Quantum emission of Kerr-type naked singularities

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(Submitted 23 September 1980)
Zh. Eksp. Teor. Fiz. **80**, 1261–1270 (April 1981)

The emission of bosons and fermions by a naked Kerr singularity is investigated. It occurs because of pair production in the gravitational field near the singularity. This process leads to "dressing" of the singularity, which is thus transformed into a black hole.

PACS numbers: 95.30.Sf, 95.30.Cq, 97.60.Lf

1. INTRODUCTION

If the Universe contains any naked singularities, i.e., singularities that are not hidden from a distant observer by an event horizon, then in the strong gravitational fields surrounding them the quantum effect may be so strong as to play an important part in the formation and evolution of the singularity. The strength of the effects depends strongly on the type of singularity. For example, the quantum emission accompanying the contraction of a charged spherical shell is large and prevents formation of a Reissner–Nordström singularity,[1] whereas there is little such emission when a long tube contracts into a filament, and the emission does not prevent the formation of a naked singularity of Kasner type.[2] In the present paper, we investigate the quantum emission of Kerr-type naked singularities, which again does not prevent their formation. However, the steady production of pairs in the field of such a singularity leads to a flux of quantum radiation from the singularity. As a result, the singularity loses mass and angular momentum.

Naturally, the singularity may emit particles of any species. However, if particle–antiparticle pairs are to be produced, there must be levels with negative energy for particles rotating around the singularity. For singularities with masses on the astronomical scale, such levels exist only for massless particles. Therefore, we shall be primarily interested in such particles.

In this paper, to estimate the intensity of the radiation and the variation in time of the parameters of the singularity due to the emission of actually existing massless particles, for example, photons, we consider the model problem of the emission of massless scalar particles. We also consider the variant in which these particles satisfy Fermi–Dirac statistics in order to estimate the emission of massless fermions, and also neutrinos, which according to the latest data may have a mass but one so small as to be ignorable in the present context. In fact, the emission of fermions becomes much weaker than the emission of bosons, which plays the principal part. In a later paper, I intend to investigate the quantum emission of photons and gravitons by a Kerr singularity, but there are grounds for hoping that the result obtained for massless scalar particles will be a good estimate for such emission, repeating the situation established in the calculation of the superradiance intensity.

2. STATES WITH NEGATIVE ENERGY

We consider the quantum emission of massless scalar particles from a Kerr naked singularity described by the metric

$$ds^2=\frac{\Delta}{\rho^2}(dt-a\sin^2\theta d\varphi)^2-\frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\varphi-adt]^2-\frac{\rho^2}{\Delta}dr^2-\rho^2d\theta^2, \qquad (1)$$

$$\rho^2=r^2+a^2\cos^2\theta,\qquad \Delta=r^2-2Mr+a^2,\qquad a^2>M^2,$$

where the source has mass M and angular momentum $J=Ma$. It has a singularity at $r=0$. A suitable coordinate transformation makes it possible to recognize the topological form of this singularity.[3] It is found that the singularity has the form of a ring, and through its interior one can continue the metric to an asymptotically flat space with negative values of the radius vector r.

This structure of the Kerr singularity leads to a qualitative difference between the quantum emission resulting from the formation of Kerr and Reissner–Nordström singularities. Thus, if we compress a charged shell with charge greater than its mass, then because of the emission of scalar particles the energy needed to compress it to Planck dimensions appreciably exceeds the mass of the shell.[1] However, as is readily shown by the method used in Refs. 1 and 2, the compression of a rotating shell with angular momentum greater than the square of the mass leads only to weak emission of massless scalar particles, this decreasing to zero when a naked singularity is formed.

Therefore, we must consider other mechanisms of quantum emission. If $-|a|<M<0$, then the metric has two event horizons situated at $r<0$. The superradiance associated with them can be calculated in virtually the same way as in Refs. 4 and 5. Therefore, we shall not consider this case.

The emission of quanta of a massless scalar field investigated in the present paper is associated with the population of the levels investigated in Ref. 6. What kind of levels are these?

