Oscillations and stability of a charged helium surface

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The oscillation spectrum of the charged surface of helium (with an arbitrary amount of charge) is investigated. In the extreme case of maximum charge on the surface, the general expression for the dispersion law goes over to the previously derived asymptotic expressions. If, however, the surface is weakly charged, some new asymptotes of the dispersion law arise and permit us to study the effect of surface charges on the oscillation dispersion law for a charged helium surface, down to a zero degree of charging. It is shown that separate, many-electron dimples may be formed upon development of instability of the helium surface under weak charging conditions. Under certain particular assumptions, the various characteristics of many-electron dimples are calculated, viz., the radius of the electron spot, the total dimple energy, the critical clamping electric field required for the appearance of a hole, etc. The experiment confirms qualitatively the existence of many-electron dimples on the surface of liquid helium. [1)] Permanent address: Physics Department E-10, Technische Universität München, West Germany].

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INTRODUCTION

The systematic study of the oscillations and stability of a charged helium surface began rather recently (during the past 3–4 years). Nevertheless, within this comparatively short period, many interesting results have been obtained regarding the details of the dispersion law and the development of instability in this system. We are speaking of the researches of Gor'kov and Chernikova,[1-3] Mima, Ikezi and Hasegava,[4-6] Williams and one of the authors of the present paper (V.Sh.),[7,8] and the experimental studies of Volodin, Khaĭkin and Édel'man,[9,10] and, finally, Wanner and one of the authors of the present paper (P.L.).[11-13]

It should be noted that practically all the experimental data available at the present time can be interpreted within the framework of the so-called metallic approximation. In this approximation, the helium surface is a surface of equal electric potential. Further, it is convenient to use the assumption that the external clamping electric field above the charged surface of the helium is entirely cancelled by the electric field of the two-dimensional system of charges. Under such conditions, the theory of oscillations of the charged surface of helium becomes maximally compact and close to the classical theory of oscillations of a surface of a conducting liquid in an electric field that is normal to the surface of this liquid.[14]

However, in the case of a charged helium surface, the metallic approximation is not the only one possible. The external parameters of the electric part of the problem, namely the clamping field $E_\perp$, the mean charge density n_s, the effective mass M and the mobility of the charges along the helium surface, all can vary independently over wide limits, creating combinations that go beyond the framework of the metallic approximation. As a result, many interesting additional effects appear that merit special attention.

First of all, according to current ideas (see Refs. 5, 7 and 8) there are two mechanisms for the onset of instability of a charged surface of liquid helium. One of these (the dynamic channel) exists because of the possibility of rapid redistribution of the mobile charges over the helium surface in places with the largest deformation of the surface (Fig. 1a). The other, static mechanism leads to instability even in the case of immobile charges, localized on the surface of an oscillating liquid. The action of this mechanism is clear from the scheme shown in Fig. 1b. A real instability of the charged surface of liquid helium is a self-consistent combination of both types of instability, which make equivalent contributions in the metallic approximation. So far as the separate investigation of each of the mechanisms mentioned, it becomes possible outside the framework of the metallic approximation.

The following interesting question pertains to the effect of a finite effective mass and mobility of the charges, in their motion along the helium surface, on the dispersion law of the oscillations of the charged helium surface. This question is significant for the case of oscillations of a charged boundary separating the phases of ^{3}He and ^{4}He in a stratified ^{3}He–^{4}He solution.

The most curious and qualitatively new possibility that arises in the study of the instability of a weakly charged helium surface is the appearance of separate

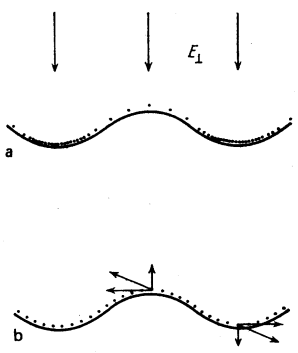

FIG. 1. Channels of instability of a charged helium surface: a—dynamic, b—static. The instability in the case b arises because of the appearance of forces of Coulomb origin, directed along the normal to the surface of the helium and indicated by arrows.

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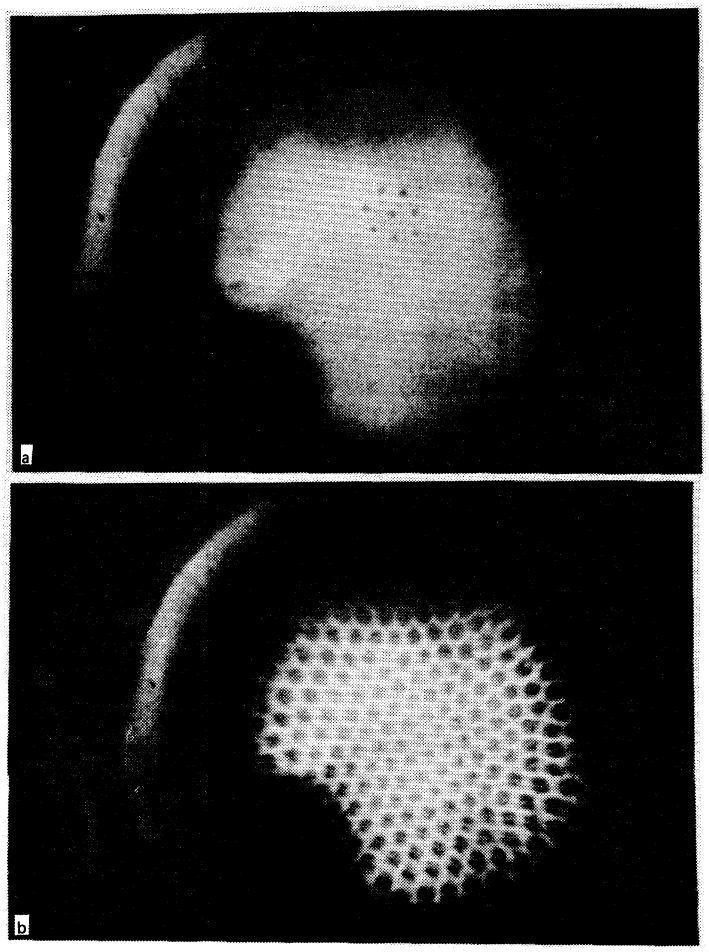

FIG. 2. Set of photographs of the helium surface under conditions of appearance of periodic structures. The diameter of the electrodes of the cell ~ 8 cm. The period of the structure is of the order of the capillary length ~ 10^{-1} cm.

dimples containing a large number of charges. Such dimples appear upon development of the instability of a weakly charged surface of liquid helium and their number is proportional to the degree of charging δ of the free surface of the helium (the degree of charging δ is defined as the ratio of given mean charge density on the helium surface to the maximum possible value of this density: $\delta = n_s / n_s^{max}$ in the case of a liquid half-space, $n_s^{max} \approx 2 \cdot 10^9$ cm^{-2}). The successive increase in the number of dimples with increase in the parameter δ is demonstrated in the two photographs shown in Fig. 2.[2)] In the region $\delta \leq 1$, the set of multicharged dimples is built up into a crystalline structure, the existence of which was predicted by Gor'kov and Chernikov[3] and studied in great detail by Ikezi.[6] The first observations of the crystal of multi-electron dimples on the interface of the phases in a ^{3}He–^{4}He solution was performed by Wanner and one of the authors.[11]

Thus, the extension of the studies of the properties of charged helium surfaces into the region of a weak degree of charging is quite promising. The aim of the present work is the description of certain simple properties of weakly charged liquid helium surfaces.

