radiating dipole along the z axis. Since photon absorption from an external field via spontaneous emission is a random process, the ensuing fluctuations of the atom momentum increase the longitudinal diffusion coefficient. The diffusion coefficient

in Eq. (7) differs numerically from the corresponding expressions obtained for a traveling wave.[9,10]

We note that the average force contains a correction that is small in terms of the parameter (3) and is due to the velocity dependence of the diffusion coefficient.

B. Standing wave

We consider now the motion of atoms in the standing wave

$$E(r)=E_0\cos kx.$$

In this case the spatial inhomogeneity of the field plays an important role and makes the solution of the equations difficult. In many important applications, however, the atom motion is slow:

$$kv_x \ll |\nu|. \tag{8}$$

This allows us to expand the system (4) and (5) in the small gradients. In addition, the recoil effect can be taken into account in the Bloch equations by perturbation theory. We put $p=p_0+p_1$ and $q=q_0+q_1$, where $p_1 \ll p_0$ and $q_1 \ll q_0$.

In the zeroth approximation the atoms are assumed to be immobile and the recoil is disregarded. Then

$$p_0=-i\Omega(x)q_0/\nu, \quad q_0=-f/(1+\chi(x)), \tag{9}$$

where $\chi(x)=2\Omega^2(x)/|\nu|^2$ is the saturation parameter in the standing wave,

$$\Omega(x)=\Omega_0\cos kx; \quad \Omega_0=dE_0/\hbar.$$

For the first-order approximation we have the equations

$$\begin{aligned} \nu p_1+i\Omega(x)q_1 &= \frac{-\hbar}{2M}\frac{\partial\Omega(x)}{\partial x}\frac{\partial f}{\partial v_x}-v_x\frac{\partial p_0}{\partial x}, \\ \gamma q_1-4\Omega(x)\,\mathrm{Im}\,p_1 &= -v_x\partial q_0/\partial x. \end{aligned} \tag{10}$$

The zeroth approximation (9) yields the averaged induced atom dipole moment that depends on the local value of the field. This determines at the same time the gradient force acting on the particle. The first-order approximation determines the nonlocal part of the dipole moment, which is proportional to $\partial\Omega/\partial x$. This leads to the appearance of a nongradient force and to diffusion of the atoms in velocity. Substituting $\mathrm{Re}(p_0+p_1)$ in (4), we arrive at the following kinetic equation:

$$\frac{df}{dt}+\frac{\partial}{\partial v_x}\left[\frac{1}{M}\left(-\frac{\partial U}{\partial x}+Gv_x\right)f+Av_x\frac{\partial f}{\partial x}\right] = \left(\frac{\partial^2}{\partial v_x^2}D+\frac{\partial^2}{\partial v_y^2}D_0+\frac{\partial^2}{\partial v_z^2}D_0/2\right)f, \tag{11}$$

$$U(x)=\frac{\hbar\Delta}{2}\ln(1+\chi(x)), \tag{12}$$

$$G=\frac{2\hbar\Delta}{\gamma|\nu|^4}\frac{\gamma^2-\chi(\gamma^2+4\Omega^2)}{(1+\chi)^3}\left(\frac{\partial\Omega}{\partial x}\right)^2, \tag{13}$$

$$A=\frac{\hbar\Delta(\gamma^2+|\nu|^2+4\Omega^2)}{\gamma|\nu|^4M(1+\chi)^2}\frac{\partial\Omega^2}{\partial x}, \tag{14}$$

$$D=D_0+\frac{1}{2}\left(\frac{\hbar}{M}\right)^2\frac{\gamma^2+8\Omega^2}{\gamma|\nu|^2(1+\chi)}\left(\frac{\partial\Omega}{\partial x}\right)^2, \quad D_0=\frac{1}{10}\gamma v_0^2\frac{\chi}{1+\chi}. \tag{15}$$

The logarithmic potential (12) was obtained earlier for slow atoms by one of us.[4]

The terms proportional to G, A, and D, obtained by perturbation theory, are small compared with the gradient force $\partial U/\partial x$. They must be taken into account, however, for these terms, unlike the average gradient force, do not average out to zero over the period of the field. The influence of these terms has therefore a systematic time-cumulative character.

