"Slow oscillations" of magnetic filaments

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(Submitted 30 July 1980)
Zh. Eksp. Teor. Fiz. **80**, 1038–1046 (March 1981)

A study is made of one kind of oscillations of magnetic filaments which are of interest in connection with energy transfer in a plasma along magnetic filaments. It is shown that taking the inhomogeneity of the plasma parameters inside the filaments into account leads to a strong damping of these oscillations, so that estimates of the transfer of mechanical energy along the magnetic filaments must be based mainly on the flexural oscillations of the filaments.

PACS numbers: 85.70. – w

1. INTRODUCTION

Under astrophysical and laboratory conditions one often meets with a situation where the whole of the magnetic flux is concentrated in separate thin and widely spaced tubes (called magnetic filaments), whereas outside these tubes the magnetic field is weak or completely absent. In particular, such a situation is typical for the atmosphere of the sun[1-3] where the oscillations of the magnetic filaments are apparently an important agent which guarantee the transfer of mechanical energy from the lower layers of the photosphere to the chromosphere. Under laboratory conditions a "mixture" of a plasma with magnetic filaments can arise near the boundary of the plasma with the magnetic field when the flute instability develops.

In connection with the problem of energy transfer along the filaments, the long-wave oscillations with a longitudinal wave vector k which is small compared to the reciprocal of the filament radius R^{-1} are of most interest: such oscillations have usually a relatively small damping rate and are easily excited by the large-scale plasma motions. Long-wave flexural oscillations of magnetic filaments have been studied before[4]; oscillations of an ensemble of such filaments were studied at the same time. The effect of a plasma inhomogeneity on the damping of flexural oscillations was elucidated in Ref. 5.

Defouw[6] indicated an interesting kind of oscillations of a magnetic filament. In his paper he dealt with axially symmetric oscillations in which the sum of the gaskinetic and magnetic pressures was not perturbed: the compression (rarefaction) of the plasma along the filament is compensated by the decrease (increase) in the longitudinal magnetic field due to a corresponding change in the cross section of the filament. We shall call these oscillations slow as they are, in essence, some kind of analog to the slow magnetosonic waves. In that sense the normal acoustic oscillations of a filament in which the gaskinetic and magnetic pressures change in phase are the analog of the fast magnetosonic waves, and at a wavelength comparable with the slow longitudinal waves they have a much higher frequency.

An important feature of the slow oscillations (however, not mentioned in Defouw's paper[6]) is connected with the fact that one can check that the absence of perturbations of the total pressure leads to the practical vanishing of the "radiative" damping of the oscillations which is connected with the emission of sound waves into external space. In that sense the slow oscillations differ strongly from the axially symmetric acoustic oscillations for which the radiative damping rate is comparable with the frequency and which hence can, in fact, not propagate along filaments (cf. Ref. 4). Slow oscillations, if they were to exist, could thus be considered to be an important agent guaranteeing the transfer of energy along the filaments.

However, it follows from qualitative considerations[7] that the slow oscillations must be very sensitive to inhomogeneities in the plasma and in the magnetic field over the cross section of the filament. As these inhomogeneities certainly exist under realistic conditions, it is necessary to consider their role in any plausible model of the slow oscillations. A study of this problem is the basic content of the present paper.

The analysis given in the present paper has shown that under conditions when the relative drop in the plasma density and/or magnetic field over the filament radius exceeds the small amount $\varepsilon \equiv (kR)^2$ (k is the wave vector of the oscillations and R the radius of the filament) the slow oscillations vanish. Apparently, this result means, in practice, that the slow oscillations can not be effective transfer agents for the energy, so that energy transfer along the filament can occur only through the flexural oscillations which were studied earlier.[4] It is also interesting to note that the damping mechanism observed here is in essence different from the absorption effect in a resonance point[8] and is connected with the fact that radiative losses again become significant when there are plasma inhomogeneities present.

