High-frequency phenomena in metals in multichannel reflection of electrons by a sample boundary

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A theory is constucted of high-frequency phenomena in a metal with a smooth boundary, with account taken of the conduction-electron spectrum that leads to multichannel specular reflection of the carriers from the sample boundary. It is shown that in a magnetic field parallel to the sample surface, umklapp processes (transitions of an electron colliding with the sample boundary to another cavity of the equal-energy surface) produce in the interior of the metal narrow bursts of high-frequency current, with intensities much higher than the field amplitude in a burst produced by electrons with extremal orbit diameters. The presence of umklapp processes leads to resonant absorption of the energy of the electromagnetic wave, with the resonant frequencies determined by the time of electron travel from the skin layer to the field burst. In conductors much thinner than the carrier mean free path, umklapp processes lead to the appearance of new frequencies of the cyclotron size-effect resonance and to anomalous transparency of the thin films. An investigation of these effects yields the umklapp probabilities, the character of electron reflection by the sample boundary, and detailed characteristics of the carrier spectrum.

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INTRODUCTION

Under the conditions of the anomalous skin effect there is produced in a metal, besides a high-frequency (HF) electromagnetic field that decreases over a distance of the order of the skin depth δ, also an HF field component that attenuates over distances of the order of the conduction-electron mean free path $l \gg \delta$ and is due to the transport of electromagnetic-field energy by the carriers into the interior of the sample. The intensity of the weakly-damped field component, however, as shown by Reuter and Sondheimer,[1] who developed a theory for the anomalous skin effect, turns out to be quite small compared with the field in the skin layer, and allowance for it introduces only small corrections in the surface impedance.

The character of the penetration of the electromagnetic waves into a metal is substantially altered in a magnetic field **H**.[2] In strong magnetic fields ($r \ll l$, where r is the electron-trajectory curvature radius), a variety of weakly damped waves is produced, and their spectrum is determined by the dynamic properties of the carriers in the metal. Narrow field bursts are also produced in the interior of the metal. In a magnetic field parallel to the surface of a bulky metallic sample the role played in the electromagnetic properties of the metal by electrons interacting with the conductor surface is different from the role of electrons that do not touch the surface (the "volume" electrons). The electrons that do not leave the narrow skin layer during the entire effective flight time τ^*, i.e., the glancing electrons incident on the metal surface at small angles φ, make the decisive contribution to the formation of the skin layer if their reflection is close to specular. The volume electrons, being effectively accelerated by the HF field in the skin layer, carry information concerning the latter into the interior of the sample and produce narrow HF field bursts at a distance that is a multiple of the extremal diameter of the electron orbit. This is the mechanism predicted by Azbel'[3] for the formation of HF-field bursts, and observed several years later by Gantmakher at radio frequencies.[4] Here $\tau^* = |\omega^*|^{-1}$, $\omega^* = \omega + i/\tau$; $\tau = l/v$, ω is the electromagnetic wave frequency, and v is the electron velocity on the Fermi surface.

The presence of field bursts leads to a large number of HF effects, particularly to anomalous transparency of thin metallic plates whose thickness is of the order of the electron-orbit diameter. In the microwave band, the volume electrons ensure resonant absorption of the electromagnetic-wave frequency at frequencies ω that are multiples of the frequency Ω of the revolution of the electron on its orbit in the magnetic field.[5] The amplitude of the field in the burst has likewise a resonant character. Under the conditions $\omega\tau > r/\delta$ of sharp resonance, this amplitude is of the same order as that of the field in the skin layer.[3]

In conductors with almost specular faces, electromagnetic-field bursts of high intensity can be produced in a much wider frequency range, including the RF band,[6] in the case of multichannel reflection of the conduction electrons from the sample boundary, i.e., if there are several nonequivalent states for the specularly reflected electrons. The only electrons that interact most effectively with the HF field are those in phase with the wave. At $\delta \ll v/\omega$, the "effective" carriers are those moving parallel to the sample boundary, and the energy they acquire near the stationary-phase points, where the velocity projection v_x on the normal to the metal surface vanishes, determines in fact the HF conductivity under conditions of the anomalous skin effect. In a magnetic field parallel to the surface of a plate with specular faces, the electron trajectory consists in the case of multichannel reflection of arcs belonging to different cavities of the Fermi surface, and the stationary-phase points are located at different depths x. The distance D_0 between them determines the position of the narrow HF-field burst in the interior of the metal on account of the umklapp

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processes—the transition of a specularly reflected electron to another cavity of the equal-energy surface.

Umklapp processes permit the glancing electrons to leave their orbit and to take part in the formation of the HF-field burst. The HF-field burst produced by glancing electron has an appreciable intensity if the umklapp is fully probable during the time of the effective range, i.e., if the following condition is satisfied

$$Q\Omega_\lambda \gg 1/\tau^*, \tag{1}$$

where Ω_λ is the frequency of the electron motion on an orbit broken up by specular reflections from the sample surface; Q is the umklapp probability. For a glancing electron that does not leave the skin layer the frequency Ω_λ is $(r/\delta)^{1/2}$ times larger than the characteristic electron revolution frequency on the orbit in a magnetic field, so that condition (1) takes the form

$$Q \gg \gamma, \quad \gamma=(\Omega\tau^*)^{-1}(\delta/r)^{1/2} \ll 1. \tag{2}$$

Since the probability of the electron remaining on an open periodic orbit is larger the smaller the carrier incidence angle, we shall regard the probability Q, just as the diffuseness parameter $(1-q)$ (see Ref. 7), to be at small φ a linear function of φ:

$$Q=b\varphi. \tag{3}$$

Since the characteristic incidence angles of an electron that does not leave the skin layer is $\varphi_1 \sim (\delta/r)^{1/2}$, the condition (2) is satisfies at low frequencies ($\omega\tau < 1$) as soon as $b > (\Omega\tau)^{-1} \approx r/l$.

If D_0 depends on the projection p_z of the electron momentum on the magnetic-field direction, then the electrons form, as a result of umklapp processes, an HF field burst at a distance D_0^{extr} from the sample surface. In this case a part is played in the formation of the burst by only a small fraction $\sim(\delta/r)^{1/2}$ of the electrons, having a spread of $D_0(p_z)$ comparable with δ. The intensity of this burst turns out to be the same as the field amplitude in the burst produced by electrons with extremal orbit diameters. At certain orientations of the magnetic field in the plane of the plate, however, D_0 may be independent of p_z, and the electromagnetic-field burst is produced by all the electrons for which umklapp is possible, so that the field amplitude in the burst is larger. This case occurs if the magnetic field coincides with the symmetry axis of the crystal. The section of the Fermi surface consists then of equal electron orbits in different momentum-space cells, and the distance between the stationary-phase points is determined by the crystallographic orientation of the plate surface. As follows from Fig. 1, which shows the cross section of a convex Fermi surface that is singly-connected within the unit cell, this distance is equal to or is a multiple of $B\sin\alpha$, where B is the period in the direction of the symmetry axis p_1, and α is the angle between the axes p_x and p_1. At rational values of $\tan\alpha = N_1/N_2$, the number of nonequivalent states for a specularly reflected electron is finite. The trajectory of an electron that collides with the surface

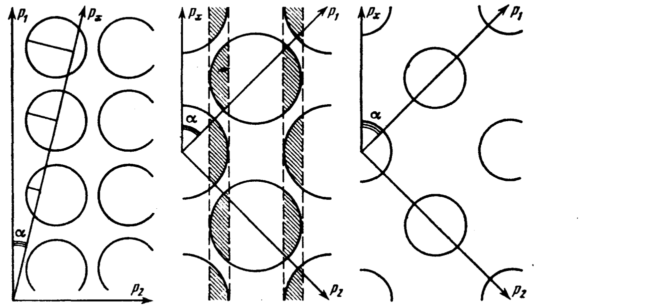

FIG. 1. Intersection of equal-energy surface and the plane p_z = const. In the presence of unklapp processes, the momentum space (p_z, t) breaks up into regions Γ_1, where such processes are possible (b, shaded regions), and regions Γ_2, where the umklapp probability is zero. The latter include electrons whose orbit diameter in a magnetic field is less that D_0 (c), e.g., electrons from the vicinity of the limiting points of the Fermi surface.

of the metal consists of the arcs shown in Fig. 1a, with a similarity coefficient c/eH, and the cusp spacing

$$D_0=\frac{N}{(N_1^2+N_2^2)^{1/2}}\frac{cB}{eH} \tag{4}$$

turns out to be the same for all p_z. Here e is the electron charge, c is the speed of light, and the integer $N \leq N_1+N_2-1$.

