# Contribution to the theory of the muon method of spin-glass investigation

A. L. Mikaelyan and V. P. Smilga

*Moscow Engineering Physics Institute*
(Submitted 9 October 1980; resubmitted 12 February 1981)
Zh. Eksp. Teor. Fiz. **81**, 2324–2333 (June 1981)

A theory is developed of the behavior of the polarization of positive muons in spin glasses at low temperatures. It follows from the results that the muon method permits, for the first time ever, to obtain a rigorous verification of the main hypothesis of the theory of S. F. Edwards and P. W. Anderson [J. Phys. F **5**, 965 (1975)], to investigate phase transitions and internal magnetic fields in spin glasses, as well as to determine whether the muons are captured by the impurity paramagnetic centers or are stopped in the crystal lattice at randomly distributed points.

PACS numbers: 61.40.Df

1. Spin glasses have been attracting in recent years an ever increasing interest, as is attested by the large number of experimental and theoretical studies. One of the central problems of spin glasses is the verification of the Edwards-Anderson theory[1] and the study of the phase transition. The principal hypothesis of the Edwards-Anderson theory has not been directly confirmed so far. Various experiments performed with the aid of the muon method[2,3] indicates only that a phase transition takes place in spin glasses at a certain temperature, but there is little said concerning its character.

It is known that the main hypothesis of the Edwards-Anderson model is that above the phase transition temperature $T_{SG}$ the spin glass behaves like an ordinary paramagnet, and at $T \ll T_{SG}$ the spins of the paramagnetic impurity centers are immobile frozen and are randomly oriented in space. The degree of freezing is characterized according to Edwards and Anderson by the known order parameter (temporal correlator)

$$\varkappa = \alpha \langle \mathbf{S}(t\to\infty)\mathbf{S}(0)\rangle;$$

here **S** is the spin of an individual magnetic center and $\alpha$ is a normalization constant ($\varkappa_{max}=1$). At low temperatures

$$\varkappa \approx 1-\left(\frac{8}{3\pi}\right)^{1/2}\frac{T}{T_{SG}}$$

and since the characteristic values of $T_{SG}$ lie near 10 K, there exists a sufficiently large temperature region in which the spins are practically completely immobile. With increasing temperature, the parameter $\varkappa$ decreases monotonically, and vanishes at $T=T_{SG}$; near the transition point ($T \leq T_{SG}$)

$$\varkappa \approx -{}^1/_2[1-(T_{SG}/T)^2].$$

We confine ourselves in this paper to the region $T \ll T_{SG}$, and show that an analysis of the behavior of the muon polarization at these temperatures makes it possible to check on the Edwards-Anderson model. The depolarization of the muons at $T \sim T_{SG}$ will be the subject of a separate paper.

2. We note first that in alloys, as shown by experiment (see, e.g., Ref. 4), and as follows from obvious considerations, muon diffusion is strongly suppressed. In any case, the muons do not diffuse at sufficiently low temperatures. We shall consider therefore hereafter the case of mobile muons, recognizing that to verify the Edwards-Anderson model the experiment can be performed at helium temperatures.

If the spins of the magnetic centers are immobile below the phase-transition point, the muons are in a static magnetic field. Consequently, the frozen-spin hypothesis can be regarded as proved if the magnetic field at the muon is shown to be static.[1)] As indicated in Refs. 5 and 6, the character of the time dependence of the local field at the muon is simplest and easiest to ascertain in experiments with a zero external field. We begin our discussion with an analysis of this case.

3. We assume in this section that the muons can be stopped in arbitrary interstices of the crystal lattice. A variant wherein the muon is captured by an impurity magnetic center will be considered in the next section. It is known that experimental investigations by the muon method yield the polarization $\mathbf{P}(t)$ averaged over the muon ensemble at a given instant of time. In a theoretical calculation of the polarization it must be borne in mind that in spin glasses the muons are in a stochastic magnetic field produced by all the paramagnetic centers. The random character of the field is determined both by the random orientation of the spins and by the random spatial distribution of the centers relative to the muon stopping point.

