Coherent Raman emission of electromagnetic waves by a wake charge in resonantly excited matter

D. B. Rogozkin and M. I. Ryazanov

Engineering Physics Institute, Moscow
(Submitted 23 December 1980)
Zh. Eksp. Teor. Fiz. **80**, 2250–2256 (June 1981)

The radiation emitted by a fast charged particle in a medium whose inhomogeneity and the nonstationarity of whose dielectric properties are due to the action of the field of an electromagnetic wave resonantly exciting the atoms of the material is considered. The nature of the radiation in the region of the combination frequencies $\omega=\omega_p\pm 2\omega_0$ (ω_p is the frequency of the longitudinal vibrations in the material and ω_0 is the frequency of the electromagnetic wave) is analyzed in detail. It is shown that, besides the Čerenkov radiation, there is emitted in this frequency region coherent radiation due to the Raman scattering of the field of the particle's wake charge by the excited atoms of the material. Furthermore, the presence of the coupled waves may lead to a restructuring of the Vavilov-Čerenkov radiation itself in the indicated frequency region. Expressions are presented for the Čerenkov- and Raman-radiation intensities, and the possibility of using the investigated phenomenon to study the spectrum of the longitudinal vibrations of various materials is noted.

PACS numbers: 78.45. + h, 78.30. − j

1. As was first pointed out by Bohr,[1] collective electron oscillations occur along the path of a fast charged particle in a material. To these oscillations correspond longitudinal waves whose frequencies are the zeros of the permittivity of the medium, i.e., the solutions to the equation $\varepsilon(\omega_p-i\gamma_p)=0$. Examples of the elementary excitations connected with the longitudinal oscillations are plasmons, longitudinal optical phonons, and longitudinal excitons.[2] Of greatest importance is the fact that the phases of the longitudinal oscillations are uniquely determined by the coordinates, the fly-by instant, and the velocity of the particle. This allows the computation of the space-time dependence of the electric-charge oscillations along the path of the particle after its transit. These oscillations along the particle's track were called wake charge, and their intensity in a plasma was first computed by Neufeld and Ritchie.[3] The existence of wake-charge oscillations has been experimentally confirmed in analyses of the separation of the products of the Coulomb disintegration of molecules after the loss of the outer electrons in thin films.[4-10]

For a charge Ze moving uniformly with velocity v in an isotropic material of permittivity $\varepsilon(\omega)$, the wake-charge density can be represented in the form[3-6]

$$\rho_k(\mathbf{r},t)=\frac{Ze}{v}\sum_p 2\left|\frac{\partial}{\partial\omega}\operatorname{Re}\varepsilon(\omega)\right|^{-1}_{\omega=\omega_p}\sin\left[\frac{\omega_p}{v}(vt-x)\right]\times\eta\left(t-\frac{x}{v}\right)\exp\left[-\gamma_p\left(t-\frac{x}{v}\right)\right],\qquad(1.1)$$

where $\eta(t)=(t+|t|)/2|t|$ is the unit step function; each term of the sum takes account of the contribution of two zeros of $\varepsilon(\omega)$ at the points $\pm\omega_p-i\gamma_p$.

Thus, a fast particle leaves behind in a material a trail of wake-charge oscillations whose length is determined by the lifetime of the longitudinal electric oscillations. The wake-charge-related change in the dielectric properties of the material can be used to extract information about the particle, the interaction of the particle with the material, and the characteristics of the longitudinal electrical excitations of the material.

One of the most interesting problems is the elucidation of the possibility of the appearance of a wake charge in interactions with a transverse electromagnetic field. Two circumstances—the longitudinal character of the wake-charge oscillations and the vanishing of the permittivity at the oscillation frequency—prevent the direct participation of the wake-charge in the emission of electromagnetic waves. But the emission becomes possible in a spatially inhomogeneous and nonstationary material because of the transformation of the longitudinal field of the wake-charge into transverse electromagnetic waves as a result of scattering. Of special interest here is the possibility of a coherent transformation of the field of the wake-charge into transverse waves. The radiation that is generated in this case is a particular case of transition (in the broad sense of the word) radiation in an inhomogeneous and nonstationary material.[11-15]

Below we consider the emission of coherent Raman radiation by a wake-charge in a material whose inhomogeneity and the time dependence of whose dielectric properties are produced by the action of a resonance plane-wave field.

