Zero-phonon ac hopping conductivity of disordered systems

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Hopping electric conductivity of a disordered system with localized states is investigated at a frequency ω. At sufficiently low temperatures, the conductivity is determined by the zero-phonon absorption of electromagnetic quanta by nonresonant pairs of states. It is shown that to calculate the electric conductivity it is necessary to take into account the Coulomb repulsion of the second electron of the pair from the first. This influences significantly the probability of the pair having only one electron, and consequently the probability that the pair can absorb a photon. As a result, $\hbar\omega$ in the previously known expressions for the electric conductivity should be replaced by $e^2/\varkappa\, r_\omega$ when $\hbar\omega \ll e^2/\varkappa\, r_\omega$ (r_ω is the characteristic arm of the pair and $\varkappa$ is the dielectric constant). Similar changes take place in the formulas for the zero-phonon electric conductivity of a one-dimensional metal, in which states are localized over the mean free path. The temperature dependence of the zero-phonon conductivity is investigated and the transition from conductivity due to relaxation absorption to zero-phonon conductivity with increasing frequency is discussed.

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1. INTRODUCTION

In systems with localized states the static conductivity at zero temperature is zero. An important theoretical question is that of the form of $\sigma(\omega)$ at low frequencies and zero temperature. Mott found that the main contribution to the conductivity is made by resonant absorption by pairs of states, one of which is occupied by an electron, and the other is empty. The overlap integral I between these states decreases with distance r between their centers in accordance with the law

$$I(r) = I_0 e^{-r/a}, \tag{1.1}$$

where a is the localization radius and I_0 is the pre-exponential factor and depends on the model of the disordered system. According to Mott, the most effective pairs are those with an arm length

$$r_\omega = a \ln(2I_0/\hbar\omega), \tag{1.2}$$

for which $\hbar\omega = 2I(r)$. For a three-dimensional system, Mott's result takes the form (see, e.g., Ref. 1)

$$\sigma(\omega) = \alpha e^2 g^2 a \hbar \omega^2 r_\omega^4, \tag{1.3}$$

where g is the state density on the Fermi level, and α is a numerical coefficient. According to Mott, $\alpha = \pi^2$, but we have obtained $\alpha = \pi^2/3$ (see Sec. 2).

In systems with a different number of dimensions, only the coefficient α and the exponent of r_ω change. This exponent is equal to three and two respectively in a two- and one- to two. A special case of a one-dimensional system is the so-called one-dimensional metal, where on the one hand the ideal-electron-gas condition $p_F l \ll 1$ is satisfied (p_F is the Fermi momentum, $l = v_F \tau$ is the mean free path, τ is the relaxation time for backward scattering, and v_F is the Fermi velocity). On the other hand, as first indicated by Mott, all the states are strictly localized because the metal is one-dimensional. The electric conductivity of such a system was calculated by Berezinskiĭ,[2] Gogolin, Mel'nikov and Rashba,[3] Abrikosov and Ryzhkin,[4] and by many others. Their result is of the form

$$\sigma(\omega) = (4/\pi\hbar) e^2 v_F \tau^3 \omega^2 \ln^2(1/\omega\tau). \tag{1.4}$$

It seems to us that this formula is a particular case of a one-dimensional version of the Mott formula.

That these formulas are identical can be easily verified by making the following substitutions in Mott's formula: the length a must be replaced by the one-dimensional localization radius, which is equal in order of magnitude to the mean free path l, the density of states should be replaced by $(\pi\hbar v_F)^{-1}$, while the quantity I_0 should be replaced by $\hbar/\tau$. The last substitution calls for an explanation. The quantity I_0 has the meaning of the straggling of the energies of localized states the distance between which is of the order of the localization radius. Therefore in the case of a one-dimensional metal the quantity I_0 can be obtained from the relation $g l I_0 \approx 1$, whence it follows in fact that $I_0 \approx \hbar/\tau$. Thus, formula (1.4) describes resonant absorption due to a transition of an electron between localized states separated from one another by a distance on the order of $r_\omega = l \ln(1/\omega\tau)$, which greatly exceeds the localization radius l.