1. OSCILLATIONS OF AN ARBITRARILY CHARGED SURFACE OF LIQUID HELIUM

A. We first consider the system shown in Fig. 3. The space between the two plates of a plane-parallel capacitor is filled with liquid helium. The symbols d, a and b denote the distance between the capacitor plates, the vacuum gap, and the thickness of the liquid helium layer in the capacitor. The z axis is directed along the normal to the liquid surface, the origin of the coordinates coincides with the position of the undisturbed liquid surface, the region $z < 0$ corresponds to the liquid phase. For simplicity, we neglect the effect of finite thickness of the helium film, i.e., we assume that b is sufficiently large; V is the potential difference applied to the plates of the capacitor, ε is the dielectric constant of liquid helium, $\sigma_0 = en_s$ is the surface charge density.

B. In the case of small oscillations, the shapes of the liquid helium surface

$$\xi(x) = \xi_q \cos qx$$

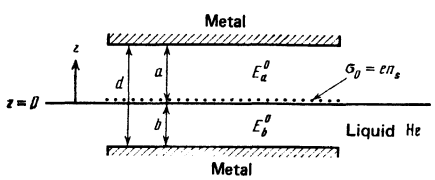

FIG. 3. Schematic diagram of the experimental cell.

the potentials Φ_a and Φ_b above and below the charged helium surface, and also the electric fields $E_a = \nabla\Phi_a$, $E_b = \nabla\Phi_b$ can be written in the form

$$\begin{aligned}
&\Phi_a = E_a^0 z + F_q e^{-qz}\cos qx, \quad z \geqslant \xi_q \cos qx,\\
&\Phi_b = E_b^0 z + G_q e^{qz}\cos qx, \quad z \leqslant \xi_q \cos qx,\\
&\sigma = \sigma_0 + \sigma_q \cos qx,\\
&E_q^{\perp} = E_a^0 - qF_q \cos qx e^{-qz}, \quad z \geqslant \xi_q \cos qx,\\
&E_b^{\perp} = E_b^0 + qG_q e^{qz}\cos qx, \quad z \leqslant \xi_q \cos qx.
\end{aligned} \tag{1}$$

The boundary conditions on the curved helium surface $z = \xi_q \cos qx$

$$E_a^{\perp} - \varepsilon E_b^{\perp} = 4\pi\sigma, \quad \Phi_a = \Phi_b, \tag{2}$$

leads to the following definition of the coefficients from (1):

$$\begin{aligned}
&E_a^0 - \varepsilon E_b^0 = 4\pi\sigma_0,\\
&q(F_q + \varepsilon G_q) = -4\pi\sigma_q, \quad G_q - F_q = (E_a^0 - E_b^0)\xi_q.
\end{aligned} \tag{3}$$

The quantities E_a^0, E_b^0 are expressed in terms of V, σ_0, ε, a, b, d:

$$\begin{aligned}
&aE_a^0 + bE_b^0 = V, \quad E_a^0 - \varepsilon E_b^0 = 4\pi\sigma_0\\
&\text{or}\\
&E_a^0 = \varepsilon V/(\varepsilon a + b) + 4\pi\sigma_0 b/(\varepsilon a + b),\\
&E_b^0 = V/(\varepsilon a + b) - 4\pi\sigma_0 b/(\varepsilon a + b).
\end{aligned} \tag{4}$$

The added pressure δP on the liquid helium surface, arising because of forces of electrostatic origin, can be obtained by variation of the energy

$$W = \int \varepsilon \frac{\mathbf{E}^2}{8\pi} d^3\mathbf{r},$$

that arises upon displacement of the charged interface from the position of equilibrium.

For simplicity in the following calculations, it makes sense to determine δP in two limiting cases.

a) The case in which $\varepsilon > 1, \sigma_0 = 0$. In this limiting case, the added pressure has the form

$$\delta P_s = \frac{q\xi_q}{4\pi}\frac{\varepsilon}{1+\varepsilon}(E_b^0)^2(\varepsilon - 1)^2. \tag{5}$$

In the case of liquid helium, this pressure is very small, since $\varepsilon - 1 \approx 0.06$.

b) The case in which $\varepsilon = 1, \sigma_0 \neq 0$. Here

$$\delta P_\sigma = (8\pi)^{-1}(E_b^{\perp} - E_a^{\perp})(E_b^{\perp} + E_a^{\perp}) = {}^1/_2\sigma(E_a^{\perp} + E_b^{\perp}). \tag{6}$$

The general expression for δP_σ (6) is divided into two terms:

$$\begin{aligned}
&\delta P_\sigma^0 = {}^1/_2\sigma_0(E_a^0 + E_b^0),\\
&\delta P_\sigma^q = (8\pi)^{-1} q[(4\pi\sigma_0)^2\xi_q + 4\pi\sigma_q(E_a^0 + E_b^0)q^{-1}],
\end{aligned} \tag{6a}$$

one of which corresponds to the static pressure on the helium surface, the other arises when the charged surface oscillates.

In the metallic approximation, when the curved charged surface of the helium is assumed to be electrically equipotential, we have $\Phi_q = 0$, where

$$\Phi_q \cos qx = \Phi_a|_{z=\xi_q\cos qx},$$

whence

$$F_q = -E_a^0\xi_q, \quad G_q = -E_b^0\xi_q. \tag{7}$$

As a result,

$$\sigma_q = \frac{q\xi_q}{4\pi}(E_a^0 + E_b^0), \quad \delta P_\sigma^q = \frac{q\xi_q}{4\pi}(E_a^{02} + E_b^{02}). \tag{7a}$$

If in addition to (7), we have the condition $E_a^0 = 0$ (the case of complete cancellation of the static electric field above the helium surface), we have from (4) $V = -4\pi\sigma_0 b$, $E_b^0 = -4\pi\sigma_0$,

$$\delta P_\sigma^q \approx q\xi_q \cdot 4\pi\sigma_0^2. \tag{7b}$$

In the opposite limiting case (immobile charges on the helium surface) when $\sigma_q = 0$, we have

$$\delta P_\sigma^q \approx {}^1/_2 q\xi_q \cdot 4\pi\sigma_0^2. \tag{7c}$$

Comparing the definitions (7b) and (7c), it is not difficult to see that in the case of immobile charges, the dynamical increment δP^q has the same structure as in the metallic approximation. Consequently, as is noted above (see the comments in Fig. 1a), even at zero mobility of the charges along the helium surface there are reasons for the development of an instability of the charged helium surface. The formal difference amounts only to the appearance of an additional numerical factor $\frac{1}{2}$, which increases the maximum density of the immobile surface charges n_s^{max} by a factor of $2^{1/2}$ relative to the case of mobile charges.

