Radiation emitted by charges in inhomogeneous and nonstationary media

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A perturbation theory is constructed for the calculation of the radiation emitted by moving charges in inhomogeneous and nonstationary media. The general expressions obtained for the angular and spectral distributions of the radiated energy can be used to calculate the radiation from a charge on a diffuse interface of two media.

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1. Electromagnetic radiation is produced when a charge moves in a medium whose permittivity varies in space and (or) in time. Although the radiation from the charge in a nonstationary medium and "ordinary" transition radiation are related effects, there are substantial differences between them. A major cause of these differences is that radiation can arise in a nonstationary medium not only on account of the kinetic energy of the moving charge, but also on account of the energy of the source responsible for the nonstationarity. This leads, in particular, to the possibility of radiation by an immobile charge in a medium with time-varying anisotropy.[1] Radiation in nonstationary and inhomogeneous media has been very little investigated because of the difficulty in solving Maxwell's equations with a permittivity $\varepsilon(\mathbf{r}, t)$. Principal attention has been paid to radiation in media whose permittivity has a traveling-wave variation.[2-4] We write down the generalized Poynting theorem for the case of nonstationary media (we assume the magnetic permeability to be unity):

$$\frac{c}{4\pi}\operatorname{div}[\mathbf{EH}]+\frac{1}{8\pi}\frac{\partial}{\partial t}(\varepsilon E^2+H^2)+\frac{1}{8\pi}E^2\frac{\partial\varepsilon}{\partial t}+\mathbf{Ej}=0, \tag{1}$$

where $\mathbf{E}$ is the electric field, $\mathbf{H}$ is the magnetic field and $\mathbf{j}$ is the current density. Equation (1) admits of the following interpretation: the change of the electromagnetic field energy density in a nonstationary medium [the second term of (1)] is due not only to the radiation and to the work of the field on the currents, but also to the energy density released or absorbed per unit time as a result of the action of the source that produces the nonstationarity in the medium [third term of (1)]. Most perspicuous in this case is the analogy with the vibrations of a weight on a spring whose rigidity K depends on the time. Indeed, at each instant of time the energy of the system (and its Hamiltonian) is $m\dot{x}^2/2+K(t)x^2/2$, and the energy change is determined by the expression $(1/2)x^2dK/dt$.

2. The perturbation theory developed below makes it relatively easy to calculate the energy radiated in nonstationary and inhomogeneous media. Let the variation of the permittivity of the medium be given by

$$\varepsilon=\varepsilon_0+\varepsilon_1(\mathbf{r},t),\quad \varepsilon_1\ll\varepsilon_0. \tag{2}$$

We write the electromagnetic-field energy density in the form

$$W=\frac{1}{8\pi}(\varepsilon_0E^2+H^2)+\frac{1}{8\pi}\varepsilon_1E^2. \tag{3}$$

Since $\varepsilon_1\ll\varepsilon_0$, the second term in (3) can be regarded as a perturbation and a rigorous perturbation theory can be constructed on the basis of the Hamiltonian

$$H_1=\frac{1}{8\pi}\int d\mathbf{r}\,\varepsilon_1(\mathbf{r},t)E^2(\mathbf{r},t),$$

which describes the "interaction" between the electric field and the permittivity that varies in time and in space.

We represent the electric field in the form $\mathbf{E}=\mathbf{E}^q+\mathbf{E}^R$, where $\mathbf{E}^q$ is the field produced by external charges or currents and $\mathbf{E}^R$ is the radiation field. For the field $\mathbf{E}^R$ we have the expansion

$$\begin{aligned}E_i^R&=-\frac{1}{c}\frac{\partial A_i^R}{\partial t}\\&=-\frac{1}{c}\frac{\partial}{\partial t}\left\{\sum_{\mathbf{k},\lambda}\left(\frac{2\pi c^2\hbar}{\omega\varepsilon_0}\right)^{1/2}e_i^\lambda[a_{\mathbf{k}\lambda}e^{i(\mathbf{kr}-\omega t)}+a_{\mathbf{k}\lambda}^+e^{-i(\mathbf{kr}-\omega t)}]\right\}\\&=-i\left(\frac{2\pi\hbar}{\varepsilon_0}\right)^{1/2}\sum_{\mathbf{k},\lambda}\omega^{1/2}e_i^\lambda(a_{\mathbf{k}\lambda}^+e^{-i(\mathbf{kr}-\omega t)}+a_{\mathbf{k}\lambda}e^{i(\mathbf{kr}-\omega t)}),\end{aligned} \tag{4}$$

