Domain-wall resonance in thin magnetic films[1)]

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A theory of linear resonance of a domain wall in a thin magnetic film is developed on the basis of an approximate calculation of the magnetic dipole interaction, in an effective-demagnetizing-factor model. Both a uniform domain wall (of Bloch type for film thickness $d > d_0$ and of Néel type for $d < d_0$) and a wall periodic along its length are considered. For the uniform domain wall, a general equation of motion for the wall center is obtained from the Landau-Lifshitz equation, and it is shown how the resonance frequency and the damping parameter vary with the film thickness. It is established that the natural frequency of oscillation of the domain wall vanishes at film thickness $d = d_0$; that is, this oscillation is a "soft mode" of the structural phase transition from a Néel wall to a Bloch wall. A domain wall periodic along its length is investigated; it consists of alternating, oppositely magnetized Bloch or Néel sections, separated by "domain walls" of internal structure. The frequency of oscillation of the "domain walls" of internal structure is studied, and the existence of a new type of resonance is established, resonance of the internal structure (RIS) of a periodic wall. The theory is compared with the experimental results of P. D. Kim, D. M. Rodichev, and I. A. Safonov [Izv. Akad. Nauk SSSR, Ser. Fiz. **36**, 1499 (1972); Bull. Acad. Sci. USSR, Phys. Ser. **36**, 1329 (1972)], who reported a resonant increase of the static susceptibility in the presence of a weak radiofrequency field oriented normal to the plane of a domain wall.

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INTRODUCTION. RESONANCE OF QUASISTATIC SUSCEPTIBILITY

The dynamics of a domain wall in ferromagnets was first considered by Döring.[3] He obtained the equation of motion of the wall center, introduced the concept of effective mass, and demonstrated the existence of resonance of uniform oscillations of a domain wall. This was all done for the domain wall characteristic of bulk materials, whose structure had been calculated earlier by Landau and Lifshitz.[4] A review of later work, theoretical and experimental, on the dynamics of the motion of a domain wall may be found in the monograph of Hubert.[5] It is appropriate to mention in particular the exact solution by Walker[6] of the problem of one-dimensional motion of a domain wall, and a series of papers by Slonczewski[7] on the structure and dynamics of domain walls of cylindrical domains. Nonuniform oscillations of domain walls in bulk materials were studied by Winter[8] and by Fartzdinov and Turov[9] within the framework of a general formulation of the problem of spin waves in magnetic materials with a domain structure.

With decrease of even a single dimension of the specimen to values corresponding in order of magnitude to the effective thickness of a domain wall, the problems of the structure of a domain wall and of its dynamics become suddenly more complex because of the increased role of the magnetic dipole interaction. In the presence of strong magnetic dipole interaction (and in thin films it often dominates), the structure of the wall is described by a system of nonlinear integrodifferential equations. Approximate (principally numerical) solutions of this static system (a review of them is given in Ref. 5) show that the actual structure of a domain wall in thin films is nonuniform in all three dimensions and is very complicated. It is natural that in such a situation even the linear dynamic problem becomes extremely complicated, since it is described by a system of integrodifferential equations in which the variable coefficients are the solution of the static problem of domain-wall structure.

The slight degree of development of the dynamic theory of domain walls in a thin film impedes their use in devices of high-frequency and of computer technology. There are a number of experimental results that cannot be understood on the basis of the dynamic theory of a wall in bulk materials. We shall discuss one such experiment,[10] which exerted a stimulating influence on the formulation of the present research.

A solitary 180-degree domain wall, oriented along the easy axis of a permalloy film, was observed under a microscope by means of the magnetooptical Kerr effect, in the form of a boundary of contrast between light and dark fields. A low-frequency ($\Omega \sim 10^2$ Hz) magnetic field was applied along the easy axis and produced displacement of the domain wall at the same frequency. The amplitude of these forced oscillations (proportional to the displacement susceptibility χ_0) was measured with a photomultiplier, responding to the periodic change of the relation between the light and dark bands in the field of view of the microscope. A weak high-frequency (HF) field was applied perpendicular to the axis; its frequency f could be varied smoothly. It was found that at a certain frequency $f = f_0$ the amplitude of the forced low-frequency oscillations of the domain wall increased abruptly. The frequency of this resonance lay in the range 30–300 MHz and depended on the film thickness d (Fig. 1).

We shall analyze the results of this experiment. First, the value of the low frequency Ω plays no role in the physical mechanism of the observed effect; the same effect could be observed also at $\Omega = 0$, since what occurred at $f = f_0$ was an increase of the quasistatic susceptibility χ_0; it is simply easier to measure χ_0 at $\Omega \neq 0$. Second, the effect of an increase of the quasistatic susceptibility on application of a small HF field

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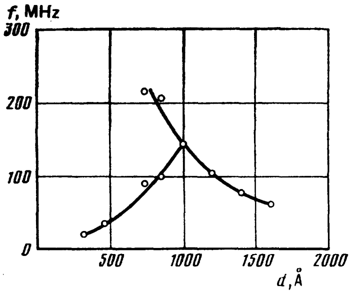

FIG. 1. Experimental variation of the resonance frequency of a high-frequency field with film thickness, according to data of Ref. 10.

is general for all hysteretic systems: such HF shaking always causes the magnetization hysteresis curve to approach the ideal curve, and the latter corresponds to maximum χ_0 and to maximum linearity of the characteristic (this circumstance is widely used in magnetic sound recording). Until now, however, all that has been observed (and used practically) is nonresonant shaking of arbitrary frequency (larger than the maximum frequency of the signal being recorded).

Thus it is a resonant increase of the quasistatic susceptibility that has been observed experimentally for the first time in Ref. 10. Qualitatively it is clear that this indicates the presence in the system of some resonance at frequency f_0, which amplified the effect of the HF shaking many times. As a result, application of a small HF field at $f=f_0$ led to such "idealization" of the magnetization curve as could have been produced in the nonresonant case only by a considerably larger amplitude of the HF field. It is clear that the primary problem in this effect is the deciphering of the nature of the resonance observed at frequency f_0 and elucidation of its degree of universality. An attempt to describe it by use of the theory of resonance of a domain wall in bulk material does not lead to either quantitative or qualitative agreement with experiment.