2. Let us now consider the polarization of an isotropic material acted upon by, besides a weak field $\mathbf{E}(\mathbf{r},t)$, a resonance pump field

$$\mathbf{E}_0(\mathbf{r},t)=\mathbf{E}_0\cos(\mathbf{k}_0\mathbf{r}-\omega_0 t).\qquad(2.1)$$

The doubled pump-field frequency $2\omega_0$ is close to the frequency ω_{10} of the atomic transition from the ground state 0 into the excited state 1:

$$|2\omega_0-\omega_{10}|\ll\omega_{10}.\qquad(2.2)$$

The polarization of the material for this case has been computed in a number of papers (see, for example, Refs. 16 and 17). We have for the Fourier transform of the polarization

$$\mathbf{P}(\mathbf{k},\omega)=\frac{1}{(2\pi)^4}\iint d^3r\,dt\,\mathbf{P}(\mathbf{r},t)\exp(i\omega t-i\mathbf{k}\mathbf{r})\qquad(2.3)$$

the relation

 0038-5646/81/061175-04$02.40

$$P_i(\mathbf{k},\omega)=\rho_{00}\chi_{ij}^{(00)}(\omega)E_j(\mathbf{k},\omega)+\rho_{11}\chi_{ij}^{(11)}(\omega)E_j(\mathbf{k},\omega)$$
$$+\rho_{10}\chi_{ij}^{(01)}(\omega)E_j(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0)+\rho_{01}\chi_{ij}^{(10)}(\omega)E_j(\mathbf{k}+2\mathbf{k}_0,\omega+2\omega_0), \quad (2.4)$$

where ρ_{00} and ρ_{11} are respectively the populations of the ground and excited states; $\rho_{01}=\rho_{10}^*$ are the off-diagonal elements of the density matrix.[16,17]

The polarizabilities $\chi_{ij}^{(rs)}(\omega)$ are defined by the formula

$$\chi_{ij}^{(rs)}(\omega)=n_0\sum_k\left(\frac{d_{rk}^i d_{ks}^j}{\omega_{ks}-\omega}+\frac{d_{rk}^j d_{ks}^i}{\omega_{kr}+\omega}\right). \quad (2.5)$$

Here n_0 is the number of resonance atoms in a unit volume, ω_{kr} is the frequency of the atomic transition from the kth into the rth state; $\mathbf{d}_{kr}$ is the matrix element of the dipole moment of the atomic transition between the kth and rth states. In the particular case in which the ground and excited atomic states are spherically symmetric ($l=0, m=0$), the polarizability tensor χ_{ij} is diagonal: $\chi_{ij}(\omega)=\chi(\omega)\delta_{ij}$.

3. Let us now find the self-field of a charge Ze uniformly moving with velocity v in a material whose polarization is described by the formulas (2.4) and (2.5). By substituting the explicit expression for the polarization into the Maxwell equations, we can derive for the case under consideration the equation

$$(k^2\delta_{ij}-k_ik_j-\omega^2\varepsilon_{ij}(\omega))E_j(\mathbf{k},\omega)=4\pi i(2\pi)^{-3}Ze\omega v_i\delta(\omega-\mathbf{kv})$$
$$+4\pi\omega^2\rho_{10}\chi_{ij}^{(01)}(\omega)E_j(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0)+4\pi\omega^2\rho_{01}\chi_{ij}^{(10)}(\omega)\times E_j(\mathbf{k}+2\mathbf{k}_0,\omega+2\omega_0). \quad (3.1)$$

Here we have taken into account the fact that the pump field makes a contribution to the material's permittivity $\varepsilon_{ij}(\omega)$, which differs from the permittivity $\varepsilon_0(\omega)\delta_{ij}$ of the unexcited material:

$$\varepsilon_{ij}(\omega)=\varepsilon_0(\omega)\delta_{ij}+4\pi\rho_{11}[\chi_{ij}^{(11)}(\omega)-\chi_{ij}^{(00)}(\omega)]. \quad (3.2)$$

In order not to introduce unimportant complications, let us find the self-field of a uniformly moving charge for the particular case of spherically symmetric 0 and 1 states of the atom, when $\chi_{ij}(\omega)=\delta_{ij}\chi(\omega)$.