To calculate the observable polarization of an ensemble of muons we use Markov's method which is standard in such cases (see, e.g., Ref. 7). The same method was used to solve a related problem—the theoretical calculation of the line shape of an NMR signal in a magnetically dilute system.[8] The results obtained there were used by the cited authors of the experiments were muons.[2,3] It must be emphasized, however, that the NMR method requires by its very nature the use of a strong external magnetic field. In addition, it was assumed in Ref. 8 that the quantization axes of the magnetic-impurity spins are parallel to the external field. Therefore the results of Ref. 8 are not applicable at all in a zero external field, and in the case of a strong field, as will be seen from Sec. 5, quantitative differences are caused by the fact that the paramagnetic-impurity spins are randomly oriented.

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The experimentally observed polarization of the muon ensemble is determined by the formula

$$P_\alpha(t)=M_{\alpha\beta}(t)P_\beta(0) \tag{1}$$

where

$$M_{\alpha\beta}(t)=\left\langle \frac{\omega_\alpha\omega_\beta}{\omega^2}+\left(\delta_{\alpha\beta}-\frac{\omega_\alpha\omega_\beta}{\omega^2}\right)\cos\omega t+e_{\alpha\beta\gamma}\frac{\omega_\gamma}{\omega}\sin\omega t\right\rangle_W. \tag{2}$$

Here $\omega=8.52\times10^4\mathbf{B}\,\mathrm{sec}^{-1}$ is the vector of the muon spin precession frequency in a magnetic field $\mathbf{B}$ (in gausses) the averaging is over the distribution function $W(\omega)$ of the local field. If the field distribution has cubic symmetry or is isotropic, formula (2) takes the simple form

$$M_{\alpha\beta}(t)={}^1/_3(1+2\langle\cos\omega t\rangle_W)\delta_{\alpha\beta}. \tag{3}$$

Obviously, it is precisely this variant which is realized in a zero external field in spin glasses, when the quantization axes of the individual spins are randomly oriented, and the crystal lattice has cubic symmetry.

Following Walstedt and Walker,[8] we write $W(\omega)$ in the form

$$W(\boldsymbol{\omega})=\int\delta\left(\boldsymbol{\omega}-\sum_j\boldsymbol{\omega}_j\right)\prod_j[(1-c)\delta(\boldsymbol{\omega}_j)+cw_j(\boldsymbol{\omega}_j)]d\boldsymbol{\omega}_j. \tag{4}$$

The summation here is over all possible positions of the magnetic centers, $c$ is the fraction of the occupied positions, and $w_j(\omega_j)$ is the distribution density of the field produced at the muon by a center is position $j$. We assume that all possible positions are crystallographically equivalent. A generalization of (4) to include the case when the paramagnetic centers have different probabilities of being in crystallographically different sites (interstices) is trivial.

We obtain now the Fourier transform of the field distribution

$$A(\mathbf{q})=\int e^{i\boldsymbol{\omega}\mathbf{q}}W(\boldsymbol{\omega})\,d\boldsymbol{\omega}.$$

We have

$$A(\mathbf{q})=\prod_j\left[1-c+c\int\exp(i\boldsymbol{\omega}_j\mathbf{q})\,w_j(\boldsymbol{\omega}_j)\,d\boldsymbol{\omega}_j\right]$$
$$=\exp\left[\sum_j\ln(1-c+c\langle\exp(i\boldsymbol{\omega}_j\mathbf{q})\rangle_{w_j})\right]. \tag{5}$$

At low impurity-center concentrations ($c\ll1$)

$$A(\mathbf{q})\approx\exp\left[-c\sum_j(1-\langle\exp(i\boldsymbol{\omega}_j\mathbf{q})\rangle_{w_j})\right]. \tag{6}$$

The fields $\omega_j$ of an individual paramagnetic center consists, as is well known, of the field due to the interaction via the Rudeman-Kittel (RK) mechanism and of the dipolar field:

$$\boldsymbol{\omega}_j=\{A\cos(2k_Fr_j+\varphi_0)\mathbf{i}_j+B[\mathbf{i}_j-3\mathbf{a}_j(\mathbf{a}_j\mathbf{i}_j)]\}\frac{m_j}{r_j^3}. \tag{7}$$

