Anisotropy of acoustoelectric effect in metals

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(Submitted 23 November 1979; resubmitted 3 October 1980)
Zh. Eksp. Teor. Fiz. **80**, 1868–1882 (May 1981)

The acoustoelectric effect at $ql > > 1$ (l is the electron mean free path) is considered for a metal with an isotropic electron dispersion law. It is shown that the anisotropy of the acoustoelectric current J^A is highly sensitive to the character of the electron scattering. In particular, when the scattering mechanism is replaced, a change can take place in the sign of the effect and in the character of the singularities (for example, logarithmic peaks can appear). This makes it possible to use the acoustoelectric effect to study a nonequilibrium electron distribution function. In scattering by phonons, the anisotropy J^A is analogous to the anisotropy of the sound absorption coefficient Γ, and in the case of scattering by impurities it is reasonably described by the τ-approximation formula and is essentially connected with the anisotropy of the deformation potential Λ. The temperature dependence of J^A is determined for an arbitrary Fermi surface, namely, J^A = const (T) for scattering by impurities; for scattering by completely dragged phonons, $J^A \sim T^{-5}$ in the case of open Fermi surface and $J^A \sim e^{T/T^*}$ for closed Fermi surfaces (including compensated metals), while if the phonons are at equilibrium (the Bloch hypothesis) $J^A \sim T^{-5}$ in all cases. Concrete examples of angular dependences of J^A are presented for alkali and noble metals.

PACS numbers: 72.50. + b, 72.15.Qm

1. INTRODUCTION

The acoustoelectric effect consists in the appearance of an emf (or an electric current, if the circuit is closed) in a sample along which an ultrasound wave propagates.[1-3] Compared with semiconductors, the magnitude of this effect in metals is very small, and it was observed only recently by Zavaritskiĭ[4] in experiments on tin, using the SKIMP procedure. These experiments have shown a rather strong anisotropy of the effect up to a reversal of the sign of the acousto-emf with changing direction of propagation of the sound wave.

Zavaritskiĭ, Kaganov, and Mevlyut[5] have undertaken an attempt to explain the anisotropy of the emf as being due to the character of the electron structure of the metal. In describing the scattering of the electrons, they confined themselves to the approximation. Kaganov, Mevlyut, and the author have later proposed a general method[6] free of this restriction.[1)] Let us recall briefly the main conclusions of Ref. 6.

Under conditions corresponding to the experimental situation, the sound wavelength $\lambda = 2\pi/q$ is much shorter than the electron mean free path l, i.e., $ql \gg 1$. In this case the sound wave can be regarded as a flux of phonons with a δ-like distribution function in wave-vector space (this is rigorously valid for longitudinal sound, while for transverse sound it is necessary to satisfy the stronger inequality $\omega\tau \gg 1$, where ω is the sound frequency and τ is the electron relaxation time). The presence of the sound wave is taken into account by adding to the field part of the kinetic equation for the electrons a term [see Ref. 6, formula (3)] which can be easily transformed into

$$U^A = -\frac{\pi\omega}{\rho\hbar s^3} W \frac{\partial}{\partial k_n}\left[\frac{|\Lambda_k|^2}{v_k}\frac{\partial f_k^0}{\partial \varepsilon_k}\delta(\cos\theta_v)\right]. \quad (1)$$

Here W is the density of the energy flux in the wave, Λ_k is the component of the deformation potential, θ_v is the angle between $\mathbf{v}$ and $\mathbf{n} = \mathbf{q}/q$, $k_n = \mathbf{k}\cdot\mathbf{n}$, s is the speed of sound, and ρ is the density of the metal. The singular character of the function U^A (which is seen to be proportional to the double layer in k-space) is physically connected with the fact that the sound wave perturbs the electrons only near the "belts" $\cos\theta_v = 0$, on which the energy and momentum laws are simultaneously satisfied.[8] As shown in Ref. 6, the fact that the collision operator $\hat{L}$ is Hermitian makes it possible to express the acoustoelectric current J^A in terms of the solution Φ_k of the kinetic equation

$$\mathbf{v}_k\mathbf{n}\partial f_k^0/\partial\varepsilon_k = \hat{L}\Phi_k, \quad (2)$$

which is used in the calculation of the electric conductivity σ. The latter, together with the current J^A is determined by the formulas

$$\sigma = \frac{2e^2}{(2\pi)^3\hbar}\int \Phi_k \cos\theta_v dS_k, \quad (3)$$

$$J_n^A = \frac{2\pi e\omega}{(2\pi)^3\hbar^2 s^3\rho} W \int \frac{|\Lambda_k|^2}{v_k^2}\frac{\partial\Phi_k}{\partial k_n}\delta(\cos\theta_v)\,dS_k. \quad (4)$$

In the case of an open circuit, the resultant acousto-emf is determined from the condition that the current in the sample $\sigma E^A = J^A$ vanish, and is then given by

$$E_n^A = \frac{\pi\omega W}{e\hbar s^3\rho}\left(\int \Phi_k\cos\theta_v dS_k\right)^{-1}\int \frac{|\Lambda_k|^2}{v_k^2}\frac{\partial\Phi_k}{\partial k_n}\delta(\cos\theta_v)\,dS_k. \quad (5)$$

The present paper is a direct continuation of the preceding one[6] and its purpose is to analyze, on the basis of Eqs. (1)–(5), the possible anisotropy of the acoustoelectric effect as a function of the character of the Fermi surface of the metal under conditions when some particular scattering mechanism predominates. This analysis is of interest because the anisotropy of the acousto-emf (or current) contains valuable information on the electron system.[4] Nonetheless, the question of the character of this information and of the possibility of its effective extraction has remained open to this day.

When considering concrete electron-scattering mechanisms, we confine ourselves to the following traditional situations:

a) scattering by impurities;

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b) scattering by phonons within the framework of the Bloch hypothesis (the assumption that the phonon system is in equilibrium);

c) scattering by phonons with allowance for their dragging.

By way of illustration we present the results of numerical calculations of J^A for metals with simple Fermi surfaces—alkali and noble metals. We shall investigate separately also the singularities in the angular dependence of J^A. In the numerical calculations, the anisotropy of the Fermi surface was taken into account by the pseudopotential method.[9] According to this method, the Fermi surface of the metal is a sphere of free electrons, distorted near the Bragg planes. If independence of the action of the different planes can be established, then the problem reduces in essence to a study of the influence of one Bragg plane. The equations of the equal-energy surfaces are determined in this case by the simple formulas[9]

$$\varepsilon_k = {}^1/_2(\varepsilon_k{}^0+\varepsilon_{k-G}^0) \pm {}^1/_2[(\varepsilon_{k-G}^0-\varepsilon_k{}^0)^2+4V_G{}^2]^{1/2}, \quad \varepsilon_k{}^0=\hbar^2k^2/2m, \tag{6}$$

in which G is the reciprocal-lattice vector, V_G is the Fourier component of the pseudopotential of the given metal. The criterion for the applicability of the method is smallness of the pseudopotential parameter $\eta = |V_G|/\varepsilon_F \ll 1$. The effects connected with the nonlocality of the pseudopotential will be neglected throughout.