2. EQUILIBRIUM STATE

We introduce a cylindrical system of coordinates (r, φ, z) with the z-axis directed along the unperturbed magnetic field. We assume the unperturbed state to be axially symmetric and uniform in z. In other words, the unperturbed density ρ, the pressure P, and the magnetic field $\mathbf{H}(0, 0, H)$ depend solely on r. Bearing in mind that the field inside the filament is much stronger than outside, we shall simply assume the ex-

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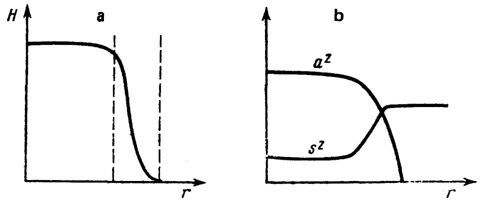

FIG. 1.

ternal magnetic field to be equal to zero (this is not a crucial assumption; one can show that taking the external magnetic field into account does not affect the final result). We note that Defouw in his paper assumed that the magnetic field, the density, and the pressure inside the filament were uniform and that the filament had a sharp boundary. We shall, on the other hand, consider a more realistic situation and assume that the magnetic field decreases smoothly from a maximum value on the axis of the filament to zero as $r\to\infty$ (Fig. 1a). We shall assume that the plasma density and pressure are also smooth functions of r.

The condition for equilibrium of the plasma gives a unique relation between the functions $H(r)$ and $P(r)$:

$$P(r)+H^2(r)/8\pi=P_*, \qquad (1)$$

where P_* is the pressure at large distances from the filament. We can express the local sound velocity $s(r)$ and the local Alfvén speed $a(r)$ in terms of the functions $P(r)$, $\rho(r)$, and $H(r)$:

$$s^2(r)=\gamma P(r)/\rho(r), \quad a^2(r)=H^2(r)/4\pi\rho(r),$$

where γ is the adiabatic index. The functions $s(r)$ and $a(r)$ are related through the equation

$$s^2(r)+\gamma a^2(r)/2=s_*^2\rho(r)/P_*$$

(we indicate by asterisks the values of the corresponding quantities at large distances from the filament). For the particular case when $\rho(r)=\text{const}=\rho_*$, we show the characteristic behavior of $s^2(r)$ and $a^2(r)$ in Fig. 1b.

3. EQUATIONS FOR SMALL FILAMENT OSCILLATIONS

We shall start from the linearized equations from single-fluid magnetohydrodynamics:

$$\rho\frac{\partial\mathbf{v}}{\partial t}=\frac{1}{4\pi}[\operatorname{rot}\mathbf{h}\times\mathbf{H}]-\operatorname{grad}\delta P, \qquad (2)$$

$$\frac{\partial\mathbf{h}}{\partial t}=\operatorname{rot}[\mathbf{v}\times\mathbf{H}], \qquad (3)$$

$$\frac{\partial\delta\rho}{\partial t}+\operatorname{div}\rho\mathbf{v}=0, \qquad (4)$$

$$\frac{\partial\delta F}{\partial t}+\mathbf{v}\operatorname{grad}F=0. \qquad (5)$$

Here $F=P\rho^{-\gamma}$, while $\mathbf{v}$, $\mathbf{h}$, $\delta\rho$ and δP are the perturbations of the velocity, magnetic field, matter density, and pressure:

$$\delta F=\rho^{-\gamma}\delta P-\gamma\rho^{-\gamma-1}P\delta\rho. \qquad (6)$$

We consider perturbations which are independent of φ and which propagate along the z-axis with a wave vector k and frequency ω, i.e., we shall assume that $\mathbf{h}$, $\delta\rho$, δP, and $\mathbf{v}$ change in proportion to $\exp(-i\omega t+ikz)$. For such perturbations we get from Eqs. (2)

$$-i\omega\rho\left(1-\frac{k^2a^2}{\omega^2}\right)v_r=-\frac{\partial}{\partial r}\left(\delta P+\frac{Hh_z}{4\pi}\right), \qquad (7)$$

$$-i\omega\rho v_z=\frac{h_r}{4\pi}\frac{\partial H}{\partial r}-ik\delta P. \qquad (8)$$

