# Absorption of sound in doped piezoelectric semiconductors

M. B. Gitis and I. A. Chaĭkovskiĭ

*Institute of Applied Physics*

Measurements were made of the acoustic absorption coefficients of CdS and CdSe samples as a function of the electrical conductivity. This conductivity was controlled by varying illumination and temperature in the range 300–1000 °K; it was measured under dc and ac conditions. The experimental results did not fit the theories of absorption of sound by homogeneous piezoelectric semiconductors. A calculation was made of the absorption coefficients of doped piezoelectric semiconductors allowing for an internal random large-scale potential resulting from fluctuations of the impurity concentration. This approach made it possible to account for the experimental results not only qualitatively, but also in respect of the order of magnitude. The possibilities of simultaneous acoustic and electrical determination of the parameters of the internal random field and chemical potential were considered.

## 1. INTRODUCTION

When an acoustic wave travels in a piezoelectric semiconductor crystal oriented in such a way that the strain caused by this wave is accompanied by a longitudinal electric field (with the field vector **E** parallel to the wave vector **k**), conduction electrons interact with the field and this gives rise to energy absorption from the elastic wave.

The absorption of sound in piezoelectric semiconductors has been investigated on many occasions both experimentally and theoretically, particularly in connection with the problems of amplification of sound (see the reviews in Refs. 1–3). The earliest investigations[4] have established that, in spite of the basic agreement between the theory and acoustic experiments, it is usually necessary to allow for additional factors. An improvement in the agreement between the theory and experiment results if allowance is made for the trapping of carriers. An equilibrium between the trapping levels and the conduction band is established at a finite rate and, therefore, it affects the processes of formation and drift of electron bunches in an electric field. This approach[4,5] accounts for the experimentally observed reduction in the absorption coefficient (gain) of sound and for the shift of the amplification threshold relative to the values predicted by the theory for ideal crystals and considered as a function of the ratio of the frequency of ultrasound $\omega$ and the time $\tau_{eq}$ taken to establish an equilibrium between the conduction band and trapping levels. However, the subsequent accumulation of experimental observations has resulted in new discrepancies between the theory and experiment.[6–11]

Some piezoelectric semiconductor crystals are characterized by an anomalously strong absorption of elastic waves, a spectral dependence of the absorption coefficient of sound on the wavelength of illumination, disagreement (with the theory) in respect of the position of the maximum of the acoustic absorption coefficient in the dependence on the electrical conductivity of a crystal, etc. This has stimulated new theoretical treatments in which the experimental results have been explained by refining earlier concepts but without affecting the main concept of the band energy structure of a crystal. For example, the possibility of existence of several types of trap with different depths is considered in Ref. 9 and this gives rise to an additional redistribution of the captured carriers between the traps in ultrasonic wave fields. The deformation interaction is allowed for in Ref. 10 in addition to the piezoelectric electron–phonon interaction. Gulyaev *et al*.[8] put forward the hypothesis that a piezoelectric semiconductor crystal may have electron impurity states with "built-in" dipole moments.

This approach makes it possible, at least qualitatively, to account for the existing experimental results with the exception of the dependence of the absorption coefficient of sound on the electrical conductivity of a crystal.

However, the following extremely important circumstance has been ignored in the interpretation of the experimental data on the propagation of elastic waves interacting with conduction electrons in a piezoelectric semiconductor. Practically all the important piezoelectric semiconductors of the II-VI type exhibit a strong photoconductivity, which is always used to optimize the conditions for the interaction of elastic waves and conduction electrons (conductivity control). The high photoconductivity is frequently accompanied by slow relaxation, whose characteristic time may range from tens of seconds to hours. None of the models which allow for the presence of trapping levels in the band gap can predict such very long relaxation times.

It can be regarded as established that the long photoconductivity relaxation times are due to collective energy barriers which appear in crystals because of various inhomogeneities.[12,13] In macroscopically homogeneous crystals the source of such inhomogeneities may be, for example, fluctuations of the donor or acceptor concentration. This accounts for many electrical and optical properties of semiconductor crystals. However, the role of these mechanisms in the acoustic processes and the possibility of invoking them to interpret the experimental results on piezoelectric semiconductors has hardly been considered, although it is quite clear that

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the physical processes responsible for electrical conduction and for the absorption of long-wavelength sound by conduction electrons (redistribution of the electron density in the electric field of an elastic wave) are analogous. Moreover, practically all II-VI crystals are strongly compensated during growth.[14] Therefore, they are necessarily characterized by a large-scale potential which modulates the spatial distribution of the carrier density.

Our aim was a more detailed investigation of the mechanism of the absorption of sound in piezoelectric semiconductors and its relationship to the electrical conductivity of a crystal and to the inhomogeneous distribution of carriers in a sample.

## 2. EXPERIMENTAL RESULTS

The earliest investigations[4,5] established that the maximum absorption coefficient (gain) of sound $\alpha$ occurs at frequencies $\omega$ which depend on the Maxwellian relaxation frequency $\omega_M = \sigma/\varepsilon$ ($\sigma$ is the electrical conductivity and $\varepsilon$ is the permittivity) and on the diffusion frequency $\omega_d = v_0^2/d$ ($v_0$ is the velocity of sound and $d$ is the electron diffusion coefficient). Since under experimental conditions it is difficult to vary $\omega_d$, it is usual to alter $\omega_M$—for a fixed value of $\omega$— by altering the electrical conductivity because of photo-excitation.