where $\mathbf{A}^R$ is the vector potential of the radiation field, $\mathbf{e}^\lambda$ is the polarization unit vector, $a_{k\lambda}^+$ and $a_{k\lambda}$ are respectively the creation and annihilation operators of a photon with wave vector $\mathbf{k}$ and polarization $\lambda=1$ or 2, and $\omega=|k|c/\varepsilon_0^{1/2}$; the summation in (4) is over all possible values of $\mathbf{k}$ and λ.

It follows from (3) that the Hamiltonian H_1 is of the form

$$H_1=\frac{1}{8\pi}\int d\mathbf{r}[\varepsilon_1(E^q)^2+\varepsilon_1(E^R)^2+2\varepsilon_1E_i^qE_i^R]. \tag{5}$$

The first term in (5) determines the change of the energy of the "external" field. The second term, which contains bilinear combinations of a^+ and a is responsible for the production and annihilation of the photon pairs, as well as for the scattering (reflection and refraction) of light by regions with variable permittivity. The last term in (5) determines the emission and absorption of photons in the field of the charges and currents (transition radiation, transition scattering, etc.). It is this last term of (5) which is of interest to us here.

In first-order perturbation theory, the scattering

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matrix is

$$S^{(1)}=-\frac{i}{\hbar}\int dt\, H_1=-\frac{i}{4\pi\hbar}\int dt\, d\mathbf{r}\, \varepsilon_1(\mathbf{r},t)E_i^{\lambda*}E_i^{q}. \tag{6}$$

We expand $\varepsilon_1(\mathbf{r},t)$ and $E_i^q(\mathbf{r},t)$ in a Fourier integral of the form

$$\varepsilon_1(\mathbf{r},t)=\int d\mathbf{k}_0\, d\omega_0 \varepsilon_1(\mathbf{k}_0,\omega_0)\, e^{i(\mathbf{k}_0\mathbf{r}-\omega_0 t)},$$
$$E_i^q(\mathbf{r},t)=\int d\mathbf{k}_1\, d\omega_1 E_i^q(\mathbf{k}_1,\omega_1)\, e^{i(\mathbf{k}_1\mathbf{r}-\omega_1 t)}. \tag{7}$$

It follows then from (6) and (7) that the matrix element corresponding to emission of a photon with wave vector $\mathbf{k}$, frequency $\omega=kc/\varepsilon_0^{1/2}$, and polarization λ is of the form

$$\begin{aligned}\langle\mathbf{k},\omega,\lambda|S^{(1)}|0\rangle=&-\frac{(2\pi)^4\omega^{1/2}}{(8\pi\hbar\varepsilon_0)^{1/2}}\int d\mathbf{k}_1\, d\omega_1\, d\mathbf{k}_0\, d\omega_0\\ &\times e_i^{\lambda}E_i^q(\mathbf{k}_1,\omega_1)\,\varepsilon_1(\mathbf{k}_0\omega_0)\,\delta(\mathbf{k}_1+\mathbf{k}_0-\mathbf{k})\,\delta(\omega_1+\omega_0-\omega)\\ =&\frac{(2\pi)^4\omega^{1/2}}{(8\pi\hbar\varepsilon_0)^{1/2}}\int d\mathbf{k}_1\, d\omega_1\, e_i^{\lambda}E_i^q(\mathbf{k}_1,\omega_1)\,\varepsilon_1(\mathbf{k}-\mathbf{k}_1,\omega-\omega_1).\end{aligned} \tag{8}$$

The probability of photon emission is determined by the square of the modulus of the matrix element (8). To calculate the energy radiation in the interval d^3k it is necessary to multiply the radiation probability by $\hbar\omega d^3k/(2\pi)^3$. We obtain

$$W_{\mathbf{k},\lambda}d^3k=\frac{(2\pi)^5\omega^2}{4\varepsilon_0}\left|\int d\mathbf{k}_1\, d\omega_1\, e_i^{\lambda}E_i^q(\mathbf{k}_1\omega_1)\,\varepsilon_1(\mathbf{k}-\mathbf{k}_1,\omega-\omega_1)\right|^2 d^3k. \tag{9}$$