Equation (3) gives

$$h_r=-\frac{kH}{\omega}v_r, \quad h_z=\frac{1}{i\omega}\frac{1}{r}\frac{\partial}{\partial r}(rv_rH), \qquad (9)$$

while Eqs. (4) and (5), if we use (6), take the form

$$-i\omega\delta\rho+\frac{1}{r}\frac{\partial}{\partial r}r\rho v_r+ik\rho v_z=0, \qquad (10)$$

$$\delta P-\gamma\frac{P}{\rho}\delta\rho=\frac{v_r}{i\omega}\left(\frac{\partial P}{\partial r}-\gamma\frac{P}{\rho}\frac{\partial\rho}{\partial r}\right). \qquad (11)$$

We eliminate $\delta\rho$ from these two equations and substitute the expression for δP thus obtained into Eq. As a result we get

$$-i\omega\rho v_z=-\frac{k}{\omega}v_r\frac{\partial}{\partial r}\left(\frac{H^2}{4\pi}+P\right)-\frac{k}{\omega}\gamma P\times\left[\frac{1}{r}\frac{\partial}{\partial r}rv_r+ikz\right].$$

The first term on the right-hand side vanishes due to the filament equilibrium condition (1) and then this equation takes the form

$$i\omega\left(1-\frac{k^2s^2}{\omega^2}\right)v_z=\frac{ks^2}{\omega}\frac{1}{r}\frac{\partial}{\partial r}rv_r. \qquad (12)$$

In order to obtain an expression for v_r we transform the expression on the right-hand side of Eq. (7), using (9), (10), (11), and the equilibrium condition (1), to the form

$$\delta P+\frac{Hh_z}{4\pi}=\rho\frac{s^2+a^2}{i\omega}\frac{1}{r}rv_r+\frac{s^2}{\omega}k\rho v_z.$$

Furthermore, substituting here the expression for v_z from Eq. (12) we find

$$\delta P+\frac{Hh_z}{4\pi}=\frac{\rho}{i\omega}\frac{\omega^2(s^2+a^2)-k^2s^2a^2}{\omega^2-k^2s^2}\frac{1}{r}\frac{\partial}{\partial r}rv_r, \qquad (13)$$

and finally we get

$$\frac{\partial}{\partial r}\rho\frac{\omega^2(s^2+a^2)-k^2s^2a^2}{\omega^2-k^2s^2}\frac{1}{r}\frac{\partial}{\partial r}rv_r+\rho(\omega^2-k^2a^2)v_r=0. \qquad (14)$$

This equation describes the small oscillations of the plasma both inside and outside the filament.

4. DISPERSION RELATION

At large distances from the filament when $r\gg R$ and where one can assume the substance to be uniform and the magnetic field to vanish, Eq. (14) is simply the Bessel equation. Bearing in mind that in the region $r\gg R$ there must only be waves going out from the filament (there are no incoming waves) one can verify that here the solution is proportional to a second-order Hankel function (emission condition):

$$v_r=AH_1^{(2)}(qr). \qquad (15)$$

Here $q=(\omega^2/s_*^2-k^2)^{1/2}$ and we choose the branch of the roots with Re $q>0$. It is understood that Re $\omega>0$; when Re $\omega<0$ one would have to put $v_r\sim H_1^{(1)}(qr)$.

At small distances (as compared to k^{-1}) from the filament axis the second term on the left-hand side of Eq. (14) is small compared to the first one (one sees easily that the small parameter is k^2R^2). We can therefore easily construct an approximate solution in that region. Neglecting the second term in the zeroth approximation we find that

$$\rho\frac{\omega^2(s^2+a^2)-k^2s^2a^2}{\omega^2-k^2s^2}\frac{1}{r}\frac{\partial}{\partial r}rv_r=B, \quad (16)$$

where B is a constant.

