Crystallization in a strong magnetic field of quasi-two-dimensional electrons at a partially filled Landau level

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The problem of the crystallization of the electrons (vacancies) in the subspace of the states of an arbitrary Landau level is solved exactly. The vibrational spectrum of the lattice does not depend on the level number, but the mean square displacement of an electron from a lattice site is proportional to this number. It is shown that the effect of the neighboring cyclotron levels on the dynamics of the system of electrons (vacancies) of a partially filled Landau level amounts primarily to a relatively large shift in the ground-state energy of the system, and is therefore unimportant in considering the stability of the lattice.

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§1. INTRODUCTION

The problem of the determination of the energy spectrum of a system of two-dimensional electrons in a perpendicular quantizing magnetic field B has recently aroused a great deal of theoretical interest. This model describes approximately the properties of real quasi-two-dimensional objects in a magnetic field (inversion layers on semiconductor surfaces, electrons on a helium-liquid surface, etc.; see Ref. 1). Fukuyama[2] and the present authors[3] have shown that if $\lambda \ll R$ [$\lambda = (c\hbar/eB)^{1/2}$ is the magnetic length and R is the mean electron spacing], then the ground state of the system is the electronic Wigner lattice (WL), and have determined the vibrational spectrum of the lattice. Kuramoto[4] and Fukuyama *et al.*[5] have later investigated the strong-magnetic-field (i.e., $e^2/\lambda \ll \hbar\omega_c$) case, in which the cyclotron Landau levels N are well defined in the system. They have assumed that the number of electrons in the system $N_e \leqslant N_0$ ($N_0 = S/2\pi\lambda^2$ is the number of possible electronic states at the Landau level and S is the area of the sample), so that at sufficiently low temperatures the electrons occupy only the lowest cyclotron level with $N = 0$. It is found[4,5] that under these conditions the WL, which exists when the population of the level is small (i.e., when $N_e/N_0 \ll 1$), goes over into a charge-density wave when the population is increased to $N_e \approx \frac{1}{2}N_0$ and, accordingly, $\lambda \sim R$.

The purpose of the present paper is to investigate another case, often encountered in experiment, in which at $T = 0$ K a few of the lowest Landau levels are completely filled and the next—to be specific, the N-th—level is only partially filled. We shall show that in the case in which the population of the N-th level is small, so that the corresponding minimum dimension of the region of localization of the electrons $\lambda(N+1)^{1/2} \ll R$, the ground state of the system is also a WL formed by the electrons of this level. If the population of the level is close to the maximum value, then a similar result is valid for the vacancy system of this level.

In our previous paper[3] we constructed the wave functions of the WL in a magnetic field as superpositions of the states of the various cyclotron Landau levels. Clearly, the weight of the states of the levels with $N \neq 0$ in this superposition will tend to zero in the limit of strong magnetic fields. Therefore, the results of this paper cannot be used in the $N > 0$ case under consideration. Furthermore, the standard method, used in Ref. 3, of diagonalizing the lattice Hamiltonian is not applicable in the present case, since the kinetic energy of the electrons (vacancies) in the subspace of the states of a given Landau level is generally a constant. Thus, to find the low-lying WL states at the N-th Landau level in first-order perturbation theory, we need to diagonalize only the potential energy in the indicated subspace. This problem is solved in §2 of the present paper with the aid of the second-quantization formalism. Further, in §3 we consider the second-order perturbation-theory corrections, the analysis of which is necessary for the elucidation of the question of the existence of the WL in the presence of filled levels in the system.

§2. DIAGONALIZATION OF THE POTENTIAL ENERGY OF THE ELECTRONS OF THE N-TH LANDAU LEVEL

Let us consider a system of two-dimensional electrons moving in the (x, y) plane in a background of uniformly-distributed neutralizing charge. The system is placed in an external perpendicular magnetic field (vector potential $\mathbf{A} = \frac{1}{2}[\mathbf{B} \times \mathbf{r}]$) so strong that the condition for the applicability of perturbation theory is fulfilled: $e^2/\lambda \ll \hbar\omega_c$. Let the number of electrons in the system be such that at sufficiently low temperatures a few of the lowest Landau levels are completely filled by electrons and the next upper—to be specific, N-th—level is occupied by N_e electrons, the mean spacing of which is considerably greater than the corresponding minimum dimension of the region of localization of an electron, i.e., for which $R \gg \lambda(N+1)^{1/2}$. If we neglect the Coulomb interaction, then the ground state of the system is multiply degenerate with respect to the different ways of disposing the N_e electrons in the states of the N-th level.