Formula (9) yields a general expression for the radiation energy in first-order perturbation theory. Planck's constant $\hbar$ drops out of the final expression and the result, as expected, is purely classical. If we are interested in the emission of an unpolarized photon, then it is necessary to sum in (9) over the polarization λ. Formula (9) can be obtained classically by solving Maxwell's equations (see the Appendix). The method described above, however, has in our opinion a number of advantages, namely, it is simple to investigate the polarization as well as to calculate higher approximations, and finally formula (9) is valid over a wider range. Indeed, since the perturbation theory is constructed on the basis of the Hamiltonian

$$H_1=\frac{1}{8\pi}\int d\mathbf{r}\varepsilon_1 E^2,$$

it follows that Eq. (9) is valid also if ε_1 is not small, provided that the change of ε_1 takes place in a sufficiently small region. We note that application of Eq. (9) to transition radiation results in the same expression for the intensity as obtained by Ginzburg and Tsytovich.[3]

3. We now apply the formulas obtained above to calculate the transition radiation produced by a charge on a charge on a moving blurred boundary between two media. Let the permittivity vary like

$$\varepsilon=\varepsilon_0+\Delta\varepsilon e^{\alpha(z-ut)}/(1+e^{\alpha(z-ut)}). \tag{10}$$

At a specified value of the time t, the permittivity changes smoothly from ε_0 as $z\to-\infty$ to $\varepsilon_0+\Delta\varepsilon$ as $z\to+\infty$. Thus, Eq. (10) for the variation of the permittivity describes a blurred boundary between two media travelling along the z axis with velocity u, $1/\alpha$ the characteristic width of the smeared zone. Radiation produced on an immobile smeared boundary between two media[5] and radiation in a smoothly nonstationary medium[6] were investigated earlier.

Let the moving boundary be crossed at the instant of time $t=0$ by a particle with charge q, moving with velocity V along the z axis. We choose the origin at the point where the charge crosses the boundary. The Fourier component of the electric field of the uniformly moving charge is

$$E^q(\mathbf{k}_1,\omega_1)=\frac{4\pi iq}{(2\pi)^3}\frac{\delta(\omega_1-\mathbf{k}_1V)}{k_1^2-\varepsilon_0\omega_1^2/c^2}\left(\frac{\omega_1\mathbf{V}}{c^2}-\frac{\mathbf{k}_1}{\varepsilon_0}\right). \tag{11}$$

We find now the Fourier component of the alternating part of the permittivity:

$$\varepsilon_1(\mathbf{k}_0,\omega_0)=\frac{\Delta\varepsilon}{(2\pi)^4}\int d\mathbf{r}\, dt\, e^{i(\omega_0 t-\mathbf{k}_0\mathbf{r})}[e^{\alpha(z-ut)}/(1+e^{\alpha(z-ut)})]. \tag{12}$$

The substitution $z-ut=\xi$ leads to the expression

$$\varepsilon_1(\mathbf{k}_0,\omega_0)=\frac{\Delta\varepsilon\delta(k_{0x})\delta(k_{0y})\delta(\omega_0-k_{0z}u)}{2\pi}\int_{-\infty}^{+\infty}d\xi\frac{e^{\xi(\alpha-ik_{0z})}}{1+e^{\alpha\xi}}. \tag{13}$$

Substituting $e^{\alpha\xi}=\xi$ in the integral (13), we get

$$\begin{aligned}\int_{-\infty}^{+\infty}d\xi\frac{e^{\xi(\alpha-ik_{0z})}}{1+e^{\alpha\xi}}&=\frac{1}{\alpha}\int_0^{\infty}\frac{\eta^{-ik_{0z}/\alpha}}{1+\eta}d\eta\\ &=\frac{1}{\alpha}B\left(1-i\frac{k_{0z}}{\alpha},\ i\frac{k_{0z}}{\alpha}\right)=\frac{\pi}{\alpha\sin(i\pi k_{0z}/\alpha)}.\end{aligned} \tag{14}$$