The wave equation for the massless scalar field

$$(-g)^{-1/2}\frac{\partial}{\partial x^i}(-g)^{1/2}g^{ik}\frac{\partial}{\partial x^k}\varphi=0 \qquad (2)$$

in the Kerr metric admits separation of the variables.[5] The eigenfunctions can therefore be sought in the form

$$\varphi_{l,m,\varepsilon}(r)=N(\omega)\Delta^{-1/2}R_{l,m,\varepsilon}(r)S_{lm}(a\varepsilon,\ \cos\theta)e^{im\varphi}e^{-i\varepsilon t}, \qquad (3)$$

 0038-5646/81/040645-05$02.40

where N is a normalization constant, and S_{lm} are spheroidal harmonics satisfying the equation

$$\left[\frac{d}{d\xi}(1-\xi^2)\frac{d}{d\xi}-\frac{m^2}{1-\xi^2}+2ma\varepsilon-(a\varepsilon)^2(1-\xi^2)+\lambda_{lm}(a\varepsilon)\right]S_{lm}(a\varepsilon,\xi)=0. \quad (4)$$

The eigenvalues $\lambda_{lm}(a\varepsilon)$ depend in a complicated manner on the quantum numbers l and m and the argument $a\varepsilon$ and have boundary value $\lambda_{lm}(0)=l(l+1)$.

Substituting (3) and (1) in (2), we obtain an equation for the radial function $R_{l,m,\varepsilon}(r)$:

$$[d^2/dr^2-V_{l,m,\varepsilon}(r)]R_{l,m,\varepsilon}(r)=0, \quad (5)$$

$$V=\frac{\lambda+1}{\Delta}-\frac{(r-M)^2}{\Delta^2}-\left(\frac{r^2+a^2}{\Delta}\right)^2\left(\varepsilon-\frac{ma}{r^2+a^2}\right)^2$$
$$=-\left(\frac{r^2+a^2}{\Delta}\right)^2(\varepsilon-\varepsilon_-)(\varepsilon-\varepsilon_+), \quad (6)$$

$$\varepsilon_\pm=\{ma\pm[\lambda\Delta+a^2-M^2]^{1/2}\}/(r^2+a^2). \quad (7)$$

States with $\varepsilon>\varepsilon_+$ describe particles, and with $\varepsilon<\varepsilon_-$ the "Dirac sea" (see Fig. 1). At $\varepsilon_-<\varepsilon<\varepsilon_+$, there is a potential barrier, through which a particle can tunnel. For large l, i.e., large λ, $\varepsilon_+<0$ (for $ma<0$) and $\varepsilon_->0$ (for $ma>0$) in a certain range of distances from the singularity.

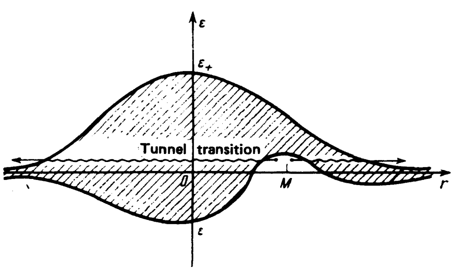
Thus, there exist particle states with negative energy. One can have the process of formation of a pair consisting of a particle in such a state and an antiparticle with positive energy, which may also be in the asymptotically flat region and travel to $r=\infty$ or $r=-\infty$. In Fig. 1, this will be described by the tunneling of particles from the Dirac sea at large r into the potential well, the particles leaving behind them a hole—an antiparticle. Accordingly, the particles in the sea for $\varepsilon_->0$ can tunnel to infinity, leaving behind an antiparticle rotating around the naked singularity. Thus, the singularity will emit a flux of massless scalar particles and antiparticles, losing mass and angular momentum. The present paper is devoted to study of this emission.