The aim of the present paper is the development of a linear dynamic theory of the motion of a domain wall in thin magnetic films. Because of the complexity of the problem, the magnetostatic interaction is taken into account in the effective-demagnetizing-factor approximation. This leads to a substantially simpler and cruder model of the structure of a domain wall. Such a model nevertheless conveys the basic features of the structure of a wall in thin films. It enables us to obtain approximate solutions of the linear dynamic problem and to determine the resonance frequencies both for a uniform domain wall and for one that is periodic along its length. The model developed has enabled us, in particular, to explain the nature of the resonance observed in Ref. 10.

1. GENERAL SYSTEM OF EQUATIONS

Both the structure and the dynamics of motion of a domain wall (as of any macroscopic formation in a ferromagnet) are described by the vector Landau-Lifshitz equation[4]

$$\dot{\mathbf{M}}=-g\,[\mathbf{M}\times\mathbf{H}^e]+\frac{\xi}{M}[\mathbf{M}\times\dot{\mathbf{M}}], \qquad (1.1)$$

where ξ is the damping parameter, g is the gyromagnetic ratio, and the effective magnetic field is connected with the phenomenological Hamiltonian $\mathscr{H}$ by the relation

$$\mathbf{H}^e=-\frac{\partial\mathscr{H}}{\partial\mathbf{M}}+\frac{\partial}{\partial x_k}\frac{\partial\mathscr{H}}{\partial(\partial\mathbf{M}/\partial x_k)}. \qquad (1.2)$$

We choose $\mathscr{H}$ and accordingly $\mathbf{H}^e$ on the form

$$\mathscr{H}=\frac{1}{2}\alpha(\nabla\mathbf{M})^2-\frac{1}{2}\beta(\mathbf{Ml})^2-\mathbf{MH}-\frac{1}{2}\mathbf{MH}^m,$$
$$\mathbf{H}^e=\alpha\nabla^2\mathbf{M}+\beta\mathbf{l}(\mathbf{Ml})+\mathbf{H}+\mathbf{H}^m, \qquad (1.3)$$

where α is the exchange constant, β is the anisotropy constant, $\mathbf{l}$ is a unit vector along the axis of anisotropy, $\mathbf{H}$ is the external magnetic field, and $\mathbf{H}^m$ is the magnetic dipole field.

The magnetic dipole field $\mathbf{H}^m$ depends on the distribution of magnetization $\mathbf{M}$ in space and is determined by Maxwell's equations; the latter at not too high frequencies (when electromagnetic wave propagation effects may be neglected) reduce to the equations for the magnetostatic potential φ:

$$\nabla^2\varphi=4\pi\,\mathrm{div}\,\mathbf{M},\quad \mathbf{H}^m=-\mathrm{grad}\,\varphi. \qquad (1.4)$$

The system of equations (1.1)–(1.4) with appropriate boundary conditions is a closed system and describes all the static and dynamic properties (in the approximation indicated) of a domain wall in an ideal magnetic crystal.

We shall further consider the following geometry (Fig. 2). A ferromagnetic layer of thickness d is located in the coordinate plane xz; the axis of easy anisotropy, determined by the unit vector $\mathbf{l}$, is oriented along the z axis; the plane of the domain wall is located in the yz plane.

If we suppose that the magnetization $\mathbf{M}$ within the layer is a function only of x and z, then the magnetostatic problem (1.4) can be solved exactly in general form. By splitting (1.4) into a system of three equations in three regions of space, with appropriate conditions for joining the potentials at the boundaries of the layer, and by expanding $\varphi(x,y,z)$ and $\mathbf{M}(x,y,z)$ as Fourier integrals in x and z, we get an expression for the Fourier transform of the potential within the layer

$$\hat{\varphi}=\frac{4\pi i}{\varkappa^2}(k_1\hat{M}_x+k_2\hat{M}_z)(e^{-\varepsilon}\operatorname{ch}\varkappa y-1)+\frac{4\pi\hat{M}_y}{\varkappa^2}e^{-\varepsilon}\operatorname{sh}\varkappa y, \qquad (1.5)$$

where

$$\varepsilon=\varkappa d/2,\quad \varkappa^2=k_1^2+k_2^2.$$

Hence the components of the magnetic field are de-

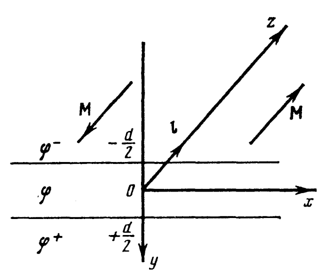

FIG. 2.

termined; after averaging over the thickness d of the plate (it is such an average that will interest us later), they have the form

$$H_x^m(x,z)=-4\pi\iint\frac{k_1}{\nu^2}(k_1\hat{M}_x+k_2\hat{M}_z)(1-V)e^{i(k_1x+k_2z)}\,dk_1\,dk_2,$$
$$H_y^m(x,z)=-4\pi\iint V\hat{M}_y\,e^{i(k_1x+k_2z)}\,dk_1\,dk_2, \qquad (1.6)$$
$$H_z^m(x,z)=-4\pi\iint\frac{k_2}{\nu^2}(k_1\hat{M}_x+k_2\hat{M}_z)(1-V)e^{i(k_1x+k_2z)}\,dk_1\,dk_2,$$

where $V=[1-e^{-\varkappa d}]/\varkappa d$, and where the integration extends between the limits $\pm\infty$.