From the condition that there be no singularity as $r\to 0$ we find

$$v_r=\frac{B}{r}\int_0^r\frac{(\omega^2-k^2s^2)r'dr'}{\omega^2(s^2+a^2)-k^2s^2a^2}. \quad (17)$$

This solution is valid for any r-dependence of ρ and H. When we evaluate the integral (17) it follows, in accordance with the causality principle, that ω has a positive imaginary part, $\mathrm{Im}\,\omega>0$.

One can take the last term in Eq. (14) for the determination of v_r in the region $r\ll k^{-1}$ into account in the framework of perturbation theory. One can, however, check that the corrections which then appear are unimportant.

When $R\ll r\ll k^{-1}$ Eqs. (15) and (17) have an overlapping region of applicability and in that region we can join the solutions; this enables us to obtain the dispersion relation. To really effect the joining we consider the behavior of the solution (17) in the region $r\gg R$. Here $a=0$ and $s=s_*=\mathrm{const}$, i.e., the integrand changes in proportion to r. Hence, the main term in the asymptotic form of v_r is proportional to r when $r\gg R$. To find the next term in the expansion we add to and subtract from the expression for v_r the term

$$\int_0^r\frac{1}{\rho}\frac{\omega^2-k^2s_*^2}{\omega^2s_*^2}r'\,dr',$$

after which we get

$$v_r=\frac{B}{r}\left\{\int_0^r\left[\frac{1}{\rho}\frac{\omega^2-k^2s^2}{\omega^2(s^2+a^2)-k^2s^2a^2}-\frac{\omega^2-k^2s_*^2}{\rho_*\omega^2s_*^2}\right]r'dr'+\frac{\omega^2-k^2s_*^2}{\rho_*\omega^2s_*^2}\frac{r^2}{2}\right\}.$$

The expression inside the square brackets decreases fast at large r and, hence, we have for $r\gg R$

$$v_r=\frac{Br}{s_*^2}\left[\frac{1}{\rho_*}\frac{\omega^2-k^2s_*^2}{2\omega^2}+\frac{D(\omega)}{r^2}\right], \quad (18)$$

where $D(\omega)$ is a quantity defined by the formula

$$D(\omega)=s_*^2\int_0^\infty\left[\frac{1}{\rho(s^2+a^2)}\frac{\omega^2-k^2s^2}{\omega^2-k^2u^2}-\frac{1}{\rho_*s_*^2}\frac{\omega^2-k^2s_*^2}{\omega^2}\right]r'dr', \quad (19)$$

while

$$u^2=\frac{a^2s^2}{a^2+s^2}.$$

We have thus found the next term in the asymptotic expression.

In accordance with the causality principle we must put $\mathrm{Im}\,\omega>0$ when evaluating $D(\omega)$. In the region $\mathrm{Im}\,\omega<0$ the function $D(\omega)$ is determined by means of the analytical continuation procedure. Using now an approximate expression for the Hankel function for small values of the argument,

$$H_1^{(2)}=qr/2-2i/\pi qr \quad (20)$$

and equating the expressions obtained from Eqs. (18) and (20), we find the dispersion relation:

$$D(\omega)=-2is_*^2/\pi\rho_*\omega^2. \quad (21)$$

The integrand in Eq. (19) for $D(\omega)$ is non-vanishing only when $r\lesssim R$, i.e., $D(\omega)\sim R^2$. As we assume that $R\ll k^{-1}$, the ratio of the left-and right-hand sides of Eq. (21), assuming that ω/ks, $\omega/ka\sim 1$, turns out to be of the order of magnitude of $k^2R^2\ll 1$. Therefore, in the class of solutions with ω/ks, $\omega/ka\sim 1$ (and the slow waves belong just to that class) one can satisfy condition (21) only through a special choice of ω, such that the denominator $\omega^2-k^2u^2$ be close to zero. That fact by itself shows the great sensitivity of the result to the inhomogeneity of the plasma, which will be confirmed by further analysis.

