Dislocations of the wave-front surface and zeros of the amplitude

N. B. Baranova and B. Ya. Zel'dovich

P. N. Lebedev Institute of Physics, Academy of Sciences of the USSR
(Submitted 12 November 1980)
Zh. Eksp. Teor Fiz. 80, 1789–1797 (May 1981)

The distribution of the complex amplitude of a monochromatic field of complicated structure is discussed in relation to the existence of points where the amplitude is equal to zero. For a uniformly polarized field the "carrier" of a zero is in general a continuous line in three-dimensional space. On this line the wave front (surface of constant phase) has a screw dislocation. Solutions are found of the parabolic wave equations that describes the behavior of the amplitude near an isolated zero-line, and also near the place where a pair of zero-lines is created or annihilated. The statistical average of the number of zeros per unit area of wave front is found for a speckle-nonuniform field with Gaussian statistics. The use of flexible mirrors in the coherent optical adaptive technique (COAT technique) to transform nonuniform beams and reverse their wave fronts is discussed.

PACS numbers: 42.10.Dy

1. INTRODUCTION

The properties of wave fields with large numbers of spatial nonuniformities have recently attracted the attention of research workers in connection with several problems, such as the application of speckle interferometry in astronomy[1] and in holography,[2] the reversal of wave fronts of light in induced scattering,[3] and so on. Because of the interference of different angular components the intensity of such a field has local maxima and minima with characteristic dimensions $\Delta r_1 \sim (k\theta)^{-1}$ transverse to the central direction of propagation and longitudinal dimensions $\Delta 1_{11} \sim (k\theta^2)^{-1}$. Here $k=2\pi/\lambda$ is the wave number, and θ is the angular divergence.

At those places where the complex amplitude has not simply a minimum but an exact zero value, the wave-front surface has a dislocation. The question of dislocations of the wave front is now being intensively studied (see, for example, Refs. 4–7). In Sec. 2 we formulate the fundamental premises concerning zeros of the amplitude of a monochromatic field. In Sec. 3 we discuss the standard solution of the parabolic wave equation near dislocations of the wave front. In Sec. 4 we find the statistical mean number of zeros per unit area of the wave front of a speckle-inhomogeneous monochromatic field with Gaussian statistics.

2. THE DIMENSIONALITY OF THE ZERO-CARRIER MANIFOLD

Let us consider a random complex field $E(x)$ which depends on a single (transverse) coordinate x. If it is formed by the interference of a large number of independent components, $\mathrm{Re}\,E(x)$ and $\mathrm{Im}\,E(x)$ are two independent random functions with Gaussian statistics. Figure 1 shows schematically a possible case of the actual forms of $\mathrm{Re}\,E(x)$ and $\mathrm{Im}\,E(x)$. Since $\mathrm{Re}\,E(x)$ takes both positive and negative values, in the intervals between them there must be points where $\mathrm{Re}\,E(x)=0$. The average interval between zeros is of the same order of magnitude as the correlation length. An exact formula can also be found (cf., e.g., Ref. 8):

$$\nu\,[\mathrm{cm}^{-1}]=\nu_+ + \nu_- = \pi^{-1}[-A''(0)/A(0)]^{1/2}, \qquad (1)$$
$$A(x)\equiv A(-x)=\langle \mathrm{Re}\,E(x_0)\ \mathrm{Re}\,E(x_0+x)\rangle,$$

where ν_+ is the average number of crossings of the axis from below to above per unit length. In the general case none of the points x_i that correspond to zeros of $\mathrm{Re}\,E(x)$ will coincide with any points x_j that are zeros of $\mathrm{Im}\,E(x)$. Therefore in general the intensity

$$I(x)=|E(x)|^2=[\mathrm{Re}\,E(x)]^2+[\mathrm{Im}\,E(x)]^2,$$

in general does not touch the axis of abscissas at any point.

