Multibeam self-channeling of plasma waves

G. L. Markov, V. A. Mironov, A. M. Sergeev, and I. A. Sokolova

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Stationary multibeam self-channeling of plasma waves of the lower hybrid frequency band $(\omega_{Hi} < \omega < \omega_{He})$ was experimentally observed under ionization nonlinearity conditions. It is shown that the proposed theoretical model admits of many-soliton solutions that explain the most characteristic features of the observed phenomenon.

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The practical utilization of the self-action of electromagnetic waves in a plasma is frequently connected with the possibility of directional and localized transfer of field energy to charged particles. One of the most suitable physical effects for this purpose is the selfchanneling of plasma waves of the lower hybrid frequency band ($\omega_{H_i} < \omega < \omega_{H_0}$, where ω_{H_0} and ω_{H_i} are the electron and ion cyclotron frequencies) under conditions of ionization nonlinearity.^{1,2} The characteristic feature of this effect is connected with the localization of the plasma by the field of an oblique Langmuir wave propagating in a plasma channel that is insulated from the chamber wall and is elongated in the direction of the magnetic field. The use of this effect seems promising for the solution of important applied problems such as the preionization of a gas and heating of plasma in magnetic traps, high-frequency pumping of lasers, and others.

A result of the experiments described in this article was the observation of a regime of stationary multibeam channeling of waves in a magnetic field, substantially different from the analogous effect in an isotropic plasma and in condensed media.

Attempts to explain the experimental results have stimulated an investigation of theoretical problems connected with the construction of multisoliton solutions of nonlinear differential equations. It has turned out that the system of equations used to describe selfchanneling of plasma waves¹⁻² is quite similar in structure to equations encountered in various branches of physics (see, e.g., Ref. 3). Therefore the results of a theoretical analysis must be regarded not only as a possible explanation of an experiment, but also as a contribution to finding the answer the question of stationary many-soliton objects ("multisolitons") in various kinds of physical systems.⁴⁻⁶ The experimental part of the work can serve then as a clear demonstration of the existence (and possibly stability) of complex soliton states.

1. The experiments were performed with a the appartus described in Ref. 2. A high-frequency discharge was excited in a glass bulb of 20 cm diameter and 180 cm length, using inductors of cylindrical (Fig. 1a), dipole (Fig. 1b), and quadrupole (Fig. 1c) type. The radiation frequency f was 60 MHz and the power W_0 ranged from 0 to 200 W. The pressure of the working gas (air) was $p \leq 10^{-2}$ Torr. The discharge was located in a longutudinal magnetic field B = 500 G whose inhomo-

geneity along the system axis did not exceed 6%.

The structure of the HF potential in the plasma was plotted using a screened post antenna connected either to an S4-27 analyzer or to an S7-8 high-speed oscilloscope, which made it possible to determine the lengths of the excited waves from the differences between the oscillation phases at different points of the system. Movable single and double Langmuir probes were used for the plasma diagnostics.

We list now the experimental results. If the excitation is symmetric,^{1,2} a characteristic conical surface is visually observed in the region of the source, and the plasma channel stretches from its apex along the magnetic field. Measurements show that oblique Langmuir waves propagate along the channel, and their longitudinal wavelength λ_{\parallel} is of the order of double the inductor dimension. The maximum plasma density and energy-flux density are reached on the system axis. Evidence of the excitation of plasma waves is the fact that cone half-apex angle θ satisfies the relation

$$\operatorname{ctg}^{2} \theta = -1 + N/N_{\mathrm{cr}}, \tag{1}$$

N is the plasma density and $N_{\rm er} = m\omega^2/4\pi e^2$.

When the plasma waves are asymmetrically excited, the structure of the plasma formation depends essentially on the supplied power and on the ratio of the electron mean free path λ_e to the plasma wavelength λ_{μ} . At relatively low powers ($W_0 \leq 20$ W) the picture of the discharge is qualitatively the same as for a symmetrical source (Fig. 2a). At $\lambda_e \geq \lambda/2$, a plasma-waveguide



FIG. 1. Types of HF inductors used to excite plasma waves: a) source of axially symmetric waves; b) source of waves with dipole symmetry; c) quadrupole source.