If the metal surface coincides with a crystallographic face having small Miller indices N_1 and N_2, so that $N_{max} \ll r/\delta$, i.e., the number of nonequivalent states for the reflected electrons is much less than r/δ, then the electromagnetic field at a distance on the order of $2r$ from the metal surface comprises a set of equidistant bursts of width of the order of the skin layer depth δ. The penetration of the electromagnetic field into the metal to a distance exceeding the maximum orbit diameter $2r_{max}$ is effected by electrons with extremal orbit diameters, and the intensity of the field burst at a depth $x > 2r_{max}$ is as a rule greatly decreased.

For a Fermi surface that is multiply connected within the limits of the unit cell or is not convex, several nonequivalent states are possible for the specularly reflected electrons even at $\alpha=0$. In this case D_0 is different for different p_z, except for some special cases when separate cavities of the Fermi surface are congruent. Although the field intensity in a burst whose position is determined by the extremal $D_0(p_z)$ is small, the determination of the magnetic-field values at which a thin metallic plate becomes anomalously transparent because of these bursts is quite important, for in addition to the magnetoacoustic oscillations in thin plates[8] they contain important information on the relative placement of the different cavities of a multiply connected Fermi surface.

In addition to formation of RF field bursts, umklapp processes lead to a new mechanism of resonant absorption of the electromagnetic-wave energy in the microwave band. A glancing electron that "splits" into two states (or several states at $N > 2$) acquires a resonant

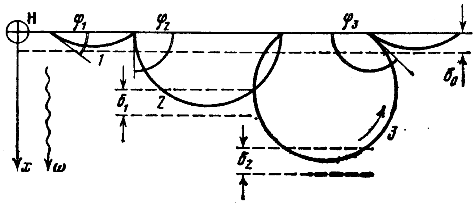

FIG. 2. Electron trajectory in specular reflection from the sample boundary, with allowance for umklapp processes. The main skin layer forms electrons that approach the metal surface at an angle $\varphi \lesssim \varphi_1$. Umklapp processes lead to formation of field bursts whose widths δ_1 and δ_2 are for the same order, but larger than δ_0.

period T_λ equal to the time of motion of the electron from the layer into the field burst, i.e., the resonance takes place at the frequencies

$$\omega T_\lambda^{extr} = \pi n. \tag{5}$$

In the considered frequency region $\Omega\tau^* \sim 1$ and the inequality (2) is satisfied at $b \sim 1$. Satisfaction of this condition, however, is generally speaking insufficient to obtain maximum resonance intensity. For resonant absorption of the energy of the electromagnetic wave it is necessary that during the effective electron travel time it pass both through the skin layer and in the field burst, as well that its motion be strictly periodic. The time of motion of the electron along the small arc that fits inside the skin layer, T_0, is smaller by a factor $(r/\delta)^{1/2}$ than the time of motion along arc 2 or 3 (Fig. 2). If the ratio R_0/T_λ is negligibly small compared with the broadening of the resonance line, i.e.,

$$\gamma_1 (r/\delta)^{1/2} \gg 1, \quad \gamma_1 = 1/\omega\tau + (1-q_2)/2\pi n, \tag{6}$$

(q_2 is the specularity parameter for the angles φ_2), then the electron motion is practically periodic. In this case, moving along small arcs in the skin layer, the electron acquires energy from the HF field, whereas the large arc determines the resonance period; at $b \sim 1$ the resonance at the frequencies (3) will be just as intense as the "ordinary" cyclotron resonance.[5]

An investigation of the resonant change of the impedance with changing magnetic field at new resonant frequencies permits a more detailed study of the interaction of the electrons with the sample boundary, determines the umklapp probability, and yields additional information on the carrier spectra in metals.

§1. SOLUTION OF KINETIC EQUATION FOR MULTICHANNEL SURFACE REFLECTION OF ELECTRONS

To determine the surface impedance and the distribution of the electromagnetic field in the metal, we must solve Maxwell's equations supplemented by the material equation that relates the electric current density $\mathbf{J}$ with alternating magnetic field intensity $\mathbf{E}(\mathbf{r})$. The last relation can be easily found with the aid of the kinetic equation for the nonequilibrium distribution function of the conduction electrons

$$f = f_0(\varepsilon) - \Psi \partial f_0/\partial\varepsilon,$$

where $f_0(\varepsilon)$ is the Fermi distribution function. The equation for Ψ, linearized in terms of the weak electric field,

$$\left(-i\omega + \frac{1}{\tau}\right)\Psi + \mathbf{v}\frac{\partial\Psi}{\partial\mathbf{r}} + \frac{\partial\Psi}{\partial t} = e\mathbf{E}(\mathbf{r})\mathbf{v}(t) \tag{7}$$

must be supplemented by a condition that takes into account the character of the reflection of the electrons by the sample boundary.

In the case of specular reflection of the carriers, this condition is

$$\Psi_i(\mathbf{p}) = w_{ik}\Psi_k(\tilde{\mathbf{p}}), \tag{8}$$

where w_{ik} is the probability of the transfer of the electron from the k-th to the i-th cavity of the Fermi surface, t is the time of motion of the electron in the magnetic field, i.e., the phase on the orbit $\varepsilon(\mathbf{p}) = \text{const}$, $p_z = \text{const}$, the momenta $\mathbf{p}$ and $\tilde{\mathbf{p}}$ are connected by the specular-reflection condition, and

$$\sum_k w_{ik} = \sum_i w_{ik} = 1.$$

We assume that the electromagnetic wave is monochromatic, so that the differentiation of Ψ with respect to time is equivalent to multiplication of Ψ by $-i\omega$. We confine ourselves to the τ approximation, which is perfectly sufficient for the description of HF phenomena under conditions of the anomalous skin effect.[2]

