Dielectron recombination

I. L. Beĭgman, L. A. Vaĭnshtein, and B. N. Chichkov

P. N. Lebedev Institute of Physics, Academy of Sciences of the USSR
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The applicability of the various methods of determining the auto-ionization probabilities and the effect of additional auto-ionization decay channels on the dielectron recombination rate are investigated. A general expression is derived for the dependence of the dielectron recombination rate on the electron density, N_e, in the plasma. It is shown that for ions with an L shell with a degree of ionization $Z = 2$–3 and electron densities in the range $N_e = 10^{-3}$–10^5 (planetary nebulae), the collisions with the electrons decrease the dielectron recombination rate for the $2s$–$2p$ transitions. For $N_e = 10^{13}$–10^{15} (the tokamak), $Z = 2$–10 and $N_e = 10^{19}$–10^{21} (laser plasma), $Z = 10$–30, the same effect occurs for all transitions from the L shell.

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INTRODUCTION

By dielectron recombination we mean the process of electron capture by an ion in which in the first phase there occurs the formation of a doubly excited quasistationary state, and in the second phase the radiative decay of this state. Schematically, this process can be represented as follows:

$$X_Z(\alpha_0)+e \to X_{Z-1}^{**}(\gamma) \to X_{Z-1}^{*}(\gamma')+\hbar\omega, \tag{1}$$

where $Z \geqslant 2$ is the spectroscopic symbol of the ion, α_0 denotes the ground state of the ion, $\gamma = \alpha nlLSJ$ denotes the intermediate doubly excited state, and γ' denotes any "stable" state of the ion (i.e., one lying below the ionization threshold) to which a radiative decay from the state γ is possible.

The importance of this process in the establishment of the ionization equilibrium in a plasma was first pointed out by Burgess.[1] It has subsequently been studied in a number of papers. Noteworthy among them are Refs. 2-7.

The total dielectron recombination rate is given by the sum:

$$\varkappa_d=\sum_{\gamma,\gamma'}\varkappa_d(\gamma,\gamma')=\sum_{\gamma,\gamma'}\varkappa_c(\gamma)\frac{A(\gamma,\gamma')}{A(\gamma)+W(\gamma)}, \tag{2}$$

where $\varkappa_c(\gamma)$ is the rate of electron capture into the γ level of the ion X_{Z-1}, while $A(\gamma)=\Sigma_{\gamma'}A(\gamma,\gamma')$ and $W(\gamma)=\Sigma_{\gamma'}W(\gamma,\gamma')$ are the total radiative-decay and auto-ionization-decay probabilities[1)] for the doubly excited state γ.

The main difficulty encountered in the determination of the dielectron recombination rate is connected with the necessity of the computation of auto-ionization decay probabilities for a large number of γ—γ' transitions and for a large number of γ states, i.e., $nlLSJ$ values.

Various approximate methods are used to simplify the computations[2-5,7]:

1. In the denominator in (2) $W(\gamma)$ is, as a rule, replaced by the mean—with respect to LSJ—value of $W(\alpha nl)$, after which the sum over LSJ and $L'S'J'$ is computed analytically.

2. Of the various decay channels for the level γ, only one—the decay into the ground state—is considered. In Ref. 4 additional channels for auto-ionization decay into excited states were taken into consideration, and this led to a significant decrease in the dielectron recombination rate for ions with $2s$ and $2p$ electrons.

3. Various approximate formulas are proposed for the auto-ionization probabilities.

In the present paper we investigate the applicability of the various methods of determining the auto-ionization probabilities. We derive simple analytical expressions in the dipole approximation for the auto-ionization probabilities. We present the results of dielectron-recombination-rate calculations for oxygen and iron ions with allowance for additional auto-ionization decay channels. The effect of these channels on the dielectron recombination rate is weaker than the effect that follows from Ref. 4.

Thus far, we have been talking about the determination of the dielectron recombination rate in the limit of low electron densities. In a real plasma, the collisions with the electrons, which destroy the highly excited states, lead to an appreciable decrease in the dielectron recombination rate, since the dominant role in the dielectron recombination rate, since the dominant role in the dielectron recombination process is played by capture into the high-lying levels n. Estimates for this effect are given in Refs. 2, 3, and 6.