2. SCATTERING BY IMPURITIES

In the calculation of the electric conductivity one frequently uses the relaxation-time approximation, i.e., Φ_k is represented in the form

$$\Phi_k = \tau_k v_k n \tag{7}$$

with a smooth function τ_k. In this approximation we have for the acousto-emf the formula[5]

$$E^A = \frac{\pi\omega}{e\rho s^2} W \left(\int \tau_k v_k \cos^2\theta_v dS_k \right)^{-1} \int \tau_k \frac{|\Lambda_k|^2}{v_k{}^2} \frac{1}{m_{nn}} \delta(\cos\theta_v)\, dS_k, \tag{8}$$

which is obtained after substituting (7) in (5). Here $m_{nn}^{-1} = \partial^2\varepsilon/\hbar^2\partial k_n^2$ is the effective-mass tensor component.

However, the τ approximation is valid only for the isotropic case, and its frequent use for metals with a complicated Fermi surface is justified mainly by the weak sensitivity of the electric conductivity to the exact form of Φ_k. On the contrary, the structure of Φ_k determines to a considerable degree the anisotropy of the acoustoelectric effect, in particular its difference from the anisotropy of the sound-absorption coefficient[10,11]

$$\Gamma(n) = \frac{2\pi\omega}{(2\pi)^3\hbar\rho s^2} \int \frac{|\Lambda_k|^2}{v_k{}^2} \delta(\cos\theta_v)\, dS_k. \tag{9}$$

Therefore the use of (7) and (8) calls for an additional justification. We shall show that in scattering by impurities expression (7) is in many cases a good approximation.

1. We consider a Fermi surface described by the weak-pseudopotential model. We begin with the case of

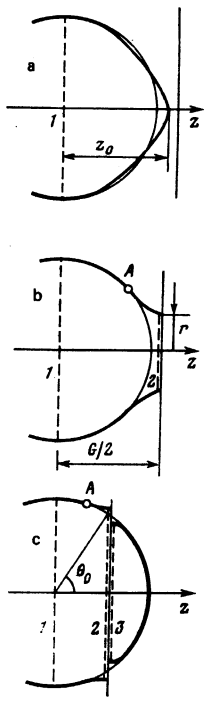

FIG. 1. Different positions of the Bragg plane relative to the free-electron sphere. The dashed lines show the belts $\cos\theta_v = 0$ at $\mathbf{n} \parallel \hat{z}$. A—parabolic point of X type.

one Bragg plane, which can be located relative to the sphere of the free electrons in one of the three ways shown in Fig. 1. The kinetic equation (2), in view of the elasticity of the scattering, is of the form

$$v_k n = \int w_{kk'}(\Phi_k - \Phi_{k'}) \frac{dS_{k'}}{(2\pi)^3\hbar v_{k'}}. \tag{10}$$

We can seek its solution by iteration in the parameter $\eta = |V_G|/\varepsilon_F \ll 1$. At $\eta = 0$, the Fermi surface is a sphere and Eq. (10) has the known solution

$$\Phi_k{}^0 = \tau_0 v_k{}^0 n = \hbar\tau_0 k n/m,$$

where

$$\frac{1}{\tau_0} = \frac{2\pi}{\hbar} \int U^2(q)\,(1-\cos\theta_{kk'}) \frac{dS_{k'}}{(2\pi)^3\hbar v_{k'}}, \tag{11}$$

and $U(q)$ is the impurity pseudopotential (in contrast to the pseudopotential V_G of the atom of the matrix), in terms of which the transition probability $w_{kk'}$ is expressed. As will be shown later on, at finite η the function Φ_k differs from Φ_k^0 only in a small vicinity ($\sim\eta k_F$) of the Bragg plane (Fig. 2). Therefore, substituting Φ_k^0 for $\Phi_{k'}$ in (10) by way of the first iteration, we obtain Φ_k with accuracy $\sim\eta$. At this accuracy we can set all the functions of $\mathbf{k}'$ in (10) equal to their values for free electrons, and carry out the integration over the unperturbed sphere. We assume that $U(q)$ is a smooth function, i.e., it varies little when the argument is changed by ηk_F; this condition is usually satisfied for all real impurities, whereas for scattering by phonons the method turns out to be perfectly unsuitable. In the calculation of $w_{kk'}$ the wave functions of the electron are taken in the form of a superposition of plane waves

$$\psi_k = \sum_g A_{k-g} e^{i(k-g)r}. \tag{12}$$

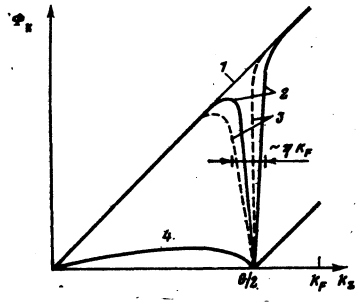

FIG. 2. Structure of the solution Φ_k of the kinetic equation for the Fermi surface of Fig. 1c in different cases: 1) solution $\Phi_k^0 \sim \mathbf{k}\cdot\mathbf{n}$ for a spherical Fermi surface; 2) solution in the τ approximation, $\Phi_k = \tau_0 \mathbf{v}_k \cdot \mathbf{n}$; 3) solution for an impurity with $U(q) = \text{const}$; 4) solution in the Bloch limit.