For a weakly rough metal surface, it is easy to obtain a boundary condition similar to that of Fal'kovskiĭ[9] or of Okulov and Ustinov,[10] with account taken of the multichannel reflection of the carrier by the sample boundary:

$$\Psi_i(\mathbf{p}) - \Psi_i(\tilde{\mathbf{p}}) = \int w_{ik}(\mathbf{p}, \mathbf{p}')[\Psi_k(\mathbf{p}') - \Psi_k(\tilde{\mathbf{p}})]d\mathbf{p}', \tag{9}$$

and then solve the kinetic equation (7) for an arbitrary scattering indicatrix.[11] In a highly inhomogeneous field, however, the nonequilibrium increment to the electron distribution function is a rapidly varying function of its arguments, and allowance for the integral term that contains $\Psi_k(\mathbf{p})$ is of little importance. In this case we return to a boundary condition of type (8), in which w_{ik} depends on the momentum of the incident electron, and $\sum_{ik} w_{ik} < 1$; this is equivalent to the approximation with a specularity parameter that depends on the angle of incidence of the charge on the sample boundary. In a magnetic field parallel to the metal surface, Eq. (7) can be easily solved by recognizing that in the case of specular reflection without umklapp the electrons move on open periodic orbits with period T_i on the i-th cavity of the Fermi surface. Condition (8) then takes the form

$$\Psi(\lambda_i, p_z; \mathbf{r}_s) = w_{ik}\exp(i\omega^* T_k)\{A_k + \Psi(\lambda_k, p_z; \mathbf{r}_s)\}, \tag{10}$$

where λ_i is the instant of reflection of the electron by the sample boundary at the point $\mathbf{r}_s$, i.e., the root of the equation

$$\int_{\lambda}^{t} \mathbf{v}_i(t')\,dt' = \mathbf{r} - \mathbf{r}_s, \quad (11)$$

$$A_k = \int_{\lambda_k}^{\lambda_k+T_k} \exp(-i\omega^*(t'-\lambda_k))\mathbf{v}(t')\mathbf{E}(\mathbf{r}_s+\mathbf{r}(t')-\mathbf{r}(t))\,dt'. \quad (12)$$

Conditions (10) constitute N algebraic equations for the functions $\Psi(\lambda_i, p_z; r_s) \equiv \Psi_i$, where N is the number of specular-reflection channels,

$$\Psi_i - U_{ik}\Psi_k = A_k U_{ik}, \quad (13)$$

$$U_{ik} = w_{ik} \exp(i\omega^* T_k), \quad (14)$$

and the solution of these equations is of the form

$$\Psi_i = V_{ij} U_{jk} A_k / \mathrm{Det}[I - \hat{U}], \quad (15)$$

where I is a unit matrix, $\hat{U}$ is a matrix with components U_{ik}, and

$$V_{ik} = \mathrm{minor}\{\delta_{ik} - U_{ik}\}.$$

For greater clarity of the calculations that follow, we confine ourselves to two-channel reflection. Generalization to the case of an arbitrary number of channels entails no difficulty. At $N=2$ we have

$$\Psi_1(t, p_z; x) = \int_{\lambda_1}^{t} \exp(-i\omega^* t_1) e v(t')\mathbf{E}(x + x(t') - x(t))\,dt' + \exp[-i\omega^*(\lambda_1 - t)]\frac{A[P_1(\beta - P_2) + Q^2] + \alpha BQ}{(\alpha - P_1)(\beta - P_2) - Q^2} \quad (16)$$

$$\Psi_2(t, p_z; x) = \int_{\lambda_2} \exp(-i\omega^* t_1) e v(t')\mathbf{E}(x + x(t') - x(t))\,dt' + \exp[-i\omega^*(\lambda_2 - t)]\frac{\beta AQ + B[P_2(\alpha - P_1) + Q^2]}{(\alpha - P_1)(\beta - P_2) - Q^2} \quad (17)$$

where

$$\alpha = \exp(-i\omega^* T_1), \quad \beta = \exp(-i\omega^* T_2), \quad T_1 = T - 2\lambda_1,$$
$$T_2 = T - 2\lambda_2(\lambda_1), \; T = 2\pi/\Omega, \; A = A_1, \; B = A_2, \; t_1 = t' - t, \quad (18)$$

and λ_1 and λ_2 are the roots of the equations

$$x(t) - x(\lambda_1) = x, \quad x(\lambda_1) - x(\lambda_2) = D_0. \quad (19)$$

The umklapp probability in specular reflection is $Q = w_{12} = w_{21}$, while the probabilities of specular reflection without umklapp are $P_1 = w_{11}$ and $P_2 = w_{22}$. All probabilities depend on the angle φ of incidence of the carriers on the sample boundary, and at small φ the probability Q and the specularity parameters $q_i = Q + P_i$ are linear functions of φ:

$$q_i = 1 - a_i\varphi \quad (i = 1, 2) \quad (20)$$

[see also (3)].

§2. ASYMPTOTIC EXPRESSIONS FOR THE HF CURRENT DENSITY IN A BULKY CONDUCTOR

Maxwell's equations

$$k^2\mathscr{E}_\mu(k) + 2\partial E_\mu(0)/\partial x = 4\pi i\omega c^{-2} j_\mu(k) \quad (21)$$

are integral equations for the Fourier components of the HF electric field

$$\mathscr{E}_\mu(k) = 2\int_0^\infty E_\mu(x)\cos kx\,dx. \quad (22)$$

In the calculation of the HF electric conductivity tensor which is the kernel of the integral operator that connects the HF field with the current:

$$j_\mu(k) = \int_0^\infty K_{\mu\nu}(k, k')\mathscr{E}_\nu(k')\,dk', \quad (23)$$

we recognize that if the carrier reflection by the sample boundary is close to specular a substantial contribution to the HF electric current is made both by electrons that collide with the conductor surface and by electrons that do not interact with the surface. The contribution of the latter to the HF electric conductivity is well known (see, e.g., Refs. 2 and 5), and for electrons repeatedly interacting with the surface the kernel $K_{\mu\nu}(k, k')$ can be represented, using Eqs. (16) and (17) for the nonequilibrium increment Ψ, in the form

$$K_{\mu\nu}(k, k') = K_{\mu\nu}^A(k, k') + K_{\mu\nu}^B(k, k'). \quad (24)$$

Here

$$K_{\mu\nu}^A(k, k') = \frac{4e^3H}{\pi ch^3}\int dp_z \int_0^{T/2} d\lambda\, v_x(\lambda) f_A(\lambda)$$
$$\times \int_\lambda^{T/2} dt\, \Phi_\mu(\omega^*, t)\cos[k(x(t) - x(\lambda))] \int_\lambda^{T/2} dt'\, \Phi_\nu(-\omega^*, t')\cos[k'(x(t') - x(\lambda))], \quad (25)$$

the integration is carried out over the entire Fermi surface, and the index corresponding to the summation over its different cavities has been left out[1)]:

$$\Phi_\mu(\omega^*, t) = v_\mu(t)\exp(i\omega^* t) + v_\mu(-t)\exp[i\omega^*(T - t)], \quad (26)$$

$$f_A(\lambda) = \frac{(q_1 - Q)(\beta - q_2) + q_1 Q}{(\alpha - q_1)(\beta - q_2) + Q(\alpha - q_1 + \beta - q_2)}, \quad (27)$$

q_1 and q_2 are the specularity parameters of the electron on orbits coupled by umklapp.

The expression for the kernel $K_{\mu\nu}^B(k, k')$ can be written in similar form by replacing λ in the integral with respect to t' by the root $\lambda_2(\lambda)$ of Eq. (19), and replacing f_A by f_B, where

$$f_B(\lambda) = \frac{Q(\alpha\beta)^{1/2}}{(\alpha - q_1)(\beta - q_2) + Q(\alpha - q_1 + \beta - q_2)}. \quad (28)$$

Under the conditions of the anomalous skin effect it suffices to know the asymptotic expressions for the HF electric conductivity tensor $K_{\mu\nu}(k, k')$ at $kr \gg 1$ and $k'r \gg 1$. The main contribution to the integrals (25) is made by the limits of the integration with respect to λ, i.e., $\lambda = 0$ and $\lambda = T/2$, which coincide with the stationary-phase points, where $v_x(\lambda) = 0$. We assume that functions of the resonant-denominator type vary slowly compared with cosines, i.e., that the following condition is satisfied:

$$|\exp(-i\omega^* T_2) - q_2| \gg (\delta/r)^{1/2}, \tag{29}$$

which is equivalent to the inequality (6) near resonance. This inequality, besides having the physical meaning mentioned in the Introduction, indicates also that a small angle change $\delta\varphi_2 \sim (\delta/r)^{1/2}$ (see Fig. 2) results in a negligibly small frequency change $\delta\Omega_\lambda$ (where $\Omega_\lambda = \pi/T_\lambda$) compared with the collision frequency $1/\tau$.