Here $A$ and $B$ are coefficients that determine the contributions of the RK fields and of the dipolar fields (with $B=\hbar\gamma_\mu\gamma_S$, where $\gamma_\mu$ and $\gamma_S$ are the gyromagnetic ratios for the muon and the magnetic center); $\hbar k_F$ is the Fermi momentum; $\varphi_0$ is the phase of the RK interaction; $\mathbf{r}_j$ is the vector connecting the muon and the magnetic center in position $j$, $\mathbf{a}_j=\mathbf{r}_j/r_j$; $i_j$ is a unit vector in the direction of the quantization axis of the spin of the magnetic center, and $m_j$ is its projection on this axis.

The actual averaging in (5) and (6) is thus over the possible directions of the quantization axes (of the vector $\mathbf{i}_j$) and over the spin projections on these axes.

When summing in (6) we recognize, following Ref. 8, that $\omega_j$ contains, besides the factor $r_j^{-3}$ that varies relatively slowly as a function of the distance from the muon, also a rapidly oscillating term that is proportional to $\cos(2k_Fr_j+\varphi_0)$ ($k_F\approx10^8\ \mathrm{cm}^{-1}$). Therefore in the summation over the positions $\mathbf{r}_j$ belonging to one period of the RK field, the factor $r_j^{-3}$ can be regarded as approximately constant. Under these conditions, the variables in fact separate and

$$\sum_{R-\pi/2k_F<r_j<R+\pi/2k_F}f[r_j^{-3},\cos(2k_Fr_j+\varphi_0)]$$
$$\approx\sum_{R-\pi/2k_F<r_j<R+\pi/2k_F}\int_0^{2\pi}\frac{d\varphi}{2\pi}f(r_j^{-3},\cos\varphi).$$

This equality holds better the larger $R$.

Since we are considering the case when $c\ll1$, the positions close to the muons obviously make a small contribution, so that this procedure can be extended to include all $R$. With this taken into account, Eq. (6) can be rewritten in the form

$$A(\mathbf{q})\approx\exp\left\{-c\sum_j\int_0^{2\pi}\frac{d\varphi}{2\pi}[1-\langle e^{i\tilde{\boldsymbol{\omega}}(\varphi)\mathbf{q}}\rangle_{\mathbf{i}_j,m_j}]\right\}, \tag{8}$$

where

$$\tilde{\boldsymbol{\omega}}_j(\varphi)=\{A\cos\varphi\,\mathbf{i}_j+B[\mathbf{i}_j-3\mathbf{a}_j(\mathbf{a}_j\mathbf{i}_j)]\}m_j/r_j^3. \tag{9}$$

Next, when summing over $j$ in (8), we can approximately go over to integration with respect to $\mathbf{r}$:

$$\sum_j\to\rho\int r^2drda,$$

where $\rho^{-1}$ is the volume per possible position of the impurity center in the lattice. Obviously, the imaginary part of the resultant integral vanishes when averaged over the directions of the vectors $\mathbf{i}$. The real part of the integral with respect to $r$ can be easily calculated by using the identity

$$\mathrm{Re}\int_0^\infty\left[1-\exp\left(\frac{i\alpha}{r^3}\right)\right]r^2dr=\frac{\pi|\alpha|}{6}.$$

As a result we obtain

$$A(\mathbf{q})=e^{-\Lambda q}, \tag{10}$$

where

$$\Lambda=\frac{\pi\rho c}{3}\langle|m|\rangle\int_0^\pi d\varphi\int_0^1dx[(A\cos\varphi+B)^2+3x^2B(B-2A\cos\varphi)]^{1/2}. \tag{11}$$

As seen from (11), the depolarization rate $\Lambda$ is directly proportional to the bulk density $\rho c$ of the paramagnetic centers and to the average modulus of the projection of the spin center on its quantization axis. Obviously, $\langle|m|\rangle$ increases with increasing $\varkappa$. In fact, in the considered low-temperature case we have $\langle|m|\rangle\approx|\langle m\rangle|\approx S$.

