Theory of absorption of sound in magnetic substances

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(Submitted 24 December 1980)
Zh. Eksp. Teor. Fiz. **80**, 2475–2479 (June 1981)

The sound absorption coefficients in ferro- and antiferromagnetic substances due to the interaction of phonons with nuclear spin waves at infralow temperatures are calculated.

PACS numbers: 75.30. – m, 62.65. + k

In this work we consider the absorption of a sound wave in ferromagnetic and antiferromagnetic substances at infralow temperatures ($T \lesssim \hbar\omega_n$, where $\omega_n = AS/\hbar$ is the precession frequency of the nuclear spin, A is the constant of hyperfine interaction, S is the spin of the electron shell). In the temperature region $T \gtrsim 1$–4 K, the sound attenuation has been calculated by a number of authors,[1,2] both microscopically at $\omega\tau \gg 1$ (where ω is the phonon frequency, τ^{-1} is the characteristic probability of a scattering process) and macroscopically at $\omega\tau \ll 1$. Under our conditions, as has been shown previously,[3,4] at temperatures $T \lesssim \hbar\omega_n$ the basic contribution to the scattering process is made by nuclear spin waves. We shall consider below the coefficient of damping due to the annihilation process—the fusion of two nuclear spin waves into one phonon. In the temperature region considered, this scattering mechanism leads to resonant absorption of the sound.

For the microscopic calculation, a necessary condition is the inequality

$$\omega\tau_{phn} \gg 1.$$

Here τ_{phn}^{-1} is the probability of scattering of a phonon by the nuclear spin wave.

1. FERROMAGNETIC SUBSTANCES

The Hamiltonian of interaction between the nuclear spin waves and the phonons has the form[3]

$$\hat{\mathcal{H}}_{nph}^{int} = \frac{1}{\mathfrak{N}^{1/2}} \sum_{123,j} \psi_{nphj}(123) \times \alpha_{nk_1}\alpha_{nk_2} b_{k_3 j}^{+} \Delta(1+2-3) + \text{c.c.}, \tag{1}$$

$$\psi_{nphj}(123) = i(\gamma\mu_e M_0) v_1 v_2 \frac{e_j^{+} k_3^{+}}{k_3^{1/2}} \left(\frac{\hbar}{2\rho a^3 c_s}\right)^{1/2}, \tag{2}$$

where ρ is the density of the magnetic material, $c_s = a\Theta_D/\hbar$ is the speed of sound, Θ_D is the Debye temperature, α_{nk}^{+} is the creation operator of a nuclear spin wave, b_{kj}^{+} is the creation operator of a phonon, γ is the magnetostriction constant, e is the polarization vector of the phonon, $j = 1, 2, 3$ are the polarizations,

$$v_i \equiv v(\mathbf{k}_i) = A(IS)^{1/2}/\hbar\omega_e(\mathbf{k}_i),$$

I is the spin of the nucleus, $\omega_e(\mathbf{k}) = \omega_e + \omega_E(ak)^2$ is the dispersion of the magnons, $\omega_e = \mu_e(H + H_a)/\hbar$ is the precession frequency of the electron spin, H_a is the field of the magnetic anisotropy, H is the external magnetic field, $\omega_E = SJ_0/\hbar$ is the exchange frequency, J_0 is the exchange integral, $SJ_0 \sim \Theta_c$ is the Curie temperature.

Using the spectral method,[5] we write out the corresponding attenuation coefficient:

$$\frac{1}{\tau_{phn}(\omega)} = \Gamma(\omega) = \frac{\pi}{\hbar} \frac{(\gamma\mu_e M_0)^2}{\Theta_D} \times \frac{\hbar}{\rho a^3 c_s} \omega(1+2N) \frac{1}{\mathfrak{N}^2} \sum_{1,2} v_1^2 v_2^2 \Delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_{nk_1}-\omega_{nk_2}). \tag{3}$$

In the case of neglect of dispersion, we obtain the result that $\Gamma(\omega)$ is proportional to $\delta(2\omega_n - \omega)$. It is clear that for elucidation of the character of the absorption curve, we must naturally take account of the dispersion of the nuclear spin wave—$\Delta\omega_n(\mathbf{k})$:

$$\omega_{nk} = \omega_n - \Delta\omega_{nk}, \qquad \Delta\omega_{nk} = A^2 IS/\hbar^2\omega_e(\mathbf{k}).$$