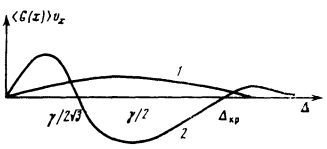

FIG. 1. Dependence of the averaged mixed-type force on the detuning. Curves 1 and 2 correspond to weak and strong fields, respectively.

Gv_x is the friction force. Depending on the detuning, the friction can be positive or negative. Just as any other dissipative force, friction cannot be represented by a gradient of a potential. From the physical point of view, a nongradient force is the result of the entanglement of spontaneous and stimulated transitions. It is natural to call it a force of mixed type.[3,4] Figure 1 shows the detuning-dependent spatially averaged mixed-type force $\langle G(x)\rangle v_x$ (the angle brackets denote averaging over the period of the field).

In weak field the force is decelerating at $\Delta<0$ and accelerating at $\Delta>0$ (curve 1). In a strong field, when $\Omega>\gamma$, G vanishes not only at zero detuning but also at finite detunings $\gamma/\sqrt{12}$ and Δ_{cr}, where $\Delta_{cr}=\Omega_0^2/\gamma$.

The diffusion coefficient D contains a contribution D_0 from the spontaneous transitions and a contribution proportional to the square of the field gradient from the stimulated transitions. In weak fields the contributions from the spontaneous and stimulated transitions differ only by a numerical factor

$$D=\frac{1}{2}\left(\frac{\hbar}{M}\right)^2\frac{\gamma}{|\nu|^2}\left[\left(\frac{\partial\Omega}{\partial x}\right)^2+\frac{2}{5}(k\Omega)^2\right], \quad \Omega\ll\gamma. \tag{16}$$

In a strong field, the stimulated diffusion coefficient is much larger than the spontaneous one

$$D=\gamma^{-1}(\hbar/M)^2\left(\frac{\partial\Omega}{\partial x}\right)^2\frac{2\chi}{1+\chi}, \quad \Omega\gg\gamma. \tag{17}$$

Thus, owing to spontaneous emission, the diffusion coefficient saturates rapidly with increasing field, while the diffusion coefficient governed by the gradient force is proportional to the field intensity (Fig. 2). The term with the mixed derivative, proportional to A, modifies somewhat the diffusion coefficient, as will be shown in § 5.

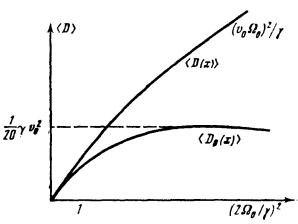

FIG. 2. Dependence of the averaged coefficient of diffusion in a standing wave on the field intensity for zero detuning. The lower curve corresponds to the spontaneous diffusion coefficient.

We shall use the kinetic equation to describe two concrete physical examples, atom-beam scattering and cooling by a standing wave.

§4. ATOM-BEAM SCATTERING BY A STANDING LIGHT WAVE

We consider scattering of a monokinetic beam of atoms moving with velocity v_y along the y axis by the field of a standing light wave directed along the x axis. There are no transverse velocities v_x in the incident beam. The problem is to find the particle distribution in the transverse velocities after passing through the field region. The case of short times τ of interaction with the field was previously studied by classical[11,12] and quantum[13,14] methods without allowance for spontaneous transitions. Scattering of atoms by a nonmonochromatic field of a standing wave at large τ was considered in Ref. 15.

It is of interest to investigate the case of long flight times, when the spontaneous emission becomes significant. Arimondo *et al.*[5] have recently observed scattering of a beam of Na atoms by the field of a monochromatic standing wave. The experimental conditions were such that $\gamma\tau \sim 10$, and the finite transverse velocities of the atoms were low, $kv_x \ll \gamma$. Under these conditions the distribution function of the scattered atoms can be found by using the kinetic equation (11). In small-angle scattering, the longitudinal velocity v_y can be regarded as constant. A particle landing in the region of a periodic potential begins to oscillate at a characteristic frequency $k(U/M)^{1/2}$ along the x axis. Superimposed on these oscillations is a certain diffusion motion.