C. Taking into account the general expression (6) for the electrostatic pressure $\delta P_\sigma(q)$ and the corresponding definitions of the electric fields (1)–(3), and solving the set of equations of motion that describe the oscillations of the surface and the motion of the charges on the helium surface:

$$\begin{aligned}
&M(\dot v_q + v_q/\tau) = eE_{\parallel}^q, \quad E_{\parallel}^q \cos qx = \partial\Phi_a/\partial x|_{z=\xi_q\cos qx},\\
&i\omega\sigma_q + i\sigma_0 q v_q + Dq^2\sigma_q = 0, \quad D \approx \tau T/M,\\
&i\omega\rho\varphi - \alpha\Delta_2\xi - \rho g\xi = \delta P_\sigma(q)\cos qx,\\
&\Delta_3\varphi = 0, \quad \partial\varphi/\partial z|_0 = \dot\xi(x),
\end{aligned} \tag{8}$$

we can obtain, in the final analysis, the following expression for the dispersion law of the coupled electrocapillary oscillations of the charged helium surface:

$$-\rho\frac{\omega^2}{q} + \alpha q^2 + \rho g = \frac{q}{8\pi}\left\{(4\pi\sigma_0)^2 + \frac{2\pi e q\sigma_0(E_a^0 + E_b^0)^2}{M(\omega - iDq^2)[2\pi e\sigma_0 q/M(\omega - iDq^2) - \omega - 1/i\tau]}\right\}. \tag{9}$$

Here ρ and α are the density and surface tension of the liquid helium, g is the acceleration due to gravity, Δ_2 and Δ_3 are two-dimensional and three-dimensional Laplace operators, ω and q are the frequency and wave number of the oscillations of the charged helium surface, M is the effective mass of the charge in its motion along the surface, τ is the effective relaxation time in the motion of the charge along the surface, D is the corresponding diffusion coefficient, which is connected with the mobility of the charges μ by the Einstein relation $\mu = eDT^{-1}$, i.e., $D \approx \tau TM^{-1}$. A dispersion equation similar to (9) was obtained earlier,[7,8] but without diffusion terms $\sim D$. As will be seen from what follows, account of the possible diffusion of the charges along the helium surface because $D \neq 0$ can

have a significant effect on the structure of the dispersion law.

In fact, we introduce the characteristic parameter

$$\gamma=Dq^2\omega^{-1} \tag{10}$$

and estimate its scale in the neighborhood of the frequencies $\omega\sim10^2$ sec^{-1} and wave numbers $q\sim10^1$ cm^{-1}, which correspond to the region of instability of the charged helium surface. For electrons over helium under the conditions $M=m_e\approx10^{-27}$ g, $\tau_e\sim10^{-8}$ sec, and $T\approx1$ K, the quantity $\gamma_e\approx10^4\gg1$. In the case of ions on the interface of ^{3}He–^{4}He, when $M_i\approx(10^5-10^6)m_e$ and $\tau_i\approx10^{-11}$ sec, the typical value of $\gamma_i\approx10^{-4}-10^{-5}\ll1$.

Taking into account the large scales of the change of the parameter γ, we can simplify the initial expression (9) for $\omega(q)$:

$$-\rho\frac{\omega^2}{q}+\alpha q^2+\rho g=\frac{q}{8\pi}\left[(4\pi\sigma_0)^2+\frac{2\pi e\sigma_0(E_a^0+E_b^0)^2}{T(q+2\pi e\sigma_0/T)}\right],\quad \gamma\gg1, \tag{11a}$$

$$-\rho\frac{\omega^2}{q}+\alpha q^2+\rho g=\frac{q}{8\pi}\left[(4\pi\sigma_0)^2+\frac{2\pi e\sigma_0 q(E_a^0+E_b^0)^2}{\omega M(2\pi e\sigma_0 q/M\omega+i/\tau)}\right],\quad \gamma\ll1. \tag{11b}$$

Thus, the dispersion relation $\omega(q)$ depends essentially on the parameter γ. Thus, in the limiting case $\gamma\gg1$ the relaxation time τ drops out completely from the general expression for ω_q. In the opposite limiting case, $\gamma\ll1$, the expression for ω_q depends explicitly on τ. This difference has a simple explanation. The fact is that in the case of light and mobile charges, similar to electrons, the local density of the electrons $n_s(x)$ can follow the oscillations of the shape of the surface practically adiabatically. In other words, we can write down the approximation

$$\sigma(x)\approx\sigma_0\exp\left(-\frac{eE_\perp\xi}{T}+\frac{e\Phi}{T}\right),\qquad E_\perp=\tfrac{1}{2}(E_a^0+E_b^0). \tag{12}$$

Here $\Phi(x)$ is the self-consistent electrostatic potential along the liquid helium surface. Linearization of the relations (12) relative to ξ and Φ leads in the given case to a direct connection between σ_q and ξ_q, replacing the more complicated connection that follows from the solution of the electron equations of motion (8). Taking this connection into account and solving the remaining equations (8), we can obtain a dispersion law that is identical with the definition of ω_q (11a).

In the case of heavy and only slightly mobile helium ions, an adiabatic connection between $\sigma(x)$ and $\xi(x)$ of the form (12) has no time to arise. As a result, it is necessary to solve the complete set of equations of motion (8), which gives the final expression (11b) for $\omega(q)$ in the limiting case $\gamma\ll1$.