For the state k in the region $k_z > 0$ we need retain in (12) two coefficients

$$A_k = V_G/[(\varepsilon_k - \varepsilon_k^0)^2 + V_G^2]^{1/2}, \quad A_{k-G} = (\varepsilon_k - \varepsilon_k^0)/[(\varepsilon_k - \varepsilon_k^0)^2 + V_G^2]^{1/2}$$

[ε_k is defined in (6)], while for the state k′ one wave is sufficient. As a result we arrive at the equation

$$\mathbf{v}_k\mathbf{n} = \frac{2\pi}{\hbar}\Phi_k \int (U_q^2 + 2A_kA_{k-G}U_qU_{G+q})\frac{dS_{k'}}{(2\pi)^3\hbar v_{k'}} - \frac{2\pi}{\hbar}\frac{\hbar}{m}\tau_0 \int \{[k_\perp' n_\perp + k_z' n_z (A_k^2 - A_{k-G}^2)]U_q^2 + 2\mathbf{k}'\mathbf{n}A_kA_{k-G}U_qU_{q+G}\}\frac{dS_{k'}}{(2\pi)^3\hbar v_{k'}}, \tag{13}$$

where $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ and $k_\perp^2 = k_x^2 + k_y^2$. We have used here the fact[2)] that in view of the strong localization of A_{k-G} near the Bragg plane we can put $k_z = G/2$ in the smooth functions U_q and U_{G+q}. It is easy to verify that for k that are far from the Bragg plane Φ_k differs from Φ_k^0 by amounts $\sim\eta$. In the immediate vicinity of the Bragg palne ($k_z \to G/2$), however, Φ_k can be written in the form (7), where τ for $\mathbf{n} \| \hat{z}$ is defined by the expression

$$\frac{\tau_\pm}{\tau_0} = \frac{\int U_q^2 d\Omega_{k'}}{\int \{U_q^2 \pm U_qU_{G+q}\,\mathrm{sgn}\,V_G\}d\Omega_{k'}},$$

and ± labels a value at $k_z = J/2 \pm 0$. Since the derivative $\Phi_k'|k_z = G/2$ turns out to be large ($\sim 1/\eta$), it follows that Φ_k differs from Φ_k^0 only at $|k_z - G/2| \lesssim \eta k_F$, thereby justifying the employed method of solution.

Expressions (13) and (11) determine Φ_k in principle. It is possible, however, to obtain Φ_k in a very simple form by using certain features of the behavior of the functions $U(q)$. Real pseudopotentials break up into two groups (Fig. 3): a) pseudopotential of homovalent (i.e., impurities having the same valence as the matrix metal), which are almost even functions of $\cos\theta_{kk'}$ (Fig. 3a); b) pseudopotentials of all the remaining impurities; they have a maximum at $q = 0$ and decrease rapidly with increasing q (Fig. 3b). We consider these two groups separately.

a) If U is an even function of $\cos\theta_{kk'}$, then the second integral in (13) vanishes and Φ_k takes the form (7) with

$$\frac{1}{\tau_k} = \frac{2\pi}{\hbar}\int (U_q^2 + 2A_kA_{k-G}U_qU_{q+G})\frac{dS_{k'}}{(2\pi)^3\hbar v_{k'}}. \tag{14}$$

For practical purposes, the sufficient criterion for the applicability of (14) is of the form[3)]

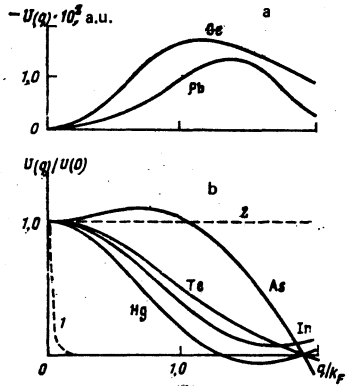

FIG. 3. Pseudopotentials of different impurities in Sn lattice (their calculation is described in Ref. 12): a) pseudopotentials of homovalent impurities are almost even functions of $\cos\theta_{kk'} = 1 - q^2/2k_F^2$ [the reason is that $U(0) = 0$, and at large q the function $U(q)$ decreases over distances of the order of the reciprocal Debye screening radius $\varkappa_D \sim k_F$]; b) the remaining pseudopotentials are bell-shaped with a maximum at $q = 0$ and a width $\sim\varkappa_D$; 1) pseudopotential corresponding to the Bloch limit, 2) $U(q) = \text{const}$.

$$\frac{\int U_q \cos\theta_{kk'}d\Omega_{k'}}{\int U_q d\Omega_{k'}} \ll 1 \tag{15}$$

and is well satisfies for homovalent impurities—a typical value of the fraction is $\approx 1/20$.

b) For most impurities of the type of Fig. 3b, the function $U(q)$ is essentially localized at $q = 0$, so that we can leave out of (13) terms containing U_qU_{q+G} (one encounters, incidentally, impurities such as As, for which this term cannot be neglected). Integrating over the directions k' and using the identities

$$\frac{\hbar}{m}k_z(A_k^2 - A_{k-G}^2) = v_z + O(\eta), \quad \frac{\hbar}{m}k_\perp = v_\perp, \tag{16}$$

which can be directly verified, we obtain after simple transformations

$$\Phi_k = \tau_0 \mathbf{v}_k\mathbf{n} + O(\eta). \tag{17}$$

Thus, for a Fermi surface described by a weak pseudopotential, the τ approximation (7) is justified for both homovalent impurities (in which case τ depends on k) and for impurities with potentials $U(q)$ that are essentially localized at $q = 0$ ($\tau = \text{const}$); these two classes cover almost all the real impurities.

2. The τ approximation (7) is valid for an arbitrary Fermi surface in one frequently employed case of a short-range impurity potential [whose Fourier transform $U(q)$ is constant], inasmuch as in this case the integral with Φ_k in (10) vanishes. This case is by itself of little interest, but together with the scattering by phonons, which will be considered below within the framework of the Bloch hypothesis (Sec. 3), it gives an idea of the dependence of the effect on the type of impurity. In fact, the Bloch case corresponds formula to scattering by an impurity with a strongly localized potential $U(q)$ [see Eq. (20) below]. This and $U(q) = \text{const}$ constitute the two limiting relations for the family of pseudopotential curves (Fig. 3b).

3. We consider on the basis of (8) the main features

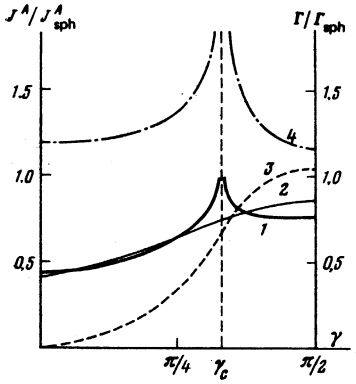

FIG. 4. Angular dependences of J^A and Γ for the Fermi surfacr of Fig. 1b with $r = 0.2k_F$ and $G = 2.2k_F$: 1) Bloch limit; 2) estimate of the contribution of the "belly" in the Bloch limit; 3) scattering by impurities with constant τ; 4) sound absorption Γ.

of the anisotropy of the acoustoelectric effect, which are typical of scattering by impurities.