To find the energy of the system in first-order perturbation theory, we must diagonalize in the subspace of the states of the degenerate N-th Landau level the potential energy operator:

$$\hat{U} = \frac{1}{2}\iint d\mathbf{r}_1\, d\mathbf{r}_2\, \hat{\Psi}_N^+(\mathbf{r}_1)\, \hat{\Psi}_N^+(\mathbf{r}_2)\, \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}\, \hat{\Psi}_N(\mathbf{r}_2)\, \hat{\Psi}_N(\mathbf{r}_1) - \frac{e^2}{2}\left(\frac{N_e}{S}\right)^2 \iint \frac{d\mathbf{r}_1\, d\mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|}. \quad (1)$$

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Here $\hat{\Psi}_N(\mathbf{r})$ is the electronic field operator of the N-th Landau level. The second term in (1) describes the interaction of the electrons of the N-th level with the uncompensated portion of the uniform background. As will be seen below, in the case of the low occupancy of the N-th level, the minimum of the energy operator $\hat{U}$ is attained when the positions of the electrons are close to the sites of some regular two-dimensional lattice (a WL) with order parameter R. Let $\mathbf{R}_j=(X_j, Y_j)$ be the sites of this lattice.

Let us construct a single-particle basis whose wave functions are linear combinations of the states of the N-th Landau level, and are centered on the WL sites. At the j-th site we set

$$\psi_{jNm}(\mathbf{r})=\exp\left\{i\frac{xY_j-yX_j}{2\lambda^2}\right\}\Phi_{Nm}(x-X_j, y-Y_j), \tag{2}$$

$$\Phi_{Nm}(\mathbf{r})=\left[\frac{n_\rho}{2\pi\lambda^2(n_\rho+|m|)!}\right]^{1/2}\exp\left\{-im\varphi-\frac{\rho^2}{4\lambda^2}\right\}\left(\frac{\rho^2}{2\lambda^2}\right)^{|m|/2}L_{n_\rho}^{|m|}\left(\frac{\rho^2}{2\lambda^2}\right),$$

$$n_\rho=N+(m-|m|)/2, \quad m\geqslant -N.$$

For the functions (2) to be a complete set in the subspace of the states of the N-th Landau level, we clearly need to take at each site as the basis functions only M of the most highly localized functions ($-N\leqslant m\leqslant M-N-1$), where $M=[N_0/N_e]$ is the number of states of the Landau level per electron, a number which is fairly high in the case of low occupancy of the level: $M\sim(R/\lambda)^2$. On the other hand, the functions (2) can be considered in our case to be orthogonal functions, since, as we shall show below, the low-lying eigenstates of the operator $\hat{U}$ are superpositions of the states (2) with magnetic-quantum-number values $m\approx -N$. Then for not too high cyclotron levels, the overlap of the wave functions of the sites j_1 and j_2 is of the order of $\exp\{-|\mathbf{R}_{j1}-\mathbf{R}_{j2}|^2/4\lambda^2\}$.

Let us expand the electronic field operator $\hat{\Psi}_N(\mathbf{r})$ in terms of the basis (2):

$$\hat{\Psi}_N(\mathbf{r})=\sum_{j=1}^{N_e}\sum_{m=-N}^{M-N-1}\psi_{jNm}(\mathbf{r})\,\hat{c}_{jNm}. \tag{3}$$

The operators $\hat{c}_{jNm}$ (we do not explicitly write out the index N in §2) satisfy the anitcommutation relations:

$$\{\hat{c}_{jm}, \hat{c}_{j'm'}\}=0, \quad \{\hat{c}_{jm}, \hat{c}_{j'm'}^{+}\}=\delta_{jj'}\delta_{mm'}. \tag{4}$$

Substituting the expansion (3) in (1), we write the operator $\hat{U}$ as a sum of different types of terms:

$$\hat{U}=\hat{U}_1(j_1=j_2=j_3=j_4)+\hat{U}_2(j_1=j_4;\ j_2=j_3;\ j_1\neq j_2)+\hat{U}_3-\text{const}. \tag{5}$$

The terms of the type $\hat{U}_1$, all of whose indices j_1 coincide, describe the interaction of two (or more) electrons at the same WL site; this corresponds to nonphonon excitations of a system with energy of the order of $e^2/\lambda(N+1)^{1/2}$.