3. INTENSITY OF THE EMISSION

In this section, we investigate emission from a Kerr singularity. We shall follow the ideas of Ref. 7. First, we find the energy spectrum of a massless scalar particle in the potential well shown in the figure. For this, we use the quasiclassical approximation

$$2\int(-V)^{1/2}dr=J(\varepsilon_n,l,m)=2\pi(n-1/2), \quad (8)$$

where the integral is taken in the region where $V<0$. We introduce a new variable and the notation



$$\alpha=a/M,\ \beta^2=\alpha^2-1,\ A=\varepsilon M,\ \varphi=(1+\alpha-m\alpha/A)/\beta,$$
$$z=r/\beta M-\beta^{-1}, \quad (9)$$

and then formula (8) takes the form

$$J(\varepsilon_n,l,m)=2\int\left\{\frac{A^2(\beta z^2+2z+\varphi)^2}{(1+z^2)^2}-\frac{\lambda}{1+z^2}-\frac{1}{(1+z^2)^2}\right\}^{1/2}dz. \quad (10)$$

As will be shown below, A is small in all the cases of interest to us, and it can be regarded as a small parameter. Accordingly, φ is a large quantity. Since the region of integration in (10) is at small z, it is easy to show that we can ignore the terms βz^2 and $2z$ in the first term in the radical, which gives

$$J=2\int_{-z_1}^{z_1}\frac{(A^2\varphi-\lambda-1-\lambda z^2)^{1/2}}{1+z^2}dz. \quad (11)$$

Levels with negative energy can exist only if the radicand is positive at $z=0$, i.e., when

$$m^2\alpha^2/\beta^2>\lambda+1. \quad (12)$$

Here, we have used the fact that $A\varphi\approx-m\alpha$.

We can now determine the classical boundaries of the motion. A particle in the well is in the interval from $-z_1$ to z_1, where

$$z_1=x_1^{1/2},\quad x_1\approx\alpha^2m^2/\lambda\beta^2-1-\lambda^{-1}-\chi A\lambda^{-1}, \quad (13)$$

$$\chi=2m\alpha\left\{2\beta^{-2}+2\beta^{-1}\left[\frac{\alpha^2m^2}{\lambda\beta^2}-1-\lambda^{-1}\right]^{1/2}+\frac{\alpha^2m^2}{\lambda\beta^2}-\lambda^{-1}\right\}. \quad (14)$$

Calculating the integral (11) by means of the substitution $y^2=x_1z^{-2}+1$, we obtain

$$J(\varepsilon_n,l,m)\approx2\pi\{[m^2\alpha^2/\beta^2-1-\chi A_n]^{1/2}-\lambda^{1/2}\}. \quad (15)$$

Hence

$$n-1/2=[m^2\alpha^2/\beta^2-1-\chi A_n]^{1/2}-\lambda^{1/2}, \quad (16)$$

$$A_n=\chi^{-1}[m^2\alpha^2/\beta^2-1-(n-1/2+\lambda^{1/2})^2]. \quad (17)$$

This result differs from the one given in Ref. 6. It was obtained there for the special case when the potential well can be approximated by a harmonic oscillator. As a result, the energy spectrum was found under a number of assumptions, one of which (in the notation of the author, it takes the form $\alpha_0\ll K^2l^2$) imposes a very stringent restriction on the parameters of the particle. This condition is not satisfied in the states that are populated in the first place in the case of emission.

The obtained energy spectrum can be used only when the condition of applicability of the quasiclassical approximation holds; in the present case, it has the form

$$x_1M\beta\lambda^{1/2}\gg1. \quad (18)$$

At small β, we have $x_1\sim\beta^{-2}$ and this condition is satisfied. At large β, we have $x_1\sim\lambda^{-1/2}$, and the condition is also satisfied.

We can now calculate the lifetime of quasidiscrete levels in the well:

$$\Gamma^{-1}|\sim|dJ/d\omega|e^{\zeta}, \quad (19)$$

where

$$\zeta=2\int V^{1/2}dr \quad (20)$$

(the integral is taken over the barrier, i.e., in the region where $V>0$). It should be noted that scalar particles are bosons, and for them the time Γ^{-1} is negative (see Ref. 7) and characterizes the time of exponential growth of the number of quanta in the level. However, we shall also consider below a situation for which the quanta of the scalar field are taken to be fermions, doing this, of course, only with a view to estimating the influence of genuine Fermi particles. In this case, Γ^{-1} is positive and can be interpreted as the decay time of the quasidiscrete level.