In the vicinity of the point $\lambda = T/2$ the relations (3) and (20) are valid for the umklapp probability and the diffuseness parameter, which turn out to be proportional to the period T_1, which vanishes at $\lambda = 0$. The function f_A has therefore a pole at this point. Separating its regular part $\tilde{f}_A$, we represent the function f_A in the form

$$f_A = 2(\Omega T_1 \eta)^{-1} + \tilde{f}_A, \tag{30}$$

$$\tilde{f}_A = (b/\eta)^2 [\exp(-2i\omega^* T_\lambda) - q_2]^{-1}, \quad \lambda \to T/2, \tag{31}$$

$$\tilde{f}_A = [\exp(-i\omega^*(T - 2\lambda)) - 1]^{-1}, \quad \lambda \to 0. \tag{32}$$

For "touching" electrons with moment of reflection close to zero, the quantities Q and $(1-q)$ also vanish, whereas the period T_1 is equal to $2\pi/\Omega$. Therefore umklapp changes very little the current of the "touching" electrons (to the extent that δ/r is small), and we may retain in f_A, with sufficient degree of accuracy, only the term (32), and neglect the contribution made to the current by the corresponding term with f_B:

$$f_B = \frac{b(\alpha\beta)^{1/2}}{\eta} [\exp(-2i\omega^* T_\lambda) - q_2]^{-1}, \quad \lambda \to \frac{T}{2}. \tag{33}$$

Here $\eta = a + b - 2i\omega^*/\Omega$, $2T_\lambda = T_2$, and the nonresonant terms of $\tilde{f}_A$ have been left out.

The first term in Eq. (30) for the function f_A corresponds to the contribution of the glancing electrons to the monotonic part of the HF electric field. Substituting it in (23) and integrating, we obtain the following expression for the kernel:

$$K_{\mu\mu}^0(k, k') = \frac{i\pi^{1/2} e^2}{\omega^* h^3} \left(\frac{eH}{c}\right)^{1/2} d_{\mu\mu} \frac{|k - k'|^{-1/2} - (k + k')^{-1/2}}{(kk')^{1/2}}, \tag{34}$$

which also depends on k and k', just as in the case of pure specular reflection of the carriers, and the sensitivity to the surface state of the sample is determined by the tensor $d_{\mu\nu}$:

$$d_{\mu\nu} = -2i\omega^* \int dp_z (\Omega\eta)^{-1} v_\mu(p_z) v_\nu(p_z) \left| \frac{m^*(p_z)}{\partial v_x / \partial\varphi} \right|^{1/2}, \tag{35}$$

where m^* is the effective mass of the electron. We assume here and below that the electric vector of the linearly polarized external wave is directed along one of the axes in terms of which the tensor $d_{\mu\nu}$ is diagonal, and we omit hereafter for simplicity the tensor indices in most equations.

The contribution made to the part of the HF current of the glancing electrons subjected to umklapp is described by the functions $\tilde{f}_A$ and f_B. In the course of the calculations that follow it will be convenient to add the electric conductivity of the volume electrons to that resonant-kernel part that is connected with the functions $\tilde{f}_A$, and break up in turn the resultant expression into two terms. The first, for which we retain the notation $K_A(k, k')$, is of the form

$$\begin{aligned} K_A(k, k') &= a_0 k_1^3 \frac{1}{k} \delta(k - k') \\ &+ \frac{\pi}{(kk')^{1/2}} \left\langle \rho\left(\frac{T}{2}\right) \int_0^{T/2} d\lambda \tilde{f}_A'(\lambda) \frac{\sin[(k - k')\Delta x(\lambda)]}{k - k'} \right\rangle \\ &- 2\left\{\left\langle \rho\left(\frac{T}{2}\right)\left(\tilde{f}_A\left(\frac{T}{2}\right) - \tilde{f}_A(\tau_1)\right)\right\rangle + \frac{1}{\pi^2} a_0 k_2^3\right\} \frac{\ln(k/k')}{k^2 - k'^2}; \end{aligned} \tag{36}$$

it describes the resonant behavior of the surface impedance. The second term, designated $K_C(k, k')$

$$\begin{aligned} K_C(k, k') &= \frac{a_0 \alpha_0 k_1^3}{(2kr)^{1/2}} \left\{ \frac{1}{k} \sin\left(2kr + \frac{s_1\pi}{4}\right) \delta(k - k') \right. \\ &\left. + \frac{1}{\pi(k'^2 - k^2)} \left[\left(\frac{k}{k'}\right)^{1/2} \cos\left(2k'r + \frac{s_1\pi}{4}\right) - \cos\left(2kr + \frac{s_1\pi}{4}\right) \right] \right\}, \end{aligned} \tag{37}$$

ensures a field burst whose distance from the surface is a multiple of $2r^{extr}$. A similar expression for the kernel $K_B(k, k')$ connected with the function f_B and responsible for the formation of an HF field burst at a depth D_0, is given by

$$\begin{aligned} K_B(k, k') &= \frac{\pi}{(kk')^{1/2}} \left\langle \rho\left(\frac{T}{2}\right) \int_{T/2 - \tau_1}^{T/2} d\lambda f_B'(\lambda) \left\{ \frac{\sin[(k - k')\Delta x(\lambda) - k'D_0]}{k - k'} \right.\right. \\ &\left.\left. + \frac{\sin[(k - k')\Delta x(\lambda) + kD_0]}{k - k'} \right\} \right\rangle - a_0 \alpha k_2^3 \frac{\sin kD_0 - \sin k'D_0}{k^2 - k'^2}, \end{aligned} \tag{38}$$

where

$$\rho_{\mu\nu}(t) = \frac{v_\mu(t) v_\nu(t)}{|v_x'(t)|}, \quad \rho\left(0, \frac{T}{2}\right) = \frac{v_\mu(0) v_\nu(T/2) + v_\mu(T/2) v_\nu(0)}{2[v_x'(0) |v_x'(T/2)|]^{1/2}}. \tag{39}$$

The quantities k_1 and k_2 characterize the spatial scale of the variation of the HF in the bursts:

$$k_1^3 = a_0^{-1} \pi^2 / 2 \langle (\rho(0) + \rho(T/2)) \operatorname{cth}(-i\omega^* T/2) \rangle, \tag{40}$$

$$k_2^3 = a_0^{-1} \pi^2 / 2 \langle \rho(0) \operatorname{cth}(-i\omega^* T/2) + \rho(T/2)(2\tilde{f}_A(\tau_1) + 1) \rangle, \tag{41}$$

the parameter α_0 is determined by the anisotropy of the Fermi surface

$$\alpha_0 k_1^3 = a_0^{-1} \frac{4\pi e^3 H}{ch^3} \left(\frac{2\pi}{|\beta_0|}\right)^{1/2} \rho\left(0, \frac{T}{2}\right) \operatorname{sh}^{-1}\left(-i\omega^* \frac{T}{2}\right), \tag{42}$$

and the parameter α is given by

$$\alpha k_2^3 = 2\pi a_0^{-1} \langle \rho(T/2) f_B(T/2) \rangle, \tag{43}$$

and also by the state of the sample boundary:

$$a_0 = (4\pi\omega c^{-2})^{-1}; \quad \beta_0 = (1/2r)\partial^2 r / \partial p_z^2; \quad s_1 = \operatorname{sign}\beta_0,$$
$$\Delta x(\lambda) = x(T/2) - x(\lambda).$$

The angle brackets denote integration over the Fermi surface along the belt where v_x vanishes with a factor $4e^3H/\pi ch^3$, and the values outside these brackets, which depend on p_z, are taken on that section of the Fermi surface where $r(p_z)$ has an extremum. Summation is implied if there are several such sections. The prime denotes differentiation with respect to t, and $\tau_1 = \lambda_2(T/2)$.