Let us now proceed to the case of two transverse coordinates x, y, for which

$$E(x,\ y)=\mathrm{Re}\,E(x,\ y)+i\,\mathrm{Im}\,E(x,\ y).$$

The zeros of the function $\mathrm{Re}\,E(x, y)$, i.e., the solutions of the equation $\mathrm{Re}E(x, y)=0$, determine a number of curves in the x, y plane; see Fig. 2. In general these curves will intersect, giving rise to multiple zeros of the function $\mathrm{Re}E(x, y)$, but this is not important for us. There is another set of curves corresponding to $\mathrm{Im}E(x, y)=0$, and now the intersections of curves of one family with those of the other gives discrete points in which we are interested, where $|E(x, y)|^2=0$; these are lines of zero intensity. If we consider the problem of the propagation of such zeros along the direction z in accordance with the wave equation, the discrete points in the x, y plane (carriers of the zeros of $|E|^2$) are converted into lines which twist about the intertwine in complicated ways ("snakes"; cf. the terminology of a previous paper[9]). It is clear that in general these lines do not intersect each other in three-dimensional space. Moreover, a given line cannot appear singly at some plane $z=\mathrm{const}$, nor can it disappear singly. Zeros in the pattern must appear or be annihilated in pairs; this simply corresponds to a change of sign of $(t)_z$ on some one line; here t is the unit vector tangent to the curve.

According to analogous considerations, in the plane case of the diffraction problem for a complex mono-

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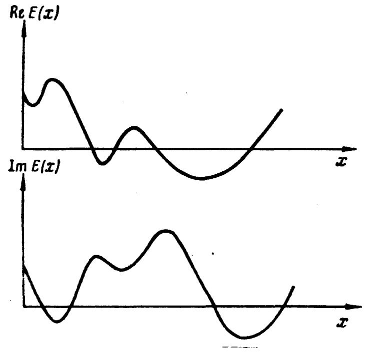

FIG. 1. Realization of random functions of one variable x: Re $E(x)$ and Im $E(x)$; in the general case zeros of these functions do not coincide.

chromatic field $E(x, z)$ with the central direction of propagation along the z axis, there must be discrete points with $|E|^2=0$, and their mean separation along x is $\sim(k\theta_x)^{-1}$, while that along z is $\sim(k\theta_x^2)^{-1}$.

Let us now consider what new features come in when the vector character of the field $\mathbf{E}$ is taken into account. In the paraxial approximation, i.e., when the divergence of the beams is small ($\theta \ll 1$), the longitudinal component E_z of the field is very small, $E_z \sim E_\perp \theta^2$, so that its contribution to the intensity is still smaller, $\delta I \sim I_0\theta^4$. Therefore in the paraxial approximation we can set $\mathbf{E}=\{E_x(x, y, z), E_y(x, y, z), 0\}$. For a field which is spatially completely polarized we have

$$\mathbf{E}(x, y, z)=\mathbf{e}E(x, y, z), \tag{2}$$

where $\mathbf{e}=(e_x, e_y, 0)$ is a constant unit vector, which in general corresponds to an elliptical polarization. Owing to this, all of the foregoing considerations were given for a completely polarized field of the form (2). In the case of a spatially inhomogeneous state of polarization $E_x(r, z)$ and $E_y(r, z)$ are linearly independent complex-valued functions, and the lines of their zeros do not in general intersect.

If we do not specify that the field is monochromatic, in the most general case of a completely polarized wave the zero lines move mainly in the direction of propagation with a speed of the order of the speed of light. For nonpolarized fields zeros appear only at

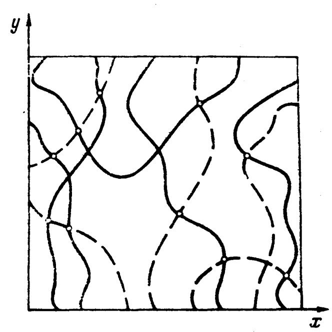

FIG. 2. Trajectories of zeros of a random function on a two-dimensional plane: Re $E(x, y)=0$ on the solid lines, and Im $E(x, y)=0$ on the dashed lines. The intersections of the solid lines with the dashed ones give points which are zeros of the total amplitude.

particular points of three-dimensional space at particular instants of time.