It is easy to see from (3.1) that the field component with frequency ω is coupled to the field components with the combination frequencies $\omega\pm 2\omega_0$. Let us consider the frequency

$$\omega\approx\omega_p+2\omega_0. \quad (3.3)$$

Whereas, generally speaking, the smallness of the quantities $\chi(\omega)$ allows us to consider the contribution of the terms with the combination frequencies to be small, this is not the case in the region of the frequencies (3.3). Indeed, in this case the field

$$\mathbf{E}(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0)\approx\mathbf{E}(\mathbf{k}-2\mathbf{k}_0,\omega_p) \quad (3.4)$$

is strong because of the small quantity $\varepsilon(\omega_p)$ contained in the denominator. This circumstance compenstates for the smallness of $\chi(\omega)$, and to determine the field $\mathbf{E}(\mathbf{k},\omega)$, we must solve simultaneously the system of two equations for $\mathbf{E}(\mathbf{k},\omega)$ and $\mathbf{E}(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0)$. (The other components with the combination frequencies can be discarded.) Allowing for the foregoing, we can derive from (3.1) the set of two equations for $\mathbf{E}(\mathbf{k},\omega)$ and $\mathbf{E}(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0)$ in the form

$$(k^2\delta_{ij}-k_ik_j-\omega^2\varepsilon(\omega)\delta_{ij})E_j(\mathbf{k},\omega)=4\pi i(2\pi)^{-3}Ze\omega v_i\delta(\omega-\mathbf{kv})$$
$$+4\pi\omega^2\rho_{10}\chi^{(01)}(\omega)E_i(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0), \quad (3.5)$$
$$[(\mathbf{k}-2\mathbf{k}_0)^2\delta_{ij}-(\mathbf{k}-2\mathbf{k}_0)_i(\mathbf{k}-2\mathbf{k}_0)_j-(\omega-2\omega_0)^2\varepsilon(\omega-2\omega_0)\delta_{ij}]E_j(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0)=4\pi i(2\pi)^{-3}Ze(\omega-2\omega_0)v_i\delta(\omega-2\omega_0-\mathbf{v}(\mathbf{k}-2\mathbf{k}_0))$$
$$+4\pi(\omega-2\omega_0)^2\rho_{01}\chi^{(10)}(\omega-2\omega_0)E_i(\mathbf{k},\omega). \quad (3.6)$$

Transforming this system, we obtain the following equation for the particle's self-field $E_i(\mathbf{k},\omega)$:

$$[k^2\delta_{ij}-k_ik_j-\omega^2\tilde{\varepsilon}_{ij}(\mathbf{k},\omega)]E_j(\mathbf{k},\omega)$$
$$=4\pi i(2\pi)^{-3}Ze\omega v_i\delta(\omega-\mathbf{kv})+4\pi\omega^2\rho_{10}\chi^{(01)}(\omega)E_i^{(0)}(\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0), \quad (3.7)$$

$$E_i^{(0)}(\mathbf{k},\omega)=4\pi i(2\pi)^{-3}Ze\frac{\omega\varepsilon(\omega)v_i-k_i}{\varepsilon(\omega)[k^2-\omega^2\varepsilon(\omega)]}\delta(\omega-\mathbf{kv}), \quad (3.8)$$

$$\tilde{\varepsilon}_{ij}(k,\omega)\approx\varepsilon(\omega)\delta_{ij}+\varkappa(\omega)e_ie_j,\quad e_i=(\mathbf{k}-2\mathbf{k}_0)_i/|\mathbf{k}-2\mathbf{k}_0|,$$
$$\varkappa(\omega)=-(4\pi)^2\rho_{10}\rho_{01}\chi^{(01)}(\omega)\chi^{(10)}(\omega-2\omega_0)\varepsilon^{-1}(\omega-2\omega_0).$$

As can be seen from Eq. (3.7), the presence of the coupled waves leads to the appearance of an effective anisotropy in the optical properties of the medium in the frequency region $\omega\approx\omega_p+2\omega_0$.