FIG. 2. Structure of plasma-waveguide channels excited by asymmetrical sources in air at B = 500 G: a) quadrupole inductor, $W_0 = 24$ W, $p = 3 \times 10^{-2}$ Torr; b) quadrupole inductor, $W_0 = 20$ W, $p = 5 \times 10^{-3}$ Torr; c) dipole inductor, $W_0 = 110$ W, $p = 10^{-2}$ Torr; d) quadrupole inductor, $W_0 = 110$ W, $p = 3 \times 10^{-3}$ Torr; 4) quadrupole inductor, $W_0 = 195$ W, $p = 3 \times 10^{-2}$ Torr. The vertical dark strips on the photographs are due to the shadows of the solenoid coils and of the mounting of the discharge bulb.

channel completely insulated from the walls can be obtained on the system axis when in the region of the source (Fig. 2b). With increasing HF power, the axial symmetry of the plasma formation is violated and the discharge splits up into individual channel (from two in the case of dipole excitation, Fig. 2c, to eight for quadrupole excitation). At sufficiently high power $(N_{max}/N_{cr} \gg 1)$ and with a quadrupole inductor $(\lambda_e \ll \lambda_{\parallel})$ focusing of the plasma waves on the system axis is observed, with successive stepwise decrease of the number of plasma channels (see Figs. 2d, e).



FIG. 3. Transverse distribution of the amplitude of the HF potential φ , measured in the vertical (solid line) and horizontal (dashed) planes at different distances L from the dipole source.



FIG. 4. Transverse distribution of plasma density for the vertical (solid line) and horizontal (dashed) symmetry planes, measured at different distances from the dipole source.

Detailed measurements were made with the discharge excited with a dipole inductor (Fig. 2b) of 65 mm diameter and of length l=35 mm, located on the bulb and narrowed down to 60 mm. The radial distribution of the amplitide of the HF potential φ at $W_0 = 100$ W and $p = 10^{-2}$ Torr is shown in Fig. 3 for two mutually perpendicular symmetry planes of the source and for various distances L from it. The plasma density distribution for this case is shown in Fig. 4. A similar distribution is possessed by the optical emission from the plasma, which was measured with an FÉU-22 photomultiplier. Figure 5 illustrates the transition from a single-beam to a two-beam discharge with increasing power. The transition from two plasmawaveguide channels to a single one in space at a given power input takes place in similar fashion.

Phase measurements have shown that the HF potential in the plasma has two components $(\varphi_{stat} + \varphi_w)$, with φ_{stat} due to the nearby static field of the source and φ_w due to the traveling plasma wave; the longitudinal scale of the latter is $\lambda_{\mu} = 13$ cm ~4*l*. A comparison of the oscillograms of the signals of the double probes used to measure the longitudinal component of the HF field has



FIG. 5. Plasma density distribution in the vertical plane, measured at $p = 10^{-2}$ Torr, B = 500 G, L = 75 cm at various levels of the HF power input W_0 , illustrating the transition from a single-beam to a two-beam discharge.

shown that the wave oscillations in neighboring plasma channels are in antiphase. The dashed curves of Fig. 3 illustrate the magnitude and the distribution of φ_{stat} in the presence of plasma. In the absence of plasma, $arphi_{
m stat}$ falls off like $1/L^2$ with decreasing distance Lfrom the source. Spectral measurements have shown that at $W_0 > 40$ W harmonics of the field frequency are generated in the vicinity of the source and propagate along the plasma-waveguide channels. If a source with dimension $l = 90 \text{ mm} (\lambda_{\mu} = 34 \text{ cm})$ is used, the qualitative pictures of the $\varphi(r)$ and N(r) distributions remain the same, apart from an insignificant increase of the longitudinal gradient. Replacement of the working gas with helium produces likewise no substantial change in the discharge structure. Similar results were obtained also with the quadrupole source.

The physical picture of the considered phenomenon appears to be the following. The (oblique Langmuir) plasma waves excited by the source cause additional ionization of the gas, and the predominantly longitudinal particle transport leads to formation of a plasma waveguide that channels the waves that produce it.