We shall make a number of comments on what has been said. Up to now we have been speaking about a random speckle pattern. It is clear, however, that the topological and dimensional arguments do not depend on the specific nature of the interfering fields. The only difference is that for regular fields zeros may not occur at all. Moreover, we have considered fields of "general form" and have discussed those of their properties that are not destroyed by a small perturbation of the initial conditions. Under these assumptions and for regular fields a zero carrier, if it exists at all, is a line in three dimensional space. Furthermore, the absence of monochromatically does not in all cases at once alter the picture.

In many cases a factorization holds:

$$E(\mathbf{r}, z=0, t)=f(t)E_0(\mathbf{r}, z=0). \tag{3}$$

This will be the case, for example, when a quasimonochromatic but plane wave passes through a fixed (unchanging in time) thin transparency. It is clear that zeros of the complete field

$$E=f(t-z/c)E_0(\mathbf{r}, z)$$

are determined by the spatial part only. We have already given another example of factorization, namely Eq. (2) for a completely polarized field. The case can occur in which the behavior $f(t)$ in time brings in a depolarization but the function $E_0(r, z)$ is still to good accuracy the same as for a monochromatic field.

Finally, for monochromatic fields a shift of the origin from which time is measured by a fraction of a period leads to an intermixing of the fields ReE and ImE. Only the properties of the function $|E|^2$ are invariant under such a shift, but not the functions $E_0(\mathbf{r}, z)$ ReE and ImE taken separately. In this respect the zeros of the fields ReE and ImE serve only for carrying through our arguments, but have no direct physical meaning. In contrast with this, the zeros of the intensity $|E|^2$ are open to direct physical observation.

3. SOLUTIONS OF THE PARABOLIC EQUATION WITH ZEROS OF THE AMPLITUDE

We shall consider completely polarized monochromatic fields with small angular divergence around the central direction of propagation z. In the paraxial approximation, as is well known, the complex amplitude $E(\mathbf{r}, x)$ of such a field, defined by the equation

$$\mathbf{E}_{\text{real}}(\mathbf{r}, z, t)=\{\mathbf{e}e^{-i\omega t+ikz}E(\mathbf{r}, z)+\mathbf{e}^*e^{i\omega t-ikz}E^*(\mathbf{r}, z)\}/2, \tag{4}$$

satisfies the parabolic equation

$$2ik\frac{\partial E}{\partial z}+\Delta_\perp E(\mathbf{r}, z)=0. \tag{5}$$

Let us assume that at some point (which we designate as $\mathbf{r}_\perp=0$) in the cross section $z=0$ the complex amplitude has an isolated zero, $E(0, 0)=0$. Owing to the continuity of the field it can be expanded in powers of the deviation $\mathbf{r}, z$:

$$E(\mathbf{r},\ z)=\mathbf{A}\mathbf{r}+B_{ik}x_ix_k+Cz+\ldots, \tag{6}$$

and it follows from Eq. (5) that

$$ikC+B_{ii}=0. \tag{7}$$

Let us first consider a small neighborhood of the zero line. Then the terms quadratic in x and y can be neglected; i.e., we can consider here $B=0$, and along with this $C\sim B\lambda/2\pi$ also can be omitted. Then a field of the form

$$E(\mathbf{r},\ z)=\mathbf{A}\mathbf{r}=(A_x'+iA_x'')x+(A_y'+iA_y'')y \tag{8a}$$

satisfies Eq. (5) exactly everywhere in space and has a zero of its amplitude on the straight line $(\mathbf{r}=0, z)$. By a rotation around the z axis the expression (8a) can be reduced to the form

$$E(\mathbf{r},\ z)=e^{i\alpha}(A_xx+iA_yy), \tag{8b}$$

where A_x, A_y are real quantities. In these coordinates the constant-amplitude surfaces are elliptic cylinders with generators along z and the ratio of semiaxes $a_x/a_y=A_y/A_x$. The phase $\varphi(r)=\arg E(r)$ increases by 2π in passing around the zero line, since

$$\varphi(\mathbf{r})=\alpha+\mathrm{arctg}\,(A_yy/A_xx). \tag{8c}$$

Since the entire field has the fast phase factor e^{ikz}, it can be said that the zero line contains a screw dislocation of the wave front. We note that for $A_x/A_y\neq\pm1$ the phase of the field depends on the angle $\gamma=\arctan(y/x)$ in a nonlinear, though monotonic, way.