The solution to Eq. (3.7) has the form

$$E_i(\mathbf{k},\omega)=\tilde{E}_i(\mathbf{k},\omega)+\Phi_{ij}(\mathbf{k},\omega)E_j\ (\mathbf{k}-2\mathbf{k}_0,\omega-2\omega_0). \quad (3.9)$$

Here we have used the notation

$$\tilde{E}_i(\mathbf{k},\omega)=4\pi i(2\pi)^{-3}\frac{Ze\delta(\omega-\mathbf{kv})}{\varepsilon(\omega)[k^2-\omega^2\varepsilon(\omega)]}\Big[\omega\varepsilon(\omega)v_i$$
$$-k_i+\frac{\varkappa(k_i\mathbf{ke}-e_i\omega^2\varepsilon)(\mathbf{ke}-\mathbf{ve}\omega\varepsilon)}{\varepsilon[k^2+(\mathbf{ke})^2\varkappa/\varepsilon-\omega^2(\varepsilon+\varkappa)]}\Big]; \quad (3.10)$$

$$\Phi_{ij}(\mathbf{k},\omega)=\frac{4\pi\rho_{10}\chi^{(01)}(\omega)}{k^2-\omega^2\varepsilon(\omega)}\Big[\omega^2\delta_{ij}$$
$$-\frac{k_ik_j(k^2-\omega^2(\varepsilon+\varkappa))-\omega^4e_ie_j\varkappa\varepsilon+\omega^2\varkappa\mathbf{ek}(e_ik_j+e_jk_j)}{\varepsilon[k^2+(\mathbf{ke})^2\varkappa/\varepsilon-\omega^2(\varepsilon+\varkappa)]}\Big]. \quad (3.11)$$

A similar expression is obtained for the particle's self-field in the frequency region $\omega=\omega_p-2\omega_0$ by making the substitution $\mathbf{k}_0\rightarrow\mathbf{k}_0$, $\omega_0\rightarrow\omega_0$; $\chi^{(01)}(\omega)\rightarrow\chi^{(10)}(\omega)$.

4. As follows from (3.9), under conditions of strong interference of the modes with frequencies $\omega\approx\omega_p\pm 2\omega_0$ and $\omega\mp 2\omega_0\approx\omega_p$, the Fourier transform $E_i(\mathbf{k},\omega)$ of the field consists of two terms. The first is proportional to the self-field of a charge uniformly moving in a medium with permittivity $\tilde{\varepsilon}_{ij}(\mathbf{k},\omega)$. The Fourier transform of this field contains the delta function $\delta(\omega-\mathbf{kv})$, which guarantees the existence of a radiation field only when the condition for the emission of the Vavilov–Cerenkov radiation is fulfilled. This implies that the first term describes the Vavilov–Cerenkov radiation, restructured in the field of the pump wave.

The second term in (3.9) contains the field produced by the uniformly moving charge at the frequency $\omega_p(\varepsilon(\omega_p-i\gamma_p)=0)$. The vanishing of the permittivity corresponds to the excitation of longitudinal electric oscillations and, consequently, to the production of a wake-charge. It is not difficult to see that, by treating $\varepsilon(\omega_p)$ as a small quantity, and retaining the leading—with respect to $\varepsilon^{-1}(\omega_p)$—terms, we can derive from the general formula (3.8) for the self-field of a uniformly moving charge the expression

$$\mathbf{E}_k^{(0)}(\mathbf{k},\omega)=-\frac{iZe\mathbf{k}\delta(\omega-\mathbf{kv})}{2\pi^2k^2(\omega-\omega_p+i\gamma_p)\partial\varepsilon/\partial\omega_p}, \quad (4.1)$$

which coincides exactly with the field produced by the corresponding component of the oscillations of the wake charge (1.1). It follows from the foregoing that the wake-charge-density oscillations can be regarded as the source of the radiation described by the second term in the expression (3.9) for the field.