It follows from the foregoing equations that the resonant kernels are small in accord with the anomaly parameter $(\delta/r)^{1/2}$ compared with the kernel $K_0(k,k')$. It is therefore advantageous to use perturbation theory and seek a solution of Maxwell's equation in the form

$$\mathscr{E}(k)=\mathscr{E}_0(k)+\Delta\mathscr{E}(k)+\mathscr{E}_{1c}(k)\cos kD_0+\mathscr{E}_{1s}(k)\sin kD_0 +\mathscr{E}_{21}(k)+\mathscr{E}_{22}(k), \tag{44}$$

where $\mathscr{E}_0(k)$ describes the main skin layer made up by the glancing electrons, and $\Delta\mathscr{E}(k)$ is a small, but resonantly dependent on the magnetic field, change of the HF field near the surface. The next two terms are responsible for the field in the burst at a depth D_0 and are also resonant. Finally, the last two terms of (44) describe an HF burst whose distance from the surface is equal to the extremal diameter of the electron orbit, and the "drawing" of the HF field into the interior of the sample to distances larger than $2r_{max}$.

The Fourier transform of $\varepsilon_0(k)$ satisfies Eq. (21) with a kernel $K(k,k')$ equal to $K_0(k,k')$. Using its known solution (see the Appendix), we obtain that part of the impedance which varies smoothly with the magnetic field

$$Z=-\frac{8i\omega}{c^2k_0}\int_0^\infty F(\xi)\,d\xi. \tag{45}$$

As a result we have

$$Z_0=-\frac{8i\omega}{c^2k_0}M_0(-1),\quad k_0=\left(\frac{4\pi^{3/2}e^2}{c^2h^3(\gamma_0-i)}\left(\frac{eH}{c}\right)^{1/2}d\right)^{2/5}, \tag{46}$$

where k_0^{-1} is the thickness of the main skin layer, and $\gamma_0=(\omega\tau)^{-1}$. We note that, depending on the relation between the parameters $(a+b)$ and $|\omega^*/\Omega|$ the impedance is proportional to $H^{1/5}$ at $(a+b)\ll|\omega^*/\Omega|$ or to $H^{1/5}$ in the opposite case.

To find the remaining terms in (44), we estimate first asymptotically, at large k and k', the characteristic integrals that appear in the right-hand side of (21). For the integral operator with kernel $K_A(k,k')$ we have

$$\int_0^\infty K_A(k,k')\mathscr{E}(k')\,dk'=\frac{\pi^2}{2}\left\langle\rho(0)\,\mathrm{cth}\left(-i\omega^*\frac{T}{2}\right)\right. \tag{47}$$
$$\left.+\rho\left(\frac{T}{2}\right)\left(2f_A\left(\frac{T}{2}\right)+1\right)\right\rangle\left\{\frac{1}{k}\mathscr{E}(k)-\frac{2}{\pi^2}\int_0^\infty dk'\frac{\ln(k/k')}{k^2-k'^2}\mathscr{E}(k')\right\},$$

$$\int_0^\infty dk' K_A(k,k')\mathscr{E}(k')\cos(k'D+\varphi)=\left\{a_0k_1^3\right. \tag{48}$$
$$\left.+\pi^2\left\langle\rho\left(\frac{T}{2}\right)\int_0^{T/2}d\lambda f_A'(\lambda)\,\theta(\Delta x(\lambda)-D)\right\rangle\right\}\frac{\mathscr{E}(k)}{k}\cos(kD+\varphi),$$

where $\mathscr{E}(k)$ is a smooth function of k and $\theta(x)$ is the unit step function.

The operator $\hat{K}_A$ thus transforms smooth functions into smooth functions and oscillating into oscillating. The operator $\hat{K}_B$, on the contrary, acting on a smooth function transforms it into an oscillating one:

$$\int_0^\infty K_B(k,k')\mathscr{E}(k')\,dk'=\pi a_0\alpha k_2^3/2k\{\mathscr{E}(k)\cos kD_0+G(k)\sin kD_0\}, \tag{49}$$

$$G(k)=\hat{G}\mathscr{E}(k)=\frac{2}{\pi}\int_0^\infty dx\frac{\mathscr{E}(kx)}{x^2-1}, \tag{50}$$

and conversely, when applied to an oscillating function yields a smooth one. The latter need not be cited here, since the corresponding $\Delta\mathscr{E}(k)$ term that describes the reaction of the HF field burst on the skin layer is small in terms of the anomaly parameter and can be left out.

Similar relations hold also for the integral operator with kernel $K_C(k,k')$:

$$\int_0^\infty K_C(k,k')\mathscr{E}(k')\,dk'=\frac{a_0\alpha_0k_1^3}{2(2kr)^{1/2}}\frac{1}{k}\left\{\mathscr{E}(k)\sin\left(2kr+\frac{s_1\pi}{4}\right)\right.$$
$$\left.-G(k)\cos\left(2kr+\frac{s_1\pi}{4}\right)\right\}. \tag{51}$$

When the local part of the operator $\hat{K}_C$ acts on a function of the type $\mathscr{E}(k)\cos(kD+\varphi)$, two oscillating functions are produced, containing in the argument both the sum $2r+D$ and the difference $2r-D$. The role of the nonlocal part of the operator in question reduces to cancellation of the latter functions so that we have as a result

$$\int_0^\infty K_C(k,k')\mathscr{E}(k')\cos(k'D+\varphi)\,dk'$$
$$=\frac{a_0\alpha_0k_1^3}{2(2kr)^{1/2}}\frac{1}{k}\sin\left[(2r+D)k+\varphi+\frac{s_1\pi}{4}\right]\mathscr{E}(k). \tag{52}$$

§3. SURFACE IMPEDANCE AND FIELD DISTRIBUTION UNDER RESONANCE CONDITIONS

The relations presented above allow us to find all the terms in formula (44) for the Fourier transform $\mathscr{E}(k)$ and investigate the resonant increment to the impedance and the distribution of the HF in the metal. An integral equation is obtained for $\Delta\mathscr{E}(k)$ by retaining in the right-hand side of Maxwell's equation (21) the terms due to the action of the operator $\hat{K}_0$ on the function $\Delta\mathscr{E}(k)$ and of the operator $\hat{K}_A$ on the Fourier component $\mathscr{E}_0(k)$. Although this equation can be solved by the method proposed by Hartmann and Luttinger,[12] knowledge of the explicit form of the solution is not essential for the determination of the resonant increment ΔZ^{res} to the impedance. The reason is that, since the kernel $K_0(k,k')$ is symmetrical, the value of ΔZ^{res} can be obtained by direct integration (see, e.g., Ref. 13). As a result we obtain for ΔZ^{res} the expression

$$\Delta Z^{res}=C\frac{8\omega}{c^2k_0}\left(\frac{b}{\eta}\right)^2\left(\frac{k_d}{k_0}\right)^3\varphi(\Delta). \tag{53}$$

Here

$$C\approx 1{,}99\cdot 10^{-2}\exp(4\pi i/5),\quad k_d^{-1}=(16\pi^2\omega c^{-2}h^{-3}p_0\rho(T/2)e^3H)^{-1/5}$$

is of the order of the skin layer thickness in the absence of a magnetic field, and near resonance, at frequencies given by Eq. (5), where T_λ has an extremum as a function of p_z, the function $\varphi(\Delta)$ takes the form

$$\varphi(\Delta)=\frac{1}{2n(2|\varkappa|)^{1/2}}\frac{(\chi+s\Delta)^{1/2}+is(\chi-s\Delta)^{1/2}}{\chi}, \tag{54}$$

where

$$\chi=(\Delta^2+\gamma_1^2)^{1/2},\quad \varkappa=\frac{p_0^2}{2T_\lambda}\frac{\partial^2 T_\lambda}{\partial p_z^2}\bigg|_{p_z=p_z^{extr}},\quad s=\operatorname{sign}\varkappa,$$

and p_0 is the Fermi momentum.