We represent the magnetization and the magnetic fields (both the external and the magnetic-dipole) in the form

$$\mathbf{M}(x,z,t)=\mathbf{M}(x,z)+\mathbf{m}(x,z,t),$$
$$\mathbf{H}(x,z,t)=\mathbf{H}(x,z)+\mathbf{h}(x,z,t). \qquad (1.7)$$

We shall consider the dynamic parts m and h small and shall neglect all powers of them above the first. After substitution of (1.7) in (1.1), the latter splits into two equations: the nonlinear static Landau-Lifshitz equation, describing the structure of the domain wall,

$$[\mathbf{M}\times\mathbf{H}^e]=0, \qquad (1.8)$$

and the dynamic equation

$$\mathbf{m}=-g\{[\mathbf{M}\times\mathbf{h}^e]+[\mathbf{m}\times\mathbf{H}^e]\}+\frac{\xi}{M}[\mathbf{M}\times\mathbf{m}], \qquad (1.9)$$

describing the linear dynamics of the domain wall. The coordinate-dependent coefficients M and $\mathbf{H}^e$ of equation (1.9) are the solutions of equation (1.8).

2. UNIFORM DOMAIN WALL

For a domain wall uniform along z, when $\mathbf{M}=\mathbf{M}(x)$, the expressions (1.6) take the form

$$H_x^m(x)=-4\pi\int\hat{M}_x(1-V)e^{ikx}\,dk,$$
$$H_y^m(x)=-4\pi\int\hat{M}_yVe^{ikx}\,dk, \qquad (2.1)$$
$$H_z^m(x)=0,\quad V=(1-e^{-|kd|})/|kd|.$$

It is clear that one cannot hope to obtain analytic solutions of the systems of differential equations (1.8) and (1.9) by substituting in them the magnetic dipole fields $\mathbf{H}^m$ and $\mathbf{h}^m$ in the form (2.1); further approximations are necessary.

A cardinal simplification of the problem is approximate replacement of the integral expressions (2.1), which relate $\mathbf{H}^m(x)$ to the unknown function $\mathbf{M}(x)$, by algebraic expressions:

$$H_x^m(x)\approx-N^xM_x(x),\quad H_y^m(x)\approx-N^yM_y(x), \qquad (2.2)$$

where the effective demagnetizing factors N^i depend on the form of the functions $M_i(x)$ and on the thickness d of the film. Such a model for approximate calculation of magnetic dipole interaction has already been used[11] for calculation of the structure of a domain wall in a thin magnetic film. The model differs from the usual introduction of demagnetizing factors in that no averaging over the width of the magnetic pole is introduced in it; therefore there remains a possibility of seeking the form of the function $\mathbf{M}(x)$ from an appropriate differential equation.

The method (2.2) is based on the assumption that for the integral characteristics of the problem—energy, frequency—the dependence of the coefficients N^i on the ratio of the effective width l of the magnetic pole to d plays a more important role than do the deviations of the functional dependence (2.2) from (2.1). This assumption was tested in Ref. 11 as follows. The energy density of a domain wall with a magnetization distribution $\sim\operatorname{sech}\sigma x$ was calculated by two methods: a) substitution of the expression (2.1) and subsequent numerical integration, and b) substitution of the expression (2.2) and direct analytic integration. Here the demagnetizing factors were approximated by the expressions

$$N^x=4\pi\frac{d}{d+l},\quad N^y=4\pi\frac{l}{d+l}. \qquad (2.3)$$

It was found that for calculation of the energy, the expressions (2.2) and (2.3) are completely satisfactory approximations to the exact expressions (2.1) over a wide range of ratios d/l, if for the width of a pole of the form $\operatorname{sech}\sigma x$ one takes the value $l\approx5/\sigma$.

On substituting the expressions (2.2) in (1.8), we get a system of nonlinear differential equations, which describe approximately the structure of a domain wall uniform along z, in the absence of external magnetic fields; either a Néel wall,

$$M_x=M\operatorname{sch}\sigma_Nx,\quad M_y=0,\quad M_z=-M\operatorname{th}\sigma_Nx,$$
$$\sigma_N^2=(\beta+N^x)/\alpha, \qquad (2.4a)$$

or a Bloch wall,

$$M_x=0,\quad M_y=M\operatorname{sch}\sigma_Bx,\quad M_z=M\operatorname{th}\sigma_Bx,$$
$$\sigma_B^2=(\beta+N^y)/\alpha \qquad (2.4b)$$

may be a solution of such a system. Because of the dependence of σ on N^i, the effective width of the pole is a function of the film thickness. On substituting $l_i=5/\sigma_i$ in (2.3), we get equations for finding the dependence of N^i on d (or of σ_i on d). For a Bloch wall,

$$N^y[5+d(\beta+N^y)^{1/2}/\alpha^{1/2}]=20\pi, \qquad (2.5)$$

for a Néel wall,

$$N^x[d+5\alpha^{1/2}/(\beta+N^x)^{1/2}]=4\pi d. \qquad (2.6)$$

The transition from one domain-wall structure to the other occurs at thickness $d=d_0$, when $N^x=N^y=2\pi$; that is,

$$d_0=l_B{}^0=l_N{}^0=5\alpha^{1/2}/(\beta+2\pi)^{1/2}. \qquad (2.7)$$

Substituting one of the two solutions (2.4) of the static problem (1.8) in (1.9), we get a dynamic system of equations for the corresponding type of domain wall. This system describes all the types of natural or forced oscillations that can occur in a uniform domain wall. It is clear that the magnetic dipole fields $\mathbf{h}^m(\mathbf{r},t)$ for most types of oscillations are described by complicated functions and cannot be approximated by expressions of the form (2.2). But such an approximation is admissible for one very important oscillation, corresponding to uniform displacement of the domain wall as a whole. In fact, in this case the oscillation $\mathbf{m}(\mathbf{r},t)$ is uniform along z and has a width of order l along x, and therefore we may assume, with the same accuracy as for the static fields, that h_x^m and h_y^m are connected with the corresponding components of **m** by relations similar to (2.2). The field h_z in this case is the mean demagnetizing field that acts in the specimen when all its domain walls are displaced from the equilibrium position by some distance x_0. For an ideal specimen, the coefficient of proportionality between h_z and x_0 can be expressed[3] in terms of the specimen geometry and the domain structure. For a real specimen there is an additional term to interaction of the domain wall with inhomogeneities; since at small x_0 this term is also proportional to x_0, we can write in the general case

$$h_x \approx -N^x m_x, \quad h_y \approx -N^y m_y, \quad h_z \approx -Mkx_0, \tag{2.8}$$

where k is the coefficient of elasticity of the domain wall.