The main difference between the process observed by us and the generation of oblique Langmuir waves by cylindrical sources, previously investigated in a number of studies,⁷ is that these waves do not propagate in a ready-made plasma, but they themselves ionize the gas and determine conditions for their existence (we recall that these waves propagate only in a sufficiently dense plasma with $N > N_{cr}$). Consequently, the parameters of the medium depend on the amplitude of the HF field, i.e., the investigated process is essentially nonlinear. It is apparently this circumstance which causes the radiation to propagate through the plasmawaveguide channel. Indeed, in accord with formula (1), the apex angle of the conical surface should decrease with increasing plasma density. This surface then stretches and goes over smoothly into a single channel, or breaks up in the case of asymmetrucal excitation into a multiple-beam structure.¹⁾

A quantitative description of the entire picture of the self-consistent distribution of the plasma and of the field is a rather complicated problem. However, the most distinctive peculiarities of the considered phenomeon are connected with the effect of multiple-beam self-channeling, the existence of which can be demonstrated by investigating the structure of the homogeneous (along the system axis) formations. Since the characteristic dimension of the system $L_{\parallel} \approx 2$ m is less than the length $\lambda_{e} \approx 6$ m of the electromagnetic wave, the plasma is concentrated in the quasistatic region of the inductor. The stationary distribution of the amplitude of the HF potential can then be described by the equation

div $(\hat{\boldsymbol{\varepsilon}} \nabla \boldsymbol{\varphi}) = 0$,

(2)

where $\hat{\epsilon}$ is the dielectric tensor of the plasma. In the case of sufficiently weak wave absorption $(\nu/\omega \ll 1, \nu)$ is the effective electron collision frequency) it is natural to assume that radiation dissipation does not play a noticeable role in the formation of the investigated structure. The propagation of the wave beam

 $\varphi_w = \psi(x, y) \exp(ik_x z)$ along the magnetic field in the employed range of parameters of the medium $(\varepsilon_{11} = \varepsilon_{22} = 1, \varepsilon_{33} = 1 - N/N_{\rm er})$ satisfies then the equation

$$\Delta_{\perp}\psi - k_z^2 (1 - N/N_{\rm cr})\psi = 0. \tag{3}$$

The model of stationary plasma production presupposes a balance between the ionization of the gas and the loss of charged particles from the volume. The main cause of the loss under the experimental conditions is the sticking of the electrons to the electronegative molecules of the air. The weak longitudinal inhomogeneity of the plasma column can be taken into account by additing to the sticking frequency an additional loss factor D_{μ}/L^2 corresponding to ambipolar diffusion to the end wall. By describing the frequency of the air-molecule ionization by electron impact with the aid of the model longitudinal-field amplitude function⁹ $\nu_1 = \alpha (k_g \psi)^{2\beta} \ (\beta > 1)$ (it is just this function which determines the temperature of the electrons in the discharge), we arrive at the equation

$$D_{\perp}\Delta_{\perp}N + (v_i - v_a)N = 0, \qquad (4)$$

where D_{\perp} is the coefficient of ambipolar diffusion transverse to the magnetic field.

In terms of the dimensionless variables $n = N/N_{\rm er}$, $r_H = r(\nu_a/D_{\perp})^{1/2}$, $\Phi = k_z \psi(\alpha/\nu_a)^{1/2\beta}$, the self-consistent system of equations (3) and (4) takes the form

$$\Delta \Phi - E(1-n) \Phi = 0,$$
 (5)
$$\Delta n - n + n \Phi^{2n} = 0.$$
 (6)

The parameter $E = k_x^2 D_1 / \nu_a$ is determined here by the ratio of the diffusion sticking length $(D_1 / \nu_a)^{1/2}$ to the characteristic scale λ_x of the field variation.

A numerical analysis of the two-dimensional separated solutions of (5) and (6), by the method proposed in Ref. 10, shows that the main laws governing the appearance of the bound state of the solitons, of the structure, and of the asymptotic behavior are the same as in the onedimensional case. A detailed investigation of the localized solutions of the one-dimensional system of equations leads to the following results.