Finding Γ_{nlm}, we can find the number of field quanta in the potential well:

$$\langle 0|N(t)|0\rangle \approx \sum_{nlm\sigma} \pm[1-\exp(\Gamma_{nlm\sigma}t)], \tag{21}$$

where the upper sign corresponds to fermions, the lower to bosons. For the fermions, we have also introduced summation over the spin index σ.

Note also that in our problem it is possible to have production of pairs with one of the particles rotating around the singularity and the other moving in the asymptotically flat region with $r<0$. In Fig. 1, this corresponds to tunneling through the left-hand potential barrier. Therefore, in what follows we shall place an arrow above the symbol to indicate the transition corresponding to the given lifetime.

In the quasiclassical approximation, we have

$$\vec{\zeta}=2\int_{z_1=x_1^{1/2}}^{z_2} F(z)\,dz, \quad F(z)=\frac{[1+\lambda+\lambda z^2-A^2(\beta z^2+2z+\varphi)^2]^{1/2}}{1+z^2}. \tag{22}$$

We shall not yet determine the upper limit of integration z_2. To obtain $\overleftarrow{\zeta}$, it is necessary to integrate the same function $F(z)$ from $z_3\approx -z_2$ to $z_4\approx -z_1$. If we regard z as a complex variable and describe on the real axis cuts from z_1 to z_2 and from z_3 to z_4, then $\vec{\zeta}$ will be equal to the integral when one passes around the first cut, and $\overleftarrow{\zeta}$ to the integral when one passes around the second. But if we deform the contour surrounding the first cut, it can be transformed into a sum of contours surrounding the second cut and the singularities at $z=i$, $z=-i$, and $z=\infty$. Hence, we have

$$\begin{aligned}\ln\frac{\overleftarrow{\Gamma}}{\vec{\Gamma}}&=\vec{\zeta}-\overleftarrow{\zeta}=2\pi i[\operatorname{res}F(z=i)+\operatorname{res}F(z=-i)+\operatorname{res}F(z=\infty)]\\&=-4\pi A-\pi\{[1-A^2(\varphi-\beta-2i)^2]^{1/2}+[1-A^2(\varphi-\beta+2i)^2]^{1/2}\}\\&\approx 4\pi A\{[1-(\beta/m\alpha)^2]^{-1/2}-1\}\approx -2\pi A\beta^2/m^2\alpha^2.\end{aligned} \tag{23}$$

Thus, we see that there is a more frequent production of pairs having one of their particles "within the singularity" with $r<0$. However, since A is small, the ratio of the lifetimes is virtually equal to unity. Therefore, we shall assume that $\vec{\Gamma}=\overleftarrow{\Gamma}=\Gamma$ and omit the transition arrows.

Let us find Γ. For this, we note that we can ignore the term $2z$ in $F(z)$. It gives only a small correction to the expression, ensuring a small difference between $\overleftarrow{\Gamma}$ and $\vec{\Gamma}$. Omitting it, we obtain

$$\zeta\approx 2A\beta\int_{z_1}^{z_2}\frac{[(z^2-z_1^2)(z_2^2-z^2)]^{1/2}}{1+z^2}dz, \tag{24}$$

where

$$x_1=z_1^2\approx m^2\alpha^2/\lambda\beta^2-1-\lambda^{-1}, \quad z_2\approx\lambda^{1/2}A^{-1}\beta^{-1}. \tag{25}$$

Going over to the variable $x=z^2$, dividing the region of integration into the two intervals (x_1, δ) and (δ, z_2^2), where $(1, x_1)\ll\delta\ll A^{-2}\beta^{-2}$, and making the substitution $y^2=1-x_1x^{-1}$ in the integral over the first interval and $y=xA^2\beta^2$ in the integral over the second interval, we obtain

$$\zeta\approx\lambda^{1/2}\ln\left[\frac{16\lambda}{x_1A^2\beta^2e^2}Q(x_1)\right], \tag{26}$$

where e is the base of natural logarithms, and

$$\begin{gathered}Q(x_1)=\left(\frac{r_1-1}{r_1+1}\right)^{r_1}, \quad r_1=(1+x_1)^{1/2},\\ Q(x_1)\xrightarrow[x_1\to\infty]{} e^{-2}, \quad Q(x_1)\xrightarrow[x_1\to 0]{} x_1/4.\end{gathered} \tag{27}$$

Substituting (26) and (15) in (19), we obtain

$$|\Gamma^{-1}|\sim\frac{32\pi Mm\lambda^{1/2}Q(x_1)[1+\beta^{-2}+(\beta^{-1}+x_1^{1/2})^2]}{e^2A^2\beta^2x_1r_1}e^{\lambda^{1/2}}. \tag{28}$$

Hence, using (21), we can find the intensity of pair production as a function of the parameters of the pairs.