Cumbersome but straightforward calculations lead us to the following formula:

$$\Gamma(\omega) = \frac{1}{2\pi} \frac{(\gamma\mu_e M_0)^2}{\hbar^2\omega_E} \frac{\hbar}{\rho a^4 c_s} (1+2N) \frac{\hbar\omega_n}{\Theta_D} \frac{A^2 IS}{\hbar^2\omega_E^2} \varphi(\omega), \tag{4}$$

where

$$\varphi(\omega) = \frac{k_0}{(k_0^2 + \omega_e/\omega_E)^2}, \quad k_0 = \left(\frac{A(IS)^{1/2}}{B\hbar\omega_E} - \frac{\omega_e}{\omega_E}\right)^{1/2},$$
$$B = \hbar \frac{2\omega_n - \omega}{A(IS)^{1/2}} - \frac{A(IS)^{1/2}}{\hbar\omega_e}.$$

It is easy to verify that for the condition of resonance the sound frequency should lie in a narrow frequency range;

$$\omega_1 \leqslant \omega \leqslant \omega_2.$$

Here

$$\omega_1 = 2\omega_n - 2A^2 IS/\hbar^2\omega_e, \quad \omega_2 = 2\omega_n - A^2 IS/\hbar^2\omega_e.$$

Investigation of the function $\varphi(\omega)$ shows that it has a maximum at

$$\omega = \omega_0 = 2\omega_n - \frac{7}{4}\frac{A^2 IS}{\hbar^2\omega_e}:$$
$$\varphi(\omega_0) = \varphi_{max} = \frac{9}{16\sqrt{3}} \left(\frac{\omega_E}{\omega_e}\right)^{1/2}; \tag{5}$$

at the end points of the interval $[\omega_1, \omega_2]$ it behaves asymptotically as

$$\varphi(\omega) = \begin{cases} \dfrac{\hbar\omega_E^{1/2}(\omega-\omega_1)^{1/2}}{\omega_e A(IS)^{1/2}} & \text{at } \omega > \omega_1,\ \omega \to \omega_1, \\ \left(\dfrac{\hbar\omega_E}{AI}\right)^{1/2} \left(\dfrac{\omega_2-\omega}{\omega_n}\right)^{1/2} & \text{at } \omega < \omega_2,\ \omega \to \omega_2. \end{cases} \tag{6}$$

Substituting φ_{max} in formula (4), we obtain the value of the resonance maximum:

$$\Gamma_{max}(\omega_0) \approx \frac{1}{\pi} \frac{(\gamma\mu_e M_0)^2}{\hbar^2\omega_E} \frac{\hbar}{\rho a^4 c_s} (1+2N) \frac{\hbar\omega_n}{\Theta_D} \frac{A^2 IS}{\hbar^2\omega_e^{3/2}\omega_E^{1/2}}. \tag{7}$$

Thus, at sound frequencies equal (with accuracy to

 0038-5646/81/061297-03$02.40

within the dispersion of the nuclear spin wave) to double the frequency of nuclear magnetic resonance (NMR), resonant absorption takes place. Although the considered mechanism is weaker than the nuclear magnetoacoustic resonance,[6] nevertheless it is relatively strong (if we take the characteristic parameters of a ferromagnetic substance $\hbar\omega_E \sim 10$ K, $\hbar\omega_e \sim 0.1$ K, $\gamma \sim 10$, $\Theta_D \sim 100$ K, $\hbar\omega_n \sim 3\cdot 10^{-2}$ K, then Γ_{max} will be approximately equal to 10^3 sec^{-1}) and should be experimentally observable at frequencies $\omega \approx 2\omega_n$.

2. ANTIFERROMAGNETIC SUBSTANCES

We now write down the corresponding Hamiltonian of nuclear spin wave–phonon interaction of an antiferromagnetic substance. It is obtained from the energy of magnetostriction in the account of the mixture of nuclear spin wave oscillations in electron spins[4]

$$\mathcal{H}_{n\text{-}ph}^{int} = \frac{1}{\mathfrak{N}^{1/2}} \sum_{123,j,\gamma\gamma'} \psi_{n\text{-}ph,j}^{\gamma\gamma'}(123)\alpha_{n\gamma k_1}\alpha_{n\gamma' k_2} b_{k_3 j}^{+}\Delta(1+2-3)+\text{c.c.} \qquad (8)$$

Here $\gamma, \gamma' = 1, 2$,

$$\psi_{n\text{-}ph,j}^{\gamma\gamma'}(123) = i\left(\frac{\hbar}{2\rho a^2 c_s}\right)^{1/2} k_3^{1/2}\gamma_0\mu_e M_0 \bar{Q}_{\gamma k_1}\bar{Q}_{\gamma' k_2}\varphi_\gamma(\mathbf{n}\mathbf{e}_j) \qquad (9)$$