If a function $E_0(\mathbf{r}, z)$ is a solution of the parabolic equation (5), then the function rotated by the two-dimensional angle $\boldsymbol{\psi}=(\psi_x, \psi_y, 0)$,

$$E_1(\mathbf{r},z)=\exp\,(ik\boldsymbol{\psi}\mathbf{r}-ik\boldsymbol{\psi}^2z/2)E_0(\mathbf{r}-\boldsymbol{\psi}z,z) \tag{9}$$

is also a solution of Eq. (5). Owing to this we can construct from Eqs. (8) the solution corresponding to the zero line $\mathbf{r}=\boldsymbol{\psi}z$ inclined at the angle ψ with the z axis.

Let us suppose that a field $E(\mathbf{r}, z)$ consists of the field $E_0(\mathbf{r}, z)$ and a small correction $u_1(\mathbf{r})$. It is not hard to see that

$$\begin{gathered}E(\mathbf{r},\ z)=e^{i\alpha}(A_xx+iA_yy)+u_1(r)\approx e^{i\alpha}[A_x(x-x_1)+iA_y(y-y_1)],\\ x_1=-\mathrm{Re}[u_1(0)e^{-i\alpha}]/A_x,\quad y_1=-\mathrm{Im}[u_1(0)e^{-i\alpha}]/A_y.\end{gathered} \tag{10}$$

In other words, a small correction u_1 added to a field $E_0(\mathbf{r}, z)$, which has a line of zeros does not remove the zeros, but merely shifts the position of the line in the x, y plane. In particular, if $u_1\sim\exp(i\Lambda z)$, then according to Eq. (9) this shift corresponds to a screw perturbatuon of the line [cf. Eq. (7)].

If in the expression (8b) one of the components of the vector $\mathbf{A}$ (for example, A_x) is equal to zero, then this solution is degenerate and describes a zero on the entire plane $y=z=0$. A zero of this type is unstable under small perturbations. Therefore we shall examine the case with $A_x=0$, taking account of the terms $\propto B_{ik}$ and $C=iB_{11}/k$ in Eq. (6). It is not hard to verify that the solution of the form (5) with $A_x=0$, $A_y\neq0$ describes a field with a line of zeros whose tangent $\mathbf{t}$ at the point $\mathbf{r}=0$, $z=0$ is directed along the x axis, i.e., $\mathbf{t}(\mathbf{r}=0, z=0)=\mathbf{e}_x$. Introducing the quantity $iA_y''e^{i\alpha}$ as a common factor, we have from Eq. (6)

$$E(\mathbf{r},\ z)=iA_y''e^{i\alpha}\{y+b_{xx}x^2+2b_{xy}xy+b_{yy}y^2+i(b_{xx}+b_{yy})z/k\}. \tag{11}$$

In this form this solution depends on a single common complex factor $iA_y''e^{i\alpha}$ and on three independent complex (i.e., on six real) parameters: b_{xx}, b_{xy}, b_{yy}. The normal vector $\mathbf{n}$ to the curve, at the point $\mathbf{r}=0$, $z=0$, the radius of curvature ρ, and the torsion T of the curve at this point are given in terms of b_{ik} by the formulas

$$\mathbf{n}=\mathbf{e}_y\cos\alpha+\mathbf{e}_z\sin\alpha,\quad \alpha=\mathrm{arctg}\left\{k\left[\frac{b_{xx}'}{b_{xx}''}(b_{xx}'+b_{yy}')+(b_{xx}''+b_{yy}'')\right]^{-1}\right\}, \tag{12a}$$

$$\rho=|b_{xx}'+b_{yy}'|\,\{k^2(b_{xx}'')^2+[(b_{xx}'+b_{yy}')b_{xx}'+(b_{xx}''+b_{yy}'')b_{xx}'']^2\}^{-1/2}/2. \tag{12b}$$