If we confine ourselves to the case

$$\tau k (U/M)^{1/2} \ll 1, \tag{18}$$

then the problem can be greatly simplified. The scattering-induced increment of the transverse momentum of the atom is proportional to τ, and the change of the coordinate along the x axis is of the order of τ^2. The condition (18) allows us to neglect the change of the transverse coordinate. In this approximation, the problem reduces to finding the transverse momenta of particles moving in the light field along straight-line trajectories. This means that we can leave out of (22) $v_x \partial f/\partial x$, $A\partial^2 f/\partial x \partial v_x$, as well as Gv_x, for in this case there is no self-averaging of the gradient force (the particle does not have enough time to move noticeably away from the initial entry coordinate along the x axis), and $\partial U/\partial x$ is larger than the mixed-type force at all detunings. The diffusion term must be retained, since the gradient force is small at small detuning and the scattering is of the diffusion type.

Thus, under the foregoing simplifications, the stationary kinetic equation takes the form

$$v_y \frac{\partial f}{\partial y} = \frac{\partial}{\partial v_x}\left\{\frac{1}{M}\frac{\partial U(x)}{\partial x} f + D(x)\frac{\partial f}{\partial v_x}\right\}. \tag{19}$$

Let the light beam span along the y axis the region $0 < y < l$. At the entrance into the field region, the distribution function is of the form

$$f(y=0) = \text{const}\,\delta(v_x). \tag{20}$$

The aperture of the beam is usually large compared with the wavelength of the light, so that the particle distribution along the x axis in the incident beam can be regarded as uniform.

Solving (19), we obtain the distribution of the scattered particles

$$f(y>l, v_x) = \text{const}\left\langle (D(x)\tau)^{-1/2} \times \exp\left[-\left(v_x + \frac{\tau}{M}\frac{\partial U}{\partial x}\right)^2 (4D(x)\tau)^{-1}\right]\right\rangle. \tag{21}$$

The angle brackets mean averaging over the spatial period of the field, since the detector registers the scattering pattern averaged over the coordinate.

We calculate now the mean squared velocity of the scattered particles $\langle v_x^2 \rangle$. In a weak field $\Omega_0 \ll \gamma$ we have

$$\langle v_x^2 \rangle = \frac{1}{8} v_0^2 \chi_0 \left[\chi_0 (\Delta\tau)^2 + \frac{14}{5}\gamma\tau\right], \quad v_0 = \frac{\hbar k}{M}. \tag{22}$$

The first and second terms in the square brackets correspond to the contributions from the gradient force and from the diffusion scattering, respectively. In a very weak field $\Omega_0 \ll (\gamma/\tau)^{1/2}$ the principal role is played by diffusion. At $(\gamma/\tau)^{1/2} \ll \Omega_0$ a more important role is assumed by scattering due to the gradient force, with the exception of the region of small detunings $|\Delta| < \Delta_1$, where

$$\Delta_1 = \frac{\gamma}{\Omega_0}\left(\frac{\gamma}{\tau}\right)^{1/2}. \tag{23}$$

These cases correspond to curves 1 and 2 of Fig. 3, which shows the dependence of the average scattering angle $\bar{\theta} = \langle v_x^2 \rangle^{1/2}/v_y$ on the detuning.

In a strong field $\Omega_0 \gg \gamma$ we obtain

$$\langle v_x^2 \rangle = v_0^2 \frac{(\xi-1)^2}{2\xi}\left[(\Delta\tau)^2 + \frac{2\xi|\nu|^2\tau}{\gamma}\right], \quad \xi = (1+\chi_0)^{1/2}. \tag{24}$$

The contribution to the scattering from the gradient force ($\sim \tau^2$) exceeds considerably the contribution from the diffusion ($\sim\tau$) at all detunings, except for a small vicinity $|\Delta| < \Delta_0$. At both $|\Delta| \sim \Delta_0$ both terms in (24) become of the same order. The frequency Δ_0 then depends strongly on the field. In not too strong a field $\gamma \ll \Omega_0 \ll \gamma\tau^2$ we have

$$\Delta_0 = (\Omega_0/\tau)^{1/2} \ll \gamma. \tag{25}$$

In a very strong field, when $\Omega_0 \gg \gamma\tau^2$, we get from (24)

$$\Delta_0 = \Omega_0/\gamma\tau \gg \gamma. \tag{25'}$$

Thus, in a strong field the plot of $\bar{\theta}(\Delta)$ (curve 3) has two characteristic frequencies, Ω_0 and Δ_0. At large