Let us now find the expression for the Fourier transform $\mathbf{E}(\mathbf{r}, \omega)$ of the field at large distances. It is convenient here to use the well-known asymptotic formula

$$\int \frac{d^3kf(\mathbf{k})}{k^2-p^2-i0} e^{i\mathbf{kr}} \approx 2\pi^2 f\left(p\frac{\mathbf{r}}{r}\right)\frac{e^{ipr}}{r}, \tag{4.2}$$

which is valid for $pr \gg 1$. Notice that, as noted above, in the case of a material in which the condition $v^2\varepsilon(\omega) > 1$ for the emission of the Vavilov–Cerenkov radiation is fulfilled, both terms in (3.9) lead to the appearance of a radiation field at large distances, while in the non-Cerenkov case ($v^2\varepsilon(\omega) < 1$) only the second term in (3.9) contributes to the radiation field.

As indicated above, the dielectric properties become anisotropic in the frequency region $\omega = \omega_p \pm 2\omega_0$. Consequently, ordinary and extraordinary waves can exist in the medium at frequencies lying in this region. Indeed, the poles of the expression (3.9) for the particle's self-field $E_i(\mathbf{k}, \omega)$ determine two types of waves: the pole

$$k^2-\omega^2\varepsilon(\omega)=0$$

describes ordinary waves, while the pole

$$k^2+(\mathbf{ke})^2\frac{\varkappa}{\varepsilon}-\omega^2(\varepsilon+\varkappa)=0$$

corresponds to extraordinary waves. Notice that the quantity $\varkappa \propto \varepsilon^{-1}(\omega \pm 2\omega_0)$ has a large imaginary part in the frequency region $\omega = \omega_p \pm 2\omega_0$, where the $E_i(\mathbf{k}, \omega)$ and $E_i(\mathbf{k} \pm 2\mathbf{k}_0,\ \omega \pm 2\omega_0)$ waves can be strongly coupled. Therefore, in the case in which the anisotropy in the dielectric properties of the medium is appreciable ($|\varkappa| \sim \varepsilon$), the extraordinary waves are strongly absorbed. On the other hand, when $|\varkappa| \ll \varepsilon$, and the absorption of the extraordinary waves is slight, the anisotropy in the dielectric properties is insignificant, and the investigation of the extraordinary waves is of no interest. Therefore, we limit ourselves below to the investigation of only the ordinary waves.

Let us, taking all the foregoing into account, represent the field at large distances as a sum of two fields, i.e., as $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$:

$$E_{1i}(\mathbf{r}, \omega)=\frac{iZe}{r}\exp(i\omega\varepsilon^{1/2}(\omega)r)\frac{1}{\varepsilon(\omega)}\left(\delta_{ij}-\frac{\psi_{ij}}{\delta_{ij}\psi_{ij}}\right)(\omega\varepsilon(\omega)v_j-k_j)\,\delta(\omega-\mathbf{kv}),$$

$$\mathbf{k}=\mathbf{n}\omega\varepsilon^{1/2}(\omega);\quad \mathbf{n}=\mathbf{r}/r;\quad \omega\approx\omega_p+2\omega_0; \tag{4.3}$$

$$\psi_{ij}=(\delta_{is}-n_in_s)\,e_se_j; \tag{4.4}$$

$$E_{2i}(\mathbf{r}, \omega)=4\pi i\frac{Ze}{r}\exp(i\omega\varepsilon^{1/2}(\omega)r)\,\omega^2\rho_{10}\chi^{(01)}(\omega)(\omega-2\omega_0-\omega_p+i\gamma_p)^{-1}\left(\frac{\partial\varepsilon}{\partial\omega_p}\right)^{-1}$$
$$\times\left(\delta_{ij}-\frac{\psi_{ij}}{\delta_{ij}\psi_{ij}}\right)(\delta_{js}-n_jn_s)\frac{(\mathbf{k}-2\mathbf{k}_0)_s}{(\mathbf{k}-2\mathbf{k}_0)^2}\delta(\omega-2\omega_0-\mathbf{v}(\mathbf{k}-2\mathbf{k}_0)). \tag{4.5}$$