On substituting these expressions in (1.9), we find that, for example, for a Bloch wall one of the solutions of the system of differential equations is the following type of oscillation:

$$m_x \sim u(t)\,\mathrm{sch}\,\sigma x, \quad m_y \sim \sigma x_0(t)\,\mathrm{sch}\,\sigma x\,\mathrm{th}\,\sigma x, \quad m_z \sim \sigma x_0(t)\,\mathrm{sch}^2\,\sigma x, \tag{2.9}$$

where u and x_0 are unknown functions of time.

On comparing the components m_y and m_z of this oscillation with the first terms of the expansion of the displaced function $\mathrm{sch}\,\sigma(x-x_0)$ and $\mathrm{th}\,\sigma(x-x_0)$ as series in σx_0, we see that the oscillation (2.9) in fact corresponds to displacement of the domain wall. On eliminating $u(t)$, we get the equation of motion of the wall center x_0. This equation has the same form for a Bloch wall and for a Néel wall, if by σ we understand either σ_B or σ_N in accordance with (2.4):

$$\ddot{x}_0+2\Gamma\dot{x}_0+\omega_0^2 x_0=\frac{g}{\sigma}[gM|N^x-N^y|h+\xi\dot{h}],$$
$$\omega_0=gM\left(\frac{k}{\sigma}|N^x-N^y|\right)^{1/2}, \quad \Gamma=\frac{1}{2}\xi gM\left(\frac{k}{\sigma}+|N^x-N^y|\right). \tag{2.10}$$

Equation (2.10) differs from the well known equation of Döring[3] for a wall in bulk material, in that the effective mass m_e of the wall depends on the film thickness and has a singularity at a critical thickness $d=d_0$, corresponding to the transition from a Néel wall to a Bloch wall:

$$m_e=2(1+\xi^2)\sigma/g^2|N^x-N^y|. \tag{2.11}$$

We have also retained in the equation a term $\propto\xi\dot{h}$ ("excitation through damping"), which is usually neglected. Here such neglect is incorrect near $d=d_0$, where this term may exceed the term describing direct excitation.

Both the characteristic frequency ω_0 of the wall oscillations and the damping parameter Γ depend strongly on the film thickness, becasue of the dependence on d of the modulus of the difference of demagnetizing factors. The values of N^x, N^y, and σ are determined from equations (2.5), (2.6), and (2.4). The elasticity coefficient k also depends on the thickness, but an explicit expression for it can be written only for an ideal film:

$$k \approx 2N_z/D \approx 8\pi d/DL_z, \tag{2.12}$$

where D is the domain width and L_z is the specimen dimension along the z axis. For a real specimen at small thicknesses, the interaction with inhomogeneities becomes dominant, and the actual dependence of k on d may disagree completely with (2.12).

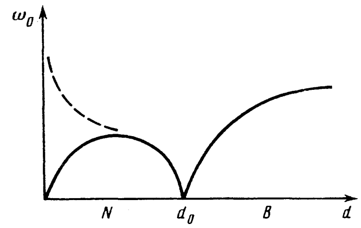

FIG. 3. Schematic variation of the characteristic frequency ω_0 of uniform oscillations of a domain wall with film thickness d. Solid curve, ideal case; dotted, allowance for the effect of inhomogeneities. N and B are the ranges of existence of Néel and of Bloch domain walls.

The dependence of ω_0 on thickness for an ideal film is shown in Fig. 3 schematically by the solid curve. The vanishing of the frequency at $d=d_0$ indicates that ω_0 is a "soft mode" corresponding to a structural phase transition, a change of the type of domain wall at a certain value of the film thickness. The vanishing of the frequency at $d=0$ is correct only for an ideal film; in this range of thicknesses, for real films, important differences from the solid curve of Fig. 3 may be observed (this is shown conventionally in Fig. 3, dotted curve).

In harmonic excitation by an external magnetic field ($h=h_0e^{i\omega t}$, $x_0=ae^{i\omega t}$), the complex amplitude a of the oscillations of the domain wall or the corresponding dynamic susceptibility χ of the material is determined by the expression

$$\chi=\frac{2M}{Dh_0}a=\frac{2gM}{\sigma D}\frac{gM|N^x-N^y|+i\xi\omega}{\omega_0^2-\omega^2+2i\omega\Gamma}. \tag{2.13}$$

Comparison of Fig. 3 with the experimental graph of Fig. 1 shows not even qualitative agreement between them. Thus the resonance of the static susceptibility observed in Ref. 10 is not related to resonance of the oscillations of the center of the domain wall.

3. PERIODIC DOMAIN WALL. RESONANCE OF THE INTERNAL STRUCTURE

A periodic internal structure of a domain wall arises in magnetic films to decrease the energy of magnetic dipole interaction. Very complicated configurations of the magnetic moment actually occur,[5] but we shall consider only the simplest model,[11] which reflects the principal properties of a periodic domain wall (Fig. 4). A domain wall periodic along the z axis, with period $2L$, consists in this model of alternating Néel and Bloch

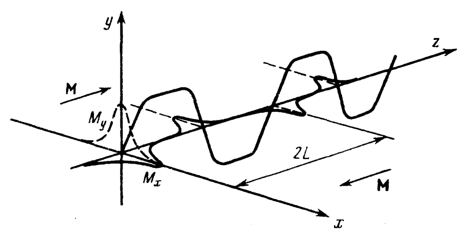

FIG. 4. Periodic domain wall. Model of the structure of a periodic domain wall according to Ref. 11.

sections of different polarities. Figure 4 represents a wall with $d \gg d_0$, when the Bloch sections form wide "domains" of internal structure, while the Néel sections form narrow "domain walls" between these domains. When $d < d_0$, the Néel sections become longer than the Bloch, and now they can be interpreted as domains of internal structure.