From the single fact that formula (8) is valid follows the statement[5] that the sign of the contribution made to E^A by a section of the Fermi surface is determined by its curvature $\varkappa_n$ in the direction n (since $m^{-1}_{nm} \propto \varkappa_n$). For example, for the surfaces of Fig. 1, the contributions of the belts 1 and 3 are positive, and that of the best 2 is negative. From the constancy of τ for impurities such as in Fig. 3b follows the independence of E^A of the type of this impurity. On the other hand, introduction of homovalent impurities can change E^A by several times.

By way of example, Fig. 4 shows the angular dependence of J^A for the surface of Fig. 1b with neck radius $r = 0.2k_F$, which is typical of noble metals[13] (curve 3). It was calculated from formula (4) with Φ_k in the form (7) and at constant τ and Λ, and is represented in the form of the ratio to the value J^A_{sph} for a spherical Fermi surface with the same τ and A. The general behavior of the function is easily understood. At $\mathbf{n} \perp \hat{z}$ ($\gamma = \pi/2$, where γ is the angle between n and the z axis) the value of J^A is close to J^A_{sph}, so that on the Fermi surface of Fig. 1b there is one belt $\cos\theta_p = 0$ passing almost entirely over the spherical part. With decreasing γ, the belt becomes deformed and breaks up at $\gamma = \gamma_c$ into two belts that occupy at $\gamma = 0$ the positions 1 and 2 (Fig. 1). The negative contribution of the best 2 leads to a considerable decrease of J^A/J^A_{sph} compared with unity. The value of $J^A_\parallel$ at $\gamma = 0$ ($\mathbf{n} \parallel \hat{z}$) can be easily estimated. The integral in (4) is transformed into (cf. Ref. 11)

$$I = \oint_{\cos\theta_p = 0} \frac{|\Lambda_k|^2}{v_k^2} R \frac{\partial \Phi_k}{\partial k_n} d\psi, \qquad (18)$$

where R is the reciprocal of the Gaussian curvature. Near the Bragg plane, R has an additional smallness $\sim\eta$ compared with the value on the spherical part, but this smallness is offset by the large value ($\sim 1/\eta$) of the derivative $\partial\Phi/\partial z$ (Fig. 2), and the contribution of belt 2 on the neck to J^A turns out to be $\sim J^A_{sph}$. Detailed calculations lead to the result

$$\frac{J^A_\parallel}{J^A_{sph}} = 1 - \frac{\Lambda_-^2 \tau_-}{\Lambda_0^2 \tau_0}. \qquad (19)$$

It is seen that $J_\parallel$ vanishes at constant τ and Λ (Fig. 4). The difference between Λ_- and Λ_0 changes the value of $J^A_\parallel$; in this case the angular dependence of J^A is qualitatively obtained from the curve 3 of Fig. 4 by dilatation or contraction relative to +1. It is interesting that zeroes of E^A in symmetrical directions were observed experimentally in tin.[4] However, the complexity of the Fermi surface of this metal does not permit a reliable interpretation.

For comparison, Fig. 4 shows also the angular dependence of the sound absorption Γ (curve 4). Since the integral of (9) does not contain the derivative $\partial\Phi_k/2k_n$, the the contribution of the Bragg plane turns out to be $\sim\eta$, and $\Gamma_\parallel/\Gamma_{sph}$, in contrast to $J^A_\parallel/J^A_{sph}$, is close to unity. Another substantial difference between the plots of Γ and J^A is the character of the singularity at $\gamma = \gamma_c$ (see Sec. 5).

For the surface of Fig. 1a, the anisotropy of J^A at constant τ and Λ has the character of small deviations from J^A_{sph} and is of little interest. For the Fermi surface of Fig. 1c, the angular dependence of J^A is on the whole similar to Fig. 4. The main dependence is that, inasmuch as a term $\Lambda_+^2\tau_+/\Lambda_0^2\tau_0$ is added to formula (9), the value of $J^A_\parallel$ is determined (at constant τ) by the ratio of Λ_+ and Λ_-, and can be either larger or smaller than J^A_{sph}. In particular, if $\Lambda_+ = \Lambda_-$, then the contribution of the Bragg plane cancels out completely and there is practically no dependence of J^A on γ.

If it is noted that the surfaces of Figs. 1a and 1c, in contrast to the Fermi surface of Fig. 1b, are topologically equivalent in the presented band scheme to the Fermi surface of almost free electrons (i.e., they undergo no topoligical changes when η decreases to zero), then the upshot of the foregoing is the following: for a Fermi surface described by a weak local pseudopotential, under conditions of scattering by impurities whose valence is other than that of the matrix atom (i.e., at constant τ), the strong anisotropy of J^A (changes $\gtrsim J^A_{sph}$) is due either to the anisotropy of Λ or to the topological differences between the Fermi surface and that obtained as a result of Harrison's

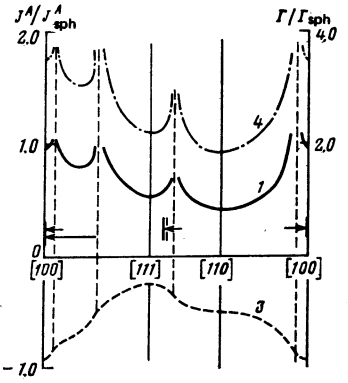

FIG. 5. Angular dependences of J^A and Γ for noble metals, plotted on the basis of Fig. 4. The notation is the same as in Fig. 4. The arrows indicate the correct positions of the singularities in Cu, determined on the basis of the Halse formula[15]: there is a singularity in the [100] direction and a "doublet" at an angle 62–63° from [100] to [110].

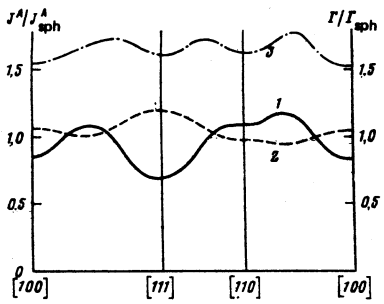

FIG. 6. Anisotropy of J^A and Γ in Li and Cs in the approximation of the independent action of the Bragg planes: 1) Bloch limit; 2) impurity scattering with constant τ; 3) Peierls limit (E^A/E_{sph}^A) and sound absorption Γ (in view of the Weinreich relation (25), the curves for these two cases coincide). J^A and E^A differ by a factor δ, which is isotropic for cubic metals.

construction (cf. Ref. 14).