Thus there are still three free real parameters. They can be characterized by considering the behavior of the phase and amplitude of the field near the lines of zeros; we shall not pursue the details. For a random field with the divergence $\Delta\theta_x\sim\Delta\theta_y\sim\theta_0$ we have in order of magnitude $b\sim k\theta_0$, and from this the estimates for ρ and T:

$$\rho\sim T\sim\lambda.$$

Along a given zero line the parameters of the ellipse that describes the surfaces of constant amplitude $|\mathbf{A}\mathbf{r}|^2=\text{const}$ or of constant phase

$$kz+\varphi(r,\ z)=\text{const},$$

will in the general case change, owing to the gradual change of the quantity A according to Eqs. (8) and also owing to the fact that the trajectory of the zero is not exactly straight. The sign of the phase change $\Delta\varphi=\pm2\pi$ on going around the zero line along a closed path is, however, a topologically stable feature. This sign is the same as that of the spiral (right or left handed) which the zero line describes when a small perturbation field u_1 is added which propagates at a small angle with the main field (8a).

The difference N_+-N_- of the numbers of zeros with positive and negative signs of $\Delta\varphi$ is conserved in the process of propagation. This means that there occur processes of creation and of annihilation only of pairs of zeros with opposite signs. On the average the numbers of "positive" and "negative" zeros in a cross section of a speckle-nonuniform field are equal. This fact is due to the assumption that the beam is statistically homogeneous. Actually, if the mean number of positive zeros N_+ (per cm^2) were larger than N_-, this would mean that on going around an area S the phase would differ a shift

$$\Delta\Phi=2\pi(N_+-N_-)S.$$

Since $S\sim L_\perp^2$ and the angle of inclination is

$$\alpha\sim\frac{\lambda}{2\pi}\nabla\varphi\sim\frac{\lambda}{2\pi}\frac{\Delta\Phi}{L_\perp},$$

where $L_\perp$ is the transverse size of the region considered, we have

$$\alpha\sim\lambda(N_+-N_-)L_\perp. \tag{13}$$

Accordingly, for a beam of unbounded cross section ($L_\perp\to\infty$) the angle of inclination α of the rays increases without bound for $N_+ \neq N_-$, which contradicts the assumed uniformity.

In principle one can imagine a bounded beam in which

$$\int (N_+ - N_-) d^2\mathbf{r} \neq 0.$$

Let us consider, for example, a field $E(x, y, z)$ of the form

$$E(x, y, 0) = r^m \exp im\psi \exp(-r^2/a^2),$$
$$r = (x^2+y^2)^{1/2}, \quad \psi = \operatorname{arctg}(y/x). \tag{14}$$

Such a field can serve as the profile of a mode of an optical resonator (cf., e.g., Ref. 10). A field of the form (14) has a zero of the m-th order on the axis. Passage of a beam of this sort, with a bounded cross section, through a nonuniform phase plate (cf. e.g., Ref. 3) gives a speckle-nonuniform field in which

$$\int (N_+ - N_-) d^2\mathbf{r} = m.$$

Questions about the statistical description of such "twisted" beams are also discussed in a paper by Shkunov and the writers.[11] Actually the total number of zeros in this field will be still larger, since pairs of zeros of opposite signs can be produced.