By application of a magnetic field along the y axis, it is possible to magnetize a periodic Bloch domain wall. The magnetization process will proceed by displacement of the Néel sections, the domain walls of internal structure; the Bloch sections oriented along the field will grow, and those oriented opposite to the field will diminish. Thus, many properties of the internal structure of a periodic domain wall are similar to the properties of the ordinary domain structure.

It is clear that a periodic domain wall will possess a number of new dynamic properties that are absent in a uniform domain wall. The most important of these properties is the occurrence of a new resonance frequency, corresponding to oscillations of domain walls of internal structure with respect to their equilibrium positions. It is the investigation of this frequency that is the goal of the present section.

The static structure of a domain wall has been investigated in a number of papers, by use both of the method of perturbation theory[11] and of numerical methods.[5] Not to mention the numerical results, even the analytic expressions[11] obtained for $d \gg d_0$ for the simplest model (Fig. 4) are too complicated for hope of solving the dynamic system (1.9), in which they occur as coefficients dependent on r.

Thiele,[12] it is true, succeeded in investigating analytically spin waves in a periodic domain wall. But he treated the problem of a periodic domain wall in an infinite material and with neglect of the magnetic dipole interaction. This is the gist of some internal contradictoriness of the model investigated in Ref. 12, since the very onset of a periodic internal structure of a domain wall is due to the finiteness of the specimen along the y axis and to the magnetic dipole interaction. For our purposes, the simplifications of Ref. 12 are inapplicable, and we shall make approximations of a different nature, not related to neglect of the magnetic dipole interaction.

In the limiting case $d \gg d_0$, a periodic domain wall can be approximated by a plate of thickness l_B and width d, split into internal domains of width L_B (Fig. 5a); in the limiting case $d \ll d_0$, by a plate of thickness d and width l_N, split into domains of width L_N (Fig. 5b). The values of L_B and L_N may be estimated by formula (7) of Ref. 13, which describes the width of a domain in a thin film:

$$L_N \approx 2d_0 l_N/d, \quad L_B \approx 2d(d_0/l_B)^{4/3}. \tag{3.1}$$

The widths l_B and l_N of the main domain wall, which enter here, are determined as functions of the film thickness d by equations (2.5) and (2.6) respectively.

In order to estimate the resonance frequency of domain walls of internal structure, we shall use the results of the preceding section, formula (2.10), but now substituting in them the characteristics of the internal domain walls. The widths of these walls (l_1 and l_2 in Fig. 5) are connected with d and l_B (or l_N) by relations analogous to (2.5) and (2.6). The coefficient k in formula (2.12) is also renormalized accordingly. As a result we get an approximate expression for the frequency of resonance of internal structure (RIS) in the form

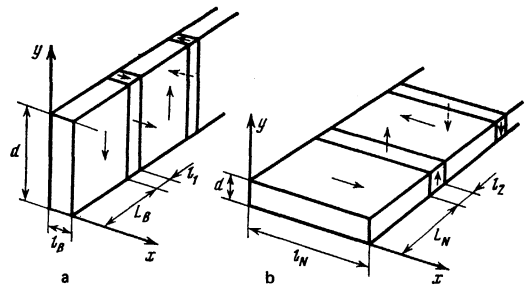

FIG. 5.

$$\omega \approx (8\pi)^{1/2} gM \begin{cases} d/l_N, & d \ll d_0 \\ l_B^{4/3}/d_0^{1/3} d, & d \gg d_0 \end{cases}. \tag{3.2}$$

It is evident that far from the point d_0, the RIS frequency behaves in qualitative accordance with the experimental results, in which resonance of the quasistatic susceptibility was observed (see Fig. 1): with increase of d, the frequency increases for $d \ll d_0$ and decreases for $d \gg d_0$.

The question of the value of the RIS frequency in the vicinity of the point $d = d_0$ of structural phase transition requires special analysis. We shall carry it out.

We consider a domain wall that is periodic along its length with period $2L$. For such a wall, the magnetization can be represented as a Fourier series with coefficients dependent on x:

$$M_j(x, z) = \sum_n M_n^j(x) e^{inqz}, \quad q = \pi/L. \tag{3.3}$$

We shall extend the effective-demagnetizing-factor model also to the magnetostatic problem for a domain wall periodic along the z axis. In the expressions (1.6), we shall neglect the antisymmetric components of the magnetic field that are induced by "foreign" poles: the contribution of the pole M_z to the field H_x^m and the contribution of the pole M_x to the field H_z^m; these effects are beyond the limits of accuracy of the model. We introduce effective demagnetizing factors N_n^j for each term of the Fourier series:

$$H_j^m(x, z) \approx -\sum_n N_n^j M_n^j(x) e^{inqz}, \quad j = x, y, z. \tag{3.4}$$

The meaning of this representation consists in the following, that we treat each half-period of the nth harmonic of the domain wall as a triaxial ellipsoid, whose dimensions along the corresponding coordinate axes are determined by the thickness l of the domain wall, the thickness d of the film, and the "harmonic length" $L/n = \pi/nq$. How legitimate is the approximation of isolated ellipsoids for description of the periodic structure of the poles? By replacing each half-period of a sinusoid by an ellipsoid, we have neglected the interaction between charges of adjacent half-periods. Such neglect is legitimate only when the half-period L/n is much larger than l and d. This inequality cannot be satisfied for all terms of the Fourier series. But we shall hereafter use only the first few terms of this series, and it is known from experiment that $L \gg l, d$; therefore the approximation (3.4) is admissible.