In the present paper we derive, with the aid of the Green function for the problem of the kinetics of highly excited levels,[8] a general expression for the dielectron recombination rate with allowance for the effect of the electronic collisions.

2. THE ZERO ELECTRON DENSITY LIMIT

Using the principle of detailed balance, we can express $\varkappa_c(\gamma)$ in terms of the probability, $W(\gamma, \alpha_0)$, for the inverse process (see, for example, Ref. 7). It is convenient to write the result in the form

$$\varkappa_d=10^{-13}B_d\beta^{3/2}e^{-\beta\chi}\ [\mathrm{cm}^3/\mathrm{c}],$$

$$B_d=10^{13}4\pi^{3/2}a_0^3Z^{-3}\sum_{\gamma\gamma'}\frac{g(\gamma)}{g(\alpha_0)}\frac{W(\gamma,\alpha_0)A(\gamma,\gamma')}{A(\gamma)+W(\gamma)}e^{\delta\beta}, \tag{3}$$

$$\beta=Z^2\,\mathrm{Ry}/T,\quad \chi=E_{\alpha\alpha_0}/Z^2\,\mathrm{Ry},\quad \delta\beta=\delta E/T\approx(Z-1)^2\,\mathrm{Ry}/n^2T,$$

where T is the electron temperature in energy units; $E_{\alpha\alpha_0}$ is the energy of the $\alpha_0-\alpha$ transition; $g(\gamma)$ and $g(\alpha_0)$ are the statistical weights of the doubly excited state, γ, of the ion X_{Z-1} and the ground state, α_0, of the

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ion X_Z: $a_0 = 0.529 \times 10^{-8}$ cm is the Bohr radius.

The transition probability $A(\gamma, \gamma')$ does not, in fact, depend on $nlLSJ$, i.e., it can be assumed that $A(\gamma, \gamma') = A(\alpha, \alpha')$. The auto-ionization probability in the LS-coupling approximation does not depend on J, but depends on LS. Allowance for this dependence significantly complicates the dielectron recombination rate calculations. We can get around this difficulty by replacing $W(\gamma, \alpha_0)$ by its mean value:

$$W(\alpha nl, \alpha_0) = \sum_{LS} \frac{(2L+1)(2S+1)}{2(2l+1)g(\alpha)} W(\gamma, \alpha_0), \tag{4}$$

where $g(\alpha) = (2L_\alpha + 1)(2S_\alpha + 1)$ is statistical weight of the excited state of the ion X_Z. The possible error associated with such a replacement is discussed in Ref. 7.

With allowance for (4) the auto-ionization probability is given in first-order perturbation theory by the expression

$$W(\alpha nl, \alpha_0) = \frac{2\pi}{\hbar} \frac{1}{g(\alpha)(2l+1)} \sum_\lambda \left| \left\langle \alpha nl \left| \frac{e^2}{r_{12}} \right| \alpha_0 E\lambda \right\rangle \right|^2. \tag{5}$$

The computation of the dielectron recombination rate with the use of the formula (5) requires the computation of the wave functions and the matrix elements for each nl. Therefore, the problem of the derivation of simpler formulas for the auto-ionization probabilities is a pressing one.

One of the most widely used methods is based on the extrapolation of the excitation cross section $\sigma_{\alpha_0\alpha}$ into the prethreshold region. In this case the auto-ionization probability is given by the expression (see, for example, Ref. 7):

$$W(\alpha nl, \alpha_0) = \frac{1}{\tau_0} \frac{(Z-1)^2}{Z^2} \frac{\chi}{\pi n^3} \frac{g(\alpha_0)}{g(\alpha nl)} \frac{Z^4 \sigma(\alpha_0, \alpha l)}{\pi a_0^2}, \tag{6}$$

where $\sigma(\alpha_0, \alpha l) = \Sigma_\lambda \sigma(\alpha_0\lambda, \alpha l)$ is the partial excitation cross section for the ion at the threshold and $\tau_0 = a_0\hbar/e^2 = 2.419 \times 10^{-17}$ sec.