(the definitions of ρ, c_s, Θ_D are given in the preceding section), γ_0 is the magnetostriction constant, $\varphi_\gamma(\mathbf{n}, \mathbf{e}_j)$ are certain functions of the angles. The function

$$F_{\gamma k} = \left(\frac{\omega_n}{4\omega_{e\gamma k}}\right)^{1/2}\frac{\omega_{T0}}{\omega_{e\gamma k}}, \quad \bar{Q}_{1k} = F_{1k}(T_k + R_k),$$
$$\bar{Q}_{2k} = F_{2k}(S_k + U_k),$$

where

$$T_k = \frac{C_k + D_k}{[4\hbar\omega_{e1k}(\varepsilon_k + B_k - \hbar\omega_{e1k})]^{1/2}},$$
$$R_k = \frac{C_k + D_k}{[4\hbar\omega_{e1k}(\varepsilon_k + B_k + \hbar\omega_{e1k})]^{1/2}},$$
$$S_k = \frac{C_k - D_k}{[4\hbar\omega_{e2k}(\varepsilon_k - B_k + \hbar\omega_{e2k})]^{1/2}}, \qquad (10)$$
$$U_k = \frac{C_k - D_k}{[4\hbar\omega_{e2k}(\varepsilon_k - B_k - \hbar\omega_{e2k})]^{1/2}},$$
$$\hbar\omega_{e\,1k} = [\mu_e^2 H(H+H_D) + \hbar^2\omega_{T0}^2 + \hbar^2\omega_E^2(ak)^2]^{1/2},$$
$$\hbar\omega_{e\,2k} = [\mu_e^2 H_D(H+H_D) + 2\hbar^2\omega_E\omega_a + \hbar^2\omega_{T0}^2 + \hbar^2\omega_E^2(ak)^2]^{1/2},$$

$\hbar\omega_E \sim \Theta_N$ is the energy of the exchange interaction, H_D is the Dzyaloshinsiii field, $\omega_{T0}^2 = 2AJ_0SI/\hbar^2$ is the frequency of the intermingling of the electron-nuclear oscillations, Θ_N is the Neel temperature, $\hbar\omega_a = \mu_e H_a$ is the anisotropy energy,

$$\varepsilon_k = \mu_e H\sin\psi + \hbar\omega_E\cos 2\psi + \mu_e H_D\sin 2\psi + (a+b)S/2 + AI,$$
$$B_k = {}^1\!/_2 S(2\sin^2\psi J_k + a - b - \mu_e H_D S^{-1}\sin 2\psi),$$
$$C_k = {}^1\!/_2 S(2\cos^2\psi J_k + a - b + \mu_e H_D S^{-1}\sin 2\psi),$$
$$D_k = S(a+b)/2, \quad \sin\psi = (H+H_D)/2H_E,$$
$$\omega_{n\gamma k} \approx \omega_n - \omega_{T0}^2\omega_n/2\omega_{e\gamma k}^2,$$

a and b are the anisotropy constants.

Using formulas (10), we find the attenuation coefficient:

$$\Gamma^{\gamma\gamma'}(\omega) = \frac{\pi}{\hbar}(1+2N)\left(\frac{S}{I}\right)^2\frac{(\gamma_0\mu_e M_0)^2}{2^6\Theta_D} \times \omega\frac{\hbar}{\rho a^3 c_s}\frac{1}{\mathfrak{N}^2}\sum_{1,2}\Delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega_{n\gamma k_1}+\omega_{n\gamma' k_2}-\omega)\left(\frac{\omega_{T0}^4}{\omega_{e\gamma' k_2}^2\omega_{e\gamma k_1}^2}\right)^2. \qquad (11)$$