4. THE STATISTICAL MEAN PROPERTIES OF THE ZEROS OF A COMPLEX GAUSSIAN RANDOM FIELD

Let us first derive the general expression for the mean number of zeros per suit area, $N = N_+ + N_-$, for a statistically homogeneous complex random process. The total number of zeros in some region of the x, y plane with area S can be written in the form

$$NS = \left\langle \int dx\, dy\, \delta(E_1(x, y))\, \delta(E_2(x, y))\, |\partial(E_1, E_2)/\partial(x, y)| \right\rangle. \tag{15}$$

Here $E(x, y) = E_1(x, y) + iE_2(x, y)$, and we do not write out explicitly the value $z = 0$. The angle brackets indicate averaging over an ensemble of random fields. It is easy to verify that Eq. (5) is correct, by noting that each zero point of the total field will give (before the averaging) a contribution unity to the right side of the equation. We note, by the way, that positive zeros (see Sec. 3) correspond to positive signs of the Jacobian $\partial(E_1, E_2)/\partial(x, y)$, and that for negative zeros $\partial(E, E)/\partial(x, y) < 0$. The absolute value of the Jacobian can be written in the form

$$|G| = \left| \frac{\partial E_1}{\partial x}\frac{\partial E_2}{\partial y} - \frac{\partial E_2}{\partial x}\frac{\partial E_1}{\partial y} \right| = |E_{1x}E_{2y} - E_{1y}E_{2x}|, \tag{16}$$

where we have introduced an abbreviated notation for the derivatives. It follows from this that

$$NS = \int dx\, dy \int dE_1\, dE_2\, dE_{1x}\, dE_{2x}\, dE_{1y}\, dE_{2y}$$
$$\times W_6(E_1, E_2, E_{1x}, E_{1y}, E_{2x}, E_{2y})\, \delta(E_1)\, \delta(E_2)\, |G|. \tag{17}$$

Here W_6 is the joint probability of the quantities E_1, E_2 and their gradients at the given point. Removing the integration over dE_1 and dE_2 and cancelling a factor S, we get

$$N = \int |G|\, W_6(0, 0, E_{1x}, E_{1y}, E_{2x}, E_{2y})\, dE_{1x} \ldots dE_{2y}. \tag{18}$$

Up to this point we have not used the assumption that the statistics of the field is Gaussian. If we do make this assumption, the quantity W_6 can be expressed in terms of the correlator of the complex field, and this last is determined by the Van Zittert-Zernicke theorem:

$$\langle E^*(\mathbf{r}_1) E(\mathbf{r}_2) \rangle = I \int j(\boldsymbol{\theta}) \exp[ik\boldsymbol{\theta}(\mathbf{r}_2 - \mathbf{r}_1)]\, d^2\boldsymbol{\theta},$$
$$\int j(\boldsymbol{\theta})\, d^2\boldsymbol{\theta} = 1, \tag{19}$$

where $j(\theta)$ is the normalized angular spectrum, and $\theta = (\theta_x, \theta_y)$. For specific calculations it is convenient to use the well known theorem that if Gaussian random quantities are uncorrelated they are independent. From Eq. (19) we have

$$\left\langle E^*(\mathbf{r}_1) \frac{\partial E(\mathbf{r}_2)}{\partial x_i} \right\rangle \Big|_{\mathbf{r}_1 = \mathbf{r}_2} = ik\langle \theta_i \rangle = ik \int j(\boldsymbol{\theta})\, \theta_i\, d^2\boldsymbol{\theta}. \tag{20}$$

By a suitable rotation of the z axis (i.e., a displacement in the θ_x, θ_y plane) we can make the two components $\langle\theta_x\rangle$ and $\langle\theta_y\rangle$ equal to zero, i.e., choose the z axis in the direction of the center of gravity of the angular distribution. Then the complex gradients $\partial E/\partial x$ and $\partial E/\partial y$ are independent of the field $E(\mathbf{r})$ itself at the same point $\mathbf{r}$. By a rotation of axes in the x, y plane the correlation matrix