In the calculation of the probability that the pair has only one electron, Mott did not take into account the potential produced by this electron. As shown by one of us,[5] this potential alters the form of the Mott-Austin formula for the conductivity connected with the relaxation absorption (with allowance for the phonons; for details see Sec. 4). We shall show below that a correct allowance for the interaction of the electrons in the problem of resonant absorption at $T = 0$ leads to the conclusion that formula (1.3) is valid only at very high frequencies $\hbar\omega \gg e^2/\varkappa r_\omega$ ($\varkappa$ is the dielectric constant), and at $\hbar\omega \ll e^2/\varkappa r_\omega$ we have

$$\sigma(\omega) = \frac{\pi^2 e^4}{3\varkappa} g^2 a \omega r_\omega^3. \tag{1.5}$$

To allow for the electron–electron interaction we must therefore multiply (1.3) by the large ratio $e^2/\varkappa r_\omega \hbar\omega$. We assume that the same holds true also in the case of the one-dimensional system discussed above, so that at

$$\hbar\omega \ll e^2/\varkappa l \ln\frac{1}{\omega\tau}$$

formula (1.4) should be replaced by

$$\sigma(\omega)=\frac{2e^4}{\pi\hbar^2}\tau^2\omega\ln\frac{1}{\omega\tau}. \tag{1.6}$$

In the next section, to be definite, we shall consider a concrete model of a disordered system with electron-electron interaction, and derive formula (1.5). It is known[6] that the electron-electron interaction produces a state-density Coulomb gap in the vicinity of the Fermi level. In Sec. 3 we shall show that formula (1.5) is valid only when the energies of the states that take part in the conductivity are outside the Coulomb gap. In this region, multielectron effects are inessential and the state density contained in (1.5) can be regarded as a nonzero constant. This region is bounded by the condition $e^2/\varkappa r_\omega \gg \Delta$, where Δ is the width of the Coulomb gap. At exponentially low frequencies this condition is violated and this, as shown in Sec. 3, leads to a change in the powers of the logarithms in (1.5).

In Sec. 4 we discuss finally the temperature dependence of the conductivity connected with the resonant absorption. At nonzero temperature, there exists also a conduction mechanism based on relaxation absorption. It consists in the fact that the transition of the electron between different states is due to phonons, and the electric field alters the equilibrium population of the levels and causes a relaxation that leads to energy dissipation (see Ref. 7). In Sec. 4 we compared these two mechanisms and ascertain in which temperature regions each of them dominates.

2. ALLOWANCE FOR TWO-SITE CORRELATIONS IN RESONANT ABSORPTION

For the sake of argument we consider a concrete model of a disordered system. It consists of sites with concentration N, randomly distributed in space. At each site there can be one electron or no electron. The total energy of the system is of the form

$$H=\sum_i \Phi_i n_i + \frac{1}{2}\sum_{i\neq j}\frac{e^2}{\varkappa r_{ij}}n_i n_j. \tag{2.1}$$

Here r_{ij} is the distance between the sites i and j, $n_i = 0$ and 1 are the occupation numbers. The energies Φ_i are randomly and uniformly distributed in a large interval $(-A, A)$. They can be connected with random violations of the short-range order in the system. We shall assume that this scatter is much larger than the energy of the Coulomb interaction at neighboring sites, i.e., that $A \gg e^2/\varkappa N^{-1/3}$.

At low temperatures, the occupation of the site i is determined by its energy

$$\varepsilon_i=\Phi_i+\sum_{j\neq i}\frac{e^2}{\varkappa r_{ij}}n_j, \tag{2.2}$$

so that $n_i = 0$ if $\varepsilon_i > \mu$ and $n_i = 1$ if $\varepsilon_i < \mu$, where μ is the Fermi energy. Since the non-Coulomb scatter of A is large, the second term in (2.2) is as a rule larger than the first, so that we can obtain a good approximation by putting

$$n_i = n_i^{(0)} = \vartheta(\mu-\Phi_i),$$
$$\vartheta(x)=\begin{cases}1, & x>0 \\ 0, & x<0\end{cases} \tag{2.3}$$

The state density $g(\varepsilon)$ obtained in this approximation is shown by the solid line in Fig. 1. In a wide energy interval, it equals $g = N/2A$, which simply describes the distribution of the energies Φ_i.