We ultimately have for $\varepsilon_1(k_0,\omega_0)$

$$\varepsilon_1(\mathbf{k}_0,\omega_0)=-i\frac{\Delta\varepsilon}{2\alpha}\delta(k_{0x})\delta(k_{0y})\delta(\omega_0-k_{0z}u)/\operatorname{sh}\left(\frac{\pi k_{0z}}{\alpha}\right). \tag{15}$$

Substituting (11) and (15) in (8) we obtain an expression for the matrix element

$$\begin{aligned}\langle\mathbf{k},\omega,\lambda|S^{(1)}|0\rangle=&\frac{(2\pi)^2q\Delta\varepsilon\omega^{1/2}}{(8\pi\hbar\varepsilon_0)^{1/2}\alpha(u-V)}e_i^{\lambda}\left(\frac{V(k_zu-\omega)}{c^2(u-V)}\mathbf{V}-\frac{\varkappa}{\varepsilon_0}\right)_i\\ &\times\left\{\left[k_x^2+k_y^2+\frac{(k_zu-\omega)^2}{(u-V)^2}-\frac{\varepsilon_0}{c^2}V^2\frac{(k_zu-\omega)^2}{(u-V)^2}\right]\right.\\ &\left.\times\operatorname{sh}\left[\frac{\pi}{\alpha}\left(k_z-\frac{(k_zu-\omega)}{(u-V)}\right)\right]\right\}^{-1},\end{aligned} \tag{16}$$

where $\varkappa=(k_x,k_y,(k_zu-\omega)/u-V))$. To calculate the radiation energy of an unpolarized photon it is necessary to sum the square of expression (16) over the polarization λ. This procedure reduces to a calculation of the following sum:

$$\begin{aligned}&\sum_{\lambda=1}^{2}\left[e_i^{\lambda}\left(\frac{V(k_zu-\omega)}{c^2(u-V)}\mathbf{V}-\frac{\varkappa}{\varepsilon_0}\right)_i\right]^2\\ &=\frac{\sin^2\theta}{(u-V)^2}\left(\frac{(k_zu-\omega)V^2}{c^2}-\frac{(k_zV-\omega)}{\varepsilon_0}\right)^2,\end{aligned} \tag{17}$$

where θ is the angle between the z axis and the direction of the radiation. In the summation of (17) we have used the transversality condition $e_i^{\lambda}/k_i=0$. Now, taking (9), (16), and (17) into account we obtain ultimately for the energy radiated at the frequency $\omega=kc/\varepsilon_0^{1/2}$ into the solid angle $d\Omega=2\pi\sin\theta\, d\theta$ $(k_z=k\cos\theta)$:

$$\begin{aligned}\frac{dW_\omega}{d\omega}=&\frac{q^2\Delta\varepsilon^2\sin^2\theta\varepsilon_0^{1/2}\omega^2}{4\alpha^2(u-V)^4c^3}\left[\frac{(uc^{-1}\varepsilon_0^{1/2}\cos\theta-1)V^2}{c^2}-\frac{(Vc^{-1}\varepsilon_0^{1/2}\cos\theta-1)}{\varepsilon_0}\right]^2\\ &\times\left\{\left[\frac{\varepsilon_0}{c^2}\sin^2\theta+\frac{(uc^{-1}\varepsilon_0^{1/2}\cos\theta-1)^2}{(u-V)^2}\left(1-\frac{V^2}{c^2}\varepsilon_0\right)\right]^2\right.\\ &\left.\operatorname{sh}^2\left[\frac{\pi\omega}{\alpha(u-V)}\left(1-\frac{V}{c}\varepsilon_0^{1/2}\cos\theta\right)\right]\right\}^{-1}d\Omega.\end{aligned} \tag{18}$$

We consider now the emission of frequencies at which the argument of the hyperbolic sign in (18) is much less than unity. In this case we have

$$\frac{dW_\omega}{d\omega}=\frac{q^2\Delta\varepsilon^2\varepsilon_0^{3/2}\sin^2\theta}{4\pi^2(u-V)^2c^3}$$
$$\times\left[\frac{(uc^{-1}\varepsilon_0^{1/2}\cos\theta-1)V^2}{c^2}-\frac{(Vc^{-1}\varepsilon_0^{1/2}\cos\theta-1)}{\varepsilon_0}\right]^2$$
$$\times\left\{\left[\frac{\varepsilon_0}{c^2}\sin^2\theta+\frac{(uc^{-1}\varepsilon_0^{1/2}\cos\theta-1)^2}{(u-V)^2}\left(1-\frac{\varepsilon_0V^2}{c^2}\right)\right]\left(1-\frac{V}{c}\varepsilon_0^{1/2}\cos\theta\right)\right\}^{-2}$$
$$\times d\Omega\left[1-\frac{1}{3}\pi^2\frac{\omega^2}{\alpha^2(u-V)^2}\left(1-\frac{V}{c}\varepsilon_0^{1/2}\cos\theta\right)^2\right]. \qquad (19)$$