In computing $W(\alpha nl, \alpha_0)$ by formula (6) for the cross sections, we used the Born–Coulomb approximation with semi-empirical, single-electron functions.[9] These same functions were also used to compute the matrix elements of the radiative transitions and the auto-ionization probabilities in the formula (5). In Table I we present the results of the auto-ionization decay probability computations for the $2pnl$ states of the Fe XXV ion in the $1sE\lambda$ state from the "exact" formula (5) and from the extrapolation formula (6). As can be seen from Table I, the divergence begins at small $n \leqslant 4$. In the case of recombination of ions with $Z \leqslant 20$ this inaccuracy in the determination of the auto-ionization probabilities does not affect the total dielectron recombination rate, since the dominant contribution to the rate of this process for ions with $Z \leqslant 20$ is made by the levels with $n \geqslant 4$.

The method considered above is the most successful of the existing approximate methods of computing auto-ionization probabilities. But the computations with the use of the formula (6) remain quite tedious; we need a modern fast computer to carry them out.

Another method is based on the use of the dipole approximation for the computation of the auto-ionization probabilities. In this case, neglecting the exchange effects, and taking into consideration in (5) only the dipole term, $1/r_{12} \approx \bar{r}_1 \cdot \bar{r}_2 / r_2^3$, in the expansion of the interelectron interaction, we can express the auto-ionization decay probability in terms of the photo-ionization cross section:

$$W(\alpha nl, \alpha_0) = \frac{3}{16\pi} \frac{(Z-1)^2 k^4}{a_0^3} f_{\alpha\alpha_0} c\sigma(nl, E), \tag{7}$$

where $f_{\alpha\alpha_0}$ is the oscillator strength of the transition, $\sigma(nl, E)$ is the effective cross section for the photo-ionization process in which the energy of the outgoing electron is $E = E_{\alpha\alpha_0}$, $k^2 = E_{\alpha\alpha_0}/(Z-1)^2$ Ry, and c is the speed of light. The expression (7) is used in Ref. 5 in the course of the calculation of the dielectron recombination rate. On the basis of this expression, we can obtain simple analytical formulas for the auto-ionization decay probabilities in the dipole approximation. To do this, we must substitute the quantum-mechanical expressions for the photo-ionization cross section[10] into (7) and go over to the limit $n \to \infty$:

$$W(\alpha nl, \alpha_0 E\lambda) = \frac{4 f_{\alpha\alpha_0}}{\tau_0 n^3 (2l+1)} \frac{D_\lambda}{k^3},$$
$$D_{\lambda=l-1} = \frac{1}{l} [k(l^2 + 1/k^2)^{1/2} F_l(1/k, 2/k) - F_{l-1}(1/k, 2/k)]^2, \tag{8}$$
$$D_{\lambda=l+1} = \frac{1}{l+1} [k((l+1)^2 + 1/k^2)^{1/2} F_l(1/k, 2/k) - F_{l+1}(1/k, 2/k)]^2,$$

where $F_l(1/k, 2/k)$ is a Coulomb wave function.

In Table II we present the auto-ionization decay probabilities for the $2pnl$ states of the Fe XXV ion in the $1sE\lambda$ state as computed from the formula (6) and, in the dipole approximation, from (8).

As can be seen from Table II, the dipole approximation by far overestimates the auto-ionization probabilities at $l \leqslant 3$. This is due to the fact that the dipole interaction potential has a singularity at the origin. Such

TABLE I. Comparison of the auto-ionization decay probabilites, $W \cdot n^3/10^{13}$, for the $2pnl$ states of the Fe XXV ion in the $1sE\lambda$ state, as computed from the "exact" formula (5) and the extrapolation formula (6).

l	n = 2	3	4	5	Formula (6)
	Formula (5)				
0	46.4	35.1	31.4	30	26.2
1	80	67.5	62	60	56.5
2	–	11.3	14	15	16.5

TABLE II. Comparison of the auto-ionization decay probabilities, $W \cdot n^3/10^{13}$, for the $2pnl$ states of the Fe XXV ion in the $1sE\lambda$ state, as computed from the formula (6) and in the dipole approximation (8), and the constants B_d corresponding to these probabilities.