It follows from the presented formulas that the main characteristics of the resonance depend substantially on the surface properties of the conductor. The width of the resonance line is determined not only by the mean free path of the carriers, but also by the specularity parameter q_2, and its amplitude increases at small b in proportion to b^2. At $b\approx 1$ and $q_2\approx 1$, the considered resonant effect is just as intense as the "ordinary" cyclotron resonance.[5,14,15]

Formula (53) for ΔZ is valid if the inequality (29) is satisfied. At $\gamma_1(r/\delta)^{1/2}\ll 1$, the main characteristics of the resonance, just as in the case of cyclotron resonance on electrons with extremal orbit diameter,[16] and as in size-effect resonances in a plate,[13,17,18] are determined by the HF-field damping depth, i.e., by the parameter δ/r. In the considered case, however, the resonant peak fine structure connected with the parameter γ_1 is much less pronounced and comes into play only in the higher derivatives of the impedance with respect to the magnetic field.

We proceed now to find the HF field in a burst of depth D_0. Using relations (48) and (49) it is easy to obtain an equation for the amplitudes $\mathscr{E}_{1e}(k)$ and $\mathscr{E}_{1s}(k)$ and express them in terms of the function

$$\mathscr{E}_{1e}(k)=g(k)\mathscr{E}_0(k),\quad \mathscr{E}_{1s}(k)=g(k)\hat{G}\mathscr{E}_0(k), \tag{55}$$

$$g(k)=\frac{\pi}{2}\frac{i\alpha k_2^3}{k^3-ik_2^3}. \tag{56}$$

As seen from these formulas, the characteristic values of the wave number k, corresponding to the poles of the function $g(k)$, are of the order of k_2 and not of k_0. Therefore the glancing electrons, which form the main skin layer of thickness $\sim k_0^{-1}$, produce after the umklapp, at a depth D_0, an HF field burst whose width is of the order k_2^{-1}.

The distribution of the HF in the metal is determined by the inverse Fourier transform

$$E(x)=\frac{1}{\pi}\int_0^\infty \mathscr{E}(k)\cos kx\,dk, \tag{57}$$

and it is easy to show, using (55) and (56), that the form of the field in the burst is described by the integral

$$E(x)=-\frac{2\partial E(0)/\partial x}{k_0}\frac{i\alpha}{4}J(\delta x,k_2), \tag{58}$$

$$J(\delta x,k_2)=\frac{1}{k_0}\int_0^\infty dk\frac{F_0(k/k_0)\cos(k\delta x)+G_0(k/k_0)\sin(k\delta x)}{(k/k_2)^3-i}, \tag{59}$$

where $F_0(k/k_0)$ is the dimensionless Fourier component of the field (see the Appendix), $G_0(k/k_0)=\hat{G}F_0$, $\delta x=D_0-x$.

Near the center of the burst $|k_2\delta x|\ll 1$ we obtain for $E(x)$

$$E(x)\approx E_0\frac{\pi i}{12}\exp\left(\frac{\pi i}{4}\right)\left(1+2\exp\left(-\frac{i\pi}{6}\right)(k_2\delta x)\right),\quad E_0=-\frac{2\partial E(0)/\partial x}{k_0}\frac{i\alpha}{4}\left(\frac{k_2}{k_0}\right)^{3/2}, \tag{60}$$

and since $E_0\sim a\sim b$, the field amplitude in the burst, in contrast to the resonant impedance increment, is proportional to the first degree of the parameter b rather than to b^2.

The expression for $E(x)$ on wings of the burst is of the form

$$E(x)\approx E_0\frac{\Gamma(3/2)}{2\sqrt{2}}\left\{\frac{\theta(-\delta x)}{(k_2|\delta x|)^{3/2}}+i\frac{\Gamma(9/2)}{\Gamma(3/2)}\frac{\theta(\delta x)}{(k_2\delta x)^{9/2}}\right\}+E_0\frac{\pi}{6\sqrt{2}}\exp\left\{\frac{i\pi}{2}(1+\theta(-\delta x))+i|k_2\delta x|\left(1+\frac{i\gamma_1}{6\Delta}\right)\right\}, \tag{61}$$

$\Gamma(z)$ is the Euler Gamma function

The oscillating term in this equation is due to the fact that the system of narrow HF-field burst ("secondary" skin layers) can serve as a source of cyclotron-wave excitation.[19] The wave appears at $\varkappa<0$, i.e., when the period T_λ has a maximum on the extremal section of the Fermi surface, on that resonance wing where the detuning $\Delta>0$, and the damping is small if $\Delta\gg\gamma_1$. Expression (54) for $\varphi(\Delta)$ then takes the form

$$\varphi(\Delta)\approx-\frac{i}{2n(|\varkappa|\Delta)^{1/2}}\left(1+\frac{i\gamma_1}{2\Delta}\right), \tag{62}$$

k_1 is almost imaginary, and the pole of the function $g(k)$ presses towards the real axis. In this case it is necessary to take into account the pole contribution in the integral (59), and it is this which leads to the second term, which describes the cyclotron-wave field, of formula (61).

The functions $\mathscr{E}_{21}(k)$ and $\mathscr{E}_{22}(k)$ responsible for the penetration of the HF field to a distance $\gtrsim 2r$ into the interior of the metal, can be easily found with the aid of (49) and (51) and represented in a compact form reminiscent of the well known formulas[2,3] that hold in the approximation in which a local connection exists between the Fourier components of the HF field and of the current. The function $\mathscr{E}_{21}(k)$, which describes a system of damped HF field bursts at distances from the surface that are multiples of $2r^{extr}$, is of the form[2)]

$$\mathscr{E}_{21}(k)=\frac{i\alpha_0k_1^3}{2(2kr)^{1/2}}\frac{\mathscr{E}_0(k)\sin(2kr+s_1\pi/4)-\hat{G}\mathscr{E}_0(k)\cos(2kr+s_1\pi/4)}{k^3-ik_1^3[1+\alpha_0(2kr)^{-1/2}\sin(2kr+s_1\pi/4)]}. \tag{63}$$

The expression for the function $\mathscr{E}_{22}(k)$, which describes the "drawing" of an HF field burst produced by umklapp processes into the interior of a conductor, is of exactly the same form as (63), except that in the numerator of the latter we must replace $2r$ in the argument of the cosine and sine by $2r+D_0$, and the field amplitudes $\mathscr{E}_0(k)$ and $\hat{G}\mathscr{E}_0(k)$ of the main skin layer must be replaced by the field amplitudes in the burst at the depth D_0, i.e., by $\mathscr{E}_{1e}(k)$ and $\mathscr{E}_{1s}(k)$ respectively.