The electrical conductivity can be also altered by varying temperature. Measurements of $\alpha$ were carried out in this way on cadmium sulfide samples at low[15] and high[11] temperatures. These investigations showed that in the determination of $\alpha$ as a function of temperature $T$ at $\omega \ll \omega_d$ there were relaxation maxima similar to those observed when $\alpha$ was determined as a function of the illumination intensity.

Comprehensive investigations of $\alpha$ as a function of the electrical conductivity for the same samples, when $\alpha$ was varied within wide limits either by heating or illumination, have not yet been carried out although they should give an experimental answer to the question how equivalent are the two approaches because, according to the theories which allow for the presence of trapping level, the absorption coefficient of sound in the $\omega \ll \omega_d$ case should depend only on the ratio of $\omega$ and $\omega_M$ and be independent of the method used to vary $\sigma$ (see below).

We investigated the absorption coefficient of sound of cadmium sulfide and selenide crystals by the echo pulse method at frequencies of $10^7$–$10^8$ Hz, which ensured that the condition $\omega \ll \omega_d$ was well satisfied ($\omega_d \approx 10^{10}$ sec$^{-1}$). The dark conductivity of CdS samples was $10^{-7}$–$10^{-10}$ $\Omega^{-1} \cdot$ cm$^{-1}$, and that of CdSe was $10^{-8}$–$10^{-9}$ $\Omega^{-1} \cdot$ cm$^{-1}$. The conductivity was measured under dc conditions. Ohmic contacts were formed by evaporation of aluminum. Variation of the illumination intensity made it possible to increase the conductivity to $10^{-3}$ $\Omega^{-1} \cdot$ cm$^{-1}$, which reversed the inequality from $1 \gg \omega/\omega_M$ to $\omega/\omega_M \gg 1$. Variation of temperature within the range 300–900 °K made it possible to alter $\sigma$ so that either of these two inequalities was satisfied. The method used in the high-temperature acoustic measurements and the apparatus employed were described in Ref. 16. The direction

of the hexagonal axis was determined by x-ray diffraction and by acoustic methods using the velocities of the longitudinal and transverse waves. The macroscopic homogeneity of the samples was checked by depositing aluminum stripe electrodes 0.5 mm wide at intervals of 1 mm and determining the potential difference between them. The selected samples were homogeneous within the limits of the experimental error. Moreover, the electromechanical coefficient, whose value governed the electron–phonon interaction, was determined. This was done by exciting elastic vibrations in a sample due to the intrinsic piezoelectric effect and measuring their amplitudes. The piezoelectric properties of CdS and CdSe samples were identical to within 15%.

Measurements of $\alpha$ were made for longitudinal and shear waves along directions ensuring an effective electron–phonon interaction via the piezoelectric potential (piezoelectrically active directions) and also along other directions. In spite of the preliminary selection of the crystals and the identity of their macroscopic properties, the nature of the dependences of the absorption coefficient of sound on the electrical conductivity depended strongly on whether the conductivity was altered by illumination or by heating.

Only some of the samples exhibited an absorption maximum when the condition $\omega/\omega_M = 1$ was satisfied, irrespective of the method used to vary the conductivity. In most cases the slow (lasting minutes) relaxation of the photoconductivity gave rise to considerable deviations. For one batch of samples there was no significant change in $\alpha$ of the piezoelectrically active waves when the conductivity was altered by illumination and this was in spite of the fact that the whole range from $1 \gg \omega/\omega_M$ to $\omega/\omega_M \gg 1$ was covered. On the other hand, when $\sigma$ was altered within the same range by an increase in temperature, there was a maximum of $\alpha$ on the $\sigma$ scale but its position did not coincide with the condition $\omega/\omega_M = 1$ (Fig. 1). For another batch of samples the absorption of sound did have an extremum in its depen-
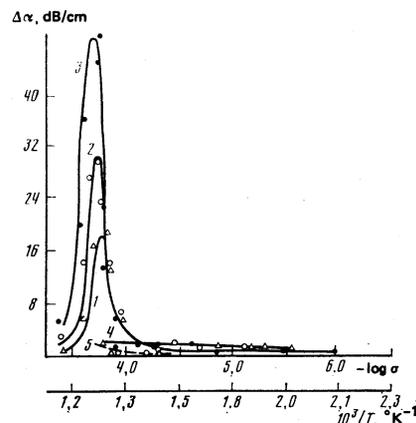


FIG. 1. Dependences of the absorption coefficient of longitudinal sound on the electrical conductivity $\sigma$ of a CdS single crystal: the continuous curves represent piezoelectrically active waves and the dashed curve represents waves which are not piezoelectrically active; 1)-3), 5) conductivity $\sigma$ was altered by varying temperature; 4) conductivity $\sigma$ was altered by illumination; $\triangle$) 15 MHz; $\bigcirc$) 25 MHz; $\bullet$) 55 MHz.
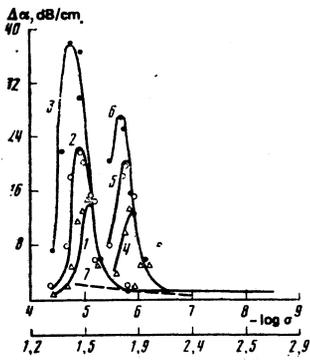
FIG. 2. Same as Fig. 1: 1)-3), 7) conductivity $\sigma$ altered by varying temperature; 4)-6) conductivity $\sigma$ altered by illumination.