D. We now track the behavior of ω_q (11) following a gradual decrease in the mean charge density. In the case when ω_q (11a) is in the region

$$q\ll2\pi e\sigma_0/T=q_T \tag{13}$$

the dispersion law has the form

$$-\rho\frac{\omega^2}{q}+\alpha q^2+\rho g=\frac{q}{8\pi}[(4\pi\sigma_0)^2+(E_a^0+E_b^0)^2]. \tag{14}$$

This expression is identical with the definition of ω_q in Ref. 5 if we set $E_\perp^2=(E_a^0+E_b^0)^2$ in the latter. Under the additional conditions

$$E_a^0=0,\ V=-4\pi\sigma_0 b,\ E_b^0=-4\pi\sigma_0$$

the expression for ω_q (14) reduces to the well-known definition of ω_q in the metallic approximation[14]:

$$-\rho\frac{\omega^2}{q}+\alpha q^2+\rho g=\frac{q}{4\pi}(4\pi\sigma_0)^2. \tag{14a}$$

In reality, however, in the problem with independent V and σ_0, the combination $E_a^0+E_b^0$ has the following meaning according to (4):

$$E_a^0+E_b^0=\frac{2V}{d}+\frac{4\pi\sigma_0(b-a)}{d}. \tag{15}$$

The stability criterion for the dispersion of ω_q (14) runs as follows:

$$(4\pi\sigma_0)^2+(E_a^0+E_b^0)^2=16\pi(\rho g\alpha)^{1/2} \tag{16}$$

or, with account of the definition (15),

$$\begin{aligned}&(2\pi\sigma_0)^2+\left[\frac{V}{d}+2\pi\sigma_0\frac{b-a}{d}\right]^2=2(2\pi\sigma_m)^2,\\ &\sigma_m^2=\frac{1}{2\pi}(\rho g\alpha)^{1/2},\quad E_\perp^{max}|_{\sigma_0\to0}=\left(\frac{V}{d}\right)^{max}=2^{1/2}\pi\sigma_m.\end{aligned} \tag{17}$$

In terms of the variables $x=V/d$, $y=2\pi\sigma_0$, $\lambda=(b-a)/d$, $L^2=2(2\pi\sigma_m)^2$ the relation (17) has the form of a curve of second order in the XY plane:

$$x^2+2\lambda xy+(1+\lambda^2)y^2=L^2. \tag{17a}$$

In the case $\lambda=0$, i.e., in the case $b=a$, this curve is a circle of radius L with center at the origin. If $\lambda\neq0$, then we are dealing with an ellipse

$$\begin{aligned}&Ax_1^2+By_1^2=L^2,\\ &A=1+\tfrac{1}{2}\lambda^2(\sqrt5+1),\quad B=1-\tfrac{1}{2}\lambda^2(\sqrt5-1),\end{aligned}$$

in the canonical system of coordinates x_1y_1. The slope k of the canonical x_1 axis in the set of coordinates xy is equal to

$$k=\tfrac{1}{2}\lambda(1+\sqrt5).$$

It should be noted that not all the area of the discussed ellipse is a region of stability of the charged helium surface. It is necessary to exclude from this area portions for which the self-field of the charges over the helium surface exceeds the external field of the capacitor V/d. The boundaries of this region are defined by the condition $E_a^0=0$ or $V=-4\pi\sigma_0 b$. In the variables x,y, such a condition corresponds to the straight lines $x=\pm2(b/a)y$. As a result, for example for the cases $\lambda=0$ and $\lambda=0.9$, the regions of stability have the form shown in Fig. 4 (the cross hatched portions). A similar picture for the case $\lambda=0$ is shown in the review, Ref. 15.

It is interesting to note that upon satisfaction of the condition $Vd^{-1}\gg2\pi\sigma_0$ along with the inequality (13), the dispersion law (14) ceases to depend on the charge density:

$$-\rho\frac{\omega^2}{q}+\rho g+\alpha q^2=\frac{q}{2\pi}\left(\frac{V}{d}\right)^2. \tag{18}$$

A similar situation is maintained with decrease in σ_0 until such time that the inequality (13) is violated.

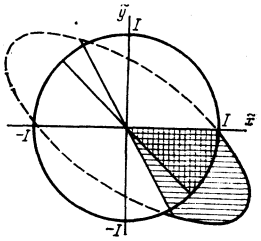

FIG. 4. Region of stability of the negatively charged helium surface upon independent variation of the potential difference V, the density of charges n_s, and the parameter $\lambda=(b-a)/d$. The graphs are drawn on the basis of (17a) in the relative variables $\tilde{x}=xL^{-1}$, $\tilde{y}=yL^{-1}$ for two values of λ: $\lambda=0$, $\lambda=0.9$. The definitions of x, y, L are given in the text.

In the transition to the region $q_T < q \ll \langle z\rangle^{-1}$, where q_T is defined in (13) and $\langle z\rangle$ is the mean thickness of the charged layer over the helium surface, the dispersion law (11a) takes a form that is qualitatively different from (14) and (18):

$$-\rho\frac{\omega^2}{q}+\alpha q^2+\rho g=V^2e\sigma_0/d^2T,\qquad q_T<q\ll\langle z\rangle^{-1}. \tag{19}$$

The definition (19) of $\omega(q)$ demonstrates how the effect of the finite charge density on $\omega(q)$ drops out in the given problem.

It must be noted that the inequalities (19) are mutually compatible under the condition $4\pi\sigma_0 \ll V/d$. If we deal with the solution of the problem under the conditions $4\pi\sigma_0 = V/d$, i.e., in the presence of complete cancellation of the external electric field over the charged surface of helium by the field of the surface electrons, then the inequalities $q > q_T$ and $q \ll \langle z\rangle^{-1}$ cannot be satisfied simultaneously. This limiting case should be investigated separately (see Ref. 16).

In the case of heavy charged particles, a "metallic" asymptote $\omega(q)$ of the form (14) is also possible. For this, as is clear from (11b), satisfaction of the inequality

$$2\pi e\sigma_0 q/M\omega > \tau^{-1} \tag{20}$$

is needed. Violation of the inequality (20) upon gradual reduction of the charge density σ_0 takes place much more simply than in the case of electrons over the helium. Thus, for $M \approx (10^5-10^6)m_e$ and $\tau_i \sim 10^{-1}$ sec, the inequality (20) does not hold in the density region $\sigma_0 \lesssim 0.2\sigma_m$ [σ_m is the maximum charge density of the electrons over the helium, defined in (17)]. However, the violation of the inequality (20) does not lead in the given case to a change in the value of the critical instability field $E_\perp^k$, since the condition for the appearance of the instability, $\omega^2 = 0$ at $q \neq 0$, automatically equates to zero the dissipative effect of the charge system on $E_\perp^k$. A more detailed investigation of the effect of heavy charged particles on $\omega(q)$ is contained in Ref. 8.

2. MANY-ELECTRON DIMPLES OVER HELIUM

As has already been noted in the introduction, the development of an instability of the charged helium surface can lead to the formation of individual stable dimples, filled with a large number of electrons, $N \approx 10^5$–10^7 (Fig. 5). Similar dimples appear under conditions of a weak charging of the initial, plane helium surface ($\sigma_0 \ll \sigma_m$) and sufficiently strong clamping field $E_\perp^k$, which exceeds a certain critical field $E_\perp^c$.

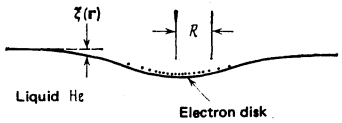

FIG. 5. Schematic form of the many-electron dimple.