In the present paper we shall limit ourselves to the consideration of only the phonon type of excitations, for which each WL site is occupied by one and only one particle:

$$\sum_{m=-N}^{M-N-1}\hat{c}_{jm}^{+}\hat{c}_{jm}=1. \tag{6}$$

In this subspace of WL states the matrix elements of the operator $\hat{U}_1$ are equal to zero. The terms of the type $\hat{U}_2$ describe the interaction of the electrons located at the various WL sites, and give rise to the excitations of the phonon type. The presence in (5) of the operator $\hat{U}_3(j_1\neq j_4$ or $j_2\neq j_3)$ is fundamentally important for the description of the exchange and hopping processes that occur in the lattice and lead to the appearance of empty sites and to the gathering of two (or more) particles at some WL sites. But the matrix elements of the terms of this type contain at least one overlap integral, which makes the contribution of the operator $\hat{U}_3$ to the energy of the system exponentially small in the limit $\lambda(N+1)^{1/2}\ll R$. On account of the foregoing, to find the low-lying WL excitations, we must diagonalize the operator $\hat{U}_2$ with allowance for the condition (6).

We carry out the diagonalization of the operator $\hat{U}_2$ in the harmonic approximation, for which purpose we expand the matrix elements of $\hat{U}_2$ in a power series up to terms of second order in the displacements of the electrons from the equilibrium positions. Taking (6) into account, we find

$$\hat{U}_2=\frac{1}{2}\sum_{j_1\neq j_2}\frac{e^2}{|\mathbf{R}_{j_1}-\mathbf{R}_{j_2}|}-\frac{e^2}{2}\left(\frac{N_e}{S}\right)^2\iint\frac{d\mathbf{r}_1\,d\mathbf{r}_2}{|\mathbf{r}_1-\mathbf{r}_2|}$$

$$+\frac{1}{2}\sum_j\sum_{\alpha,\beta}G_{jj}^{\alpha\beta}(\widehat{r_\alpha r_\beta})_j+\frac{1}{2}\sum_{j_1\neq j_2}\sum_{\alpha,\beta}G_{j_1j_2}^{\alpha\beta}(\hat{r}_\alpha)_{j_1}(\hat{r}_\beta)_{j_2}. \tag{7}$$

In this expression the difference between the first two terms is the Madelung energy E_M of the two-dimensional lattice and the $G_{j_1j_2}^{\alpha\beta}$ are the elements of the phonon tensor[3] ($\alpha, \beta=x, y$). In (7) we have set

$$(\hat{r}_\alpha)_j=\sum_{m_1,m_2=-N}^{M-N-1}\langle m_1N|r_\alpha|m_2N\rangle\hat{c}_{jm_1}^{+}\hat{c}_{jm_2}, \tag{8a}$$

$$(\widehat{r_\alpha r_\beta})_j=\sum_{m_1,m_2=-N}^{M-N-1}\langle m_1N|\widehat{r_\alpha r_\beta}|m_2N\rangle\hat{c}_{jm_1}^{+}\hat{c}_{jm_2}. \tag{8b}$$

The subsequent transformations are based on the properties of the matrix elements of the operators (8). It can be shown that, in the subspace of the states limited by the condition (6), the operators (8b) can be expressed in terms of the operators (8a):

$$(\hat{r}_\alpha^2)_j=\lambda^2(N+1/2)+(\hat{r}_\alpha)_j^2, \quad (\widehat{r_xr_y})_j=(\widehat{r_yr_x})_j$$

$$=[(\hat{r}_x)_j(\hat{r}_y)_j+(\hat{r}_y)_j(\hat{r}_x)_j]/2. \tag{9}$$

The operators (8a) can in turn be expressed in terms of the standard operators

$$\hat{a}_j=\sum_{m=-N}^{M-N-1}(\text{sign}\,m)\,(N+m+1)^{1/2}\hat{c}_{jm}^{+}\hat{c}_{j,m+1} \tag{10}$$

with the aid of the formulas

$$(\hat{r}_x)_j=2^{-1/2}\lambda(\hat{a}_j+\hat{a}_j^{+}), \quad (\hat{r}_y)_j=2^{-1/2}i\lambda(\hat{a}_j-\hat{a}_j^{+}). \tag{11}$$