Method of computation	l = 0	1	2	4	8	B_d
Formula (6)	26.2	56.6	16.5	0.16	$0.68 \cdot 10^{-7}$	4.02
Formula (8)	841	249	36.7	0.18	$0.68 \cdot 10^{-7}$	18

a difference between the auto-ionization probabilities can lead to an appreciable error in the calculation of the dielectron recombination rate (see Table II). Nevertheless, formula (8) is of practical interest, since it gives the correct values of the auto-ionization probabilities at $l > 3$.

Simpler formulas for the λ-integrated auto-ionization decay probability can be derived on the basis of the quasiclassical expression for the photo-ionization cross section[11] in the $n \to \infty$ limit:

$$W(\alpha nl, \alpha_0) = (6\pi\tau_0 n^3)^{-1} k^2 f_{\alpha\alpha_0} (l+{}^1/_2)^4 (K_{1/3}^2(\xi) + K_{2/3}^2(\xi)), \tag{9}$$

where $\xi = k^2(l + \frac{1}{2})^3/6$; $K_{1/3}$ and $K_{2/3}$ are modified Bessel functions.

The formulas (7)-(9) have a limited region of applicability: their use in the entire n and l ranges in dielectron recombination rate calculations may lead to substantial errors. Therefore, everywhere below, we use the formula (6) to determine the auto-ionization probabilities.

In the majority of papers on dielectron recombination only one channel for the auto-ionization decay of the doubly excited state—the decay into the ground state—is taken into account. In the case of the recombination of hydrogen- and helium-like ions, this assumption is justified. But for more complex ions it is necessary to take into consideration additional channels for auto-ionization decay into excited states, since the probability of such a decay (if it is energetically possible) significantly exceeds the probability for auto-ionization decay into the ground state. Using (6), and taking all the possible auto-ionization decay channels into account, we can write the expression for B_d from (3) in the form

$$B_d = C\frac{Z-1}{n^4} f_{\alpha_0\alpha} \sum_{n \geqslant n_1} B_n, \tag{10}$$

$$B_n = \sum_{l<n} \frac{(2l+1)}{B+(n/n_S)^3} e^{\delta\beta}, \quad n_1^2 = \frac{(Z-1)^2 \mathrm{Ry}}{E_{\alpha\alpha_0}},$$

$$C = 10^{13} \frac{4\pi^{3/2}}{137^3} \frac{a_0\hbar}{m} \left(\frac{Z-1}{Z}\right)^3 = 0.53 \left(\frac{Z-1}{Z}\right)^3,$$

$$B = \sum_{\alpha'} \frac{E_{\alpha\alpha'}}{E_{\alpha\alpha_0}} \frac{g(\alpha')}{g(\alpha_0)} \frac{\sigma(\alpha', \alpha l)}{\sigma(\alpha_0, \alpha l)},$$

$$n_S = 137 \left[\frac{n_1^2 \sigma(\alpha_0, \alpha l)}{\pi^2 a_0^2 (2l+1) f_{\alpha_0\alpha}}\right]^{1/3}.$$

TABLE III. Results of computations of the constants B_d for the dielectron recombination rates in the $T \to \infty$ limit for oxygen and iron ions with allowance for additional auto-ionization decay channels.

Transition	Oxygen			Iron		
	Z	1 channel	3 channels	Z	1 channel	3 channels
		B_d	B_d		B_d	B_d
$1s-2p$	8	31.1	–	26	3.95	–
$1s-2p$	7	43.2	–	25	3.8	–
$2s-2p$	6	4.9	–		0.366	–
$2s-3p$		8.83	1.79	24	2.6	1.3
$2s-2p$	5	30.6	–		1.16	–
$2s-3p$		19.2	2.26	23	5.2	1.82
$2s-2p$	4	32.7	–		1.42	–
$2p-3d$		27.8	8.9	22	10.5	10.5
$2s-2p$	3	40.5	–		1.56	–
$2p-3d$		48.6	7.9	21	21.6	21.4
$2s-2p$	2	42.9	–		1.56	–
$2p-3d$		47.7	3.9	20	33	32.3