For E < 1 there exists only one localized field distribution that does not differ qualitatively from the exact analytic solution of (5) and (6) at $E = 1/4\beta^2$:

$$\Phi = (\frac{3}{2})^{1/2\beta} \operatorname{ch}^{-1/\beta}(x/2), \quad n = (\beta + 1) \operatorname{ch}^{-2}(x/2).$$
(7)

The amplitude Φ_{max} of a single soliton decreases to 1 as $E \rightarrow 0$, and the distribution of the density *n* becomes narrower than the corresponding distribution of the HF potential.

If the scale $(D_{\perp}/\nu_a)^{1/2}$ of the density change exceeds the characteristic dimension of the field (i.e., E>1), there exists an infinite number of localized solutions, which are produced by successively adding a single soliton, so that the phases of the oscillations in the individual field clusters differ by π (Fig. 6). At $E\gg1$ the multiple-hump structure of the "almost" sinusoidal spatial oscillations of the amplitude of the HF potential turns out to be contained in a single distribution of the plasma density. As $E \rightarrow 1$, the distance between the individual "humps" increases. Unfortunately, it was im-



FIG. 6. Self-consistent distribution of the plasma density n (solid line) and of the HF potential Φ (dash-dot line) at E = 3 and $\beta = 2$. The dashed line shows a single soliton.

possible to determine the canonical variables of the considered autonomous system of equations (5) and (6) and determine the analytic characteristics of the solitons, in analogy with what can be done for a Hamiltonian with similar properties.⁶

Similarly, the system of equations (5) and (6) admits at E > 1, besides the single (axially symmetric) soliton, also of multiple-hump separated distributions. Such solutions correspond to the possibility of multiple-beam self-channeling of plasma waves. By way of example, Fig. 7 shows the distribution of the amplitude of the HF potential in a bisoliton (E = 4).

3. We discuss now the agreement between the experimental data and the foregoing theoretical concepts. We see first of all a qualitative agreement in the distribution of the amplitude of the HF potential (see Figs. 3 and 7). It is unfortunately difficult to determine the limiting value of $E(k_z^2 D_1/\nu_a = 1)$ under the conditions of our experiment, since the corresponding wavelength $\lambda_{\mu} = 2\pi(\nu_a/D_1)^{-1/2}$ is of the order of the dimensions of the system. However, special measures taken to strengthen the magnetic field (to $B \approx 1.5$ kG) have led, as expected, to results that point to the existence of a bifurcation value of E. Thus, for a single source of length $l \approx 3$ cm (E = 1.5, $W \leq 100$ W, $p = 10^{-2}$ Torr) two channels are observed, and at $l \approx 10$ cm (E = 0.5) the discharge emission is axially symmetrical.

Another characteristic feature of multisoliton formations is the phase difference equal to π , between the wave fields in neighboring plasma-waveguide channels. To obtain a more accurate value of this quantity under conditions $\varphi_{st} \approx \varphi_w$, the phase of the longitudinal electric field was measured with an electric double probe. Since the amplitudes of the fields at which multibeam self-channeling is realized are quite large ($v \approx v_T$), near the inductor, harmonics that are multiples of ω are generated. At high powers, the time dependences of the fields on the axes of neighboring channels have a



FIG. 7.

rather complicated form (the amplitude of the second harmonic becomes of the order of the first), but it is easy to ascertain that the signals from the probes are shifted by π . In addition, attempts to produce (by introducing various changes in the excitation system) multiple-hump on-phase field distributions were unsuccessful.

Allowance for the weak absorption explains also the successive decrease of the number of channels to one (Fig. 2e). Indeed, the total energy flux decreases as a result of dissipation, and consequently the number of plasma channels can change in discrete fashion.

The reported experimental results demonstrate thus the feasibility of stationary multibeam self-channeling of oblique Langmuir waves in a plasma, and the derived multisoliton solutions explain the most characteristic features of the observed phenomenon.

- ¹⁾ The deformation of conical surfaces near a cylindrical source was recently investigated⁸ under conditions of striction nonlinearity. The authors observed experimentally an increase of the apex angle on approaching the focal region. In contrast to our investigation, this attests to a decrease of the plasma density in the region of a strong HF field.
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