The expression in the next-to-last line of (19) is the energy of the transition radiation of a charge on a moving "abrupt" boundary between two media. From (19) we obtain the criterion for the applicability of the abrupt-boundary approximation

$$\frac{L}{L_f}\frac{V}{|u-V|}\ll 1, \qquad (20)$$

where $L=1/\alpha$ is the characteristic width of the transition layer, $L_f=(V/\omega)(1-Vc^{-1}\varepsilon_0^{1/2}\cos\theta)^{-1}$ is the length over which radiation of frequency ω is formed in a medium with permittivity ε_0. Thus, when a relativistic particle moves in a direction opposite to the moving boundary, the frequency spectrum, which can be calculated by assuming the boundary to be abrupt, broadens substantially in the short-wave direction. In the opposite case, if

$$\frac{L}{L_f}\frac{V}{u-V}\gg 1,$$

the radiation energy is exponentially small.

In the limit $u\to\infty$ we obtain from (18)

$$\frac{dW_\omega}{d\omega}=\frac{q^2V^4\Delta\varepsilon^2\sin^2\theta\cos^2\theta d\Omega}{4\pi^2c^5\varepsilon_0^{1/2}(1-Vc^{-1}\varepsilon_0^{1/2}\cos\theta)^2(1-V^2c^{-2}\varepsilon_0\cos^2\theta)^2}, \qquad (21)$$

which coincides, as expected, with the expression for the energy radiated in a nonstationary medium when the permittivity is instantaneously changed from ε_0 to $\varepsilon_0+\Delta\varepsilon$ ($\Delta\varepsilon\ll\varepsilon_0$).[7] In the limit as $u\to 0$ we obtain the transition radiation produced on an immobile blurred boundary between two media, first calculated by Amatuni and Korkhmazyan.[5]

If the charge is at rest ($V=0$), the expression for the radiated energy is

$$\frac{dW_\omega}{d\omega}=\frac{q^2\Delta\varepsilon^2\sin^2\theta\omega^2\,\mathrm{sh}^{-2}(\pi\omega/\alpha u)\,d\Omega}{4\alpha^2c^3\varepsilon_0^{3/2}(u^2c^{-2}\varepsilon_0-2uc^{-1}\varepsilon_0^{1/2}\cos\theta+1)^2}. \qquad (22)$$

As follows from (22), the radiated energy vanishes at $u=0$ and $u\to\infty$. Let us obtain the value of u at which the radiated energy is a maximum. We obtain it by solving the equation

$$\frac{d}{du}\left(\frac{dW_\omega}{d\omega}\right)=0. \qquad (23)$$

The solution of Eq. (23) for the case of an abrupt moving boundary is $u=c/\varepsilon_0^{1/2}$. Thus, the radiation from an immobile charge is maximal if the boundary velocity is equal to the phase velocity of light in the "unperturbed" medium.

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APPENDIX

Classical derivation of equation (9)[1)]

Let the permittivity of the medium vary in accord with (2). We seek the solution of Maxwell's equations in the form $\mathbf{E}=\mathbf{E}_0+\mathbf{E}_1$ and $\mathbf{H}=\mathbf{H}_0+\mathbf{H}_1$, where $\mathbf{E}_0$ and $\mathbf{H}_0$ are the solutions of Maxwells equations with the permittivity ε_0 while $\mathbf{E}_1$ and $\mathbf{H}_1$ are small corrections to the field intensities, due to the alternating part of the permittivity. Then, discarding terms of order smaller than the first, we obtain from Maxwell's equations the following system for $\mathbf{E}_1$ and $\mathbf{H}_1$:

$$\varepsilon_0\,\mathrm{div}\,\mathbf{E}_1=-\mathrm{div}(\varepsilon_1\mathbf{E}_0),\ \mathrm{div}\,\mathbf{H}_1=0,$$
$$\mathrm{rot}\,\mathbf{E}_1=-\frac{1}{c}\frac{\partial}{\partial t}\mathbf{H}_1,\ \mathrm{rot}\,\mathbf{H}_1=\frac{\varepsilon_0}{c}\frac{\partial}{\partial t}\mathbf{E}_1+\frac{1}{c}\frac{\partial}{\partial t}(\varepsilon_1\mathbf{E}_0). \qquad (24)$$