The integral in (11) can be calculated in elementary fashion in two limiting cases: that of a pure dipolar field ($A \ll B$)

$$\Lambda^{dip} = {}^1/_3\pi^2\rho c\langle|m|\rangle|B|[1+{}^1/_2\cdot 3^{-1/2}\ln(2+3^{-1/2})]$$
$$\approx 4.19\hbar\gamma_\mu\gamma_S\rho c\langle|m|\rangle \tag{12}$$

and in the opposite case of a pure $RK$ field ($A \gg B$)

$$\Lambda^{RK} = {}^2/_3\pi\rho c\langle|m|\rangle|A|. \tag{13}$$

Estimates show that the characteristic values of the depolarization rate should be $10^6$ to $10^7$ sec$^{-1}$.

Using (10), we easily obtain

$$\langle\cos\omega t\rangle_W = \int d\omega\int\frac{d\mathbf{q}}{(2\pi)^3}\cos\omega t e^{-\Lambda q}e^{-i\omega q} = (1-\Lambda t)e^{-\Lambda t}. \tag{14}$$

Substituting (14) in (3), we get ultimately

$$\mathbf{P}(t) = {}^1/_3[1+2(1-\Lambda t)e^{-\Lambda t}]\mathbf{P}(0). \tag{15}$$

A plot of $P(t)$ is shown in Fig. 1. It is seen that over long times the polarization tends to the value $P(0)/3$. At $t=2\Lambda^{-1}$ the polarization goes through a minimum equal to

$$P_{min} = {}^1/_3(1-2e^{-2})P(0) \approx 0.24P(0).$$

A most important fact is that the minimum does not depend on the density of the magnetic centers, whereas the instant of time at which the polarization is a minimum is inversely proportional to the density [see Eq. (1)].

We estimate now the limits of the validity of (25). As already noted, it was derived with the summation in (6) replaced by integration over all of space. Actually, however, it is necessary to integrate over the region $r > r_0$, where $r_0$ is the minimum possible distance from the muon to the magnetic center. The main contribution to an integral of the form

$$\int_0^\infty [1-\exp(i\alpha/r^3)]r^2 dr$$

is made by the region $|\alpha|r^{-3} \lesssim 1$. Therefore the extension of the integration over all distances is legitimate if the condition $|\alpha|r_0^{-3} \gtrsim 1$ is satisfied. Assuming $\alpha \approx \gamma_\mu\mu_B q$, we find that expression (10) for the Fourier transform of the distribution density of the local field is valid at $q \gtrsim r_0^3\gamma_\mu^{-1}\mu_B^{-1}$. Accordingly, Eq. (15) describes correctly the time dependence of the polarization when the similar condition $t \gtrsim r_0^3\gamma_\mu^{-1}\mu_B^{-1}$ is satisfied. The characteristic values of $r_0$ amount to 1–2 Å. We conclude thus that Eq. (15) is valid at times $t \gtrsim 10^{-8}-10^{-9}$ sec, i.e., practically in the entire region accessible to observation.

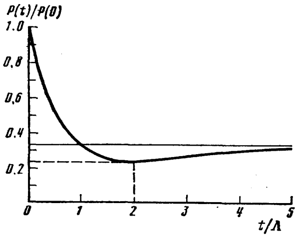


FIG. 1. Behavior of the polarization when the muon stopping points are uniformly distributed.

At short times $t \lesssim 10^{-8}$ sec the $\mathbf{P}(t)$ dependence can be determined by expanding (3) in powers of the time:

$$\mathbf{P}(t) \approx (1-{}^1/_3\langle\omega^2\rangle t^2)P(0) = (1-\sigma_0^2t^2)\mathbf{P}(0), \tag{16}$$

where

$$\sigma_0^2 = {}^1/_3 c\sum_j \omega_j^2. \tag{17}$$

in crystals with cubic symmetry

$$\sigma_0^2 = \frac{1}{3}c\sum_j[A^2\cos^2(2k_F r_j+\varphi_0)+2B^2]\frac{\langle m^2\rangle}{r_j^6}, \tag{18}$$

where $\langle m^2\rangle \approx \varkappa S^2 \approx S^2$.