For simplicity, we assume that there is no coupling between branches, i. e., we set $\gamma = \gamma'$; then simple calculations, similar to those of the ferromagnetic case, lead us to the following formula for $\Gamma^\gamma(\omega)$:

$$\Gamma^\gamma(\omega) = \frac{1}{2^6\pi}(1+2N)\frac{\hbar}{\rho a^4 c_s}\left(\frac{S}{I}\right)^2\frac{(\gamma_0\mu_e M_0)^2}{\hbar\Theta_D}\left(\frac{\omega_{T0}}{\omega_E}\right)^6\varphi_\gamma(\omega), \qquad (12)$$
$$\varphi_\gamma(\omega) = \frac{k_{0\gamma}}{(k_{0\gamma}^2 + \omega_{e\gamma}^2/\omega_E^2)^2}, \quad k_{0\gamma} = \left(\frac{\omega_{T0}^2}{2\omega_E^2}\frac{\omega_n}{\Omega} - \frac{\omega_{e\gamma}^2}{\omega_E^2}\right)^{1/2},$$
$$\Omega_\gamma = 2\omega_n - \omega - \frac{\omega_{T0}^2}{2\omega_{e\gamma}^2}\omega_n,$$

and the frequency ω belongs to the interval $\omega_{1\gamma} \leq \omega \leq \omega_{2\gamma}$ where

$$\omega_{1\gamma} = 2\omega_n - \omega_{T0}^2\omega_n/\omega_{e\gamma}^2, \quad \omega_{2\gamma} = 2\omega_n - \omega_{T0}^2\omega_n/2\omega_{e\gamma}^2,$$
$$\varphi_{max} = \frac{9}{16\sqrt{3}}\left(\frac{\omega_E}{\omega_{e\gamma}}\right)^3$$

meaning that the resonance peak is

$$\Gamma_{max}^\gamma \approx \frac{1}{\pi}(1+2N)\frac{\hbar}{\rho a^4 c_s}\left(\frac{\gamma_0\mu_e M_0 S}{I}\right)^2\frac{1}{\hbar\Theta_D}\frac{\omega_{T0}^6}{\omega_E^3\omega_{e\gamma}^3}. \qquad (13)$$

The behavior of $\varphi(\omega)$ at the end points of the interval $\omega_{1\gamma} \leq \omega \leq \omega_{2\gamma}$ turns out to be

$$\varphi(\omega) = \begin{cases} \left(\frac{\omega_E}{\omega_{T0}}\right)^3\left(\frac{\omega_{T0}}{\omega_{e\gamma}}\right)^2\left(\frac{\omega-\omega_{1\gamma}}{\omega_n}\right)^{1/2} & \text{at } \omega > \omega_{1\gamma},\ \omega \to \omega_{1\gamma}, \\ \left(\frac{\omega_E}{\omega_{T0}}\right)^3\left(\frac{\omega_{2\gamma}-\omega}{\omega_n}\right)^{1/2} & \text{at } \omega < \omega_{2\gamma},\ \omega \to \omega_{2\gamma}. \end{cases}$$

For such an easy-plane antiferromagnetic substance as $MnCO_3$, the formula (13) shows that Γ_{max} is approximately equal to 10^6 sec^{-1}.

The existence of two characteristic frequencies of precession of the electron spins in the antiferromagnetic substances, ω_{e1} and ω_{e2}, leads to two resonant maxima. This can easily be verified from the fact (we recall that $\omega_{e2} > \omega_{e1}$) that the frequencies of the end points of the intervals satisfy the following inequality ($\gamma = \gamma'$):

$$\omega_{11} < \omega_{21} < \omega_{12} < \omega_{22}. \qquad (14)$$

Account of the intermingling ($\gamma \neq \gamma'$!) of the low-frequency and high-frequency branches of the oscillations leads naturally to the result that both maxima become coupled with one another and then the middle part of the inequality (14) simply drops out.

Comparison of the sound absorption coefficients in ferromagnetic and antiferromagnetic substances indicates definitely that in the antiferromagnetic case [see formula (13) at $\gamma = 1$], due to the increase in the exchange interaction between the nuclear spin wave and the phonons, the resonance will be approximately $(\omega_E/\omega_e)^{3/2}$ times more powerful.

As has already been pointed out earlier,[3,4] in the temperature range $T \leq \hbar\omega_n$ activationless mechanisms of dissipation are the more important. This factor plays also an important role here in the isolation and finding of the greatest contribution to the background of the sound absorption curve from the various scattering processes. It can be shown that Landau-Rumer mechanisms[1] and sound scattering by impurities[7] lead to the greatest probabilities at the temperatures considered. However, the fact that the phonon frequency in the given situation (we recall that $\omega \approx 2\omega_n$) is not large in order of

magnitude (10^8–10^9 sec^{-1}) leads to the result that even these mechanisms do not give any significant contribution to the background. Namely, in the frequency ranges $\omega < \omega_1(\omega_{11})$ and $\omega > \omega_2(\omega_{22})$, $\Gamma(\omega)$ will be practically equal to zero.

The author takes this occasion to thank A. F. Andreev and M. I. Kaganov for numerous discussions.

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Translated by R. T. Beyer