The operator $\hat{a}_j$ has the meaning of a lowering operator at the j-th site. On account of the relations (6) and (4) we have

$$[\hat{a}_j, \hat{a}_{j'}]=0, \quad [\hat{a}_j, \hat{a}_{j'}^{+}]=\delta_{jj'}(1-M\hat{c}_{j,M-N}^{+}\hat{c}_{j,M-N}). \tag{12}$$

The matrix elements of the operator $\hat{c}_{j,M-N}^{+}\hat{c}_{j,M-N}$ in (12) are equal to zero in our approximation, since we choose as the basis the functions (2) with $m\leqslant M-N-1$

and neglect the overlap integrals involving the wave functions of different sites. Consequently, the operators (10) satisfy the Bose commutation relations. The Hamiltonian (7) can be reduced to a quadratic form in the operators $\hat{a}_j$ with the aid of (9) and (11). By going over in this expression to the phonon operators

$$\hat{a}_{\mathbf{k}}=\left(\frac{1}{N_c}\right)^{1/2}\sum_j \hat{a}_j e^{-i\mathbf{k}\mathbf{R}_j}, \tag{13}$$

we can reduce(7) to a sum of commuting operators $\hat{U}_{\mathbf{k}}$, each of which is a linear combination of the products $\hat{a}_{\mathbf{k}}^+\hat{a}_{\mathbf{k}}$, $\hat{a}_{\mathbf{k}}\hat{a}_{-\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^+\hat{a}_{-\mathbf{k}}^+$. The operators $\hat{U}_{\mathbf{k}}$ are then diagonalized with the aid of the Bogolyubov transformation. Finally, we find

$$\hat{U}_2=E_M+\frac{\hbar(N+1)}{4\omega_c}\sum_{\mathbf{k}}[\omega_1^2(\mathbf{k})+\omega_2^2(\mathbf{k})]+\sum_{\mathbf{k}}\hbar\frac{\omega_1(\mathbf{k})\omega_2(\mathbf{k})}{\omega_c}(\hat{A}_{\mathbf{k}}^+\hat{A}_{\mathbf{k}}+1/2), \tag{14}$$

where the Bose operators $\hat{A}_{\mathbf{k}}$ are linear combinations of the operators $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{-\mathbf{k}}^+$; $\omega_1(\mathbf{k})$ and $\omega_2(\mathbf{k})$ are the frequencies, determined in Ref. 6, of the longitudinal and transverse WL vibrations in zero magnetic field.

Thus, in the harmonic approximation the eigenvalues of the operator $\hat{U}_2$ do not depend on the Landau-level number, and form one phonon-type vibrational branch, which coincides with the leading term of the expansion of the low-frequency ω_- branch (see Ref. 3) in the strong magnetic field limit. The $\omega_+\approx\omega_c$ type vibrational branch does not arise in the present case, since vibrations of this type are connected with particle transitions between the various cyclotron levels. The second term in (14) is a quantum correction to the Madelung lattice energy, and arises as a result of the finiteness of the particle size.

The mean square displacement of an electron at a WL site can be computed with the aid of the formulas expressing the operators $\hat{a}_j$ in terms of the operators $\hat{A}_{\mathbf{k}}$:

$$\langle\mathbf{r}^2\rangle=\lambda^2\left\{2N+1+\frac{1}{N_c}\sum_{\mathbf{k}}\left(\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}\right)(\bar{n}_{\mathbf{k}}+1/2)\right\}, \tag{15}$$

where the $\bar{n}_{\mathbf{k}}=\langle\hat{A}_{\mathbf{k}}^+\hat{A}_{\mathbf{k}}\rangle$ are the mean occupation numbers for the phonon states. For the lowest Landau level, this expression, as was to be expected, coincides with the displacement, found earlier,[3] of an electron at a WL site in the limit of strong magnetic fields. The estimate given in Ref. 3 for the numerical coefficient in (15) gives the value $\langle\mathbf{r}^2\rangle\approx 2\lambda^2(N+1)$ for $T=0$ K. This circumstance justifies the assumption made above that the low-lying eigenstates of the operator $\hat{U}$ are superpositions of the states (2) with values of $m\approx -N$. (Let us recall that