To avoid using tables for determination of N^j, it is convenient to have analytical expressions that approximate the demagnetizing factors of a triaxial ellipsoid. We propose as such an approximation, for an ellipsoid with axes a, b, and c, the expressions

$$N^j = 4\pi k_j/(k_x+k_y+k_z), \tag{3.5}$$

where $k_x = 1/a, k_y = 1/b, k_z = 1/c; j = x, y, z$.

The expression (3.5) describes accurately all the limiting cases (sphere, cylinder, thin plate); for $k_z = 0$, it becomes the well-known elliptic-cylinder approximation (2.3); in intermediate cases it reproduces satisfactorily (with maximum error $\lesssim 30\%$) the demagnetizing-factor values given in Ref. 14.

On substituting (3.4) in (1.8), we get a system of nonlinear partial differential equations describing the structure of a domain wall in our simplified model. It is easy to show by direct substitution that at the point d_0, when $N^x = N^y$, the exact solution of this system, satisfying the boundary conditions, has the form

$$M_x = M \operatorname{sch} \sigma x \cos qz, \quad M_y = M \operatorname{sch} \sigma x \sin qz, \quad M_z = M \operatorname{th} \sigma x, \tag{3.6}$$

where the characteristic parameter σ of the domain wall is determined by the expression

$$\sigma^2 = q^2 + (\beta + N)/\alpha, \quad N = N_1^x = N_1^y. \tag{3.7}$$

Thus when $d = d_0$, the magnetization distribution in the domain wall along the z axis has a simple sinusoidal form. Physically, this result is understandable, since when $d = d_0$ in our model, there is complete magnetic symmetry in the xy plane for magnetization in the domain wall: the same value of the energy of magnetic dipole interaction (because $N_1^x = N_1^y$) and the same value of the anisotropy energy for any orientation of the vector $\mathbf{M}$ in the xy plane.

The relation (3.7) connects σ with q and with the physical parameters of the magnetic film. Because of the simplified nature of the model, a second relation, which should determine q, is missing, and the period $L = \pi/q$ remains undetermined.

On departure of d from d_0, the symmetry in the xy plane is destroyed, and other harmonics appear in the magnetization distribution. Thus an approximate estimate shows that at small $\Delta d = d - d_0$, the solution should have the following structure:

$$\begin{aligned} M_x &\approx M \operatorname{sch} \sigma x [(1-a) \cos qz + a \cos 3qz], \\ M_y &\approx M \operatorname{sch} \sigma x [(1+a) \sin qz + a \sin 3qz], \end{aligned} \tag{3.8}$$

where $a \sim \Delta d/d$. The form of this solution already significantly recalls Fig. 4: the projection M_x forms the walls, while the projection M_y forms the domains of internal structure.

We shall consider the dynamics of internal structure at the single point $d = d_0$. On substituting (3.6) in (1.9), neglecting damping, and introducing the cylindrical projections

$$m^\pm = m_x \pm i m_y, \quad h^\pm = h_x \pm i h_y, \tag{3.9}$$

we get a dynamic system of three equations in the form

$$\begin{gathered} \pm\varepsilon m^\pm + t[\nabla^2 m^\pm + (2\sigma^2 s^2 - \beta/\alpha) m^\pm + \alpha^{-1}(h_m^\pm + h^\pm)] \\ - s[\nabla^2 m_z + 2\sigma^2 s^2 m_z + \alpha^{-1} h_z^m] e^{\pm i\varphi} = 0, \\ s(m^+ e^{-i\varphi} + m^- e^{-i\varphi}) + 2t m_z = 0, \end{gathered} \tag{3.10}$$

where the following notation has been introduced:

$$\begin{gathered} s \equiv \operatorname{sch} \sigma x, \quad t \equiv \operatorname{th} \sigma x, \\ \varepsilon = \omega/\alpha g M, \quad \varphi = qz. \end{gathered} \tag{3.11}$$

On representing $m_j(s, z)$ as a Fourier series of the form (3.3) and h_j^m in the form (3.4), substituting in (3.10), and equating to zero the coefficient of each harmonic, we obtain an infinite chain of ordinary differential equations. Cutting off the chain at terms corresponding to $n = 2$, we have

$$\begin{gathered} (\pm\varepsilon + t\mathscr{L}_0) u_0^\pm - s\mathscr{L}_z m_1^\pm = 0, \quad (\mp\varepsilon + t\mathscr{L}_2) u_2^\pm - s\mathscr{L}_z m_1^\pm = 0, \\ (\pm\varepsilon + t\mathscr{L}_2) v_2^\pm = 0, \quad s(u_0^\pm + u_2^\pm) + 2t m_1^\pm = 0, \end{gathered} \tag{3.12}$$

where

$$\mathscr{L}_i = d^2/dx^2 + 2\sigma s^2 - \sigma_i^2.$$

The functions that occur in these equations are connected with $m^\pm$ and m_z by the relations

$$\begin{gathered} m^\pm = u_0^\pm + u_2^\mp e^{\pm 2i\varphi} + v_2^\pm e^{\mp 2i\varphi}, \\ m_z = m_1^+ e^{-i\varphi} + m_1^- e^{+i\varphi}. \end{gathered} \tag{3.13}$$

The quantities σ_i have the form

$$\begin{gathered} \sigma_0^2 = (\beta + N_0)/\alpha, \quad \sigma_2^2 = (2q)^2 + (\beta + N_2)/\alpha, \\ \sigma_z^2 = q^2 + N_1^z/\alpha, \quad N_0 = N_0^x = N_0^y, \quad N_2 = N_2^x = N_2^y. \end{gathered} \tag{3.14}$$

We solve for $m_1^\pm$ in the last equation (3.12) and substitute it in first and second. Introducing the new functions

$$t\mu^\pm = u_0^\pm + u_2^\pm, \quad \nu^\pm = u_0^\pm - u_2^\pm, \tag{3.15}$$

we get a system for the components due to $h^\pm$, in the form

$$\begin{gathered} \nu^{\pm\prime\prime} + [\sigma^2(2s^2-1) - q^2]\nu^\pm + (ct \pm \varepsilon)\mu^\pm = -h^\pm/\alpha, \\ \mu^{\pm\prime\prime} + [\sigma^2(2s^2-1) - q^2 + s^2(q^2 - \sigma_z^2)]\mu^\pm + (ct \pm \varepsilon)\nu^\pm = -th^\pm/\alpha, \end{gathered} \tag{3.16}$$

where $c = (\sigma_2^2 - \sigma_0^2)/2$. This system of equations describes the resonance of oscillations of "domain walls" of internal structure. This type of "domain wall" displacement leads to a bias magnetization, periodic in time, of the main domain wall in the xy plane. For $\varepsilon = 0$, the system (3.16) describes the initial linear section of the process of magnetization of a domain wall by a static magnetic field h.