FIG. 4. Temperature dependences of the absorption coefficient of shear waves of a CdS single crystal. The notation is the same as in Fig. 1.

dence on the electrical conductivity, but the values of the conductivity corresponding to the maximum $\alpha$ varied strongly depending on the method used to control $\sigma$ (Fig. 2). The values of $\sigma_T$ at the temperature maxima of $\alpha$ were always several times higher than the values of $\sigma_I$ corresponding to the maxima of $\alpha$ when the illumination intensity was varied. The other samples had acoustic properties intermediate between the two groups just described. The results plotted in Figs. 1 and 2 were obtained for CdS crystals. Similar results were recorded also for CdSe. One of the dependences is plotted in Fig. 3 for longitudinal piezoelectrically waves. The results are independent of the nature of the piezoelectrically active wave. By way of example, Fig. 4 shows the dependence of $\alpha$ on the conductivity obtained for shear waves in a sample of CdS showing no maximum of $\alpha$ as a function of the illumination intensity.

The electrical conductivity of a crystal did not influence the absorption of elastic waves traveling along non-piezoelectrically active directions (dashed curves in Figs. 1-3). An increase in temperature caused $\alpha$ to rise monotonically within a narrow range and the frequency dependence of the absorption coefficient was very weak. The nature of the changes observed was very similar to the high-temperature absorption of sound in silicon reported earlier.[17] In other words, in the investigated range of frequencies, conductivities, and temperatures the mechanisms of the absorption of sound not as-
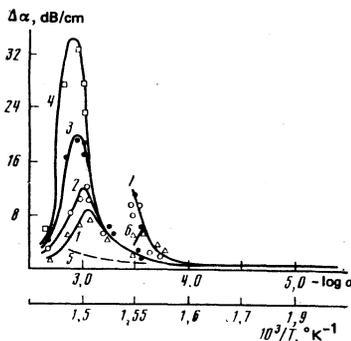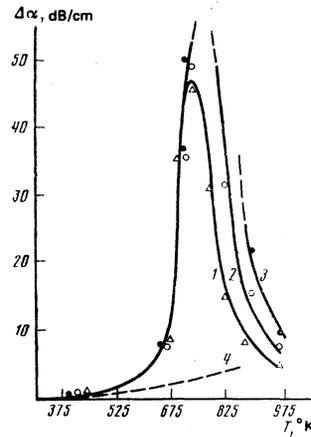
sociated with the electron−phonon interaction via the piezoelectric potential were not very effective and could be ignored.

In addition to the acoustic investigations, we also carried out special measurements of $\sigma$ of different samples and at different frequencies. The measurements were made by an indirect method involving determination of the $Q$ factor of a circuit in which a sample was connected in parallel; an E4-7 $Q$ meter was used. When the condition $R_{0eq} \gg \sigma^{-1}$ was obeyed ($R_{0eq}$ is the equivalent resistance of the circuit) the experimental error did not exceed 10%. This condition limited the frequency range of the measurements.

Figure 5 shows typical dependences $\sigma(T)$ obtained at frequencies $10^5$ and $10^6$ Hz, and also under dc conditions deduced from the current-voltage characteristic. In all cases the contacts were ohmic (prepared by the evaporation of aluminum) and this was checked by measuring the electrical conductivity for both directions of the voltage. It is clear from the figure that at sufficiently high temperatures corresponding to the maxima of $\alpha$ in the acoustic measurements the value of $\sigma$ depended exponentially on temperature. The argument of the exponential function $\Delta E_\sigma$ occurring in the temperature dependence of the electrical conductivity was maximal for the dc measurements. It decreased somewhat on increase in the frequency. For the results shown in Fig. 5, the dc value was $\Delta E_\sigma = 0.7$ eV, whereas at $10^6$ Hz it was $\Delta E_\sigma$



FIG. 3. Same as Fig. 1, but for a CdSe single crystal: 1)-5) conductivity $\sigma$ altered by varying temperature; 6), 7) conductivity $\sigma$ altered by illumination; $\square$) 75 MHz.
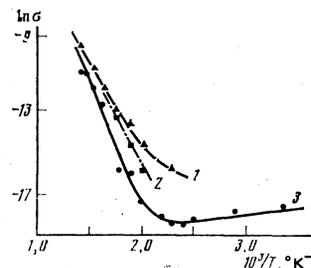


FIG. 5. Temperatures dependences of the electrical conductivity of a CdS single crystal: 1) $10^6$ Hz; 2) $10^5$ Hz; 3) dc.