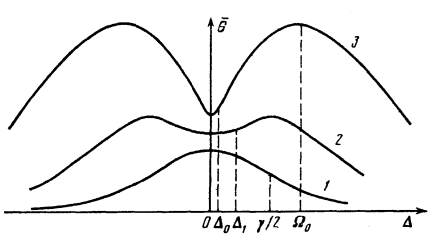

FIG. 3. Dependence of the average scattering angle $\bar{\theta}$ on the detuning.

detunings $|\Delta| \gtrsim \Omega_0$ the scattering picture coincides with the results of the theory of pure potential scattering (without allowance for spontaneous emission). In particular, the characteristic scattering angle obtained from (24) at $|\Delta| \sim \Omega_0$, is $\bar{\theta} \sim \tau k d E_0 / M v_y$. If we substitute here τ from (18), in which we replace the inequality sign by equality, we obtain

$$\bar{\theta} \sim (d E_0 / M v_y^2)^{1/2}. \tag{26}$$

This agrees with the theoretical estimates[11,12] and with experimental results.[5] At $\Delta < \Omega_0$ the spontaneous emission alters the scattering picture substantially. Thus, in the absence of spontaneous emission the scattering peak is reached at the resonant frequency, and a dip is produced in this region at $\gamma\tau > 1$. Unfortunately, no measurements of the frequency dependence of the scattering angle was made in Ref. 5.

§5. COOLING OF ATOMS BY LIGHT

The rate of cooling or heating of atoms (depending on the sign of the detuning) in a standing wave was estimated earlier for the case of strong[4] and weak[16] saturation. Here we consider the final stage of cooling of atoms in a standing wave, obtain the stationary solution of the kinetic equation, and calculate the temperature of the cooled gas. We consider uniform motion along the x axis, assuming that the distribution function does not depend on v_y and v_z. This means actually that the energy of the transverse motion is considerably higher than that of the longitudinal motion. In the absence of collisions, such a strong anisotropy of the temperature is perfectly feasible.

Thus, the one-dimensional stationary equation (11) for the distribution function takes the form ($v_x \equiv v$)

$$v \frac{\partial f}{\partial x} + \frac{\partial}{\partial v}\left\{\frac{1}{M}\left[-\frac{\partial U}{\partial x} + Gv\right] f + Av \frac{\partial f}{\partial x}\right\} = \frac{\partial^2}{\partial v^2}(Df). \tag{27}$$

We use hereafter the smallness of the terms with G, A, and D compared with the gradient force. To this end we change from the variables x and v to the variables x and $\varepsilon = Mv^2/2 + U(x)$, where ε is the total energy of the particle. Equation (27) then takes the form

$$\frac{\partial f}{\partial x} + \frac{\partial}{\partial \varepsilon}\left\{v(\varepsilon, x)\left[Gf + MA\frac{\partial f}{\partial x} + M\left(A\frac{\partial U}{\partial x} - MD\right)\frac{\partial f}{\partial \varepsilon}\right]\right\} = 0,$$

$$v(\varepsilon, x) = \pm (2(\varepsilon - U)/M)^{1/2}. \tag{28}$$

We seek the solution of this equation by perturbation theory, representing the distribution function in the form $f(\varepsilon, x) = f_0(\varepsilon) + f_1(\varepsilon, x)$, with f_1 much smaller than f_0 by virtue of the smallness of the parameters A, G, and D. From (28) we have

$$\frac{\partial f_1}{\partial x} + \frac{\partial}{\partial \varepsilon}\left\{v(\varepsilon, x)\left[Gf_0 + M\left(A\frac{\partial U}{\partial x} - MD\right)\frac{\partial f_0}{\partial \varepsilon}\right]\right\} = 0. \tag{29}$$