In Table III we present the results of dielectron recombination rate calculations for oxygen and iron ions, and illustrate the effect of the additional auto-ionization decay channels. As can be seen from Table III, allowance for the additional channels leads to a significant decrease in the dielectron recombination rate. And the smaller the atomic number and degree of ionization of the ion under consideration, the more strongly this effect is manifested. Thus, for example, for the $2p-3d$ transition of the OIV and FeXXII ions, the additional channels cause the dielectron recombination rate for oxygen to decrease by a factor of three, while they do not have any effect in the case of iron. This is explained by the fact that those values of n that make a contribution to the dielectron recombination rate are higher in the case of oxygen ions than in the case of iron ions. The additional $\alpha nl - \alpha' E\lambda$ auto-ionization decay channels actually reduce the contribution of the high n ($n > ((Z-1)^2 \mathrm{Ry}/E_{\alpha\alpha'})^{1/2}$). Therefore, their effect shows up more strongly in the case of oxygen ions than in the case of iron ions.

The various dielectron recombination rate calculations for the FeXVII ion (the $2p-3d$ transition) are compared in Fig. 1. As can be seen from Fig. 1, the analytical Burgess formula,[1] which does not take the individual characteristics of the recombining ion into consideration, gives a result that is higher by a factor of one and a half than the result obtained in our calculation, in which only one (i.e., the $2p^5 3dnl - 2p^6 E\lambda$) auto-ionization decay channel is taken into account. Allowance for the additional channels for auto-ionization decay to the levels $2p^5 3s$ and $2p^5 3p$ decreases by more than an order of magnitude the contribution to the dielectron recombination rate of the states $2p^5 3dnl$ with $n \geqslant 8$. This leads to a roughly 30% decrease in the total dielectron recombination rate as compared to the results[2)] obtained in Ref. 4.

3. EFFECT OF COLLISIONS ON THE DIELECTRON RECOMBINATION RATE

In the case of an extremely tenuous plasma, which is considered above, the ion $X^{*}_{Z-1}(\gamma')$ (we assume for simplicity that $\gamma' = \alpha_0 nl$) produced as a result of the process (1) will decay to the ground state $X_{Z-1}(\gamma_0)$ with a probability equal to one. In a real plasma it is necessary to further take into consideration the possibility of ionization by outer electrons. This can be done by introducing into the formula (10) a dimensionless factor

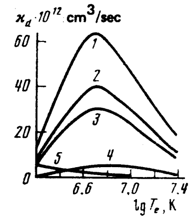

FIG. 1. Comparison of the dielectron recombination rates $\varkappa_d$ for the Fe XVII ion (the $2p-3d$ transition). 1) Calculation with the Burgess analytical formula,[1] 2) and 3) calculation without, and with, allowance for additional auto-ionization decay channels (present paper), 4) calculations carried out in Ref. 4 with allowance for additional decay channels, 5) the photorecombination rate.

$j(\alpha_0 nl) \equiv j_n \leqslant 1$, which represents the probability that the ion will make a transition into the ground state γ_0 from the state γ'. In the formula (10), instead of the old expression for B_d, we shall now have

$$B_d = C\frac{Z-1}{n_1^4} f_{\alpha_0\alpha} \sum_{n \geqslant n_1} B_n j_n.$$

If the main process competing with the radiative decay downwards were direct ionization by electron impact, then the factor j_n would have been equal to

$$j_n = A(\gamma', \gamma_0)/(A(\gamma') + N_e\langle v\sigma_i\rangle),$$

where $\langle v\sigma_i\rangle$ is the ionization rate. But there also occurs step-by-step ionization, which, in the case of large n, is more effective than the direct ionization. In the general case the factor j_n can be expressed in terms of the Green function G_{nn_p} of the balance equations for the highly excited levels:

$$j_n = \sum_{n'} A(\alpha_0 n', \gamma_0) G_{n'n},$$

$$\sum_{n' \neq n} A(\alpha_0 n, \alpha_0 n') G_{nn_p} + N_e(\langle v\sigma_{nn'}\rangle + \varkappa_i(n)) G_{nn_p} - A(\alpha_0 n', \alpha_0 n) G_{n'n_p} - N_e\langle v\sigma_{n'n}\rangle G_{n'n_p} = \delta_{nn_p}, \tag{11}$$

where $\Sigma_{n'}$ denotes summation over all the states of the discrete spectrum of the ion X_{Z-1}, $A(\alpha_0 n', \alpha_0 n)$ and $N_e\langle v\sigma_{n/n}\rangle$ are the probabilities for radiative and collision-induced transitions (if $n' \leqslant n$, then $A(\alpha_0 n', \alpha_0 n = 0)$, $\varkappa_i(n')$ is the rate of ionization by electron impact, and δ_{nn_p} is the Kronecker symbol.