$$\langle \theta_i \theta_k \rangle = \frac{1}{k^2} \left\langle \frac{\partial E^*(\mathbf{r}_1)}{\partial x_i} \frac{\partial E(\mathbf{r}_2)}{\partial x_k} \right\rangle \Big|_{\mathbf{r}_1 = \mathbf{r}_2} = \int j(\boldsymbol{\theta})\, \theta_i \theta_k\, d^2\boldsymbol{\theta} \tag{21}$$

can be transformed to principal axes, after which all three of the complex quantities (E, $\partial E/\partial x$, and $\partial E/\partial y$) are independent at the given point. In this system of coordinates, in which $\langle\theta\rangle = 0$, and the matrix $\langle\theta_i\theta_k\rangle$ is diagonal, the probability distribution of the parameters $E_1, E_2, E_{1x}, E_{2x}, E_{1y}, E_{2y}$ is given by

$$W_6(E_1, E_2, E_{1x}, E_{2x}, E_{1y}, E_{2y}) = \pi^{-3}(k^4\langle\theta_x^2\rangle\langle\theta_y^2\rangle)^{-1} I^{-3} \exp\{-I^{-1}(E_{1x}^2 + E_{2x}^2)/k^2\langle\theta_x^2\rangle - I^{-1}(E_{1y}^2 + E_{2y}^2)/k^2\langle\theta_y^2\rangle\} \exp\{-I^{-1}(E_1^2 + E_2^2)\}. \tag{22}$$

After this the integral in Eq. (18) can be calculated easily and gives, in the specified coordinates system

$$N = \frac{k^2}{2\pi}(\langle\theta_x^2\rangle\langle\theta_y^2\rangle)^{1/2}. \tag{23a}$$

Going back to the original (arbitrary) coordinate system, we can write the expression (23a) in the form

$$N = \frac{k^2}{2\pi}(\det \hat{C})^{1/2}, \quad C_{ik} = \langle\theta_i\theta_k\rangle - \langle\theta_i\rangle\langle\theta_k\rangle. \tag{23b}$$

We note that the probability distribution (22) enables us to discuss the statistical mean properties of the ellipses of constant amplitude [cf. Eq. (8)]. Since the complex vector $\mathbf{A}$ of Eq. (8a) is the same as the gradient $\nabla_\perp E(\mathbf{r}, z)$ of the complex field. Thus the large semiaxis of this ellipse is most often oriented in the direction in which the angular divergence of the beam is smallest.

Other interesting quantities are the average number M (per cm^3) of points of creation and annihilation of pairs of zeros and their distribution $m(\alpha)$ (cm^{-3}rad^{-1}) in the direction of the tangent $\mathbf{n} = (\cos\alpha, \sin\alpha, 0)$ to the zero line at the point of creation or annihilation. For this purpose it is convenient to introduce the gradient of the complex field in the direction $\mathbf{n}$:

$$E_n = E_{1n} + iE_{2n} = \left(\cos\alpha \frac{\partial}{\partial x} + \sin\alpha \frac{\partial}{\partial y}\right)(E_1(\mathbf{r}, z) + iE_2(\mathbf{r}, z)). \tag{24}$$

The creation or annihilation of a pair of zeros with a given corresponds to the satisfaction of four real conditions:

$$E_1(\mathbf{r}, z)=0, \quad E_2(\mathbf{r}, z)=0, \quad E_{1\alpha}(\mathbf{r}, z, \alpha)=0, \quad E_{2\alpha}(\mathbf{r}, z, \alpha)=0. \tag{25}$$

The quantity $m(\alpha)$ is given by the expression

$$V\int m(\alpha)\,d\alpha=\left\langle \int d^2\mathbf{r}\,dz\,d\alpha\,\delta(E_1)\,\delta(E_2) \times \delta(E_{1\alpha})\,\delta(E_{2\alpha})\left|\frac{\partial(E_1,E_2,E_{1\alpha},E_{2\alpha})}{\partial(x,y,z,\alpha)}\right|\right\rangle. \tag{26}$$

If we are interested only in the total number

$$M=\int_0^{2\pi} m(\alpha)\,d\alpha,$$

the problem can be stated somewhat differently. Namely, the tangent vector to the zero line passing through a given point can be written in the form

$$\mathbf{t}=\mathbf{L}/|\mathbf{L}|, \quad \mathbf{L}=[\nabla_3 E_1, \ \nabla_3 E_2]. \tag{27}$$

Then the point of production of a pair of zeros corresponds to the satisfaction of three conditions:

$$E_1(\mathbf{r}, z)=0, \quad E_2(\mathbf{r}, z)=0, \quad L_z(\mathbf{r}, z)=0. \tag{28}$$

The calculations are rather cumbersome, however, and we do not give them.