5. STUDY OF THE DISPERSION RELATION

First we use Eq. (21) to reproduce Defouw's result for a stepwise distribution of the plasma parameters (uniform filament with a sharp boundary). In that case

$$D(\omega)=\frac{s_*^2R^2}{2(s^2+a^2)}\frac{\omega^2-k^2s^2}{\omega^2-k^2u^2},$$

and we have from (21)

$$z(z-s^2/u^2)/(z-1)=-iQ/k^2R^2, \quad (22)$$

$$z=\omega^2/k^2u^2, \quad Q=4s_*^2a^2/\pi u^4. \quad (23)$$

As there stands a large quantity ($Q\sim 1$, $kR\ll 1$) on the right-hand side of Eq. (22) one can easily find both solutions of this equation. One of them corresponds, in agreement with what was said at the end of the preceding section, to an almost exact vanishing of the demoninator of the left-hand side:

$$z_S\approx 1-i\frac{k^2R^2}{Q}\left(\frac{s^2}{u^2}-1\right),$$

and the second ($|z|\gg 1$)

$$z_F\approx -iQ/k^2R^2.$$

The indexes S and F indicate that the solutions found correspond to the slow and fast magnetosonic oscillations.

Using the remark following Eq. (15) we find ω_S and ω_F from the expressions for z_S and z_F:

$$\omega_S\approx ku\left\{1-i\frac{k^2R^2}{2Q}\left(\frac{s^2}{u^2}-1\right)\right\}, \quad (24)$$

$$\omega_F\approx\frac{1}{\sqrt{2}}\frac{u}{R}(1-i). \quad (25)$$

The real part of ω_S is the same as Defouw's result while the imaginary part describes the radiative damping (an effect neglected by Defouw). It is clear that when $kR\ll 1$ the radiative damping is, indeed, small.[1)] As to the fast oscillations they have a very high frequency and turn out to be rapidly damped. We note that for them $q\sim R^{-1}$ so that for them the expansion (20) is not valid; equation (25) for ω_F has therefore only a qualitative meaning.

In order to formulate the conditions under which the plasma inhomogeneity starts to show an appreciable effect on the slow oscillations we consider a model situation when the quantity

$$u^2=\frac{s^2a^2}{s^2+a^2}$$

changes along the radius of the filament by a small amount ε:

$$u^2=\begin{cases} u_0^2(1-\varepsilon r^2/R^2), & r<R \\ 0, & r>R \end{cases}.$$

It is clear that when $\varepsilon \ll 1$ the remaining quantities in the integral for D are independent of r inside the tube. We thus have for D

$$D(z)=\frac{R^2}{2}\frac{u_0^2 s_*^2(z-a_0^2/u_0^2)}{s_0^2 a_0^2}\int_0^1\frac{d\zeta}{z-1+\varepsilon\zeta},$$

where z is defined by Eq. (23). This integral determines an analytical function $D(z)$ in the complex z-plane with a cut between the points $z=1-\varepsilon$ and $z=1$ (Fig. 2):

$$D(z)=\frac{R^2}{2}\frac{u_0^2 s_*^2}{a_0^2 s_0^2}\frac{z-a_0^2/u_0^2}{\varepsilon}\ln\frac{z-1+\varepsilon}{z-1}.$$

The imaginary part of the logarithm changes between the limits $-i\pi$ and $i\pi$.

In the framework of the model considered the dispersion relation has the form

$$z\left(z-\frac{a_0^2}{u_0^2}\right)\ln\frac{z-1+\varepsilon}{z-1}=-iQ\frac{\varepsilon}{k^2R^2}. \qquad (26)$$

As $\varepsilon\to 0$ we can perform an expansion in powers of ε on the left-hand side and we are led to the dispersion equation (22), which has the solution $z\approx 1$. One can check that this result is valid when $\varepsilon \ll k^2R^2$. By contrast, when $\varepsilon \gg k^2R^2$ (even though still $\varepsilon \ll 1$) Eq. (26) cannot have a solution $z\approx 1$. Indeed, as we noted earlier, the imaginary part of the logarithm changes in the limits from $-i\pi$ to $i\pi$. Thus, if the absolute magnitude of the right-hand side of (26) is large compared to unity, the modulus of z must also be large compared to unity in order that we can satisfy Eq. (26). We can easily find that solution. When $|z|\gg 1$

$$\ln\left(1+\frac{\varepsilon}{z-1}\right)\approx\frac{\varepsilon}{z},$$

and from (26) we get $z=-iQ\varepsilon/k^2R^2$, i.e., a solution with $|z|\gg 1$, corresponding to a fast wave [cf. (25)].