At low frequencies, the resonant absorption is determined by pairs consisting of an amply and occupied site with energies that are very close to each other. In addition, these energies should be close to the Fermi energy. Because of the large average scatter, these soft pairs are very rare and allocated far from one another in space. The overwhelming majority of sites that surround a given pair have high energies, and their occupation can be described by expression (2.3), which neglects the Coulomb interaction. (The condition for the applicability of this approximation is discussed in detail in Sec. 3. Its gist is in fact that the energies of the sites that form a soft pair should lie outside the Coulomb gap.) However, as we shall show now, it is necessary to take into account the Coulomb correlation of the occupation numbers of the sites of the softest pair. In this approximation we arrive at the following two-site Hamiltonian:

$$H_{1,2}=\varphi_1 n_1+\varphi_2 n_2+\frac{e^2}{\varkappa r}n_1 n_2+I(r)(a_1^+ a_2 + a_2^+ a_1), \tag{2.4}$$

where r is the distance between sites 1 and 2; a_i^+ and a_i are the creation and annihilation operators; $n_i = a_i^+ a_i$, $I(r)$ is the overlap integral defined by (1.1), and

$$\varphi_1=\Phi_1+\sum_{j\neq 1,2}\frac{e^2}{\varkappa r_{1j}}n_j^{(0)}, \quad \varphi_2=\Phi_2+\sum_{j\neq 1,2}\frac{e^2}{\varkappa r_{2j}}n_j^{(0)}. \tag{2.5}$$

The last term in (2.4) describes the quantum state overlap necessary for the calculation of the probability of the resonant transition. It is clear that since the sites that surround the soft pair have energies that differ greatly from the energies of the sites of this pair, the quantum overlap must be taken into account only inside the pair. (In the three-dimensional case this is equivalent to stating that the system is far from the Anderson transition, so that a pair of sites capable of collectivizing an electron is a rare phenomenon.)

The Hamiltonian (2.4) describes four states.

1. There are no electrons on the pair. The energy $E_0 = 0$.

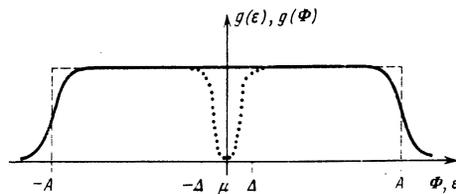2. The pair has one electron. There are two states with energies

FIG. 1. Dependence of the state density on the energy. The dashed line is the distribution of the energies Φ, the solid line is the state density $g(\omega)$ in the approximation (2.5), while the dash-dot line is the Coulomb gap near the Fermi level.

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$$E_1^{\pm}=(\varphi_1+\varphi_2)/2\pm\Gamma/2, \quad \Gamma=[(\varphi_1-\varphi_2)^2+4I^2(r)]^{1/2}. \quad (2.6)$$

3. The pair has two electrons. There is one state with energy

$$E_2=\varphi_1+\varphi_2+e^2/\varkappa r. \quad (2.7)$$

Resonant absorption is connected with the transition of the electron from the state with energy E_1^- into a state with energy with E_1^+. The energy absorbed by one pair per unit time under the influence of the electric field

$$\vec{\mathscr{E}}={}^{1}/_{2}[\vec{\mathscr{E}}_0e^{i\omega t}+\vec{\mathscr{E}}_0e^{-i\omega t}], \quad (2.8)$$

is of the form

$$Q=\frac{2\pi}{\hbar}\frac{\hbar\omega}{4}|e\vec{\mathscr{E}}_0\langle-|\mathbf{r}|+\rangle|^2\delta(\hbar\omega-\Gamma)\exp\left(\frac{\Omega-E_1^-+\mu}{kT}\right)\left[1-\exp\left(-\frac{\hbar\omega}{kT}\right)\right], \quad (2.9)$$

$$\exp\left(-\frac{\Omega}{kT}\right)=1+\exp\left(-\frac{E_1^--\mu}{kT}\right)+\exp\left(-\frac{E_1^+-\mu}{kT}\right)+\exp\left(-\frac{E_2-2\mu}{kT}\right),$$

where $e\vec{\mathscr{E}}_0\langle-|\mathbf{r}|+\rangle$ is the matrix element of the transition between the states with energies E_1^- and E_1^+. It is easy to find that

$$|e\vec{\mathscr{E}}_0\langle-|\mathbf{r}|+\rangle|^2=(e\vec{\mathscr{E}}_0\mathbf{r})^2I^2(r)/\Gamma^2. \quad (2.10)$$

Summing over all the pairs in a unit volume and changing over to electric conductivity, we obtain

$$\sigma(\omega)=\frac{2\Sigma Q_i}{\mathscr{E}_0^2}=\frac{g^2}{\mathscr{E}_0^2}\int_{-A}^{A}d\varphi_1\int_{\varphi_1}^{A}d\varphi_2\int Q\,d^3r=\frac{4\pi^2e^2g^2[1-e^{-\hbar\omega/kT}]}{3\hbar^2\omega}\int_{-A}^{A}d\varphi_1$$

$$\times\int_{\varphi_1}^{A}d\varphi_2\int_0^{\infty}drr^4I^2(r)\delta(\hbar\omega-\Gamma)\exp\left(\frac{\Omega-E_1^-+\mu}{kT}\right). \quad (2.11)$$