The Kramers approximation was used in the computation of the quantities $A(\alpha_0 n', \alpha_0 n)$, while the Born approximation was used in the computation of the quantities $\langle v\sigma_{n/n}\rangle$ and $\varkappa_i(n')$. (These approximations are analyzed in Ref. 8.) By using the analytical expression for the Green function given in Ref. 8, we can show that for low N_e, $j_n \sim 1 - C_1 x$, while in the limiting case of large N_e, $j_n \sim C_2 x^{-5/7}$, where $x = (N_e n^7/10^{16} Z^7)\sqrt{\beta}$ and C_1 and C_2 are some constants. With allowance for these limiting cases, the general approximate formula for the factor j can be represented in the form

$$j_n = \left(1 + \frac{ax}{1 + bx^{5/7}}\right)^{-1}. \tag{12}$$

A satisfactory agreement with the numerical computations of the factor j_n is obtained if we set $a = 0.247$ and $b = 0.537$ (Fig. 2). For comparison, we also show in Fig. 2 the factor j_n without allowance for the step-by-step ionization.

In Ref. 4 the effect of the electron collisions on the dielectron recombination rate was taken into account by cutting off the sum over n in the formula (3) at some n_c. The value of n_c is determined from the condition that the probabilities for collision-induced transitions between neighboring levels be equal to the probabilities for the radiative processes. In Fig. 3 we compare the values of n_c as a functions of Z from Refs. 4 and 6 with those values of n for which the factor j_n (12) is equal to $\frac{1}{2}$. It can be seen that the effect of the electron collisions is overestimated in Ref. 4, but underestimated in Ref. 6.

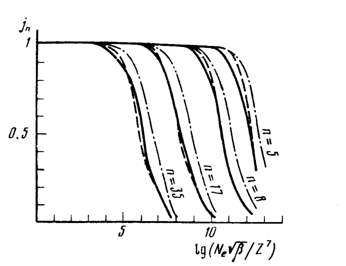

FIG. 2. Effective probability, j_n, for transition from the level n into the ground state. The solid curves are plots of the approximation (12), the dashed curves are the results of a numerical calculation, and the dot-dash curves are the results of calculations in which no allowance was made for the step-by-step ionization.

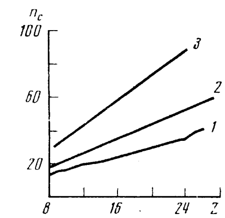

FIG. 3. Comparison of the effective collision limits n_c for $N_e = 10^{14}$ cm^3 T/Z^2 Ry = 0.1. 1) Taken from Ref. 4, 2) n_c obtained from the condition $j_n = 1/2$ (the present paper), 3) n_c from the condition $j_n = 1/2$ (Ref. 6).

In Fig. 4 we present the results of dielectron recombination rate calculations for hydrogenlike ions as functions of $N_e Z^{-7}\sqrt{\beta}$. As can be seen from Fig. 4, the smaller Z is, the greater is the effect of the collisions with electrons on the dielectron recombination rate.

The situation for the recombination of ions with an L shell turns out to be more complex. On the one hand, in this case the higher n values contribute to the dielectron recombination process, and this leads to the strengthening of the dependence on density (especially for the $2s-2p$ transitions). On the other hand, allowance for the additional auto-ionization channels reduces the role of the large n, i.e., leads to an effective increase in the density starting from which the effect of the electron collisions come into play. In Fig. 5 we show the density dependence of the dielectron recombination rates for the $2s-2p$ and $2p-3d$ transitions in the O IV and FE XXII ions.