Setting $h^\pm = 0$, we consider the problem of characteristic oscillations. When $d = d_0$ and $k_z = nq/\pi \ll k_x, k_y$, we have from (3.5)

$$\begin{gathered} N_n^x = N_n^y \approx 2\pi - nqd_0, \quad N_n^z \approx 2nqd_0, \\ \sigma_z^2 \approx 2d_0 q/\alpha, \quad c \approx -d_0 q/\alpha. \end{gathered} \tag{3.17}$$

For $q \to 0$, the homogeneous equations corresponding to the system (3.16) reduce to degenerate ($k = 1$, where k is the modulus of the elliptic integral) Lamé equations. The solutions of these equations satisfying the boundary conditions of the problem have the form

$$\nu = a \operatorname{sch} \sigma x, \quad \mu = b \operatorname{sch} \sigma x. \tag{3.18}$$

For $q \neq 0$, equations (3.16) differ from Lamé equations by the parameter $q/\sigma \ll 1$. Now a and b are no longer constants but are functions of x. Approximate values of them are conveniently sought in the form of power series in $(\tanh \sigma x)^n$ or $(\sigma x)^n$. In both cases one gets the same first approximation (corresponding to $n=0$) and the same order of magnitude for the coefficients of the subsequent terms. Restricting ourselves to the first two terms of the series, we obtain two characteristic frequencies of the system: a low-frequency $\varepsilon_1 = \sigma_z q$ and a high-frequency $\varepsilon_2 \approx 2\sigma^2$. The high-frequency oscillation ε_2 does not disappear even when $q=0$ and corresponds to oscillations of the domain-wall thickness. The oscillation ε_1, proportional to q, describes resonance of domain-walls of internal structure. If we use the expansions

$$a = 2\sum_{n=0}^{\infty} a_n (\sigma x)^n, \quad b = 2\sum_{n=0}^{\infty} b_n (\sigma x)^n, \tag{3.19}$$

the characteristic oscillation at frequency ε (with allowance for only the first two terms of each series) has the form

$$\begin{aligned} m_x &= 2 \operatorname{sch} \sigma x (a_0 \sin^2 qz + b_0 \operatorname{th} \sigma x \cos^2 qz) \\ &\quad - \frac{d_0}{L_0} \sigma x \operatorname{sch} \sigma x (a_0 \operatorname{th} \sigma x \cos^2 qz + b_0 \sin^2 qz), \\ m_y &= \operatorname{sch} \sigma x \sin 2qz \left[-a_0 + b_0 \operatorname{th} \sigma x - \frac{d_0}{L_0} \sigma x (a \operatorname{th} \sigma x - b) \right], \\ m_z &= \operatorname{sch}^2 \sigma x \cos qz \left(-b_0 + \frac{d_0}{2L_0} a_0 \sigma x \right). \end{aligned} \tag{3.20}$$

Here the coefficients a_1 and b_1 have already been expressed in terms of a and b from the relations between the amplitudes for $\varepsilon = \varepsilon_1$; from these same relations we have

$$\frac{b_0}{a_0} = \frac{q}{\sigma_z} \approx \left(\frac{\alpha q}{2d_0} \right)^{1/2} = \frac{\pi}{5} \left(\frac{d_0}{L_0} \right)^{1/2}. \tag{3.21}$$

The first terms in the expressions for m_x and m_y, proportional to $\operatorname{sch} \sigma x$, describe the displacement of the domain walls of internal structure. This motion occurs at frequency ε_1 with amplitude a_0. The following two terms in the expressions for m_x and m_y and the first term in m_z describe bending oscillations of the main domain wall. In fact, their form with respect to x corresponds to the expression (2.9), describing displacement of a domain wall; but now there is also a periodic dependence on z, which in sum leads to a bending motion of the domain wall. The amplitude of this motion is smaller than a_0 by a factor $(d_0/L_0)^{1/2}$.

The subsequent terms, with still smaller amplitude $(a_0 d_0/L_0)$, correspond to oscillations of the thickness of the main domain wall [this can be shown by comparing their structure with the expansion in $\Delta\sigma x$ of functions of the form $\operatorname{sech}(\sigma + \Delta\sigma)x$, where $\Delta\sigma$ is a correction due to change of thickness of the domain wall]. Finally, the last terms in the expressions for m_x and m_y are already small corrections to the bending oscillations. Thus in actuality, the oscillation of the spins in a domain wall with characteristic frequency ε_1 has a very complicated structure; but when $d_0/L_0 \ll 1$, the basis of this oscillation consists in displacement of domain walls of internal structure.

If we seek an approximate solution of the system (3.16) by expanding $a(x)$ and $b(x)$ as series in powers of $\tanh \sigma x$, the expressions obtained differ from (3.20) by replacement of σx by $\tanh \sigma x$, and by a coefficient $\frac{1}{2}$ in front of the small terms proportional to d_0/L_0. But the same form and frequency of the basic motion, oscillation of domain walls of internal structure, are obtained by use of any expansion.