$$\langle mN|\mathbf{r}^2|mN\rangle=2\lambda^2(2N+m+1).)$$

The foregoing arguments are, of course, valid provided $[\langle\mathbf{r}^2\rangle]^{1/2}\ll R$. But Fukuyama and Yoshioka[7] believe that even when $\lambda\sim R$ there exists in the system a charge-density wave whose structure does not differ in fact from the WL structure, so that the peaks of the electron-density wave coincide with the sites of the WL corresponding to the same number of electrons at the level. If that is so, then the solution found in the present section can apparently serve as a good first approximation for the investigation of charge-density waves.

We have considered above the case of low occupancy of the Landau level. If the population of the level is close to the highest possible value, then the results are valid for the system of vacancies of this level. It is only necessary to go over in (1) to the vacancy field operators $\hat{\Phi}_N(\mathbf{r})=\hat{\Psi}_N^+(\mathbf{r})$, and place the $\hat{\Phi}_N^+(\mathbf{r})$ operators to the left of $\hat{\Phi}_N(\mathbf{r})$. This transformation leads in first-order perturbation theory to only a shift in the ground-state energy of the system. Therefore, the vibrational spectrum of the WL formed by the vacancies of the N-th level and the mean square displacement of a vacancy from a site of the WL are also given by the formulas (14) and (15).

§3. EFFECT OF THE FILLED LEVELS

The authors of a number of papers[4,7] assume that the effect of the neighboring cyclotron levels on the dynamics of the system of electrons (vacancies) of a given Landau level is negligible in the limit of strong magnetic fields. But in the presence of filled levels in the system, this question needs to be considered separately. As an example, let us consider the problem of the crystallization of the vacancies of the lowest Landau level.

Let the population of the lowest Landau level be close to the highest possible value. By performing in the total Hamiltonian of the system the operator transformation described at the end of §2, we can establish that, in second-order perturbation theory, the main contribution to the energy of the system of vacancies of the lowest Landau level is made by the operator

$$\hat{H}'=\sum_{j_1}\sum_{m_1=0}^{M-1}\sum_{X,N\geqslant 1}\iint d\mathbf{r}_1\,d\mathbf{r}_2\,\Psi_{NX}^*(\mathbf{r}_1)\varphi_{j_1m_1}^*(\mathbf{r}_2)$$
$$\times\frac{e^2}{|\mathbf{r}_1-\mathbf{r}_2|}\varphi_{j_2m_2}^*(\mathbf{r}_1)\varphi_{j_3m_3}(\mathbf{r}_2)\hat{c}_{NX}^+\hat{b}_{j_1m_1}^+\hat{b}_{j_2m_2}^+\hat{b}_{j_3m_3}+\text{H.c.} \tag{16}$$

Here the $\Psi_{NX}(\mathbf{r})$ are the wave functions of the electrons of the N-th level in the Landau representation; X is the quantized position of the center of the cyclotron motion; $\hat{c}_{NX}$ is the electron annihilation operator; the $\varphi_{jm}(\mathbf{r})$ are the wave functions of the vacancies of the lowest Landau level in the basis (2); $\hat{b}_{jm}$ is the vacancy annihilation operator. The Hamiltonian (16) does not conserve the electron and vacancy numbers separately, but describes the creation of the electrons and vacancies from vacuum and their annihilation with allowance for the equality of the two numbers.

It is easy to see that the matrix elements of the operator (16) are of the order of e^2/λ. Therefore, in second-order perturbation theory the contribution of this operator to the energy of the system of vacancies is of the order of $\Delta E^{(2)}\sim(e^2/\lambda)^2/\hbar\omega_c=e^2/a_B$ (a_B is the Bohr radius). If $a_B\lesssim R$, then this quantity exceeds the vacancy-WL binding energy $E_M\sim e^2/R$. Let us show, however, that in spite of their relatively large values the corrections under consideration turn out to be weakly dependent on the disposition of the vacancies at the Landau level.