=0.6 eV. The most important feature of these results was the frequency dependence of the conductivity at fairly low (compared with the reciprocal of the electron momentum relaxation time) frequencies. Increase in temperature weakened the frequency dependence of $\sigma$. The photoconductivity relaxation time measured under dc conditions for CdS and CdSe samples was 5–20 min at $T = 300$ °K.

The main experimental results requiring special analysis were thus as follows.

1. In the case of the samples exhibiting slow relaxation of the photoconductivity the behavior of $\alpha$ depended strongly on the method used to vary $\sigma$, by altering temperature or illumination intensity, although the condition $\omega/\omega_d \ll 1$ was satisfied by a large margin.

2. The amplitudes of the maxima of $\alpha$ considered as a function of the electrical conductivity at the same frequency were different for different cadmium sulfide single crystals with the same electromechanical coupling coefficient.

3. The conductivities $\sigma_T$, corresponding to the maxima of $\alpha$ considered as a function of temperature, were shifted toward higher values compared with the values of $\sigma_I$ corresponding to the maxima of $\alpha$ as a function of the illumination intensity.

4. The values of the electrical conductivity measured directly by the dc method at temperatures corresponding to the maxima of $\alpha$ did not agree with the values calculated from the condition $\omega/\omega_M = 1$.

## 3. DISCUSSION OF RESULTS

We can easily show that these experimental observations cannot be interpreted within the framework of the existing theory of the absorption of sound in piezoelectric semiconductors allowing only for the presence of discrete trapping levels in an undistorted band gap. We can do this using the results of Ref. 5, because all the other calculations[4,10,6] carried out in the $\omega \ll \omega_d$ approximation give analogous results.

It follows from the experimental results that the deformation electron–phonon interaction is ineffective in the investigated range of frequencies and this is in full agreement with the general theoretical estimates.[2]

According to Ref. 5, the absorption coefficient of elastic waves accompanied by a longitudinal piezoelectric field is

$$\alpha_0 = \eta^2 \frac{\omega}{v_0} \left[ \frac{\omega}{\omega_M} + a \left( \frac{bf_0\omega^2}{\omega_M\omega_d} + \frac{\omega}{\omega_M} a \right) \right] \left[ \left( \frac{\omega}{\omega_M} - a \right)^2 + \left( 1 + \frac{bf_0\omega^2}{\omega_M\omega_d} + \frac{\omega}{\omega_M} \right)^2 \right]^{-1}, \quad (1)$$

where $\eta$ is the electromechanical coupling coefficient, $v_0$ is the velocity of the appropriate piezoelectrically active elastic wave, $f_0$ is the unbound fraction of the space charge created by an ultrasonic wave

$$a = \frac{\omega\tau_{eq}(1-f_0)}{f_0 + \omega^2\tau_{eq}^2}, \quad b = \frac{f_0^2 + \omega^2\tau_{eq}^2}{f_0(\omega^2\tau_{eq}^2 + f_0)}.$$

We can easily show that in the case of a homogeneous semiconductor the condition $\omega^2/\omega_M\omega_d = k^2\Lambda^2 \ll 1$ is satisfied by frequencies not exceeding $10^8$ Hz ($\Lambda$ is the Debye screening radius). Numerical estimates show that in the case of CdS and CdSe crystals with any values of $\tau_{eq}$ and $f_0$ in the range $\omega/\omega_M \gtrsim 0.1$ at frequencies up to $10^8$ Hz we have $bf_0\omega/\omega_d \ll a$ [graphs of the dependences $a(\omega\tau_{eq}, f_0)$ and $b(\omega\tau_{eq}, f_0)$ are plotted in Ref. 5 for a wide range of these parameters]. Therefore, in the model of a homogeneous piezoelectric semiconductor containing trapping levels in the band gap, we find that in the investigated range of frequencies

$$\alpha_0 \approx \eta^2 \frac{\omega}{v_0} \frac{\omega/\omega_M}{1 + \omega^2/\omega_M^2}, \quad (2)$$

i.e., the dependence of the absorption coefficient of sound on $\omega/\omega_M$ is of the relaxation type.

The dependence of the absorption coefficient of sound on the electrical conductivity given by Eq. (2) disagrees with the experimental results both quantitatively and qualitatively. In fact, according to Eq. (2), a maximum of $\alpha$ should be observed always at $\omega = \omega_M = \sigma/\varepsilon$ irrespective of the method used to reach the necessary value of $\sigma$, but the experimental results show that this is not always so. Moreover, the amplitude of the maximum should be $\frac{1}{2}\eta^2\omega/v_0$, i.e., for a fixed value of the electromechanical coupling coefficient, it should be governed only by the frequency at which the measurements are made. However, our experiments (Figs. 1–3) on various CdS and CdSe samples showed that the amplitudes of the maxima were different when $\eta \approx$ const. Moreover, the experimental maxima in the dependences of $\alpha$ on $\sigma$ were sharper than predicted by the theory of the absorption of sound in homogeneous samples.