4. We have considered so far a Fermi surface with one Bragg plane. However, since the difference between Φ_k and the solution for the isotopic case Φ_k^0 has a local character, the interference between the planes is significant only in a small part of k-space and usually does not affect the value of J^A (see, however, Sec. 5). Therefore, using the independent-plane approximation (i.e., summing those deviations of J^A/J^A_{sph} from unity which are connected with each of them), we can plot the angular dependences for the more complicated cases. The results of such a construction for noble metals are shown in Fig. 5 [in Cu, Ag, and Au there are eight necks[13] oriented in the directions of the vertices of the cube ([111])]. A similar construction was made for alkali metals (Fig. 6). We first calculated the angular dependences for the Fermi surface of Fig. 1a with 5% "swellings" (this corresponds to Cs and Li, Ref. 13), and then took into account that such swellings are present in the directions of the twelve edges of the ([110]) cube.

3. SCATTERING BY PHONONS WITHIN THE FRAMEWORK OF THE BLOCH HYPOTHESIS

We consider now a situation wherein the electron-phonon scattering predominates, and assume that the phonon system is at equilibrium (the Bloch limit); the influence of phonon dragging will be analyzed later (Sec. 4).

1. For an arbitrary Fermi surface we can clarify only the temperature dependence of J^A and E^A as $T \to 0$. By the Bloch method (Ref. 6, Sec. 37) it is proved that Φ_k is almost independent of energy and Eq. (2) is reduced, accurate to $(T/\Theta)^2$, to the form (10). The function $w_{kk'}$ takes the simplest form in the Debye approximation for phonons

$$w_{kk'} = \frac{2\pi q^2 |\Lambda_k|^2}{\rho\omega_q} \frac{Z}{(e^z-1)(1-e^{-z})}, \quad Z = \frac{\hbar\omega_q}{T}, \quad q = |k'-k|. \qquad (20)$$

If the temperature is so low that the wave vector of the phonon is much less than all the characteristic dimensions of the Fermi surface, then in view of the localization of $w_{kk'}$ near $q=0$ we can expand in powers of q all the quantities in (10) which depend on $k' = k+q$ (Ref. 6, p. 220; Ref. 17). It turns out then that $\Phi_k \sim T^{-5}$ and satisfies a diffusion-type differential equation.[17] We then obtain from (3)–(5) that $\sigma \sim T^{-5}$, $J^A \sim T^{-5}$, and $E^A = \text{const}(T)$.

2. We investigate now the anisotropy of J^A, using as an example metals with a thin bridges, $r \ll k_F$ (Fig. 1b). We use the fact that part of the Fermi surface adjacent to the neck has rotational symmetry; in this case, when n is directed along the neck axis $\hat{z}$, the diffusion equation (see Ref. 17) can be integrated for any profile of the $\rho(z)$ surface:

$$\Phi(z) = \frac{1}{2}\int_0^z dz \frac{\rho(1+\rho_z'^2)^{1/2}}{D} + C_1 k_F^2 \int_0^z \frac{(1+\rho_z'^2)^{1/2}}{D\rho} dz + C_2 \qquad (21)$$

(D is the coefficient of electron diffusion over the Fermi surface). The constants C_1 and C_2 are determined from the boundary condition, which for the Fermi surface of Fig. 1b take the form $\Phi(0)=0$ and $\Phi(G/2)=0$. In the presence of several necks, the first condition is replaced by the condition of "matching" on the boundary of the axial-symmetry region. It is easily seen that in the general case $C_1 \sim 1$. From this we obtain the estimate

$$\frac{|\Phi_z'|_{G/2}}{|\Phi_z'|_0} \sim \frac{C_1}{C_1+1/2}\frac{D_0 k_F}{Dr} \sim \frac{\Lambda_0^2}{\Lambda^2}\frac{v^2}{v_F^2}\frac{k_F}{r} \qquad (22)$$

(the quantities with the zero subscript pertain to the spherical part of the Fermi surface—the "belly," and those without the subscript, to the neck). Recognizing that according to (16) we have $v = v_F r/k_F$ and $\Lambda \sim \Lambda_0$, we obtain $|\Phi_z'|G/2 \ll |\Phi_z'|_0$.

Thus, the derivative $\partial\Phi_k/\partial k_n$ on the neck turns out to be much smaller than in the case of impurity scattering. This leads to two consequences: 1) the contribution of the belt 2 on the neck turns out to be small, and the anisotropy of J^A is determined mainly by the contribution of the "belly"; 2) Φ_k differs from the solution Φ_k^0 for the isotropic case on the entire Fermi surface and not in a small vicinity of the Bragg plane (Fig. 2), therefore the contribution of the "belly" differs greatly from the value of J^A_{sph} for a spherical Fermi surface. To estimate this contribution we can approximate Φ_k on the "belly" in the vicinity of the belt 1 by a linear function. The form of the latter, in view of the linearity of Eq. (2) in n, can be established from symmetry considerations; thus, for the Fermi surface of Fig. 1b we have

$$\Phi_k \approx Ak_\perp n_\perp + Bk_z n_z,$$

whence the contribution of the "belly" is $\sim A\sin^2\gamma + B\cos^2\gamma$. Figure 4 shows the angular dependence of J^A, obtained as a result of integrating the diffusion equation, and also an estimate of the contribution of the "belly." It is seen that the contribution of the neck is small in the entire angle interval, except for the vicinity of γ_c where there is a logarithmic singularity (its existence is established from general considerations, see Sec. 5). When the remaining necks present in Cu, Ag, and Au are added, the Fermi surface requires a cubic symmetry; then the form of the solution on the "belly" is $\approx A(k \cdot n)$, and its contribution to J^A is $\sim A$ and is constant. The total angular dependence of J^A (Fig. 5) is obtained by adding to this constant con-

tribution (chosen arbitrarily in Fig. 5) the logarithmic singularities connected with each neck; the resultant anisotropy of J^A is similar to Γ (Fig. 5). We note that in the Bloch limit the anisotropy of the deformation potential Λ_k has practically no effect on the angular dependence of J^A, since the ratio of the contributions of the "belly" and of the neck does not depend on Λ/Λ_0 [cf (4) and (22)].

We take particular notice of the case $\ln(1/\eta) \gg 1$ (in this case the existence of the neck calls for $\ln[k_F/(G/2 - k_F)] \gg 1$). The behavior of the function $\Phi(z)$ in this case is not at all trivial. First, $C_1 \sim 1/\ln(1/\eta) \ll 1$, i.e., there is an additional smallness Φ_s' on the neck [see (22)]. Second, at a distance $\sim \eta k_F$ from the neck the function $\Phi(z)$ increases rapidly and goes over into the solution $\Phi^0(z)$ for a spherical Fermi surface.[4)] Thus, on the "belly" Φ_k coincides with Φ_k^0 and the contribution of the "belly" to J^A is equal to J^A_{sph} (i.e., the constant contribution on Fig. 5 is equal to unity).