In contrast to the instability under the conditions $\sigma_0 \lesssim \sigma_m$, which leads to the formation of a periodic deformation of the helium surface with characteristic period of the order of the inverse capillary constant, and which admits of a description in terms of nonlinear perturbation theory, the instability in the case $\sigma_0 \ll \sigma_m$ is accompanied by a significant rearrangement of the electron distribution over the helium surface. As a result, the formal problem turns out to be much more nonlinear in the given case and the problem of the self-consistent calculation of the critical conditions necessary for the formation and stable existence of many-electron dimples remains open at the present time. Nevertheless, using the experimental fact of the presence of such dimples with a fixed number of electrons on the liquid helium surface, and also the additional information on the uniformity of the electron distribution at the bottom of the dimple, we can attempt to find certain important parameters of the individual many-electron dimple: its characteristic geometric dimensions, the effective mass and mobility in motion of the dimple along the helium surface and so on.

A. Taking the above into account, we write out the total excess energy associated with the formation of a many-electron dimple on the helium surface:

$$W=\int d^2\mathbf{r}\left\{\alpha\left[(1+(\nabla\xi)^2)^{1/2}-1+\frac{\varkappa^2}{2}\xi^2\right]+eE_\perp n_s(\mathbf{r})\xi(\mathbf{r})+{}^1/_2 en_s(\mathbf{r})\varphi(\mathbf{r})\right\}, \tag{21}$$

$$\varkappa^2=\rho g/\alpha,\quad \int n_s\,d^2\mathbf{r}=N.$$

Here $\xi(\mathbf{r})$ is the self-consistent deformation of the helium surface under the action of the electron pressure, $n_s(\mathbf{r})$ is the local electron distribution along the surface of the dimple, N is the total number of electrons in the dimple, $\varphi(\mathbf{r})$ is the electrostatic potential in the vicinity of the charged dimple. The factor $\frac{1}{2}$ in front of the term $en_s\varphi$ implies the taking of the integral only with respect to one of the surfaces of the charged disk. The remaining notation in W (21) has been encountered previously.

The first two terms in the energy W arise because of the bending of the helium surface, which has a finite surface tension α, and because of the accompanying change in the gravitational energy. The surface term is written down for the general case of not too small curvature of the helium surface, when the derivative $\nabla\xi$ can take on values $\gtrsim 1$. Here the capillary term ceases to be a quadratic function of the deformation.

The third term determines the connection between the mechanical and electron parts of the problem and is written in the approximation of a sufficiently strong clamping field, when the increments of polarization origin, which are important in the general variant of the theory, can be neglected. Usually, these increments become unimportant in fields $E_\perp \gtrsim 300$ V/cm.[17] In our case we are dealing with clamping fields $E_\perp \gtrsim 3000$ V/cm. Therefore the absence of polarization terms in the interaction between the electrons and the helium surface is fully justified. Still another detail that needs comment is the difference between the general definition (6) of the pressure $\delta P_\sigma = \frac{1}{2}\sigma(E_a + E_b)$ of electrostatic origin on the bent charged surface of the helium and the particular definition $\delta P_\sigma = \sigma E_\perp$, $\sigma = en_s$ that follows from (21) when W is varied with respect to ξ. The reason for the apparent difference is that in the problem of one charged dimple with dimensions $R \ll a, b, d$, the values of the homogeneous electric field over (E_a^0) and under (E_b^0) the helium surfaces are identical: $E_a^0 = E_b^0 = E_\perp$. As a result, according to (3), the possible nonuniform fields above and below the dimples should satisfy the condition $G_q = F_q$. Using this equality and the definitions of E_a and E_b (1), it is not difficult to establish the fact that

$$^1/_2(E_a+E_b) = {}^1/_2(E_a^0+E_b^0) = E_\perp.$$

Thus, the general expression for δP_σ (6) is equivalent in the given case to the definition $\delta P = \sigma E_\perp$.

We assume that

$$\nabla\xi < 1, \quad 0 \leqslant r < \infty. \tag{22}$$

This allows us to linearize the surface term in the total energy W (21). The validity of such an assumption can be verified in what follows. As a result,

$$W = \int d^2\mathbf{r}\left\{\frac{\alpha}{2}[(\nabla\xi)^2 + \varkappa^2\xi^2] + eE_\perp n_s(r)\xi(r) + \frac{1}{2}en_s(r)\varphi(r)\right\}. \tag{23}$$

The variation of W (23) with respect to $\xi(\mathbf{r})$, $\varphi(\mathbf{r})$ and $n_s(\mathbf{r})$ under the condition

$$\int n_s\, d^2\mathbf{r} = N$$

leads to the equations of mechanical equilibrium for ξ, to Poisson's equation for φ and to the connection between ξ and φ on the surface of the dimple, which has the meaning of the condition that the chemical potential be constant along this surface:

$$\begin{gathered}
|\Delta_2\xi - \varkappa^2\xi = \alpha^{-1}eE_\perp n_s(r), \\
\xi'(0) = 0, \quad \xi(\infty) \to 0, \quad \Delta_3\varphi = 0, \\
\varphi\big|_{\substack{z\to\pm\infty \\ r\to\infty}} \to 0, \quad \frac{\partial\varphi}{\partial z}\bigg|_{+0} - \frac{\partial\varphi}{\partial z}\bigg|_{-0} = -4\pi e n_s(r), \\
eE_\perp\xi(r) + e\varphi(r) = \lambda, \quad 0 \leqslant r \leqslant R, \quad r = |\mathbf{r}|.
\end{gathered} \tag{24}$$

The origin of the cylindrical coordinates is identical with the center of the dimple, Δ_2 and Δ_3 are two-dimensional and three-dimensional Laplace operators, λ is the Lagrangian multiplier which ensures the conservation of the total number of electrons N, and R is the assumed radius of the electron spot in the dimple (in the region $r \leqslant R n_s > 0$).

Using the Bessel transform for $\xi(r), \varphi(r), n_s(r)$:

$$\xi(r) = \int_0^\infty \xi(q) J_0(qr)\, q\, dq,$$

$$\varphi(r) = \int_0^\infty n_s(q) \exp(-|z|q) J_0(qr)\, dq,$$

$$n_s(q) = 2\pi e \int_0^\infty n_s(r) J_0(qr)\, r\, dr,$$

we can rewrite the system (24) in the following form:

$$\begin{gathered}
e^2 \int_0^\infty dq\, J_0(qr) n_s(q) \left(1 - \frac{E_\perp^2}{2\pi\alpha}\frac{q}{q^2+\varkappa^2}\right) = \lambda, \quad r \leqslant R, \\
\int_0^\infty dq\, J_0(qr) n_s(q) = 0, \quad r > R,
\end{gathered} \tag{25}$$

where $J_0(x)$ is a Bessel function of order zero. Thus, the problem of the many-electron dimple reduces to the solution of paired integral equations in $n_s(q)$. The methods of solution of such equations have been described, for example, in Ref. 18.