It follows that the experimental results on the electron–phonon interaction in piezoelectric semiconductors, like the measurements of the electrical properties (slow relaxation of the photoconductivity, frequency dependence of the electrical conductivity), cannot be interpreted within the framework of the theory which allows only for the presence of discrete trapping levels in the band gap which has flat boundaries.

The slow relaxation of $\sigma$ and its frequency dependence are explained well by introducing the concept of collective barriers, so that similar inhomogeneities may affect the absorption of sound in piezoelectric semiconductors. Since in the case of macroscopically homogeneous crystals an internal electric field creating energy barriers and, consequently, inhomogeneities in the carrier distribution is due to fluctuations in the donor and acceptor concentrations, it is appropriate to consider a piezoelectric semiconductor doped with a donor impurity in a concentration $N_D$, which also contains an acceptor impurity in a concentration $N_A$. As pointed out earlier, such a model is quite realistic because the growth of crystals of the semiconductors exhibiting the strongest piezoelectric effect is accompanied by the appearance of a large number of structure defects giving rise to donor and acceptor levels.[14] These levels may originate from impurity atoms, vacancies in the host lattice, host atoms in interstices, and complexes of these defects. Therefore, real crystals of piezoelectric semiconductors are usually strongly compensated.

Typical values of the donor and acceptor concentrations in high-quality crystals range up to $10^{16}$ cm$^{-3}$, whereas the carrier densities are $10^9$–$10^{11}$ cm$^{-3}$ when the electrical conductivity is such that a maximum of $\alpha$ can be observed. It should be stressed that both CdS and CdSe crystals have a wide band gap (exceeding 2 eV) and, therefore, practically all the free carriers are due to doping and not due to thermal band–band transitions. We shall consider the specific case when $N_D > N_A$. In principle, impurities may be distributed in the bulk of a sample in a correlated or uncorrelated manner, which naturally influences the final results. However, since no information was available on the nature of the distribution of impurities in our CdS and CdSe crystals, we assumed that this distribution was uncorrelated.

A random distribution of donors and acceptors is known to give rise to a random potential[18] which is dominated by large-scale (compared with the average distance between impurities) fluctuations. The existence of such a coordinate-dependent potential creates a corresponding spatial distribution of the electron density and this alters the conditions for the formation of electron bunches in the course of propagation of a piezoelectrically active elastic wave in a piezoelectric semiconductor, although the elastic wavelength is much greater than the typical radius of such fluctuations. Clearly, the influence of fluctuations in the donor and acceptor concentrations on the absorption of sound should increase with the value of the rms fluctuations of the large-scale potential, which depend strongly on the impurity concentration and on the compensation.

It should be pointed out that for typical values of the donor concentration and electron density in piezoelectric semiconductors ($N_D \approx 10^{16}$ cm$^{-3}$ and $n = 10^{10}$ cm$^{-3}$) the large-scale potential is considerably greater than the thermal energy of carriers. In fact, estimates of typical values of the potential give $\sim 0.40$ eV for temperatures of the order of 300 °K.

Since we shall be interested in the case of low frequencies ($\omega \ll \omega_d$), the influence of the large-scale potential on the absorption coefficient of sound should be manifested only by an inhomogeneous distribution of electrons in the conduction band, whereas localized carriers should not participate directly in the absorption of sound, exactly as in the homogeneous case in the absence of an external electric field. These localized carriers simply influence (together with donors and acceptors) the chemical potential $\xi$ of the system.

In the weak compensation case the amplitude of the rms fluctuations of the potentials is low compared with the Fermi energy and this is true irrespective of the degree of doping.[18] Consequently, the rms fluctuations of the carrier density in the conduction band $\delta n^2$ are small compared with the average density $\langle n_0 \rangle$. Therefore, the propagation of sound in such a piezoelectric medium can be considered using the theory of an effective medium.[19] However, the corrections to the absorption coefficient of sound compared with the homogeneous case are of the required sign and are proportional to $\delta n^2 / \langle n_0 \rangle^2$.

In the case of strong compensation of a piezoelectric semiconductor one should consider separately the cases of light and heavy doping, although in both cases the random potential is strong, the distribution of electrons in the conduction band is very inhomogeneous, and the absorption of high-frequency sound is entirely due to electrons whose energies exceed the percolation level. However, in the latter case we have to allow also for the polarization of electron drops (formed as a result of heavy doping) on the distortion of the bottom of the conduction band.[20]

Single crystals of CdS and CdSe grown from the melt and characterized by typical donor and acceptor concentrations of the order of $10^{16}$–$10^{19}$ cm$^{-3}$ can be regarded as lightly doped because the Bohr radius is 30 and 60 Å for the two compounds (the effective electron mass in CdS is $\sim 0.2 m_0$, whereas in the case of CdSe it is $0.13 m_0$, where $m_0$ is the mass of a free electron—these values are taken from Ref. 14). We shall now estimate the absorption coefficient of sound for this case.