If there is no phase transition into the frozen state, the sample remains an ordinary paramagnet all the way to $T \approx 0$. The dependence of the muon polarization on the time can then have a qualitatively different character. In fact, in paramagnets the muons turn out to be located in a field that fluctuates in time. In order of magnitude, the characteristic local field at the muon is the same as the field at any impurity center. The correlation time for the local fields in the sample is $\tau_c \approx \omega_S^{-1}$ ($\omega_S$ is the average precession frequency of the impurity-center spins in the local field). It is obvious that the muon spin precession frequency $\omega_\mu$ in the very same fields is smaller by two or three orders than $\omega_S$. In paramagnets the frequency $\omega_S$ is practically independent of temperature and is determined entirely by the averaging modulus of the local field. Trivial estimates show that at a density $c=10^{-2}$ we have $\tau_c \approx 10^{-8}-10^{-9}$ sec. Since $\omega_\mu\tau_c \ll 1$, the known condition that the correlation time be small is satisfied. Then, as is well known,

$$\mathbf{P}(t) = e^{-\Lambda' t}\mathbf{P}(0), \quad \Lambda' \approx {}^2/_3\langle\omega_\mu^2\rangle\tau_c \approx 2\sigma_0^2\tau_c$$

(see, e.g., Ref. 9, p. 13 of original, p. 684 of the translation). Thus, the muon polarization in paramagnets decreases monotonically to zero.

According to the Edwards-Anderson theory,[1] the order parameter $\varkappa$ in spin glasses is equal to unity only at absolute zero. Therefore at any finite temperature a fluctuating field is present in addition to the static magnetic field. At low temperature, the amplitude of the fluctuating field component is small; its mean square is given, obviously, by

$$\langle\omega_v^2\rangle \approx (1-\varkappa)\langle\omega_\mu^2\rangle = 3(1-\varkappa)\sigma_0^2.$$

The interaction of the muon spin with the fluctuating part of the field leads to the appearance of another depolarization mechanism. As a result, Eq. (15) takes the form

$$\mathbf{P}(t) = {}^1/_3[1+2(1-\Lambda t)e^{-\Lambda t}]\exp(-\Lambda_v t)\mathbf{P}(0),$$
$$\Lambda_v \approx 2(1-\varkappa)\sigma_0^2\tau_c. \tag{19}$$

This formula is valid if $1-\varkappa \ll 1$. It is clear that the additional factor $\exp(-\Lambda_c t)$ that appears in (19) is only a small correction to (15) at times of the order of the muon lifetime. In fact, we have

$$\Lambda_v \approx {}^2/_3(1-\varkappa)\langle\omega_\mu^2\rangle\tau_c \approx (1-\varkappa)\Lambda\gamma_\mu\gamma_S^{-1} \lesssim 10^{-3}\Lambda \sim 10^3-10^4 \text{ sec}^{-1}.$$

Uemura *et al.*[10] have recently reported an investigation of the depolarization of positive muons in AuFe and CuMn spin glasses in a zero external field. The experiment was performed at temperatures $T > \frac{1}{2} T_{SG}$. The parameter $\varkappa$ is in this case not close to unity, so that their results cannot be compared with our theory. In the theoretical interpretation of the behavior of the muon polarization, Uemura *et al.*[10] propose that in spin glass the local field fluctuates as one whole with a frequency $\nu = \tau_c^{-1}$, the only consequence of a temperature change is a change in $\nu$. This model does not agree with either the Edwards-Anderson theory or with the behavior of the paramagnetic states of the system. In spin glass, especially near the phase-transition point, the decisive factor should be the change of the amplitude of the fluctuating part of the field and, as indicated above, this amplitude should vanish as $T \to 0$. In Ref. 10 is given, in addition, a formula for the $P(t)$ dependence in the static case (with a reference to a private communication from Kubol), which agrees with our Eq. (15) but in which the parameter $\Lambda$ is not calculated.

Equations (11), (15), (16), (18), and (19) provide a complete description of the low-temperature behavior of the polarization of positive muons in spin glasses in the absence of an external field and under the condition that the muons are not captured by impurity magnetic centers.