We confine ourselves for the time being to the case $T=0$. It is easily seen that in this case it is necessary to replace in (2.11) the Gibbs exponential by unity, and to integrate over a region satisfying the conditions

$$E_1^--\mu<0, \quad E_1^--\mu<E_2-2\mu, \quad (2.12)$$

which mean that the pair is in a state with energy E_1^-. We introduce new variables $x=(\varphi_1-\varphi_2)/2$ and $E_1^-=(\varphi_1+\varphi_2-\hbar\omega)/2$. Then

$$\sigma(\omega)=\frac{8\pi^2e^2g^2}{3\hbar^2\omega}\int_0^{\infty}dr\int_0^{\hbar\omega/2}dx\int dE_1^-r^4I^2(r)\delta(\hbar\omega-\Gamma). \quad (2.13)$$

Substituting (2.6) and (2.7) in (2.13) and recognizing that $\hbar\omega=\Gamma$, we find that integration with respect to E_1^- must be carried out within the limits

$$\mu-e^2/\varkappa r-\hbar\omega<E_1^-<\mu. \quad (2.14)$$

Thus,

$$\int dE_1^-=\hbar\omega+e^2/\varkappa r. \quad (2.15)$$

Integrating with respect to x with the aid of the δ function, we obtain

$$\sigma(\omega)=\frac{4\pi^2e^2g^2}{3\hbar^2}\int_0^{r_\omega}\left(\hbar\omega+\frac{e^2}{\varkappa r}\right)\frac{r^4I^2(r)\,dr}{[\hbar^2\omega^2-4I^2(r)]^{1/2}}. \quad (2.16)$$

From this, accurate to the number under the logarithm sign in (1.2), we obtain the final result

$$\sigma(\omega)={}^{1}/_{3}\pi^2e^2g^2a\omega r_\omega^4[\hbar\omega+e^2/\varkappa r_\omega]. \quad (2.17)$$

If we discard the second term in the square bracket, which takes into account the electron-electron interaction, then we obtain the Mott formula (1.3) with $\alpha=\pi^2/3$.

We explain now qualitatively the meaning of the factor in the square bracket. Disregarding the electron-electron interaction, Mott has assumed that the pair has only one electron and can consequently participate in the absorption only if $E_1^-<\mu$ and $E_1^+>\mu$ [Fig. 2(a)]. Thus, according to Mott, the only pairs of significance are those with $\mu-\hbar\omega<E_1^-<\mu$, so that the integral with respect to E_1^- yields $\hbar\omega$. In fact, the energy of ionizing the second electron of the pair to the Fermi level is not $\mu-E_1^+$ but $\mu-(E_2-E_1^-)$, and it is precisely this last quantity which should be negative in order that there be no second electron [Fig. 2(b)]. The level E_1^+ can in this case dip below the Fermi level to a depth up to $e^2/\varkappa r$, and the level E_1^- to $\hbar\omega+e^2/\varkappa r$. It is this which explains the results (2.15) and (2.17).

Repeating the derivation of (2.7) for a one-dimensional system immersed in a medium with dielectric constant $\varkappa$, we obtain

$$\sigma(\omega)={}^{1}/_{2}\pi e^2ag^2\omega r_\omega^2[\hbar\omega+e^2/\varkappa r_\omega]. \quad (2.18)$$

As stated in the introduction, in the case of a one-dimensional metal we have $g=(\pi\hbar v_F)^{-1}$ and $I_0=\hbar/\tau$. It seems natural to us to assume that in this case, at large distances $x\gg l$ from the localization center, the wave functions decrease like $\exp(-x/a)\exp(ipx)$, and in the vicinity of the Fermi level the quantity a can be regarded as constant. We are interested in the overlap of states whose centers are separated by a distance of the order of $r_\omega=l\ln(1/\omega\tau)$, and the energies differ by $\hbar\omega$. If $\omega\tau\ll1$, the condition $\Delta pr_\omega\ll1$ is satisfied, where $\Delta p=\hbar\omega/v_F$ is the difference between the momenta of the overlapping states. Then the oscillating factor does not play any role in the calculation of the overlap integral, and formula (1.1) is valid as before, where $a=Cl$ and C is a numerical coefficient. We did not find in the literature an unequivocal statement concerning the quantity C. We have therefore chosen C such that at $\hbar\omega\gg e^2/\varkappa r_\omega$ formula (2.18) coincides exactly with the Berezinskii formula (1.4). This yields $C=2$, i.e., $a=2l$. For a one-dimensional metal formula (2.18) then takes the form