At ultrasound frequencies not exceeding $10^8$–$10^9$ Hz the elastic wavelength is considerably greater than the size of inhomogeneities and, therefore, the propagation of sound can be described by a system of equations from the theory of elasticity of piezoelectric media and the Maxwell equations in which allowance is made for the internal electric fields. This system should be supplemented by the recombination equation describing possible electron transitions from a free to a localized state and vice versa. Moreover, if the dimensions of the inhomogeneities exceed greatly the mean free path of carriers, the scattering of carriers by the large-scale potential can be ignored compared with the scattering by optical phonons which dominates the calculated mobility in cadmium sulfide and selenide.[14] Although the dimensions of the fluctuation inhomogeneities have not yet been determined experimentally for piezoelectric semiconductors, the above assumption is supported by the fact that the carrier mobilities in CdS and CdSe calculated for the scattering by optical phonons are in good agreement with the measurements carried out on a large number of crystals prepared under different conditions.[14] Consequently, the carrier mobility $\mu$ can be regarded as independent of the coordinates.

Since the quantities occurring in the system of equations described above are functions of the coordinates, the calculated absorption coefficient of sound should be averaged over the volume. However, calculations are simplified if integration over the volume is replaced with averaging over the energies corresponding to all possible impurity configurations. This can be done by assuming an electron distribution described by the functions $n(\mathscr{E})$ and $\nu(\mathscr{E})$ representing respectively, the densities of electrons in the conduction band and at energy levels at those points where the distortion of the bottom of the conduction band is of energy $\mathscr{E}$ relative to its value in a homogeneous crystal.

Bearing this point in mind, we can write down the system of equations describing the propagation of sound in a compensated lightly doped piezoelectric semiconductor in the case when $\omega \ll \omega_d$ (since piezoelectric semiconductors have a special piezoelectrically active direction, all the equations are expressed in terms of

projections along this direction):

$$j = q\mu[n_0(\mathscr{E}) + \bar{n}(\mathscr{E})]E, \tag{3}$$

$$q\frac{\partial \bar{n}(\mathscr{E})}{\partial t} = \frac{\partial j}{\partial x} - q\frac{\partial \bar{\nu}(\mathscr{E})}{\partial t}, \tag{4}$$

$$D = \varepsilon E + \beta \partial u/\partial x, \tag{5}$$

$$\partial D/\partial x = -4\pi q[\bar{n}(\mathscr{E}) + \bar{\nu}(\mathscr{E})], \tag{6}$$

$$\rho\frac{\partial^2 u}{\partial t^2} = c\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial E}{\partial x}. \tag{7}$$

Here, $j$ is the current density; $\mu$ is the carrier mobility; $n_0(\mathscr{E})$ and $\nu_0(\mathscr{E})$ are the equilibrium densities of free (with energies above the percolation level) and localized electrons; $\bar{n}(\mathscr{E})$ and $\bar{\nu}(\mathscr{E})$ are the nonequilibrium electron densities in the field of an acoustic wave; $D$ is the electric induction vector; $\bar{E}$ is the electric field associated with the acoustic wave; $u$ is the mechanical displacement in the wave; $\rho$, $\varepsilon$, $\beta$, and $c$ are the density, permittivity, piezoelectric modulus, and elastic modulus of a crystal.

It should be stressed that the condition $\omega \ll \omega_d$ allows us to ignore the gradient term in Eq. (3) describing the piezoelectric current, whereas Eqs. (4) and (6) contain only the total electron density $\varkappa = n(\mathscr{E}) + \nu(\mathscr{E})$. This ensures the absence of the influence of the recombination kinetics on the absorption coefficient of sound in the low-frequency case.

If we ignore the nonlinear effects, the simultaneous solution of the system (3)–(7) gives rise to a wave equation with an undetermined elastic modulus

$$\rho\frac{\partial^2 u}{\partial t^2} = c'\frac{\partial^2 u}{\partial x^2}. \tag{8}$$

Here,

$$c' = c\left\{1 + \frac{\beta^2}{c\varepsilon}\left[1 + i\frac{4\pi q n_0(\mathscr{E})\mu}{\omega\varepsilon}\right]^{-1}\right\}. \tag{9}$$

The wave equation (8) can be solved by substituting $u = u_0 \exp[i(kx - \omega t)]$ and allowing for the smallness of the electromechanical coupling coefficient ($\eta^2 = \beta^2/c\varepsilon \ll 1$). Then, $k = \omega/v_0 + i\alpha$ and

$$\alpha = \omega\rho^{1/2} \operatorname{Im}(c')^{-1/2}. \tag{10}$$

Substituting Eq. (9) into Eq. (8) we find that

$$\alpha(\mathscr{E}) = \omega\left(\frac{\rho}{c}\right)^{1/2}\left(\frac{\beta^2}{c\varepsilon}\right)\frac{q\omega\mu n_0(\mathscr{E})/\varepsilon}{\omega^2 + q^2\mu^2 n_0^2(\mathscr{E})/\varepsilon^2}. \tag{11}$$

The expression for the observed value of the absorption coefficient of sound is found by averaging over all possible values of the random potential corresponding to different fluctuations of the impurity concentration:

$$\alpha = \int_{\mathscr{E}_c}^{\infty} \alpha(\mathscr{E})F(\mathscr{E})d\mathscr{E}, \tag{12}$$

where $F(\mathscr{E})$ is the distribution function of the random potential[18] and $\mathscr{E}_c$ is the energy corresponding to the percolation level. We can easily see that if $F(\mathscr{E}) = \delta(\mathscr{E})$, Eq. (12) gives the value of $\alpha$ for a homogeneous piezoelectric semiconductor.[4]