For thick necks ($\gamma \sim k_F$), and also for surfaces of the type of Fig. 1c, the analysis is carried out similarly on the basis of (21). Now there are no small parameters whatever, and $|\Phi_s'|G/2 \sim \Phi_s'|_0$. The contribution of the Bragg plane remains small ($\sim\eta$) as before compared with the contribution of the "belly," and the main conclusions are the same as in the considered case. For alkali metals (Fig. 6), in view of the smallness of the deviations of J^A/J^A_{sph} from unity, the construction can be carried out in the same manner as for impurity scattering.

4. INFLUENCE OF PHONON DRAGGING

1. For an ideally pure metal with a closed Fermi surface, under conditions of phonon dragging, there is realized as $T \to 0$ the so-called Peierls limit, in which the resistance depends exponentially on the temperature. In this case an analytic solution can be obtained for an arbitrary Fermi surface.

It is known (Ref. 18, Sec. 81) that, neglecting umklapp processes, the homogeneous kinetic equation has a "drift" solution[5)]

$$\Phi_{\mathbf{k}}^{dr} \sim \mathbf{kn}, \tag{23}$$

which is not orthogonal in the left-hand side, takes the form

$$U^E = e\mathbf{v_k}\mathbf{E}\frac{\partial f_k^0}{\partial \varepsilon_k}$$

in the presence of an electric field, and is equal to U^A [see (1)] in the case of passage of a sound wave. Consequently, the inhomogeneous equation has no solution for any finite E in the former case and finite W in the latter. Nonetheless, even at $E=0$ and $W=0$ the solution (23) carries a perfectly finite current. This means infinite conductivity and an infinite acoustoelectric current. The acousto-emf is nevertheless quite finite. Indeed, in the case of simultaneous presence of U^E and U^A in the left-hand side of the kinetic equation, it is possible to make it orthogonal to (23) by suitable choice of E and W. This yields the relation

$$-(U^A, \Phi_{\mathbf{k}}^{dr}) = (U^E, \Phi_{\mathbf{k}}^{dr}),$$

which must be solved for E. Formally the result is the same as if (23) were directly substituted in (5). Since $\partial(\mathbf{k}\cdot\mathbf{n})/\partial k_n = 1$, the same integral appears in the numerator of (5) as in the absorption coefficient (9); the integral in the denominator is a constant equal to the volume V_{FS} contained under the Fermi surface. Consequently

$$E^A = \frac{\pi\omega W}{e\rho\hbar s^3 V_{FS}} \int \frac{|\Lambda_k|^2}{v_k^2}\delta(\cos\theta_v)\, dS_k. \tag{24}$$

Thus, in this case the anisotropy of E^A coincides with the anisotropy of Γ. With the aid of (9) we can write

$$E^A = \frac{W}{N_e e s}\Gamma \tag{25}$$

(N_e is the electron density). This is none other than the Weinreich relation, which is well known for the isotropic case. It is obtained from the condition that the total forces acting on the electrons by the sound wave and by the electric field be equal [3]. Since the correct conditions for the determination of E^A is the vanishing of the total current through the sample, (25) is incorrect in the general case. In the Peierls limit these two conditions are equivalent, in view of the absence of dissipation of the quasimomentum.

The presence of umklapp processes causes σ and J^A to become finite but to increase exponentially with decreasing temperature. Formulas (24) and (25) remain in force accurate to exponentially small corrections, i.e., the acousto-emf in this region is practically independent of temperature.

If a closed Fermi surface consists of several isolated electron and hole cavities, formulas (24) and (25) can be generalized in natural fashion. In place of (25) we have

$$E^A = \frac{W}{(N_h - N_e)|e|s}\Gamma, \tag{26}$$

where N_h is the number of holes. For a compensated metal ($N_e = N_h$), E^A is made finite by the umklapp processes, i.e., in this case the acousto-emf depends exponentially on temperature. The physical reason is that without umklapp processes the quasimomentum of the system increases continuously under the action of the sound wave; the electric field cannot establish equilibrium, since the total force it exerts on the charged particles is equal to zero.

2. In the presence of an open Fermi surface, for directions parallel to all the band boundaries that intersect the surface (if such directions exist), formula (26) is valid as before. For the remaining directions we can trace only the temperature dependences of J^A and E^A, which turn out to be the same as in the Bloch case, i.e., $J^A \sim T^{-5}$ and $E^A = \text{const}(T)$ (since the proportionality $\Phi_k \propto T^{-5}$ is preserved, although the diffusion equation becomes integrodifferential[17]). It is curious that in the case of an open Fermi surface and $N_e = N_h$, E^A can have different temperature dependences for different sound-propagation directions.

3. The anisotropy predicted for E^A by formula (24) differs substantially from that obtained in the τ ap-

proximation [see (8)]. In particular, the conclusion that the sign of E^A is connected with the curvature of the Fermi surface is more incorrect: according to (24), all the sections of a singly connected electron surface (see, e.g., the "dumbbell" in Ref. 5) make positive contributions. It is possible to cite an even clearer although somewhat artificial example. If the Fermi surface consists of two identical cavities, one electron and the other hole, then in the τ approximation $E^A = 0$ and in the Peierls limit $E^A = \infty$.

4. The anisotropy of E^A in the Peierls case for alkali metals is shown in Fig. 6, curve 3. A comparison with the dependence corresponding to the Bloch limit shows that the dragging of the phonons leads to a certain change in the anisotropy of the effect, although qualitatively the dependences are perfectly analogous.

In the case of open surfaces, the dragging of the phonons likewise does not lead to a substantial change in the anisotropy of J^A. An estimate of the derivative $\partial\Phi_k/\partial k_n$ is simplest to carry out with the aid of the relation

$$|\Phi_s'|_{a/2} - a = \text{const}/Dr,$$

which follows from the quasimomentum balance[17] (a is an integral term that takes into account the phonon dragging). By shifting the Brillouin cell by $G/2$, we obtained a similar relation for $\Phi_s'(0)$, with a constant of the same order. Recognizing that a is of the order of the mean value of Φ_s' on the Fermi surface and that $Dr \gg D_0 k_F$ [see (22)],[6] we find that generally speaking $|\Phi_s'|_{G/2} \sim |\Phi_s'|_0$. However, the contribution of the belt 2 on the neck is small as before compared with the contribution of the "belly" ($\sim\eta$), and all the conclusions of the preceding section remain in force. No special calculation of the anisotropy of J^A was made for this case, since the calculations turn out to be quite cumbersome.