4. A number of recent experiments (see, e.g., Ref. 4) have shown that in many metals muons should be captured by impurities. One cannot exclude this possibility in spin glasses, too. The field at a muon captured by a magnetic impurity center consists of two parts, $\omega = \omega_1 + \omega_2$, where $\omega_1$ is the field from the center closest to the muon, and $\omega_2$ is the field of all the remaining centers, which are randomly distributed in the sample. At low impurity density, the distributions $W_1(\omega_1)$ and $W_2(\omega_2)$ of the fields are practically independent. The distribution of the summary field is obviously given by

$$W(\omega) = \int \delta(\omega - \omega_1 - \omega_2) W_1(\omega_1) W_2(\omega_2) d\omega_1 d\omega_2. \tag{20}$$

This yields for the Fourier transform

$$A(q) = \int e^{i\omega q} W(\omega) d\omega = \left[\int e^{i\omega_1 q} W_1(\omega_1) d\omega_1\right] \cdot \left[\int e^{i\omega_2 q} W_2(\omega_2) d\omega_2\right] = A_1(q) A_2(q). \tag{21}$$

The distribution of the field due to the randomly located centers was investigated in Sec. 3. It was shown there that $A_2(q) \approx \exp(-\Lambda q)$, where $\Lambda$ is given by (11). In the calculations that follow we confine ourselves for simplicity to the case when the distribution $W_1(\omega_1)$ [and correspondingly $A_1(q)$] is spherically symmetrical. For a complete determination of the polarization it is then sufficient, according to (3), to calculate $\langle \cos \omega t\rangle_W$. Using (21), the assumption that $A_1(q)$ has spherical symmetry, and the explicit form of $A_2(q)$ it is easy to show that

$$\langle \cos \omega t\rangle_W = e^{-\Lambda t}(\langle \cos \omega t\rangle_{W_1} - \Lambda t \langle \sin(\omega t)/\omega t\rangle_{W_1}). \tag{22}$$

It is obvious furthermore that the overwhelming contribution to the polarization is made by the center that captured the muon. Indeed, as seen from (11), the

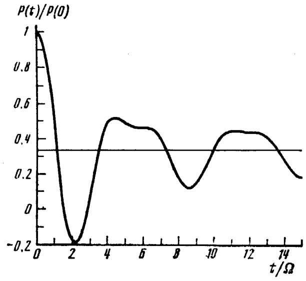


FIG. 2. Behavior of the polarization when muons are captured by impurity parmagnetic centers (case of a pure dipolar field).

parameter $\Lambda$ contains a smallness of order $c$. The contribution of all the remaining centers can therefore be neglected in first-order approximation, i.e., we can put $\Lambda = 0$ in (22).

We calculate now $\langle \cos \omega t\rangle_W$. We assume that a muon can be captured with equal probability by one of the $n$ positions near an impurity center characterized by the vector $\mathbf{r}_j$, with $r_j = r_0$. Then

$$\langle \cos \omega t\rangle_{W_1} = \frac{1}{n} \sum_{j=1}^{n} \langle \cos[\omega_j(\mathbf{r}_j, \mathbf{i}_j, m_j)t]\rangle_{\mathbf{i}_j, m_j}, \tag{23}$$

where $\omega_j(\mathbf{r}_j, \mathbf{i}_j, m_j)$ is given by (7). We average over the spin projections on the quantization axis under the assumption that the magnetic centers are completely polarized (this corresponds to the case of low temperatures). As a result we get

$$\langle \cos \omega t\rangle_{W_1} = \int_0^1 \cos\left\{\frac{St}{r_0^3}\left[[A\cos(2k_F r_0 + \varphi_0) + B]^2 + 3B[B - 2A\cos(2k_F r_0 + \varphi_0)]x^2\right]^{1/2}\right\} dx, \tag{24}$$

where $S$ is the spin of the magnetic center.

Figure 2 shows by way of illustration the $P(t)$ dependence for the case of a pure dipole field ($A = 0$). The polarization is then determined according to (3) and (24) by the expression

$$\mathbf{P}(t) = \frac{1}{3}\left\{1 + 2\int_0^1 \cos[\Omega t(1 + 3x^2)^{1/2}] dx\right\} \mathbf{P}(0), \tag{25}$$

where

$$\Omega = SBr_0^{-3} = \hbar \gamma_\mu \gamma_S S r_0^{-3}.$$

It is seen from the figure that before it reaches the asymptotic value $P(0)/3$ the polarization $P(t)$ executes many oscillations and even reverses sign. The first minimum $P_{min} \approx -0.19P(0)$ is reached at $\Omega t \approx 2.2$. Thus, besides the quantitative difference (faster damping) there is also a qualitative difference between the behavior of the polarization in the case of muon capture by an impurity and in the case of a uniform distribution of the stopping points. An experimental discrimination between these situation should therefore be easily realized.