In the case of an uncorrelated impurity distribution,[18] we have

$$F(\mathscr{E}) = \frac{1}{\Delta}\exp\left(-\frac{\mathscr{E}^2}{\Delta^2}\right)$$

($\Delta$ is determined by the impurity concentration and degree of compensation), and it also follows from Ref. 21 that

$$n_0(\mathscr{E}) = N_c \exp\left(\frac{\xi - \mathscr{E}}{T}\right)$$

($N_c$ is the effective density of states in the continuous spectrum near the percolation level). Calculating the integral in Eq. (12) by parts and bearing in mind that the function $\tan^{-1}\lfloor\mathscr{E}\exp(-\mathscr{E}/T)/q\mu N_c\rfloor$ obtained as a result of integration is smooth compared with the second factor $\exp(-\mathscr{E}^2/\Delta^2)$, which has a sharp maximum, we find that the coefficient of absorption of low-frequency sound in a compensated lightly doped semiconductor is

$$\alpha = \frac{\omega T}{\Delta}\left(\frac{\rho}{c}\right)^{1/2}\eta^2\exp\left(-\frac{\mathscr{E}_c^2}{\Delta^2}\right)\operatorname{arctg}\frac{\omega\tau_c[\exp\{\Delta\sqrt{2}-\mathscr{E}_c)/T\} - 1]}{1 + \omega^2\tau_c^2\exp\{(\Delta\sqrt{2}-\mathscr{E}_c)/T\}}, \tag{13}$$

where

$$\tau_c = \frac{\varepsilon}{q\mu N_c}\exp\left(-\frac{\xi - \mathscr{E}_c}{T}\right) = \tau_0\exp\left(-\frac{\xi - \mathscr{E}_c}{T}\right).$$

Before we analyze the formula (13), it is worth mentioning that in the $\omega \ll \omega_d$ approximation, which is insensitive to the kinetics of the establishment of an equilibrium between localized and free carriers, the system (5)–(7) describes the absorption of sound also in a heavily doped piezoelectric semiconductor. The only difference is that the electric induction vector should also include the term $\gamma\bar{E}$, which allows for the polarization of electron drops in the electric field of an elastic wave ($\gamma$ is the polarization coefficient of such drops). Then, since the electron density in a drop $n_d$ is very different from the electron density $n_c$ at the percolation level,[18] the contribution of the two groups of electrons to the absorption of sound of a given frequency can be treated independently, i.e., the polarization of the drops is effective at higher frequencies ($n_d \gg n_c$).

We shall analyze Eq. (13) for the absorption coefficient of sound in two limiting cases.

1. Case $\Delta/T \ll 1$, which corresponds to a weak influence of the potential relief. For a Gaussian random potential[22] we have $\mathscr{E}_c = -0.68\Delta$. Therefore,

$$\alpha \approx 0.7\omega\left(\frac{\rho}{c}\right)^{1/2}\eta^2\frac{\omega\tau_c}{1 + \omega^2\tau_c^2}, \tag{14}$$

i.e., the absorption coefficient of sound depends on $\omega\tau_c$ in the same way as in the homogeneous case, but it is slightly smaller. The position of a maximum of $\alpha$ is governed by the condition $\omega\tau_c = 1$, i.e., by the density of electrons with energies above the percolation level (in a nondegenerate semiconductor). Since $n_c$ determines also the electrical conductivity of a sample ($\sigma = q n_c \mu$), the values of the conductivity measured under dc conditions and calculated from $\omega\tau_c = 1$ should be identical. In other words, Eq. (14) obtained in the approximation of weak compensation cannot explain the experimental results.

2. The case $\exp(\Delta/T) \gg 1$ corresponds to a moderate compensation and a strong influence of the potential relief. Then,

$$\alpha \approx 0.7\frac{\omega T}{\Delta}\left(\frac{\rho}{c}\right)^{1/2}\eta^2\tan^{-1}\frac{\omega\tau_c\exp(2,1\Delta/T)}{1 + \omega^2\tau_c^2\exp(2,1\Delta/T)} \tag{15}$$

M. B. Gitis and I. A. Chaĭkovskiĭ

i.e., the amplitude of the maximum of $\alpha$ is governed not only by the electromechanical coupling coefficient but also by the rms value of the random potential.

We can readily see that in this case the behavior of $\alpha$ as a function of the frequency, temperature, and carrier density differs considerably from the behavior in the case of a homogeneous material. Moreover, the absorption coefficient of sound for a homogeneous piezoelectric semiconductor is a function of $\omega\tau_M$ and variation of $\omega$ or $\tau_M$ is entirely equivalent, whereas in the case of a compensated sample the dependences on $\omega$ and $\tau_c$ are different. This is due to the fact that a change in $\tau_c$ can be produced either by altering temperature or the carrier density (this changes the energy position of the energy potential and percolation level).