The system (24) coincides formally with the system of Maxwells equations if we put

$$\rho=-\frac{1}{4\pi}\mathrm{div}(\varepsilon_1\mathbf{E}_0),\ \mathbf{j}=\frac{1}{4\pi}\frac{\partial}{\partial t}(\varepsilon_1\mathbf{E}_0).$$

We note that the "charge and current densities" defined in this manner satisfy the continuity equation.

To calculate the radiation, we use the method described in the book by Landau and Lifshitz (Ref. 8, §66). The Fourier component of the vector potential of the radiation field at large distances is of the form

$$\mathbf{A}_\omega=\frac{e^{ikR_0}}{cR_0}\int\mathbf{j}_\omega e^{i\mathbf{k}\mathbf{r}}\,d\mathbf{r},\ k=\omega\varepsilon_0^{1/2}/c, \qquad (25)$$
$$\mathbf{j}_\omega=\frac{1}{2\pi}\int dt e^{i\omega t}\frac{1}{4\pi}\frac{\partial}{\partial t}(\varepsilon_1\mathbf{E}_0). \qquad (26)$$

Expanding $\varepsilon_1\mathbf{E}_0$ in a Fourier integral of the form

$$\varepsilon_1\mathbf{E}_0=\int d\omega_1 d\mathbf{k}_1 e^{i(\mathbf{k}_1\mathbf{r}-\omega_1 t)}(\varepsilon_1\mathbf{E}_0)_{\omega_1,\mathbf{k}_1}, \qquad (27)$$

we obtain from (26)

$$\mathbf{A}_\omega=-\frac{ie^{ikR_0}}{R_0}\frac{\omega}{2c}(2\pi)^2(\varepsilon_1\mathbf{E}_0)_{\omega,\mathbf{k}}$$
$$=-\frac{ie^{ikR_0}}{R_0}\frac{\omega}{2c}(2\pi)^2\int d\mathbf{k}_1 d\omega_1\mathbf{E}_0(\mathbf{k}_1,\omega_1)$$
$$\times\varepsilon_1(\mathbf{k}-\mathbf{k}_1,\omega-\omega_1). \qquad (28)$$

From (28) we obtain the Fourier component of the magnetic field intensity

$$\mathbf{H}_\omega=\frac{e^{ikR_0}}{R}\frac{\omega(2\pi)^2}{2c}[\mathbf{k}\times(\varepsilon_1\mathbf{E}_0)_{\omega,\mathbf{k}}]$$
$$=\frac{e^{ikR_0}}{R_0}\frac{\omega^2(2\pi)^2}{2c^2}\varepsilon_0^{1/2}[\mathbf{n}(\varepsilon_1\mathbf{E}_0)_{\omega,\mathbf{k}}], \qquad (29)$$

where $\mathbf{n}$ is the unit vector in the direction of $\mathbf{k}$. We represent n in the form

$$\mathbf{n}=[\mathbf{e}_1\mathbf{e}_2], \qquad (30)$$

where $\mathbf{e}_{1,2}$ are independent unit vectors of the polarization. The energy radiated into a solid angle $d\Omega$ in the frequency interval $d\omega$ is determined by the square of the modulus of (29). Taking (30) into account we have

$$W_\omega=\frac{(2\pi)^4\omega^4\varepsilon_0^{1/2}}{4c^3}\sum_{\lambda=1}^{2}|\mathbf{e}_\lambda(\varepsilon_1\mathbf{E}_0)_{\omega,\mathbf{k}}|^2d\omega d\Omega, \qquad (31)$$

which coincides with expression (9) summed over the polarizations, if we replace d^3k in it by $\omega^2c^{-3}\varepsilon_0^{3/2}d\omega d\Omega$.

[1)]Performed with the author jointly with V. E. Rok.

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