On substituting into the expression for the characteristic frequency of these oscillations the values of σ_z, q, and α, expressed in terms of d_0 for $\beta \ll 2\pi$ from the relation (2.7), we have

$$\omega_0 \approx 4gM (d_0/L_0)^{1/2}. \tag{3.22}$$

For $g = 1.7 \cdot 10^7$ s^{-1}/Oe, $M \approx 10^3$ G, $d_0 \approx 10^{-5}$ cm, and $L_0 \sim 10^{-3} - 10^{-4}$ cm, we have

$$f_0 = \omega_0/2\pi \sim 10^7 - 3 \cdot 10^8 \text{ Hz},$$

which corresponds to the frequency range of the experiment.[10]

CONCLUSION

The theory developed here for linear resonance of a domain wall in a thin magnetic film is based on an approximate calculation of the magnetic dipole interaction, in the effective-demagnetizing-factor model. The results that follow from this model for the structure of a domain wall—both uniform and periodic—are a crude representation of the actual complicated situation; the model conveys only the main structural features of a domain wall and does not describe such details as the broad "wings" that extend beyond the main maximum in the magnetization distribution, proportional to $\operatorname{sech} \sigma x$, as "binding," etc. (see Ref. 5). But it enables us to solve the dynamic problem over a wide range of thicknesses, both for uniform and for periodic domain walls.

For a uniform wall, a general equation of motion (2.10) for the wall center was obtained, and it was shown how the resonance frequency and the damping parameter depend on the film thickness. It was established that the frequency of this oscillation vanishes when $d = d_0$; that is, this oscillation is a "soft mode" of the structural phase transition from a Néel wall to a Bloch wall.

For the periodic domain wall, the existence of resonance of a new type was established: resonance of "domain walls" of internal structure (RIS) of a periodic wall. [This new resonance frequency exists in a periodic wall along with the old one, due to oscillation of the wall as a whole; the latter frequency, for a periodic wall, differs little from the corresponding expression (2.10) for a uniform wall, if $d/L \ll 1$.] Formulas (3.2), which show the trend of the dependence of the RIS frequency on the film thickness for the limiting cases $d \ll d_0$ and $d \gg d_0$, and formula (3.22) for the value of the frequency at $d = d_0$ enable us to assert that it is this resonance that was first observed in Ref. 10 by an in-direct method, as resonance of the increase of the static susceptibility in the presence of a weak HF field.

There is still another argument in favor of this supposition. In Ref. 10 the HF field was applied in the plane of the film at various angles to the direction of the domain wall. It was found that the maximum of the effect corresponds to orientation of **h** along the x axis and the minimum to orientation along the z axis; as is evident from (3.16), precisely this result should be characteristic of resonance of the internal structure (for resonance of the whole wall, the direct opposite).

Thus the effect observed in Ref. 10 is apparently due to the resonance, described here theoretically, of the internal structure of a periodic domain wall. This resonance should be observed whenever the domain wall has a periodic internal structure; and this is characteristic also of certain types of cylindrical domains.

It would be interesting to investigate the RIS phenomenon experimentally in various situations, and principally to elucidate the question: does an abrupt increase of the static susceptibility always result from this resonance? This last effect might find application in practical devices that use thin magnetic films.

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[1)]The content of this paper was reported briefly in Ref. 1; see also the preprint, Ref. 2.

[1]V. A. Ignatchenko and P. D. Kim, Abstracts of Conference of CMEA (Council for Mutual Economic Assistance, Sovet ėlpm, ocjeslpo vzao, ppomoschchi, SÉV) Countries on Physics of Magnetic Materials, Jaszowiec, Poland, 1980 (Wroclaw, 1980), p. IV–17.

[2]V. A. Ignatchenko and P. D. Kim, Rezonans domennoĭ stenkĭ v tonkikh magnitnykh plenkakh (Domain-Wall Resonance in Thin Magnetic Films), Preprint IFSO (Institute of Physics, Siberian Branch)-115F, Krasnoyarsk, 1979.

[3]W. Döring, Z. Naturforsch. **3a**, 373 (1948) [Russ. transl. in: Ferromagnitnyĭ rezonans (Ferromagnetic Resonance), ed. S. V. Vonsovskiĭ, M.:IIL, 1951, p. 312.

[4]L. D. Landau and E. M. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935) (Reprinted in L. D. Landau, Collected Works, Pergamon, 1965, No. 18 and in D. ter Haar, Men of Physics: L. D. Landau, Vol. 1, Pergamon, 1965, p. 178).

[5]A. Hubert, Theorie der Domänenwände in geordneten Medien, Springer, 1974 (Russ. transl., Mir, 1977).

[6]L. R. Walker, quoted by J. F. Dillon, in: Magnetism (ed. G. T. Rado and H. Suhl), Vol. 3, Academic Press, New York, 1963, p. 450.

[7]J. C. Slonczewski, J. Appl. Phys. **44**, 1759 (1973); **45**, 2705 (1974).

[8]J. M. Winter, Phys. Rev. **124**, 452 (1961).

[9]M. M. Farztdinov and E. A. Turov, Fiz. Met. Metalloved. **30**, 1064 (1970) [Phys. Met. Metallogr. **30**, No. 5, 170 (1970)].

[10]P. D. Kim, D. M. Rodichev, and I. A. Safonov, Izv. Akad. Nauk SSSR, Ser. Fiz. **36**, 1499 (1972) [Bull. Acad. Sci. USSR, Phys. Ser. **36**, 1329 (1972)]; P. D. Kim, D. M. Rodichev, I. A. Safanov, and V. I. Zlobin, Proc. Int. Conf. on Magnetism ICM-73, Vol. 4, p. 190.

[11]V. A. Ignatchenko and Yu. V. Zakharov, Zh. Eksp. Teor. Fiz. **49**, 599 (1965) [Sov. Phys. JETP **22**, 416 (1966)].

[12]A. A. Thiele, Phys. Rev. **B14**, 3130 (1976).

[13]V. A. Ignatchenko and Yu. V. Zakharov, Zh. Eksp. Teor. Fiz. **43**, 459 (1962) [Sov. Phys. JETP **16**, 329 (1963)].

[14]J. A. Osborn, Phys. Rev. **67**, 351 (1945).

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