Since $f_1(\varepsilon, x)$ is a periodic function of the coordinate under stationary conditions, we obtain after averaging over the period

$$\frac{\partial}{\partial \varepsilon}\left\{\langle vG\rangle f_0 + \left\langle vM\left(A\frac{\partial U}{\partial x} - MD\right)\right\rangle \frac{\partial f_0}{\partial \varepsilon}\right\} = 0. \tag{30}$$

The expression in the curly brackets must be set equal to zero, for otherwise the function f_0 will not be normalizable. From this we get

$$f_0(\varepsilon) = \text{const} \exp\left\{-\int_0^{\varepsilon} d\varepsilon' \langle v(\varepsilon', x) G(x)\rangle \times \left[M\left\langle v(\varepsilon', x)\left(A(x)\frac{\partial U}{\partial x} - MD(x)\right)\right\rangle\right]^{-1}\right\}. \tag{31}$$

The exact distribution function (31) can be simplified by approximating the average of the products by the product of the averages. Then the distribution function takes the standard form $f_0 \sim \exp(-\varepsilon/T)$, where

$$T = M\left\langle A\frac{\partial U}{\partial x} - MD\right\rangle \Big/ \langle G\rangle. \tag{32}$$

It is easy to verify that the numerator in (32) is negative. A stationary distribution exists therefore if $\langle G\rangle < 0$.

We consider now some cases at limiting field intensities. In a weak field $\Omega_0 \ll \gamma$ we have

$$T = 7\hbar |v|^2 / 20\Delta. \tag{33}$$

The minimum temperature $7\hbar\gamma/20 \approx \hbar\gamma/3$ is reached at $\Delta = \gamma/2$. This agrees in order of magnitude with results by others.[17,18] The depth of modulation of the potential $U(x)$ is small in this case compared with the temperature, so that the fraction of particles trapped in the potential wells is small.

At $\Omega_0 \gg \gamma$ and $\chi_0 \sim 1$ we get from (32)

$$T = \frac{\hbar\Delta}{2}\chi_0 \frac{1+4\xi}{(1+\xi)(1+3\xi)}, \quad \gamma \ll \Delta \ll \Delta_{cr}. \tag{34}$$

Finally, in a strong field $\chi_0 \gg 1$

$$T = |2\sqrt{2}\hbar\Omega_0 v^3 / \Delta(|v|^2 - 4\Delta^2)|. \tag{35}$$

The condition that the temperatures be positive leads to the following restriction on the detuning: $-\gamma/\sqrt{12} < \Delta < 0$ or $\gamma/\sqrt{12} < \Delta \ll \Omega_0$. It is understood that the detuning is not too close to the boundaries of the indicated region, for otherwise the condition (8) that the atoms be slow may be violated. We have seen that in a strong field the atom temperature turns out to be of the order of the Rabi frequency and is comparable with the depth of the potential wells. Therefore the fraction of the bound atoms becomes of the order of unity. We note that cooling of sodium atoms with the aid of resonant light pressure has been recently observed.[19]

§6. CONCLUSION

To describe the drift of atoms in an optical field it suffices to take into account only the average light-pressure forces. In those cases, however, when the average effective force is small, account must be taken of the fluctuations of the light-pressure force, which lead to diffusion of the atoms in velocity. This situation takes place as a rule at small detunings from resonance and for long times of interaction with the field, $\gamma\tau \gg 1$. To describe the motion of the atoms in the field in this case it is necessary to use the kinetic equation. The kinetic equation for purely induced transitions in a nonmonochromatic field of a standing wave was considered earlier in Refs. 8 and 15. The atom diffusion due to spontaneous transitions in a traveling-wave field was estimated in Refs. 9, 10, 20, and 21.

In the present paper we calculate the coefficient of diffusion in a monochromatic standing wave; this coefficient is governed by a certain combination of spontaneous and induced transitions. In addition, a more accurate spontaneous diffusion coefficient for a traveling wave is obtained.