4. BACK REACTION OF THE EMISSION ON THE PARAMETERS OF THE NAKED SINGULARITY

Emitting quanta of the massless scalar field, the singularity loses mass and angular momentum. It can readily be seen from Eq. (21) that

$$dM/dt=-\sum_{n,m,l}\varepsilon_n|\Gamma_n|\exp\{-2\Gamma_{n,m,l}t\}, \tag{29}$$

$$dJ/dt=-\sum_{n,m,l}m|\Gamma_n|\exp\{-2\Gamma_{n,m,l}t\}. \tag{30}$$

The 2 in front of Γ_n in the exponential appears because the quanta are emitted into both the region $r>0$ and the region $r<0$. Although emission of the latter does not change the parameters of the naked singularity for a distant observer, it nevertheless changes the population numbers of the levels. From (29) and (30), we have

$$d\alpha/dt=-M^{-2}\sum_{n,m,l}(m-2\alpha A_n)|\Gamma_n|\exp\{-2\Gamma_nt\}. \tag{31}$$

Using the inequality

$$A_n<[m\alpha-(\lambda+1)^{1/2}\beta]/(1+\alpha^2), \tag{32}$$

which was obtained from the condition $A_n<\varepsilon_-$ for $r=M$, we have

$$m-2\alpha A_n>\beta(1+\alpha^2)^{-1}[2(\chi+1)^{1/2}\alpha-m\beta]>0. \tag{33}$$

It can be seen from this that $d\alpha/dt<0$, i.e., the reaction of the emission can only decrease the parameter α. When it has decreased to unity, the singularity acquires a horizon, and is transformed into a black hole with $a=M$. It is also necessary to show that this transition occurs before the mass of the singularity has decreased to zero.

We consider the case when only the level with parameters A and m is populated. Then from (29) and (31), we have

$$M\frac{dM}{dt}\approx\frac{A}{m}M^2\frac{d\alpha}{dt},\qquad(34)$$

whence

$$M=M_0\exp\left[A(\alpha-\alpha_0)/m\right].\qquad(35)$$

We see that when α decreases to unity, M is still positive. Moreover, the change in the mass is slight. The ratio M/M_0 is minimal for values $\alpha_0\approx 1.25$ (we took the ratio A/m, which is different for different levels, equal to this value for the level with the minimal Γ), for which it is of order 0.98. Therefore, in the following calculations we shall ignore the variation of the mass.

We now consider what levels will be populated fastest. It can be seen from (28) that the characteristic population times Γ^{-1} increase exponentially fast with increasing l. Therefore, we must consider in the first place the levels with the minimal value of l that is possible for the given α. We consider first the case $\beta\ll 1$. For $\beta<0.7$, there exists a level with $l=1$, and this will be populated first. In this limit, Eq. (28) becomes

$$|\Gamma^{-1}|\sim K_nM\beta^{-1},\quad K_n=\frac{K}{1-\beta^2(n+l)^2m^{-2}},$$
$$K=8\gamma^4[1+(1+\gamma^{-1})^2]^3\exp(\lambda^{1/2}-1/2),\quad \gamma=\lambda^{1/2}/|m|.\qquad(36)$$