5. CONCLUSION

The problem of reversing a wave front of laser radiation has recently attracted much attention. Besides the methods of nonlinear optics (Mandel'shtam-Brillouin induced scattering, four-wave and three-wave processes, reversal at a surface), there are also discussions of the possibility of creating a flexible adaptive surface a coherent optical adaptive technique (COAT). To reverse a wave front with a mirror it is necessary that a surface of constant phase of the light field coincide with the surface of the mirror. We wish here to call attention to the fact that when there is a simple zero in the incident field its wave front is a many-sheet surface with a singularity of the type of a circular stairs near the point of zero amplitude. This makes it clear that it is impossible with a smooth bending of a mirror surface to make it coincide with such a wavefront, not only in the neighborhood of the zero, but also far away from it. Essentially the same remark applies to the idea of using a COAT technique to make perfect astronomical images.

From our point of view it is extremely interesting to elucidate the answer to the question: Is it possible to use two (or more) COAT mirrors, placed successively, to reverse a wave front of a field with amplitude zeros? For this the field must have equal numbers of positive and negative zeros. Then with the first mirror one can try to make all the zeros of opposite signs annihilate each other on the way to the second mirror, and it will have to reverse only a smooth wave front. We recall that also in the geometric-optics approximation there exist problems of beam transformation that require not fewer than two mirrors [cf. Ref. 12].

In conclusion the writers thank V.V. Shkunov for valuable discussions of the properties of speckle-inhomogeneous fields.

[1]A. A. Tokovinin and P. V. Shcheglov, Usp. Fiz. Nauk **129**, 645 (1979) [Sov. Phys. Uspekhi 22, 960 (1979)].
[2]N. G. Vlasov, and A. E. Shtan'ko, Opt. Spektrosk. **43**, 192 (1977) [Opt. Spectrosc. (USSR) **43**, 109 (1977)].
[3]B. Ya. Zel'dovich, V. I. Popovichev, V. V. Ragul'skiĭ, and F. S. Faĭzullov, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 160 (1972) [JETP Lett. **15**, 109 (1972)].
[4]J. F. Nye and M. V. Berry, Proc. Roy. Soc. London **A336**, 165 (1974).
[5]F. J. Wright, in Structural stability in physics, Eds. W. Güttinger and H. Eikemeier, Springer Verlag, Berlin, 1979, p. 141.
[6]M. V. Berry, Singularities in waves and rays, Lectures in Les Houches Summer School, 1980 (to be published by North Holland Pub. Co.).
[7]M. V. Berry, J. F. Nye, and F. J. Wright, Phil. Trans. Roy. Soc. London **A291**, 453 (1979).
[8]V. I. Tikhonov, Vybrosy sluchaĭnykh protessov (Dispersion in random processes). Moscow, Nauka, 1970, p. 123.
[9]Ya. B. Zel,dovich and V. V. Shkunov, Kvant. Elektron. **5**, 36 (1978) [Sov. J. Quantum Electron. **8**, 15 (1978)].
[10]George Birnbaum, Optical masers, New York, Academic Press, 1964.
[11]N. B. Baranova, B. Ya. Zel'dovich, and V. V. Shkunov, Kvant. Elektron. **5**, 973 (1978) [Sov. J. Quantum Electron. **8**, 559 (1978)].
[12]B. E. Kinber, Radiotekh. Elektron. **7**, 973 (1962).

Translated by W. H. Furry