Equations (57) and (63) yield the distribution of the HF field in the interior of the metal. The characteristic scale of variation of the HF field in the bursts is connected with the quantities k_1^{-1} and k_2^{-1}, and their form depends essentially on the character of the extremum of the electron-orbit diameter as a function of p. Thus, an HF field burst whose distance from the surface is close to $2r^{\max}$ turns out to be almost symmetrical, with a small shift of the maximum of the field amplitude, amounting to k_0^{-1}, relative to the point $x=2r^{\max}$, and can be described by the following interpolation formula:

$$E(x)\approx E_1c_0\exp\left(\frac{i\pi}{6}\right)\frac{1+c_1u(k_1/k_0+c_2u)\ln|k_1/k_0+c_2u|}{1+c_3u^6\ln|u|}+E_1c_0\frac{\sqrt{3}}{2}\exp\{i|u|(1+i\gamma_0/6\Delta)\}, \tag{64}$$

where

$$E_1=-\frac{2\partial E(0)/\partial x}{k_0}\frac{\alpha_0}{(2k_1r)^{1/2}}\left(\frac{k_1}{k_0}\right)^{3/2},\quad u=k_1(2r-x),$$
$$c_0=\frac{\exp(i\pi/6)}{12\sqrt{6}},\quad c_1=\frac{9\cdot 3^{1/2}\exp(-i\pi/3)M_0(-1)}{2\pi^2},$$
$$c_2=\frac{\pi}{6M_0(-1)},\quad c_3=\frac{1}{2\Gamma(4)}, \tag{65}$$

which goes over into the exact expressions at $|u| \ll k_1/k_0$, $k_1/k_0 \ll u \ll 1$, and $1 \ll |u| \ll k_1r$.

The high-frequency field burst at a depth $x\approx 2r^{\min}$, on the contrary, is antisymmetric;

$$E(x)\approx -E_1c_0\frac{u+c_4k_1/k_0}{1+c_5u^2}+E_1c_0\frac{i\sqrt{3}}{2}\operatorname{sign}(\delta x)\exp\left\{i|u|\left(1+\frac{i\gamma_0}{6\Delta}\right)\right\}, \tag{66}$$

where

$$c_4=\frac{3M_0(-1)}{\pi},\quad c_5=-\frac{2\pi i}{3\sqrt{3}}\exp\left(\frac{i\pi}{6}\right).$$

The oscillating terms in these formulas, which describe the field of the cyclotron wave, must be retained under the same conditions as in formula (61).

A comparison of formulas (64)–(66) with the corresponding expressions for the field in the burst produced by glancing electrons subjected to umklapp shows that at $b\approx 1$ the burst intensity at a depth D_0 is larger by $(k_1r)^{1/2}$ times than the field amplitude at a distance $2r^{extr}$ from the surface.

The resonant character of the amplitude and of the width of the HF field burst at a depth D_0 is different from that at a depth $x\approx 2r^{extr}$. The resonance in a burst produced by electrons with extremal orbit diameter sets in at frequencies ω that are multiples of the frequency of revolution of these electrons in a magnetic field: $\omega=n\Omega_e$. If the extrema of the effective mass m^* and of the diameter $2r$ coincide, then the quantities k_1^3 and α_0 turn out to be proportional to the function $\varphi(\Delta)$, which is given by formula (54) in which γ_1 must be replaced by γ_0. Therefore the width of the burst k_1^{-1} near resonance is "compressed" in proportion to $(\varphi(\Delta))^{-1/2}$ and its amplitude increases like $(\varphi(\Delta))^{4/3}$. If m^* has no extremum on the Fermi-surface section whose electrons form the HF field burst, then k_1^{-1} is practically independent of the detuning from resonance, and its amplitude is proportional to the parameter

$$\alpha_0\sim\{\exp(-i\omega^*T_e)-1\}^{-1}.$$

For the burst at a depth D_0 we find that at $\omega T_\lambda^{extr}=\pi n$ the parameter α does not depend on the detuning from resonance: $\alpha=2\pi^{-1}(-1)^nb/\eta$, and $k_2^3\sim\varphi(\Delta)$. Therefore the width of the burst is $k_2^{-1}\sim(\varphi(\Delta))^{-1/3}$, and its amplitude $\sim(\varphi(\Delta))^{1/2}$. On the other hand if the frequency ω is a multiple of the revolution frequency of the electrons with the extremal effective mass, then the multiple return of these electrons to the narrow field burst at the depth D_0 leads again to a resonant character of the amplitude and of the width of the burst. In this case k_2^{-1} is also proportional to $(\varphi(\Delta))^{-1/3}$, and the amplitude $E(D_0)$ at the resonance is $\sim(\varphi(\Delta))^{-1/2}$, and is consequently minimal.

The shapes of the bursts at distances $2r+D_0$, $4r$, and $4r+D_0$ from the surface can be analyzed in similar fashion.

§4. RESONANT PHENOMENA IN THIN CONDUCTORS

Umklapp processes give rise also to new resonance phenomena in thin conductors, of thickness d smaller than the maximum diameter of the electron orbit. They lead to additional broadening of the size-effect cyclotron-resonance lines, a broadening due to the electrons that return to the skin layer upon reflection from the plate surface opposite to the skin layer.[17,18,22] In addition, umklapp processes lead to the appearance of new resonant frequencies. The shapes of the resonance curves can be analyzed in analogy with the procedure used above for a bulky conductor. For electrons that collide with the plate surface $x=d$, the high-frequency electric conductivity tensor is of the form[3)]

$$K_{\mu\nu}^A(k,k')=\frac{4e^3H}{\pi ch^3}\int dp_z\theta(2r(p_z)-d)\int_0^{\tau_d}d\lambda\, v_x(\lambda)f_A(\lambda)\int_0^{\lambda}dt\,\varphi_\mu(\omega^*,t)$$
$$\times\cos[k(d+x(t)-x(\lambda))]\int_0^{\lambda}dt'\varphi_\nu(-\omega^*,t')\cos[k'(d+x(t')-x(\lambda))],$$
$$\varphi_\mu(\omega^*,t)=v_\mu(t)\exp(-i\omega^*t)+v_\mu(-t)\exp(i\omega^*t), \tag{67}$$

which is similar to expression (5a) from Ref. 18, with the function f_A replacing the resonance denominator. Here τ_d is the root of the equation $x(\tau_d)-x(0)=d$, while in the exponentials α and β of (25) the periods $T_1(\lambda)$ and $T_2(\lambda)$ are defined as follows:

$$T_1(\lambda)=2\lambda,\quad T_2(\lambda)=2\lambda_2(\lambda).$$

Integrating in (67), we obtain an asymptotic expression for $K_A(k,k')$:

$$K_A(k,k')=\pi^2\langle\rho(0)f_A(\tau_d)\rangle\frac{1}{(kk')^{1/2}}\left\{\delta(k-k')-\frac{1}{\pi(k+k')}\right\}, \tag{68}$$

which is valid if the condition $|\exp(-i\omega^*T_1)-q|\gg\delta/r$ is satisfied. At small Q, the function $f_A(\tau_1)$ can be represented after simple transformation in the form

$$f_A(\tau_d)=\frac{q}{\exp(-2i\omega^*\tau_d)-q+Q}+\frac{Q^2}{\exp(-2i\omega^*\tau_d')-q+Q}. \quad (69)$$

As seen from this formula, umklapp processes lead to the appearance of new frequencies of the size-effect cyclotron resonance. These frequencies are given by

$$\omega\tau_d^{\prime\,extr}=\pi n, \quad (70)$$

$$x(\tau_d)-x(\tau_d')=D_0, \quad \left.\frac{\partial\tau_d'}{\partial p_z}\right|_{p_z=p_z^{extr}}=0.$$

At the "old" frequencies $\omega\tau_d^{extr}=\pi n$ for the resonant increment to the impedance, we obtain

$$\Delta Z_{res}\approx 0.97\cdot 10^{-2}\frac{8\omega}{c^2k_0}\left(\frac{\tilde{k}_d}{k_0}\right)^3\varphi(\Delta), \quad (71)$$

where in Eq. (54) for $\varphi(\Delta)$ we must make the substitution $\gamma_1\to\gamma_1+Q/2\pi n$, and $\varkappa$ replaced by

$$\frac{1}{2\tau_d}\left.\frac{\partial^2\tau_d}{\partial p_z^2}\right|_{p_z=p_z^{extr}}, \quad \tilde{k}_d^{-1}=\left(\frac{16\pi^2\omega p_a\rho(0)e^2H}{c^3h^3}\right)^{-1/3}.$$

At frequencies that are multiples of $\pi/\tau_d^{\prime\,extr}$, the resonant increment to the impedance is multiplied by the small factor Q^2.