We see from (17) that for levels with the smallest l the values of A_n are bounded above: $A_{n,l,m}<A^{(0)}_{l,|m|}$, where $A^{(0)}_{11}\approx 0.12$, $A^{(0)}_{21}\approx 0.23$, $A^{(0)}_{22}\approx 0.3$. This confirms for the case $\beta\ll 1$ the assumption that A in the cases in which we are interested can be regarded as a small parameter. Bearing this in mind, we can expand the expression for λ in powers of the small parameter $a\varepsilon=\alpha A$. Since the coefficient of the linear term is small (note that all the coefficients of the powers of the parameter αA obtained in Ref. 8 for the case of a vector field did not exceed unity), we can restrict ourselves to the zeroth term, setting $\lambda=l(l+1)$. Hence, using (36), we can find the coefficient K for any level. For example, $K\sim 7000$ for $l=m=1$, $K\sim 8000$ for $l=m=2$, and $K\sim 5\times 10^4$ for $l=2$, $m=1$. In the well, there are then about $m\beta^{-1}$ levels with the given l, m, which can be seen from (17). However, the levels with $l=m$ and small l are filled first, since they have a much shorter lifetime Γ^{-1} than the others. Therefore, we shall consider only them.

In considering times shorter than Γ^{-1} for these levels, we can expand the exponential in a series and retain the first two terms. Then for both bosons and fermions we find that in levels with $l=|m|$ there are $n(t)$ particles:

$$n(t)\approx 2\int_0^t dt\sum_{i=1}^{m\beta^{-1}}\left(1-\frac{i^2\beta^2}{m^2}\right)^2\beta M^{-1}K^{-1}_{l=|m|}=\frac{16mt}{15MK_{l=|m|}}.\qquad(37)$$

From this, we readily find that the parameters of the singularity vary with time as follows:

$$\beta^2=\beta_0^2-t\frac{32}{15M^3}\sum_{m=1}^{\infty}m^2K^{-1}_{l=|m|},\quad t<K_{l=m=1}M\beta_0^{-1}.\qquad(38)$$

Finding the sum by means of (36), we obtain the estimate

$$\beta^2\approx\beta_0^2-2\cdot 10^{-3}\,M^{-3}t.\qquad(39)$$

If $\beta_0\sim M^{-2/3}$ or a smaller quantity, the singularity can become a black note before the population numbers of the levels become comparable with unity. But if this is not so, we must consider separately the case of bosons and fermions.

Bosons occupy the levels for $t>\Gamma^{-1}_{l=m=1}$ with exponentially increasing rate. Therefore, the main part is played by levels with the smallest Γ^{-1}, i.e., with $l=m=1$. We shall consider only these levels. Assuming, as before, that β is small, we obtain

$$\beta^2\approx\beta_0^2-\frac{16}{15}M^{-2}\beta^{-1}\exp\frac{2\beta t}{K_{l=m=1}M}.\qquad(40)$$

Bearing in mind that the rate of decrease of β increases all the time, we can replace on the right-hand side all the β by β_0, because when β begins to differ appreciably from β_0 the process has virtually ended—its duration is determined basically in the stage in which β hardly changes. Therefore, the time of "dressing" of the naked singularity can be estimated as

$$t_{\text{dress}}\sim\frac{K_{l=m=1}M}{2\beta_0}\ln(M^2\beta_0^3).\qquad(41)$$

The quantum emission of bosons can also transform a singularity with $\beta\gg 1$ into a black hole. In this case, the well contains only states with angular momenta greater than some minimal $l\sim 2\alpha^2$. Because of the exponential growth of the number of particles in the level, we should consider only population of levels with $l=m\sim 2\alpha^2$. In this case $x_1\sim m^{-1}$, and the quasiclassical treatment is also correct. In the limit $\beta\gg 1$, the expression (28) reduces to

$$|\Gamma^{-1}|\sim\frac{80M\alpha^9\exp(2\alpha^2)}{(m-2\alpha^2)^2}\sim 100M\alpha^9\exp(2\alpha^2).\qquad(42)$$

Here, we assume that $m-2\alpha^2\sim 1$. This quantity appeared in the expression (17) from the expression for A_n:

$$A_n<A_1=(m-2\alpha^2)/2\alpha^3\ll 1.\qquad(43)$$

Thus, in this case too $A\ll 1$ for the levels in which we are interested. Due to the emission of bosons in the case $\alpha\gg 1$, this parameter of the singularity changes by

$$\Delta\alpha\approx-\alpha^2M^{-2}e^{2\Gamma t}.\qquad(44)$$

Therefore, the dressing time of the singularity is of order

$$t_{\text{dress}}\sim 50M\alpha_0^9\exp(2\alpha_0^2)\ln(M^2/\alpha_0).\qquad(45)$$