5. SINGULARITIES IN THE ANGULAR DEPENDENCE OF THE ACOUSTO-ELECTRIC EFFECT

1. It is seen from Figs. 4 and 5 that the angular dependence of J^A has singularities that arise, according to Ref. 5, in those sound-propagation directions n at which a change of the topology of the belts takes place. An analysis of these singularities can be carried out in the general case in analogy with the procedure used in Ref. 20 for sound absorption (see also the review[8]).

According to Ref. 20, the change of the topology of the belts always takes place at a parabolic point of the Fermi surface in one of two ways: a) the belt degenerates into a point and vanishes (in accordance with the terminology of Ref. 20, this is called an O-type point); b) the belt acquires a self-intersection point, after which it breaks into two (X-type point). The O-type point is connected with a jump and the X-type is connected with a logarithmic singularity in the angular dependence of Γ.

Let the change of the topology of the belt take place at a point A, at which one of the principal curvatures $\varkappa_1$ vanishes. We connect with A a local coordinate system with the z axis directed normal to the Fermi surface and with the x axis in the direction of the line of the curvature $\varkappa_1$. In these coordinates, the equation of the surface near A is of the form

$$z = f(x, y) = {}^1/_2\varkappa_2 y^2 + \alpha_1 x^3 + \alpha_2 y^3 + \beta_1 x^2 y + \beta_2 x y^2 + \ldots.$$

The direction of the x axis is chosen such that $\alpha_1 > 0$. The vector n is assumed to lie in the xz plane at an angle γ to the x axis; then the singularities arise at $\gamma = 0$. The point A is of the O-type at $3\alpha_1\beta_2 - \beta_1^2 > 0$ and of the X-type at $3\alpha_1\beta_2 - \beta_1^2 < 0$.

The singularities in the acousto-emf are determined by the integral [see (18)]. In the general case the behavior of Φ_k near the point A is not remarkable in any way, and in particular, $(\partial\Phi_k/\partial k_n)_A \neq 0$, and the singularities are the same as in Γ:

$$\begin{aligned} &O\text{-type} \quad I_{\text{sing}}^{(O)} = \frac{\pi}{(3\alpha_1\beta_2 - \beta_1^2)^{1/2}} \left(\frac{\Lambda^2}{v^2}\Phi_x'\right)_A \Theta(\gamma), \\ &X\text{-type} \quad I_{\text{sing}}^{(X)} = \frac{1}{(\beta_1^2 - 3\alpha_1\beta_2)^{1/2}} \left(\frac{\Lambda^2}{v^2}\Phi_x'\right)_A \ln\frac{1}{|\gamma|}. \end{aligned} \tag{27}$$

[$\Theta(x)$ is the Heaviside function]. However, under conditions when the τ approximation is valid [formula (8)], $\partial\Phi_k/\partial k_n = \tau\hbar/m_m$ and vanishes at the parabolic point. Consequently, the singularities become weaker.

$$\begin{aligned} I_{\text{sing}}^{(O)} &= -\frac{\pi}{(3\alpha_1\beta_2 - \beta_1^2)^{1/2}} \left(\frac{\tau\Lambda^2}{v}\right)_{xA}' \gamma\Theta(\gamma), \\ I_{\text{sing}}^{(X)} &= -\frac{1}{(\beta_1^2 - 3\alpha_1\beta_2)^{1/2}} \left(\frac{\tau\Lambda^2}{v}\right)_{xA}' \gamma\ln\frac{1}{|\gamma|}. \end{aligned} \tag{28}$$

It follows therefore that it is possible to change the character of the singularity by changing the scattering mechanism. As is clear from Sec. 2, singularities of the type (28) take place in scattering by impurities. A transition to scattering from phonons, via changing the temperature or the impurity density, can lead to the onset of singularities of the type (27).[7]

2. One Bragg plane usually leads to the onset of parabolic points of the X-type (point A on Fig. 1b or 1c), therefore the singularities on Figs. 4 and 5 take the form $\ln|\gamma - \gamma_c|$ and $(\gamma - \gamma_c)\ln|\gamma - \gamma_c|$. The position of the critical direction can be easily obtained for the surface of Fig. 1c under the conditions $\eta \ll G/k_F$ and $\eta \ll (G - 2k_F)/k_F$:

$$\gamma_c = \pi/2 - \theta_0 + O(\eta^{1/3}). \tag{29}$$

According to this formula, the determination of the critical directions for an arbitrary complicated Fermi surface, described by a weak pseudopotential, is a purely geometric problem; it is in these directions that the belt $\cos\theta_v = 0$ on the unperturbed sphere is tangent to the Bragg planes.

O-type points can possibly appear in the case of Fig. 1a, but according to the available data[13] they do not appear on Fermi surfaces of alkali metals.

3. As indicated in Sec. 2, the interference between the planes is significant only in small regions of k-space and usually does not manifest itself in the anisotropy of J^A. However, if in such a small region there is a parabolic point responsible for a singularity in a certain direction, then the influence of the interference near this direction can also be not weak. Consequences of such an interference can be a change in

the position of the singularity, its vanishing, the coalescence of several singularities into one, etc. In those cases when there is available a Fermi-surface picture sufficient to describe clearly its topology, all these deviations from the independent-plane approximation can be established visually.

In noble metals, the interference between the necks can cause the belts of the parabolic points around neighboring necks (which are present in the absence of interference) to merge into a single closed curve; as a result the three singularities of Fig. 5 degenerate into one, which turns out to be in the symmetrical direction [100] (see Ref. 21 for details). An analysis of the topology of the Fermi surface of noble metals with the aid of the Halse analytic representation[15] shows that this merging actually takes place in Cu and Au, but not in Ag. The correct positions of the singularities for Cu are shown in Fig. 5 by arrows.

6. CONCLUSION

It is clear from the foregoing analysis that the anisotropy of the acoustoelectric effect is analogous to the anisotropy of sound absorption in the case of scattering of electrons by phonons, but differ substantially from it in the case of scattering by impurities; in the latter case the anisotropy of E^A is reasonably described by formula (8), obtained in the τ approximation.

We return now to the question raised at the start of the article: what information can be obtained with the aid of the acoustoelectric effect. It is most natural to use it to reconstruct directly from experiment, on the basis of formula (5), the distribution functions Φ_k [or the anisotropic relaxation time τ_k introduced in accordance with (7)], which is of great importance for the kinetics of metals. To this end, to be sure, it is necessary to determine independently the parameters of the Fermi surface and the deformation potentials Λ_k.