4. CONCLUSION

The extrapolation formula (6) used in Refs. 2-4 and 7 is valid for $n \geqslant 4$. The dipole approximation[5,6] is applicable for $l > 3$, overestimating the auto-ionization decay probability for low l values by more than an order of magnitude. Simple analytical expressions, valid for $l > 3$, have been obtained in the dipole approximation for the auto-ionization decay probabilities.

Dielectron recombination rate calculations are quite tedious even if we use the extrapolation formula (6) for

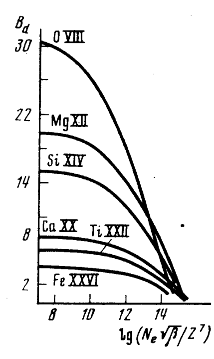

FIG. 4. The coefficients B_d in the formula, (3), for the dielectron recombination rate for hydrogenlike ions; $\beta = Z^2$Ry/T.

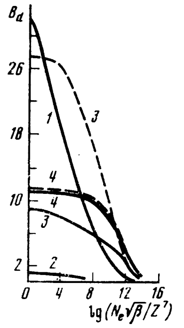

FIG. 5. Density dependence of the coefficients B_d in the formula, (3), for the dielectron recombination rate. 1) The $2p-2p$ transition, O IV, 2) the $2s-2p$ transition, O IV (the solid curve is the result of the computation with allowance for additional channels), 4) the $2p-3d$ transition, Fe XXII the solid and dashed curves pertain to the same types of calculation as in the case 3).

the auto-ionization probabilities. Therefore, the development of approximate methods of describing the auto-ionization probabilities and the derivation of simple asymptotic formulas of the type (8), that are valid for any $\alpha_0-\alpha$ transition, with no limitations imposed on n and l, are essential.

In the case of the recombination of ions with an L shell, it is necessary to take the additional auto-ionization decay channels into account. The smaller the atomic number and degree of ionization of the ion under consideration, the stronger the influence of these channels.

In real physical objects, it is necessary to take into consideration the dependence of the dielectron recombination on the density. In ions with an L shell and $Z = 2-3$, this dependence for the $2s-2p$ transitions turns out to be important even for low densities $N_e \sim 10^3-10^5$ cm^{-3} (planetary nebulae). In the density range from 10^{13} to 10^{15} cm^{-3}, which is characteristic of the tokamak, the dielectron recombination rate for transitions involving a change in the principal quantum number ($2s-3p$, $2p-3d$), and occurring in ions with $Z \leq 10$, is also affected by collisions. Of all the high-temperature objects the laser plasma and the vacuum spark possess the highest density ($N_e \gtrsim 10^{19}$ cm^{-3}). Under these conditions the dielectron recombination corresponding to the $2s-2p$ transitions can, in general, be neglected; for $2s-3p$- and $2p-3d$-type transitions of ions with $Z \lesssim 30$ the dielectron recombination rate decreases appreciably, while for $Z \lesssim 20$ the density dependence begins to have an effect even for transitions of the type $1s-2p$.

Unfortunately, a direct comparison of the theoretical dielectron recombination rates with experiment is impossible at present. We can obtain only indirect information from a comparison of the theoretical calculations with the spectra, simultaneously observed in a plasma, of ions of different degrees of ionization. These calculations include ionization equilibrium calculations (the ionization equilibrium being determined by the dielectron-recombination, photorecombination, and ionization rates) and line-intensity calculations, for which data on the excitation rates and the decay probabilities are necessary. Such a comparison with the K spectra for iron ions with $Z = 18$-25 in the tokamak[12] indicates satisfactory agreement of the theoretical calculations with experiment for temperatures of 800-1000 eV. For higher temperatures (1200-1500 eV) the experimental intensities of the satellites corresponding to ions of low degree of ionization turn out to be higher than the theoretical intensities, a discrepancy which may be due to the nonisothermy of the plasma.

[1)]The auto-ionization decay process as a result of which the electron returns to the continuous spectrum is an alternative channel for the process (1).

[2)]The causes of the discrepancy are not quite clear. But if we carry out the computation under the assumption that the auto-ionization decay of the $2p^53dnl$ states to the $2p^53s$ and $2p^53p$ levels is possible for any n, then we obtain results close to the results obtained in Ref. 4.

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