Noting that the first of the relations (25) is satisfied in the region $0 \leqslant r \leqslant R$, we can define the quantity λ as the value of the left side of this equation at $r = 0$. As a result, Eqs. (25) take the form

$$\begin{gathered}
\int_0^\infty [J_0(qx) - 1]\left(1 - \frac{q}{q^2+\varkappa^2R^2}\right) n_s(q)\, dq = 0, \quad x \leqslant 1, \\
\int_0^\infty J_0(qx) n_s(q)\, dq = 0, \quad x > 1, \\
x = r/R, \quad R = 2\pi\alpha/E_\perp^2.
\end{gathered} \tag{25a}$$

In the limiting case $\varkappa R \ll 1$, which is of real interest, the combination $f(q) = q/(\varkappa^2R^2 + q^2)$ in the first of Eqs. (25a) can be simplified to $f(q) \to q^{-1}$. Here, as is easy to show, taking into account the limiting behavior

$$n_s(q)\,|_{q\to 0} \to \text{const},$$

the integral remains convergent. As a result, the set of equations (25a) becomes nondimensional. This means that if the many-electron dimple exists, then in the limiting case $\varkappa R \ll 1$ the radius of the dimple R does not depend on N and, in accord with (25a), it is determined by the expression

$$R_0 = c\alpha/E_\perp^2, \quad c \approx 1, \tag{26}$$

where the constant c depends on the details of the solution of the set of equations (25a).

B. In the variational solution of the problem of the many-electron dimple, proposed in Ref. 19, an appropriate, for example, Gaussian, distribution of the electron density $n_s(\mathbf{r})$ is postulated on the surface of the dimple:

$$n_s(r) = \frac{N}{\pi R^2}\exp\left(-\frac{r^2}{R^2}\right). \tag{27}$$

The calculation of the deformation and of the Coulomb parts of W (23) gives the following final result in this case:

$$W = \frac{Q^2E_\perp^2}{8\pi\alpha}\exp\left(\frac{\varkappa^2R^2}{2}\right)\mathrm{Ei}\left(-\frac{\varkappa^2R^2}{2}\right) + \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}\frac{Q^2}{R}. \tag{28}$$

Here $Ei(x)$ is integral exponential function and $Q = eN$.

The determination of the radius R now reduces to the solution of the equation $\partial W/\partial R \equiv W' = 0$ under the condition $W'' > 0$. It is not difficult to show that such a solution exists only in the region $\varkappa R < 1$. Using the asymptote of $Ei(x)$ at small x, we find that the definitions of R, W and $\xi(0)$ in the region $\varkappa R \ll 1$ have the following form:

$$R_0 = \frac{(2\pi)^{1/2}\alpha}{2E_\perp^2}, \quad W = -\frac{Q^2E_\perp^2}{4\pi\alpha}\left(\ln\frac{2^{1/2}\gamma^{-1/2}}{\varkappa R_0} - 1\right),$$
$$\xi(0) \approx -\frac{QE_\perp}{2\pi\alpha}\ln\frac{1}{\varkappa R_0}, \quad \varkappa R_0 \ll 1, \quad \gamma = 1.78, \tag{29}$$
$$W'' = Q^2E_\perp^2/4\pi\alpha R_0^2 > 0.$$

It is obvious that the variational definition of R_0 in (29) correlates with the general result for R_0 from (26) with accuracy to within a number ~1.

In the region $\varkappa R \lesssim 1$, the asymptotes (29) become inaccurate and it becomes necessary to use numerical methods for the determination of the corresponding quantities. It is convenient here to represent the energy (28) in the following form:

$$\tilde{W} = 2\left(\frac{2}{\pi}\right)^{1/2}\frac{W}{Q^2\varkappa} = \left[s\exp\left(\frac{x^2}{2}\right)\mathrm{Ei}\left(-\frac{x^2}{2}\right) + \frac{1}{x}\right], \quad s = \frac{1}{2\varkappa R_0}, \quad x = \varkappa R, \tag{28a}$$

where R_0 is defined in (29). Introducing s and constructing the system of equations $\tilde{W}(s, x)$, we can find the position of the minimum X of this function and its value at the minimum point. This information suffices to determine the properties of R_0 and $\tilde{W}$ in the vicinity of $\varkappa R \lesssim 1$. Thus, the minimum of the function $\tilde{W}$ first appears at $s = s_{\min} \approx 0.90$ at the point $X_{\max} \approx 1.14$. The function $\tilde{W}$ first goes to zero at $s = s^* = 1.05$ at the point $X^* = 0.72$. The dependence of X on the parameter $1/2s = \varkappa R_0$ in the region $s \geqslant s_{\min}$ is shown in Fig. 6. This dependence gives an idea of the value of the true dimensionless radius of the dimple X in comparison with its asymptotic value $\varkappa R_0$.

There is also sense in calculating the total energy of the charged surface of helium under conditions of occupation by electrons of all the available area under the electrodes of the capacitor. In this case, we need to assume that the gradients $\nabla\xi$ are insignificant, that the quantity n_s does not depend on r, that the total area S occupied by the electrons is sufficiently large, $S \gg \varkappa^{-2}$, that the self-field of the electrons is small in comparison with the applied field of the capacitor, $2\pi e n_s \ll E_\perp$,

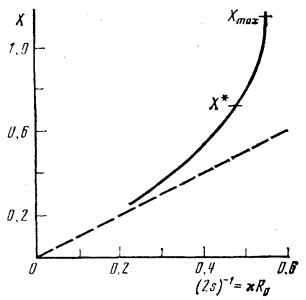

FIG. 6. Behavior of the dimensionless radius X as a function of the parameter $(2s)^{-1} = \varkappa R_0$. The definitions of X, $X_{\max}$, X^*, s, R_0 are given in the text.

and that the charged helium surface is located for simplicity at the median plane of the capacitor, i.e., $a = b$ (see Fig. 2).

The corresponding total energy W turns out here to be equal to

$$W \approx -3E_\perp^2Q^2/8\rho g S, \quad Q = en_sS. \tag{30}$$

Equating the energies W (28a) and (30) leads to an equation which can be used as the definition of the critical field $E_\perp^{\min}$ for the transition of the electrons from the nonlocalized state to the dimple:

$$\tilde{W}(s, x) = -\pi s/x^2 S,$$

$\tilde{W}(s, x)$ and s are defined in (28a). Under the conditions $\varkappa^2 S \gg 1$, this equality reduces to the requirement $\tilde{W} \approx 0$. As was shown above, the function W vanishes at $s = s^* \approx 1.05$ and $X^* = 0.72$. It can then be concluded that the formation of the dimple becomes advantageous if

$$E_\perp > E_\perp^{\min}, \quad (E_\perp^{\min})^2 = (2\pi)^{1/2}s^*\varkappa\alpha = (16.52 \pm 0.005)\varkappa\alpha. \tag{31}$$

The definition of $E_\perp^{\min}$ (31) becomes literally identical with the other possible definition of $E_\perp^{\max}$, which follows from analysis of the behavior of the dispersion law of the charged helium surface under conditions of a small degree of charging [see the definition of $E_\perp^{\max}$ (17)], but the numerical coefficient in the definition of $E_\perp^{\max}$ of (31) is somewhat larger.