The reconstruction of other quantities (for example, Λ_k) from J^A is made difficult by the dependence of J^A on the character of the scattering: this is where J^A falls short compared with Γ. However, the contribution made to Γ by the most interesting regions of k-space (the vicinities of the Bragg planes) is weakened to the extent that the factor $\eta = |V_G|/\varepsilon_F$ is small, whereas the contribution to J^A (in scattering by impurities) is of the usual order of magnitude. Therefore, if the measurements are made with the same accuracy, the anisotropy of J^A contains more significant information; on the other hand, certain quantities (the effective masses) cannot be obtained at all from Γ. The realization of this program calls for a realistic estimate of Φ_k. For example, as is clear from the foregoing analysis, interpretation on the basis of the τ approximation is reliable to the extent that E^A does not depend on the sort of impurity. It is more advantageous to determine τ_k from other experimental data, but one must see to it that this quantity be introduced in identical fashion. We emphasize that the foregoing pertains only to scattering by impurities; in scattering by phonons, the contribution made to J^A by the Bragg planes is small and the acoustoelectric effect has no advantages whatever compared with sound absorption.

The author thanks N. V. Zavaritskiĭ for calling his attention to the topic, M. I. Kaganov and Yu. K. Dzhikaev for reading the paper in manuscript and for a number of valuable remarks and also A. A. Altukhov and Sh. T. Mevlyut for numerous discussions.

[1)] A similar approach was used by Kagan[7] for an isotropic electron-dispersion law.

[2)] Use was made also of the function $U_q = U(q)$, which follows from the proposed spherical symmetry of the pseudo-potential $U(r)$ in coordinate space. Deviations from this approximation can be substantial for crystals with covalent and strongly polar bonds, while for metals it is assumed to be insufficient.[9]

[3)] This criterion is obtained if the first three terms are retained in the expansion of $U(\cos\theta_{kk'})$ in spherical harmonics.

[4)] The author thanks R. N. Gurzhi for pointing out this circumstance.

[5)] In cases when the dragging is substantial, the operator $\hat{L}$ in (2) is understood to be the operator arising in the right-hand side of the equation for the electrons after eliminating from it the phonon distribution function (Ref. 18, §82).

[6)] No account was taken in Ref. 16 of the change of the Fermi velocity on the neck ($v \sim v_F r/k_F$), and it was therefore concluded that Φ_k' is large on the neck.[19] At $\ln(1/\eta) \gg 1$, however, the results of Ref. 17 remain valid, since Φ_k' becomes large at a distance $\sim \eta k_F$ from the neck (see the end of Sec. 3). In the general case, when the parameters of the neck are determined not by one but by several Bragg planes, the situation $v \sim v_F$ is possible for long necks (see the estimate in Ref. 14).

[7)] Formally, even at very small deviations from the τ approximation, the singularities take the form (27), but their power is decreased to the extent of the indicated smallness, and even slight smearing makes them completely indistinguishable. The power of the singularitites of type (28), however, has the usual order of magnitude, and it is this which makes it possible to reveal them.

[1]P. H. Parmenter, Phys. Rev. **89**, 990 (1953).
[2]E. I. Blount, Phys. Rev. **114**, 418 (1959).
[3]G. Weinreich, Phys. Rev. **107**, 317 (1957).
[4]N. V. Zavaritskiĭ, Zh. Eksp. Teor. Fiz. **75**, 1873 (1978) [Sov. Phys. JETP **48**, 942 (1978)]; Pis'ma Zh. Eksp. Teor. Fiz. **25**, 61 and **26**, 44 (1977) [JETP Lett. **25**, 55 and **26**, 39 (1977)].
[5]N. V. Zavaritskiĭ, M. I. Kaganov, and Sh. T. Mevlyut, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 223 (1978) [JETP Lett. **23**, 205 (1978)].
[6]M. I. Kaganov, Sh. T. Mevlyut, and I. M. Suslov, Zh. Eksp. Teor. Fiz. **78**, 376 (1980) [Sov. Phys. JETP **51**, 189 (1980)].
[7]V. D. Kagan, Fiz. Tverd. Tela (Leningrad) **9**, 3293 (1967) [Sov. Phys. Solid State 9, 259 (1968)].
[8]M. I. Kaganov and I. M. Lifshitz, Usp. Fiz. Nauk **129**, 487 (1979) [Sov. Phys. Usp. **22**, 904 (1979)].
[9]W. A. Harrison, Pseudopotentials in the Theory of Metals, Benjamin, 1966.
[10]A. I. Akhiezer, M. I. Kaganov, and G. Ya. Lyubarskiĭ, Zh. Eksp. Teor. Fiz. **32**, 837 (1957) [Sov. Phys. JETP **5**, 685 (1957)].
[11]A. B. Pippard, Proc. Roy. Soc. **A257**, 165 (1960).
[12]A. A. Altukhov, Yu. K. Dzhikaev, N. V. Zavaritskiĭ, and I. M. Suslov, Zh. Eksp. Teor. Fiz. **75**, 2256 (1978) [Sov. Phys. JETP **48**, 1137 (1978)].
[13]A. P. Cracknell and K. C. Wong, The Fermi Surface, Clarendon Press, Oxford, 1973, Secs. 4.2 and 5.2.

[14]A. A. Altukhov, N. V. Zavaritskiĭ, and I. M. Suslov, Zh. Eksp. Teor. Fiz. **79**, 1518 (1980) [Sov. Phys. JETP **52**, 765 (1980)].
[15]M. R. Halse, Phil Trans. Roy. Soc. **A265**, 507 (1969).
[16]H. Bethe and A. Sommerfeld, Elektronentheorie der Metalle, in Handbuch d. Physik, Vol. 24, part 2, Springer, 1933, p. 333.
[17]R. N. Gurzhi and A. I. Kopeliovich, Zh. Eksp. Teor. Fiz. **61**, 2514 (1971) [Sov. Phys. JETP **34**, 1343 (1972)].
[18]E. M. Lifshitz and L. P. Pitaevskiĭ, Fizicheskaya kinetika (Physical Kinetics), Nauka, 1979, §§81, 82.
[19]V. N. Flerov, Fiz. Tverd. Tela (Leningrad) **16**, 55 (1974) [Sov. Phys. Solid State **16**, 33 (1974)].
[20]G. T. Avanesyan M. I. Kaganov, and T. Yu. Lisovskaya, Zh. Eksp. Teor. Fiz. **75**, 1786 (1978) [Sov. Phys. JETP **48**, 900 (1978)].
[21]I. M. Suslov, Fiz. Tverd. Tela (Leningrad) **23**, (1981) [*sic*].

Translated by J. G. Adashko