Considered as a function of the frequency, the coefficient $\alpha$ has a maximum when the condition $\omega\tau_c = e^{-\Delta/T}$ is satisfied, in contrast to $\omega\tau_M = 1$ in the case of a homogeneous piezoelectric semiconductor. A change in temperature alters the condition for a maximum. The temperature of a maximum $T_M$ is governed by the condition

$$T_M = (2\mathscr{E}_c - 2\xi + 2.1\Delta)\left\{\ln\left[\frac{1}{\omega^2\tau_0^2}\left(1 + \frac{2.1\Delta}{\mathscr{E}_c - \xi}\right)\right]\right\}^{-1} \quad (16)$$

in contrast to the homogeneous case

$$T_M{}^0 = -\xi/\ln\omega\tau_0. \quad (e)$$

Since $\xi < 0$, the temperature position of the maximum of the acoustic absorption coefficient of a compensated piezoelectric semiconductor is shifted relative to its value for an uncompensated material in the direction of higher temperatures. Therefore, the values of the dc conductivity corresponding to the acoustic absorption maximum are, in agreement with the experimental results, higher than those calculated from the condition $\omega\tau_M = 1$ (homogeneous sample). Moreover, whereas in the case of a homogeneous piezoelectric semiconductor the absorption at the maximum of $\alpha$ is determined uniquely by the electrical conductivity of a sample ($\tau_M = \varepsilon/\sigma$), in the case of an inhomogeneous crystal there is no such relationship because the conductivity is proportional to $\exp[(\xi - \mathscr{E}_c)/T]$ [compare with Eq. (13)]. It is worth mentioning that the condition (16) for observing a temperature maximum of $\alpha$ can be rewritten in the form

$$\omega\tau_0 = \left(1 + \frac{2.1\Delta}{\mathscr{E}_c - \xi}\right)^{1/2}\exp\left(-\frac{2\mathscr{E}_c - 2\xi + 2.1\Delta}{2T_m}\right). \quad (16a)$$

Then, having measured $\alpha$ as a function of temperature at different frequencies $\omega_1$ and $\omega_2$, we obtain

$$\ln(\omega_1/\omega_2) = (\mathscr{E}_c - \xi + \Delta)\frac{T_{m1} - T_{m2}}{T_{m1}T_{m2}}, \quad (17)$$

where $T_{m1}$ and $T_{m2}$ are the temperatures of the maxima at the frequencies $\omega_1$ and $\omega_2$ (if $\omega_1 < \omega_2$, then $T_{m2} > T_{m1}$).

The condition (17), together with measurements of the electrical conductivity as a function of temperature, makes it possible to determine the rms potential. For example, an analysis of the results of measurements of the conductivity of a sample of CdS plotted in Fig. 1 gives the argument of the exponential function in the temperature dependence of $\sigma$ as 0.8 eV, whereas for the sample of Fig. 2 the corresponding value is 0.6 eV. An analysis of the acoustic measurements in accordance

with Eq. (17) gives, for the same samples, the arguments of the exponential function in (16a) as 1.5 eV and 1.2 eV. Hence, it follows that in the case of the first sample of CdS we have $\Delta = 0.7$ eV, and for the second we have $\Delta = 0.6$ eV when the band gap of the homogeneous crystal is 2.4 eV. Similar results are obtained also for CdSe samples. It should be stressed that the very existence of a difference between the arguments of the exponential functions representing the temperature dependences of the electrical conductivity and those deduced from the shift of the $\alpha(T)$ maximum at different frequencies cannot be interpreted within the framework of the theory of the absorption of sound in homogeneous piezoelectric semiconductors.[11]

Allowance for the large-scale potential in piezoelectric semiconductors makes it possible to explain in a natural manner the experimentally observed differences between the dependences of the absorption coefficient of sound on temperature and illumination intensity, because in the case of inhomogeneous piezoelectric semiconductors the condition for observing a maximum of $\alpha$ is controlled not only by the conductivity but also by the ratio $\Delta/T$ [via the factor $\exp(2.1\Delta/2T)$]. Therefore, the attainment of a given value of the electrical conductivity by illumination at room temperature or by heating is not equivalent from the point of view of the absorption of sound and the observation of a maximum of $\alpha$ may require both higher or lower values of the conductivity, compared with that obtained by increasing the temperature. The actual form of the dependences $\alpha(l)$ is outside the scope of the present paper ($l$ is the illumination intensity).

It follows that allowance for the compensation of crystals of piezoelectric semiconductors and, consequently, for the large-scale potential makes it possible to interprete practically all the main experimental results. Moreover, the acoustic methods may become an important source of information on the parameters of the internal random field.

We shall conclude by noting that, as demonstrated by Eq. (13), if $\Delta/T \gg 1$, then $\alpha \to 0$. Physically, this means that at very high values of the random rms potential, there are no electrons at the percolation level and this absorption mechanism of sound does not apply to lightly doped compensated piezoelectric semiconductors. In the case of heavily doped compensated semiconductors the value of $\alpha$ differs from zero because of the polarization of electron drops.[20]

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146    Sov. Phys. JETP 54(1), July 1981

M. B. Gitis and I. A. Chaĭkovskiĭ    146

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