In pure samples with perfect surfaces, when the following inequalities hold

$$\gamma_1\ll Q\ll 1,$$

the width of the resonance curve is determined by the umklapp probability Q. Since the angle of incidence of the electron on the plate surface depends on the relation between $2r^{extr}$ and d, an investigation of this resonance effect can yield important information on umklapp processes for electrons incident on the sample surface at arbitrary angle, and determine the dependence of Q on φ.

To observe the size-effect resonance described above it is obviously necessary that D_0 be less than d (Fig. 3a). In a very thin plate, with thickness much less than the curvature radius r of the electron orbit, umklapp processes make it possible for electrons whose orbit center is outside the sample to take part in the high-frequency size effects. Indeed, if the center of the orbit of a glancing electron is located after the umklapp exactly at the middle of the plate (Fig. 4), i.e.,

$$D_0=r+d/2, \quad (72)$$

then such an electron has only two choices: either remain on its orbit, or turn again into an electron glancing along one of the plate surfaces. Umklapp leads thus to a substantial correlation between the opposite faces of the plate and its anomalous transparency under the condition (72). In such a plate, the glancing electron acquires a resonant period equal to the time $\tilde{\tau}_d$, of motion from one surface to another, i.e., resonance takes place under the condition

$$\omega\tilde{\tau}_d^{extr}=\pi n. \quad (73)$$

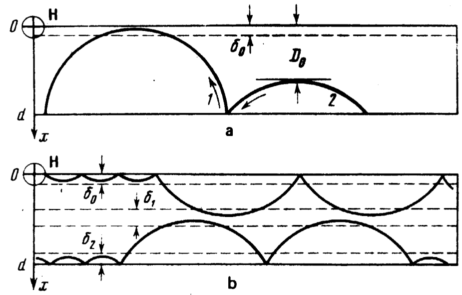

FIG. 3. a) Electron umklapp in a thin plate and formation of a new resonance frequency determined by the time of motion along the arc 2. b) Formation of HF-field bursts in a plate whose both faces reflect carrier charges with umklapp.

The most interesting fact is that resonance is possible in a magnetic field so weak that the bending of the electron trajectory is significant only in the skin layer. That is to say, if only the inequality $(\delta r)^{1/2}\ll d$ is satisfied, the electron-trajectory segment that crosses the plate is an arc with small curvature $\sim d/r$, and the period $\tilde{\tau}_d$ is practically independent of the magnetic field. The resonance frequencies are determined in this case by the extremal electron velocities on the Fermi surface along the normal to the plate boundary.

The results can be easily generalized to the case of an arbitrary number N of channels of specular reflection of the electrons. The electrons produced in this case, in the interior of the metal, at a distance on the order of the extremal diameter of the electron orbit $(N-1)$, a narrow electromagnetic-field burst. New resonance frequencies and their harmonics are also produced, viz.,

$$\omega T_\lambda^N=\pi n,$$

where T_λ^N is the extremal time of motion of the electron from the skin layer to the N-th field burst. In addition, combination resonance frequencies of the type $\omega(T_\lambda^N+T_\lambda^M)=2\pi n$, determined by the time of motion of the charge from one electromagnetic-field burst to another.

It should be noted that the formation of field bursts at distances that are multiples of D_0 from the surface, is possible also in the presence of only two scattering channels. In fact, let Eq. (19) have a solution τ_2 at $\lambda=\tau_1$, then the electrons reflected at the instant $\lambda=\tau_1$ absorb HF-field energy at a depth D_0, and after umklapp form a new burst at a depth $2D_0$, etc. The shape of the field in the burst, in contrast to the case when it is produced by direct umklapp from the skin layer, then

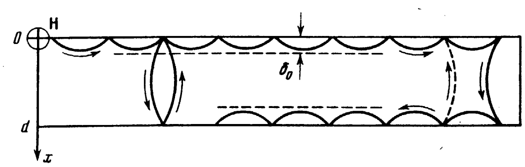

FIG. 4. Trajectories of glancing electrons in a plate with specularly reflecting faces; the center of the electron orbit is located at the center of the plate after umklapp.

depends on the distribution of the field in the preceding burst.

In a plate with specularly reflecting faces, unilateral excitation of an electromagnetic wave produces a peculiar doubling of the number of HF-field bursts (Fig. 3b). The reason is that the electrons colliding with the surface $x=0$ produce field bursts at a depth $\lesssim 2r_{max}$, while the electrons that interact with the surface $x=d$ fill, as a result of umklapp the region $2r_{max} \lesssim x \lesssim 4r_{max}$ with bursts. If the sample thickness d satisfies the condition $d=2ND_0$, then the next burst emerges to the plate surface opposite to the skin layer, and this surface turns out to be transparent to the electromagnetic waves.

APPENDIX

The equation for the Fourier transform $\varepsilon_0(k)$, in the dimensionless coordinates

$$F(\xi)=-(2\partial E(0)/\partial x)^{-1}k_0^2\mathscr{E}(k),\quad \xi=k/k_0, \tag{A.1}$$

is of the form

$$\xi^2F_0(\xi)-i\int_0^\infty d\xi'(\xi\xi')^{-1/2}[|\xi-\xi'|^{-1/2}-(\xi+\xi')^{-1/2}]F_0(\xi')=1. \tag{A.2}$$

If we seek the solution of this equation, following Hartmann and Luttinger,[12] in the form of a contour integral

$$F_0(\xi)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dz\xi zM_0(z), \tag{A.3}$$

then the Mellin transform

$$M_0(z)=\int_0^\infty d\xi F_0(\xi)\xi^{-z-1} \tag{A.4}$$

can be obtained in explicit form 15:

$$M_0(z)=\left(\frac{4}{25\sqrt{2\pi}}\right)^{2(z+2)/5}\Gamma^{-1}\left(\frac{7}{5}\right)\exp\left[\frac{\pi i(z+2)}{5}\right] \times\cos\frac{\pi z}{2}\Gamma(z+1)\Gamma\left(\frac{1-2z}{5}\right)\Gamma\left(\frac{3-2z}{5}\right). \tag{A.5}$$

The impedance of the metal, in the principal approximation in an anomaly parameter that depends smoothly on the magnetic field, is expressed in terms of the values of $M_0(z)$ at the point $z=-1$:

$$Z_0=-\frac{8i\omega}{c^2k_0}M_0(-1), \tag{A.6}$$

$$M_0(-1)=\frac{1}{2}\left(\frac{5}{4\pi}\right)^{2/5}\sin\frac{2\pi}{5}\Gamma^2\left(\frac{3}{5}\right)\exp\left(\frac{\pi i}{5}\right). \tag{A.7}$$

1) We use standard notation: e is the electron charge, h is Planck's constant, c is the speed of light, and m^* is the conduction-electron effective mass.

2) We are forced to cite here the exact formula for $\mathscr{E}_{21}(k)$ and discuss the shape of a field burst at $x\approx 2r^{extr}$ in a metal with a real boundary, since the results of many investigations of the anomalous penetration of an electromagnetic field in a metal (see, e.g., Refs. 20 and 21) are contradictory and do not agree with formula (63) for the Fourier component of the electromagnetic field.

3) The kernel $K^B_{\mu\nu}(k, k')$ does not play an important role in the impedance.

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