Having the definitions of $\xi(0)$ and R_0 (29), it is not difficult to determine the region of applicability of the results in the sense of the applicability of inequality (22) or in more appropriate terms $\xi(0)/R < 1$. Written in explicit form, this inequality becomes

$$\frac{QE_\perp^3}{\pi(2\pi)^{1/2}\alpha^2}\ln\frac{1}{\varkappa R_0} < 1. \tag{32}$$

At a fixed value of $E_\perp$ the relation (32) is bounded from above by the total number N of charges in an individual multi-electron dimple. For an electric field $E_\perp \gtrsim E_\perp^{\min}$ [$E_\perp^{\min}$ from (31)] and for the remaining parameters that are characteristic of liquid helium, the quantity $N_{\max}$ has the scale $N_{\max} \approx 10^6$.

Thus, over a wide range of values N, $10 \lesssim N \lesssim 10^6$, autolocalization of the system of surface charges is possible in the multi-electron dimples and also linearized description of this localization.[3)] The localization radius R_0 from (29) does not depend on N here and satisfies the inequality $\varkappa R_0 < 1$. The maximum displacement $\xi(0)$ satisfies the inequality $\xi(0)/R < 1$ right up to $N \lesssim N_{\max}$. The energy decrease associated with the appearance of the multi-electron dimple at $E_\perp \gtrsim E^{\min}$ and $N \sim 10^5$ has, according to the definition of W (29) the scale $W \sim 10^{-7}$–10^{-8} erg, or $W \sim 10^4$–10^5 eV. This energy is macroscopically large in comparison with the characteristic helium temperatures, and therefore the multicharged dimple, although it is in a metastable state, is little subject to thermal fluctuations and can exist a rather long time. A brief communication on the properties of the multi-electron dimple on the helium surface has been published in Ref. 19.

C. The analogy mentioned above between the one- and many-electron dimples is preserved in the calculation of the dynamic characteristics of the dimple along the helium surface. For example, the determination of the effective mass M is obtained in the following way[20]:

$$M=\rho v_d^{-2}\int \varphi_s \frac{\partial \varphi_s}{\partial z}\Big|_0 d^2\mathbf{r},$$
$$\nabla^2\varphi_s=0,\quad \frac{\partial\varphi_s}{\partial z}\Big|_0 = v_d\cos\theta\frac{\partial\xi}{\partial r},\quad \varphi_s|_{z\to\infty}\to 0. \tag{33}$$

The angle θ of the cylindrical coordinate system located at the center of a dimple moving with a velocity v_d is measured from the direction of motion, $\xi(r)$ is the deformation of the surface of helium under the action of the electron pressure. Under completely adiabatic conditions, when the deformation of the surface has time to adjust itself to the motion of the electron spot, we can use for $\xi(r)$ the expression obtained in the static problem. The condition of applicability of such an approximation is

$$Mv_d^2<W, \tag{34}$$

where W is defined in (28).

The solution of the problem (33), carried out in Ref. 20 in connection with the calculation of the effective mass of a single-electron dimple, leads to the following final expression for M:

$$M=\rho Q^2E_\perp^2/16\alpha^2\varkappa. \tag{35}$$

It is interesting to note that the value of M does not depend on R. It is obvious also that, in the case of a surface dimple, the value of the attached mass M does not bear a direct relation to the total volume of the dimple, as occurs, for example, in the case of a body moving in the volume of a liquid. Actually, the volume of the dimple

$$v=2\pi\int\xi(r)r\,dr=QE_\perp/\alpha\varkappa^2 \tag{36}$$

is proportional to the first power of the clamping and to the second power of the capillary constant: $v\approx\xi(0)\varkappa^{-2}\approx QE_\perp\alpha^{-1}\varkappa^{-2}$, where $\xi(0)$ is defined in (29). At the same time, the quantity M is quadratic in $E_\perp$ and contains the single power of $\varkappa$. The numerical value of M at $N\approx10^5$ and $E_\perp\approx3000$ W/cm is $M\approx3\times10^{-9}$ g.

A similar generalization to the case of a many-electron dimple is possible also in the problem of its mobility along the helium surface. The final result for the mobility μ equivalent to the Stokes mobility of the helium ion in a liquid-helium volume has the form

$$\mu=2(2\pi)^{1/2}\alpha^2R_0/\eta QE_\perp^2. \tag{37}$$

Here η is the coefficient of the first viscosity of liquid helium.

A comparison of the mobility μ of the many-electron dimple with the Stokes mobility of an isolated helium ion in a liquid-helium volume

$$\mu_\pm=e/6\pi\eta R_\pm$$

shows that the effective mobility of the dimple is much greater than $\mu_\pm$:

$$\mu/\mu_\pm=18\pi^2(2\pi)^{1/2}\alpha^2R_\pm/Ne^2E_\perp^2.$$

Thus, at $N\approx10^5$ and $E_\perp\approx3000$ V/cm, the ratio $\mu/\mu_\pm\sim10^3$–10^4. This is not surprising, since the effective charge of the dimple is very large, and the geometry is significantly different from the geometry of an ordinary sphere, which promotes the increase in the mobility of the dimple.

It is of interest to estimate the role of the wave resistance in the motion of the dimple along the helium surface. Taking into account the explicit form of the ripplon spectrum

$$\omega^2=gq+\alpha q^3/\rho,$$

it is not difficult to determine that the minimum group velocity v_{min} for such a spectrum is approximately equal to

$$v_{min}\approx(g\varkappa^{-1})^{1/2}.$$

The numerical value of $v_{min}\approx10$ cm/sec.

Thus, the wave resistance turns out to have a threshold and arises only when the dimple goes along the surface with a velocity $v_d>v_{min}$. Under similar conditions, the kinetic energy $W_{kin}\approx Mv_d^2$ of the moving dimple becomes comparable with the energy of localization of the electrons in the dimple, $W_{kin}\lesssim W$, where W is defined in (29). As a result, a self-consistent theory of wave resistance should take into account the effect of motion of the dimple on the localization of the electron cloud in the dimple. This problem will not be discussed here.