The solution corresponding to a slow wave exists thus when $\varepsilon \lesssim k^2R^2$, while it disappears when $\varepsilon \gtrsim k^2R^2$. We now find the critical value of ε for which the solution vanishes. To do this we construct the logarithm of the function

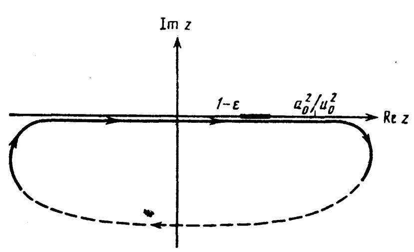

FIG. 2.

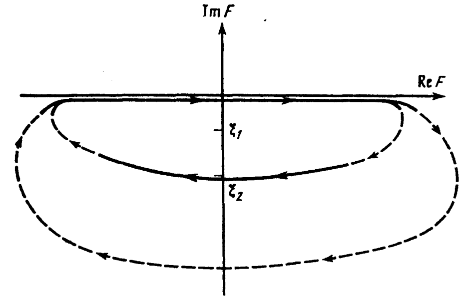

Fig. 3. $\xi_1=-iQ\varepsilon/k^2R^2$, $\xi_2=i\pi(1-\varepsilon/2)(1-\varepsilon/2-a_0^2/u_0^2)$

$$F(z)=z\left(z-\frac{a_0^2}{u_0^2}\right)\ln\frac{z-1+\varepsilon}{z-1},$$

mapping the contour shown in Fig. 2 onto the complex F-plane. In accordance with the Nyquist criterion the number of roots of Eq. (26) lying in the lower z-half-plane[2)] is equal to the number of times the hodograph of F goes round the point $-iQ\varepsilon/k^2R^2$. We show the shape of the hodograph in Fig. 3. It is clear from the figure that the critical value of ε is determined from the relation

$$\pi\left(1-\frac{\varepsilon}{2}\right)\left(\frac{a_0^2}{u_0^2}+\frac{\varepsilon}{2}-1\right)=Q\frac{\varepsilon}{k^2R^2}.$$

When $kR\ll 1$ it follows from this that

$$\varepsilon_{cr}=\frac{\pi k^2R^2}{Q}\left(\frac{a_0^2}{u_0^2}-1\right).$$

6. CONCLUSION

Physically, the origin of the effect considered in the present paper is connected with the fact that an inhomogeneity in the plasma parameters leads to an impossibility to satisfy the condition that nowhere across the cross section of the tube [except that point where the local value of u equals ω/k; see Eq. (14)] is the total pressure perturbed only a little; therefore, in the case of an inhomogeneous filament the radial pulsations of the surface become large and cause a catastrophic growth of the radiative losses. In that sense the effect differs strongly from the one considered earlier[5] of the absorption of flexural oscillations of a filament in the Alfvén resonance point $\omega=ka$ (see also Timofeev's review article[8]).

As it is difficult to expect that the plasma inside a magnetic filament would be uniform to a high degree, we are led to the conclusion that in actual fact the slow oscillations of a magnetic filament do not exist. For estimating the transfer of mechanical energy along magnetic filaments one must thus base oneself solely upon flexural (and, maybe, torsional) oscillations of the filaments.

[1)]The statement in our earlier paper[4] that there are no weakly damped oscillations with $m=0$ is connected with the fact that in that paper right from the start we studied the case of a zero-temperature plasma inside the filaments ($s=0$); in that case $u=0$ and the slow oscillations disappear indeed.

2)As it is clear that the system is stable, we are interested only in roots with Im $z<0$.

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Translated by D. ter Haar.