It is not difficult to derive from (4.3) and (4.5) an expression for the energy emitted into the solid-angle element $d\Omega$ in the direction $\mathbf{n}$ in the frequency range $d\omega$:

$$d\mathscr{E}=(|\mathbf{E}_1(\mathbf{r}, \omega)|^2+|\mathbf{E}_2(\mathbf{r}, \omega)|^2)r^2\varepsilon^{1/2}(\omega)\,d\omega d\Omega. \tag{4.6}$$

The angular and spectral distribution of the Vavilov–Cerenkov radiation, retuned in the field of the pumping wave, has the form (T is the total transit time):

$$\frac{d\mathscr{E}_1}{d\omega d\Omega}=\frac{Z^2e^2}{\varepsilon^{1/2}(\omega)}\omega^2\frac{T}{2\pi}|M_1|^2\delta(\omega-\mathbf{kv}),$$
$$M_{1i}=(\delta_{ij}-\psi_{ij}/\delta_{ij}\psi_{ij})\,(n_j-\varepsilon^{1/2}(\omega)v_j). \tag{4.7}$$

The wake-charge-density oscillations yield for the radiation an angular–frequency distribution of the form

$$\frac{d\mathscr{E}_2}{d\omega\, d\Omega}=Z^2e^2\omega^4\varepsilon^{1/2}(\omega)\frac{T}{2\pi}|\mathbf{M}_2|^2\,\delta(\omega-2\omega_0-\mathbf{v}(\mathbf{k}-2\mathbf{k}_0)), \tag{4.8}$$

$$M_{2i}=4\pi\rho_{10}\chi^{(01)}(\omega)\left(\delta_{ij}-\frac{\psi_{ij}}{\delta_{ij}\psi_{ij}}\right)(\delta_{js}-n_jn_s)$$
$$\times\frac{(\mathbf{k}-2\mathbf{k}_0)_s}{(\mathbf{k}-2\mathbf{k}_0)^2}(\omega-2\omega_0-\omega_p+i\gamma_p)^{-1}\left(\frac{\partial\varepsilon}{\partial\omega_p}\right)^{-1}$$

5. Let us now discuss the cause of the generation of the additional radiation (4.8). As has already been noted, it is due to the presence of wake-charge-density oscillations. The wake-charge field of frequency ω_p can be Raman scattered by an excited atom of the material, changing in the process its frequency by $2\omega_0$ (the latter circumstance is due to the fact that the pump-field frequency ω_0 is lower than the resonance-transition frequency ω_{10} by a factor of two). As a result of this Raman scattering, the longitudinal field of the wake-charge is transformed into a transverse emitted plane-wave field of frequency $\omega \approx \omega_p + 2\omega_0$. The atom of the material is first excited by the pump field, and, after the Raman scattering, undergoes a transition back into the ground state. The process leading to the generation of the radiation with frequency $\omega \approx \omega_p - 2\omega_0$ occurs in similar fashion.

Thus, the atom changes neither its energy nor its momentum, and its participation in the process is not reflected in the laws of conservation of energy and momentum. Notice also that the wake-charge oscillations can transfer the energy ω_p and the momentum $\mathbf{q}$ related by the condition $\omega_p = \mathbf{q}\cdot\mathbf{v}$. Therefore, using the conservation laws

$$2\mathbf{k}_0+\mathbf{q}=\mathbf{k},\quad 2\omega_0+\omega_p=\omega, \tag{5.1}$$

we can derive the relation

$$\omega-2\omega_0=(\mathbf{k}-2\mathbf{k}_0)\mathbf{v}, \tag{5.2}$$

which is in fact guaranteed by the argument of the delta function in (4.5). The process under consideration occurs without energy or momentum transfer, which ensures its coherence with respect to the various atoms, i.e., the coherent addition at the observation point of the radiation fields arising in the Raman scattering of the field of the wake-charge by the various atoms. The coherence of the process ensures the growth of its intensity in comparison with the possible noncoherent processes.

Naturally, for the radiation to be observed, it is necessary that the radiation frequency ω lie in the transmission band $\mathrm{Im}\,\varepsilon(\omega) \to 0$ of the material.