D. In concluding this section, we determine the form of the interaction between two dimples, located at a distance $r=|\mathbf{r}_i-\mathbf{r}_j|$ from each other (here $\mathbf{r}_i$ and $\mathbf{r}_j$ are the coordinates of the centers of two dimples). For calculation of W_{ij}, we need to substitute in W [Eq. (23)] the deformation and the distribution of the charges for the two individual dimples, and to separate the interference terms. It is convenient to carry out the concrete calculations by using a somewhat transformed expression for W. Namely, taking into account the equation of mechanical equilibrium (24) and the identity transformations in the definition of W in (23), we bring the energy to the form

$$W=\frac{1}{2}\int QE_\perp\xi(\mathbf{r})n_s(\mathbf{r})d^2\mathbf{r}+V_e. \tag{38}$$

Assuming now that the deformation of the helium surface is created by the superposition of two solutions

$$\xi(\mathbf{r})=\xi_1(\mathbf{r})+\xi_2(\mathbf{r}),\quad n_s(\mathbf{r})=n_1(\mathbf{r})+n_2(\mathbf{r}),$$

where

$$\xi_i(\mathbf{r})=-\frac{QE_\perp}{2\pi\alpha}K_0(\varkappa|\mathbf{r}-\mathbf{r}_i|),\quad n_i(\mathbf{r})=\frac{1}{\pi R^2}\exp\left(-\frac{|\mathbf{r}-\mathbf{r}_i|}{R^2}\right),$$

we find from (38) the energy of interaction of two dimples of like charge

$$W_{ij}=-\frac{Q^2E_\perp^2}{4\pi\alpha}K_0(\varkappa r)+\frac{Q^2}{r} \tag{39}$$

or

$$W_{ij}=Q^2\varkappa\left[-\frac{3\pi}{8\varkappa R}K_0(x)+\frac{1}{x}\right],\quad x=\varkappa r,$$

where $K_0(x)$ is a Bessel function of imaginary argu-

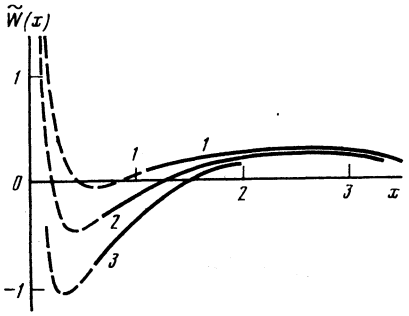

FIG. 7. Graphs of the dimensionless energy $\tilde{W}(x) = W_{ij}/\varkappa Q^2$ as functions of $x = \varkappa r$ for three values of $\varkappa R$: 0.56 (curve 1); 0.43 (curve 2); 0.33 (curve 3). The dashed curves indicate the regions in which the determination of $\tilde{W}(x)$ is inaccurate.

ment, $r = |\mathbf{r}_i - \mathbf{r}_j|$, and R is defined in (29). At small distances, this interaction has the character of a Coulomb repulsion, followed by a region of attraction of deformation origin, and finally, in the region $r \gg \varkappa^{-1}$, the Coulomb repulsion again dominates. In the case of oppositely charged dimples, the interaction has a form similar to (39), but with opposite sign. The graph of the dependence of the dimensionless energy of interaction $\tilde{W}(x)$ of two like-charged dimples, $\tilde{W} = W_{ij}Q^{-2}\varkappa^{-1}$, on the dimensionless distance $x = \varkappa r$ for different $\varkappa R$ (0.56, 0.43, 0.33) is shown in Fig. 7. At distances $r > 2R$, where R is the radius of the individual dimple, the definition of W_{ij} from (39) loses meaning. The corresponding parts of the graphs of Fig. 7 are denoted by dashed lines. It follows from Fig. 7 that the minimum of the functions $\tilde{W}(x)$, which determines the stationary distance between like-charged dimples, is located in the region marked by the dashes. Consequently for the description of the bound state of a complex of two like-charged dimples, we need a more accurate calculation of the function $\tilde{W}$ at distances $r \sim R$. As for the problem of the bound state of two oppositely charged dimples, the location of the equilibrium is determined in this case by the extremum of the function $\tilde{W}(x)$ on Fig. 7 in the region $x \approx 2.5$ (the maximum in Fig. 7). This extremum is located in the region of applicability of the definition of W_{ij} (39) and therefore has a precise meaning.

CONCLUSION

We now present some summaries. We studied in this paper the spectrum of oscillations of a charged helium surface under the conditions of an arbitrary degree of charging δ of this surface. In the limiting case $\delta \lesssim 1$, the general expression (9) for the dispersion of ω_q goes over into the previously known asymptote. If now $\delta \ll 1$, then new dependences $\omega(q)$ arise which allow us to investigate the effect of surface charges on ω_q in the entire interval of δ down to $\delta - 0$ [see the definitions (14), (18) and (19) of ω_q].

It was shown that upon development of an instability of a charged helium surface under the conditions $\delta \ll 1$, the formation of individual many-electron dimples is possible. Using the assumption that the density of charges n_s on the bottom of the dimple has a homogeneous distribution [see the definition (27) of n_s], for which there are experimental grounds, we can calculate various characteristics of an individual many-electron dimple: the radius R of the electron spot, the coupling energy W and the critical field $E_\perp^{min}$ necessary for the formation of a many-electron dimple, the effective mass M and the mobility of the dimple μ in its motion along the helium surface, and the interaction energy W_{ij} of two dimples with like and opposite charges located at a finite distance of one from the other. In the calculation of W_{ij} it turned out that this energy, contains besides the Coulomb term an additional term of deformation origin. The deformation term has a sign that is opposite to that of the Coulomb term, and leads to a significant renormalization of the interaction. As a result, in the region of sufficiently strong applied fields $E_\perp$ having a scale $E_\perp > E_\perp^{min}$, formation of a bound complex of oppositely charged dimples is possible. The problem of a bound state of a complex of two like-charged dimples needs additional study.

The bulk of the research was completed during the visit of one of the authors (V.Sh.) at the Munich Technical University. He takes this occasion to express his deep gratitude to Professor H. Kinder and this university and also to Dr. P. Leiderer for his kind invitation, hospitality and fruitful collaboration.

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[2)]The details of the corresponding experiment, carried out by one of the authors (P. L.) and the quantitative information following from it will be published separately. Here there is sense in noting that we are dealing with a cell of diameter ~8 cm. distance between the plates of the capacitor $d \approx 1.5$ cm, a lower glass plate, which is covered with gold is used additionally as a mirror, upper plate, also of glass, is made semitransparent (covered by a conducting, transparent layer of InO), which allows one to observe the state of the helium surface along the normal to the surface through the upper plate; the helium surface is located in the middle part of the cell, the number of electrons in a single dimple $N \approx 10^6$–10^7, the temperature $T \approx 4°$ K, the distance between neighboring dimples $\sim 10^{-1}$ cm.

[3)]A similar phenomenon appears also in the case of a single electron located on the surface of helium in a strong field $E_\perp$. The difference is only that the Coulomb interaction between the electrons is replaced by the energy of the zero-point oscillations of a single electron. [20]

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