If the various types of diffusion are considered from the point of view of the mechanism of the scattering of the external-field photons then, just as in the case of the average forces, there are three types of diffusion: spontaneous, stimulated, and mixed.[3] The greatest difference between these diffusion coefficients takes place in a strong saturating field. The spontaneous diffusion coefficient is then independent of the field, while the mixed-type diffusion coefficient, as well as the induced-diffusion coefficient,[8,15] increases in proportion to the field intensity.

Next, the kinetic equation is used to find the distribution function of atoms scattered in a standing wave. It is shown that the dependence of the average scattering angle on the detuning in a strong field has a nonmonotonic character. It is of interest to note that the frequency width of the dip (25) is less than the natural line width of the atom.

We have also obtained the temperature of the atoms cooled by a standing light wave, and investigated its dependence on the field parameters.

[1]G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys. JETP **15**, 1088 (1962)].
[2]A. Ashkin, Phys. Rev. Lett. **24**, 156 (1970); **25**, 1321 (1970).
[3]A. P. Kazantsev, Usp. Fiz. Nauk **124**, 113 (1978) [Sov. Phys. Usp. **21**, 58 (1978)].
[4]A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **66**, 1599 (1974) [Sov. Phys. JETP **39**, 784 (1974)].
[5]A. Arimondo, H. Lew, and T. Oka, Phys. Rev. Lett. **43**, 753 (1979).
[6]A. P. Kol'chenko, S. G. Rautian, and R. I. Sokolovskiĭ, Zh. Eksp. Teor. Fiz. **55**, 1864 (1968) [Sov. Phys. JETP **28**, 986 (1969)].
[7]F. A. Vorob'ev, S. G. Rautian, and R. I. Sokolovskiĭ, Opt. Spektr. **27**, 728 (1969) [Opt. Spectrosc. **27**, 398 (1969)].
[8]A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **67**, 1660 (1974) [Sov. Phys. JETP **40**, 825 (1975)].
[9]A. Yu. Pusen, Zh. Eksp. Teor. Fiz. **70**, 851 (1976) [Sov. Phys. JETP **43**, 441 (1976)].
[10]E. V. Baklanov and B. Ya. Dubetskiĭ, Opt. Spektrosk. **41**, 3 (1976) [Opt. Spectrosc. **41**, 1 (1976)].
[11]A. P. Kazantsev and A. P. Surdutovich, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 346 (1975) [JETP Lett. **21**, 158 (1975)]; B. L. Zhelnov, A. P. Kazantsev, and G. I. Surdutovich, Kvant. Elektron. (Moscow) **4**, 893 (1977) [Sov. J. Quantum Electron. **7**, 499 (1977)].
[12]G. A. Delone, V. A. Grinchuk, A. P. Kazantsev, and G. I. Surdutovich, Opt. Commun. **25**, 399 (1978).
[13]R. J. Cook and A. F. Bernhardt, Phys. Rev. **A18**, 2533 (1978).
[14]A. P. Kazantsev, G. I. Surdutovich, and V. P. Yakovlev, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 542 (1980)] [JETP Lett. **31**, 509 (1980)].
[15]A. P. Botin and A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **68**, 2075 (1975) [Sov. Phys. JETP **41**, 1038 (1975)].
[16]T. W. Hänsch and A. L. Schawlow, Opt. Commun. **13**, 68 (1975).
[17]V. S. Letokhov, V. G. Minogin, and B. D. Pavlik, Zh. Eksp. Teor. Fiz. **72**, 1328 (1977) [Sov. Phys. JETP **45**, 698 (1977)].
[18]Yu. I. Klimontovich, and S. P. Luzgin, Zh. Tekh. Fiz. **48**, 2217 (1978) [Sov. Phys. Tech. Phys. **23**, 1269 (1978)].
[19]V. I. Balykin, V. S. Letokhov, and V. I. Mishin, Zh. Eksp. Teor. Fiz. **78**, 1376 (1980) [Sov. Phys. JETP **51**, 692 (1980).
[20]I. V. Krasnov and N. E. Shaparev, Zh. Eksp. Teor. Fiz. **77**, 899 (1979) [Sov. Phys. JETP **50**, 453 (1979)].
[21]L. Mandel, J. Optics **10**, 51 (1979).

Translated by J. G. Adashko