Let us retain in the operator (16) only the terms for which $j_1=j_3$, thus neglecting the overlap of the wave functions of different sites. Then there are two pos-

sibilities; either $j_1 = j_3 = j_2$, or $j_1 = j_3 \neq j_2$. In the first case the different WL sites interact independently of each other. The corresponding energy shift for a vacancy is of the order of $\Delta E^{(2)}$, and practically does not depend on the presence of the small number of other vacancies at the level. Moreover, this shift turns out to be also independent of the quantum number m of the wave function of the isolated vacancy. Otherwise it would have had to be treated as an additional effective single-particle potential that should be taken into account in considering the WL vibrations. It is convenient to perform the calculations in the Landau representation, in which the wave function of the isolated vacancy is $b_X^+|0\rangle$. The second-order vacancy-energy correction arising from the operator (16) does not depend on the quantum number X, and is equal to

$$\Delta E^{(2)}(X) = -\frac{(e^2/\lambda)^2}{2\hbar\omega_c}\sum_{N=1}^{\infty}\frac{1}{2^N N^2}\left\{1-\frac{\Gamma^2(N+1/2)}{N\Gamma^2(N)}F(1/2,1/2,N+1;-1)\right\}, \quad (17)$$

where $F(\alpha, \beta, \gamma; z)$ is a hypergeometric function. It is also easy to establish that any linear combination of the type

$$\sum_X \eta_X b_X^+|0\rangle$$

with arbitrary coefficients η_X leads to the same energy shift (17). Since the wave functions (2) are linear combinations of this type, the energy shift for the isolated vacancy does not depend on the form of the vacancy's wave function, and, consequently, does not affect the spectrum of the vacancy-WL vibrations.

In the second case, in which $j_1 = j_3 \neq j_2$, the contribution to the energy of the system of vacancies depends on the disposition of the vacancies at the level, but it turns out to be much smaller than the contribution from the anharmonic terms of the operator $\hat{U}_2$ discussed in §2. Indeed, allowance for the anharmonic terms of the operator $\hat{U}_2$ leads to a shift in the normal-mode frequencies of the WL by an amount of the order of

$$(\Delta\hbar\omega)_1 \sim \frac{(\Delta U)^2}{\Delta E^{(1)}} \sim \left[\frac{e^2}{R}\left(\frac{\lambda}{R}\right)^3\right]^2 \Big/ \hbar\frac{\omega_D^2}{\omega_c} \approx \frac{e^2}{R}\left(\frac{\lambda}{R}\right)^4,$$

where $\Delta\hat{U}$ is a characteristic value of the anharmonic terms of the operator $\hat{U}_2$ and $\Delta E^{(1)}$ is a characteristic quantum of the WL vibrations [$\omega_D \approx (e^2/mR^3)^{1/2}$ plays the role of a Debye frequency for the WL].

To estimate the WL natural-frequency shift arising from the structure-sensitive terms of the operator (16), let us expand the corresponding matrix elements of this operator in a power series in the small parameter λ/R. The zeroth-order terms in this expansion vanish on account of the orthogonality of the wave functions of the various cyclotron levels, and the first-order terms vanish as a result of the fact that the WL is assumed to be in equilibrium. The second-order terms lead to a shift in the normal-mode frequencies of the WL by an amount of the order of

$$(\Delta\hbar\omega)_2 \sim \left[\frac{e^2}{R}\left(\frac{\lambda}{R}\right)^2\right]^2 \Big/ \hbar\omega_c = \frac{e^2}{R}\left(\frac{\lambda}{R}\right)^4 \frac{e^2/R}{\hbar\omega_c}.$$

In our approximation $\lambda \ll R$ and $e^2/\lambda \ll \hbar\omega_c$; therefore, $(\Delta\hbar\omega)_2 \ll (\Delta\hbar\omega)_1$.

A similar analysis of the second-order corrections to the energy of the electron (vacancy) system of an upper level that arise on account of the filled lower Landau levels shows that these corrections lead to a relatively large shift in the self-energy of the electrons of the partially filled Landau level, but that they manifest themselves weakly in the interaction of these electrons. Therefore, the corrections under consideration almost do not affect the vibrational spectrum, determined in §2 of the present paper, of the electron (vacancy) WL.

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