To estimate the time in which the singularity becomes a black hole through the emission of massless scalar particles behaving like fermions (to estimate such a time for neutrinos, for example), we use the following device. We assume that at a certain time all levels are populated that have a definite set of quantum numbers l and m and all values of n that are then possible, the remaining levels being completely empty. We denote by $P=\Sigma m^2$ the sum taken over all pairs (l, m) for the filled levels. Then we have

$$\beta^2\approx\beta_0^2-PM^{-2}\beta^{-1}.\qquad(46)$$

The value of β decreases to zero when P reaches the value

$$P_{\text{crit}} = \beta_0^3 M^2/6\sqrt{3}. \tag{47}$$

For an estimate, we assume that all levels with $l \leqslant l_{\max}$ and all possible values of m are populated. We shall also assume that $l_{\max} \gg 1$, since otherwise the singularity would be transformed into a black hole in a time of order Γ^{-1}. Then we obtain the estimate

$$P_{\text{crit}} \sim l_{\max}^4/6. \tag{48}$$

Hence

$$l_{\max} \sim \beta_0^{3/4} M^{1/2}. \tag{49}$$

Therefore, the singularity "dresses" in a time of order

$$t_{\text{dress}} \sim \Gamma^{-1}_{l=m=l_{\max}} \sim 1000 M \beta_0^{-1} \exp(\beta_0^{3/4} M^{1/2}). \tag{50}$$

Note that this estimate is very rough and certainly an underestimation, since levels with $|m| < l$ have much longer population times. However, it is needed only to estimate the results of a different problem, and therefore we shall not consider this case more accurately, being content if the estimate gives us the correct order of magnitude of t_{dress}. This quantity could be found more accurately, but there is no point in doing so, since in real processes the population of the levels with negative energy is due to photons and gravitons, fermions playing an insignificant part. It was to estimate the smallness of their contribution to the "dressing" of the singularity that we found the approximate expression for t_{dress}.

It is helpful to find the values of the parameters of a naked singularity for which the time of its transition into a black hole does not exceed the time of existence of the Universe, which is estimated at 10^{62} Planck units. Taking the mass of the singularity equal to the solar mass (10^{38} Planck units), we find that during the time of existence of the Universe singularities with parameters $\beta < 10^{-28}$ can "dress themselves" through the emission of massless scalar particles; here, the particles can be assumed to be either bosons or fermions. In addition, for bosons, which true scalar particles are, we obtain one further interval of parameters in which a singularity can "dress itself" through exponential increase in the number of particles in the levels. It extends from $\beta \sim 10^{-20}$ to $\alpha \approx 4$. Thus, we see that the quantum emission can also have astronomical consequences. At the same time, the fermion emission is virtually negligible. For example, a singularity with $\beta = 0.1$, i.e., $\alpha \approx 1.005$, can become a black hole during the lifetime of the Universe only provided its mass does not exceed 2g.

We also note that, through the emission of bosons, naked singularities with $\alpha = 3$ can "dress themselves" during the lifetime of the Universe if their mass does not exceed 6×10^7 solar masses, while singularities with $\alpha = 5$ can do so if their mass does not exceed 3×10^{25} g, which is appreciably less than the mass of the moon. For $\alpha = 6$, we obtain $M < 2 \times 10^{15}$ g, and for $\alpha = 7$ the limiting mass is 7 kg. Since real Kerr singularities, if they exist, will hardly have large values of the parameter m, the dressing process is important for them.

5. CONCLUSIONS

Thus, for the model with scalar bosons and fermions we have found that the quantum emission from Kerr singularities is important. Naturally, the bosons play the main part. This emission leads to a decrease in the ratio a/M, which continues until the ratio reaches unity and the singularity is transformed into a black hole. During the time of this process, its mass hardly changes. An estimate shows that for reasonable parameters of Kerr singularities that could exist in nature the "dressing" time is shorter than the time of existence of the Universe, so that the dressing process can actually take place.

I should like to thank I.M. Khalatnikov and A.A. Starobinskiĭ for valuable discussions.

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Translated by Julian B. Barbour