To estimate the intensity of the process, let us compare (4.8) with the intensity of the normal Cerenkov radiation emitted in a material with permittivity $\varepsilon - 1 \sim 1$. The ratio of the intensity (4.8) to the intensity $\mathscr{E}_{VC}$ of the Vavilov–Cerenkov radiation has, in order of magnitude, the form

$$\frac{d\mathscr{E}_2}{d\mathscr{E}_{VC}} \sim (4\pi\chi)^2 \left(\frac{\omega_p}{\gamma_p}\right)^2 \sim (4\pi)^2 (n_0 a^3)^2 \left(\frac{\omega_p}{\gamma_p}\right)^2,$$

where a is of the order of the atomic dimension and n_0 is the number of resonance atoms in a unit volume.

According to Pine's estimates,[2] for the long-wave longitudinal electric oscillations, which are precisely the oscillations that participate in the considered process, the ratio $\omega_p/\gamma_p \sim 10^3$. In this case the intensity of the radiation emitted by the wake-charge is comparable to the intensity of the Cerenkov radiation when $n_0 \sim 10^{20}$ cm^{-3}.

In conclusion, let us note that the above-considered effect of Raman emission of electromagnetic waves by a wake-charge can be used to investigate the spectrum of the longitudinal vibrations [i.e., the zeros of $\varepsilon(\omega)$] of various materials.

[1]N. Bohr, Penetration of Atomic Particles through Matter, Danska Vidensk. Selsk. Mat.-fys. Medd. **18**, No. 8 (1948) 1950 p. 53 (Russ. Trans. IIL, Moscow, 1950, p. 53).

[2]D. Pines, Elementary Excitations in Solids, Benjamin, New York, 1963 (Russ. Transl., Mir, Moscow, 1965), Chaps. 1, 3, and 4.

[3]J. Neufeld and R. H. Ritchie, Phys. Rev. **98**, 1632 (1955).

[4]W. Brandt, A. Ratkowski, and R. H. Ritchie, Phys. Rev. Lett. **33**, 1325 (1974).

[5]V. H. Neelavathi and R. H. Ritchie, Atomic Collisions in Solids, Vol. 1, Plenum Press, New York, 1975, p. 289.

[6]R. H. Ritchie, W. Brandt, and P. M. Echenique, Phys. Rev. B **14**, 4808 (1976).

[7]Z. Vager and D. S. Gemmell, Phys. Rev. Lett. **37**, 1352 (1976).

[8]W. Brandt and R. H. Ritchie, Nucl. Instrum. Methods **132**, 43 (1976).

[9]Z. Vager, D. S. Gemmell, and B. J. Zabransky, Phys. Rev. A **14**, 638 (1976).

[10]Yu. Kagan, Yu. V. Kononets, and N. K. Dzhamankyzov, Zh. Eksp. Teor, Fiz. **74**, 288 (1978) [Sov. Phys. JETP **47**, 148 (1978)].

[11]K. A. Barsukov and B. M. Bolotovskiĭ, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **7**, 291 (1964).

[12]V. L. Ginzburg and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **65**, 132 (1973) [Sov. Phys. JETP **38**, 65 (1974)].

[13]V. L. Ginzburg and V. N. Tsytovich, Phys. Rep. **49**, No. 1 (1979).

[14]M. I. Ryazanov, Zh. Eksp. Teor. Fiz. **65**, 123 (1973) [Sov. Phys. JETP **38**, 61 (1974)].

[15]Yu. S. Kalashnikova, Zh. Eksp. Teor. Fiz. **71**, 2085 (1976) [Sov. Phys. JETP **44**, 1096 (1976)].

[16]P. A. Apanasevich, Osnovy teorii vzaimodeĭstviya sveta s veshchestvom (The Principles of the Theory of the Interaction of Light with Matter), Pt. II, Nauka i Tekhnika, Minsk, 1977, Chap. 2.

[17]V. S. Butylkin, A. E. Kaplan, Yu. G. Khronopulo, and E. I. Yakubovich, Rezonansnye vzaimodeĭstviya sveta s veshchestvom (Resonance Interaction of Light with Matter), Nauka, Moscow, 1977, Chap. 2.

Translated by A. K. Agyei