5. We consider now the case of a strong external field. It is known that in this case the polarization component perpendicular to the external field is damped

much more rapidly than the parallel component. We assume as usual the external field to be directed along the $z$ axis and introduce the notation $P_+ = P_x + iP_y$. The complex polarization $P_+$ is described by the formula

$$P_+(t) = e^{i\omega_0 t} G(t) P_+(0),$$

where $\omega_0$ is the frequency of the muon spin precession in the external field, and $G(t)$ is the relaxation function.

If the muon stopping points are uniformly distributed, we can calculate $G(t)$ by using the results of Ref. 8, subject to an additional averaging over the directions of the quantization axes of the spins of the magnetic centers. This averaging does not change qualitatively the relaxation function

$$G(t) = e^{-\Lambda t}, \qquad (26)$$

but the numerical value of the depolarization rate $\Lambda$ is different and is determined by Eqs. (11)–(13).

We mark with a prime the value of $\Lambda$ as given in Ref. 8. We then have in the pure dipole and in the pure Ruderman-Kitel cases

$$\Lambda^{dip}/\Lambda'^{dip} \approx 0.9, \quad \Lambda^{RK}/\Lambda^{RK} \approx 0.5. \qquad (27)$$

The region where formula (26) is valid is the same as that of (15), i.e., $t \gtrsim 10^{-8}$ sec. For short times $G(t) \approx 1 - \sigma_1^2 t^2$, with $\sigma_1^2 = \frac{1}{2}\sigma_0^2$, where $\sigma_0^2$ is given by (17).

If the muons are captured by impurities, then we easily obtain, in full analogy with the derivation of (22) and (23),

$$G(t) = e^{-\Lambda t} \left\{ \frac{1}{n} \sum_{j=1}^{n} \langle \exp[i\omega_{jz}(\mathbf{r}_j, \mathbf{i}_j, m_j) t] \rangle_{\mathbf{i}_j, m_j} \right\}, \qquad (28)$$

where the summation is over the positions closest to the paramagnetic centers, and $\omega_{jz}(\mathbf{r}_j, \mathbf{i}_j, m_j)$ is the projection of the field defined by Eq. (7) on the direction of the external field. Averaging under the same assumptions as in the derivation of (23), we obtain ultimately

$$G(t) = e^{-\Lambda t} \int_0^1 dx \sin\left\{ \frac{St}{r_0^3} [[A\cos(2k_F r_0 + \varphi_0) + B]^2 + 3B[B - 2A\cos(2k_F r_0 + \varphi_0)]x^2]^{1/2} \right\} \left\{ \frac{St}{r_0^3} [[A\cos(2k_F r_0 + \varphi_0) + B]^2 + 3B[B - 2A\cos(2k_F r_0 + \varphi_0)]x^2]^{1/2} \right\}^{-1}. \qquad (29)$$

We see therefore that the relaxation function oscillates many times but, in contrast to the zero-field case, it tends to zero at long times.

6. Our results show that the muon methods offer wide prospects for the study of the properties of spin glasses. Most information will be provided by experiments in the absence of an external field. If it is observed that at low temperatures the polarization tends asymptotically to the value $\mathbf{P}(0)/3$, and to zero with rising temperature, then this will be evidence of the pressure of a phase transition and that the spins of the magnetic centers are immobile below the phase transition point. The form of the $\mathbf{P}(t)$ dependences leads to unambiguous conclusions concerning the character of the distribution of the muon stopping points.

1) We assume that the nuclei of the matrix have zero spin. Depolarization by nuclear spins has been thoroughly investigated (see, e.g., Ref. 9) and can be easily taken into account.


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Translated by J. G. Adashko