$$\sigma(\omega)=\frac{4}{\pi\hbar^2}e^2v_F\tau^3\omega\left[\hbar\omega\ln^2\frac{1}{\omega\tau}+\frac{e^2}{2\varkappa v_F\tau}\ln\frac{1}{\omega\tau}\right]. \quad (2.19)$$
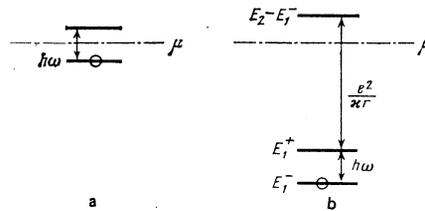

FIG. 2. Energy scheme of resonant absorption by a pair with arm length r: a—neglecting the Coulomb interaction; b—with allowance for the Coulomb interaction of the second electron with the first. As a result of this interaction, the energy of the second electron $E_2-E_1^-$ exceeds by $e^2/\varkappa r$ the energy of the term E_1^+ and can be larger than μ, even though $E_1^+<\mu$.

3. TWO-SITE APPROXIMATION AND INFLUENCE OF THE COULOMB GAP

The preceding calculations were based on the two-site Hamiltonian (2.4). Two assumptions were made.

1. We have neglected the correlation between the occupation numbers of the soft pair and the other occupation numbers. In particular, the probabilities of a pair having one electron and two electrons were calculated with the same values of the other occupation numbers, and consequently with the same values of φ_1 and φ_2, whereas the appearance of a second electron in a pair could cause a change in the neighboring occupation number such that the double occupancy turns out to be more probable than in the considered "hard" version.

2. For all the sites that do not enter in the soft pair, we used the approximation (2.3). This approximation makes, in particular, the state density on the Fermi level a quantity different from zero.

It is known[6] that in the three-dimensional case the second approximation is patently incorrect in the vicinity of the Fermi level, since a Coulomb gap is present with a width $\Delta \approx e^3 g^{1/2}/\varkappa^{3/2}$, and the state density vanishes at the Fermi level (Fig. 1). On the other hand, as shown by one of us,[8] at energies higher than Δ the Coulomb interaction influences the occupation numbers little, and the approximation (2.3) is valid. It is therefore necessary to compare the values of φ_1 and φ_2, which are of importance in our problem, with the value of Δ. As seen from (2.13)–(2.15), in the most important case $e^2/\varkappa r_\omega > \hbar\omega$, the condition $\varphi_1, \varphi_2 \gg \Delta$ is of the form

$$e^2/\varkappa r_\omega \gg \Delta. \qquad (3.1)$$

It is satisfaction of this condition which makes it possible to regard the state density as constant.

The same condition allows us to disregard the change of the surrounding occupation numbers when the charge of the soft pair is changed. Assume, e.g., that we have changed the occupation number n_1 of site 1 belonging to the pair. Then all the surrounding sites j will be acted upon by an additional potential of the order of $e^2/\varkappa |r_j - r_1|$. At a sufficiently large distance from the site 1 there are inevitably sites whose energy goes through the Fermi level when an additional potential is added, as a result of which their charge changes. The distance R to the nearest site of this kind can be estimated from the condition $gR^3 e^2/\varkappa R \approx 1$. This yields

$$R = r_0 = (ge^2/\varkappa)^{-1/2} = e^2/\varkappa\Delta. \qquad (3.2)$$

It is now necessary to estimate whether the potential produced by the charge at a distance R from the pair influences the characteristic energies of the pair. To this end it is necessary to compare $e^2/\varkappa r_0$ and $e^2/\varkappa r_\omega$. From this we see that when condition (3.1) we can disregard the rearrangement of the neighboring occupation numbers when the charge composition of the pair is changed.

Let us discuss briefly the form of the electric conductivity when a condition inverse to (3.1), i.e., $r_\omega \gg e^2/\varkappa\Delta$ is satisfied. In this case it is necessary to substitute in (2.5) the set of occupation numbers corresponding to the ground state of the system, and to take into account the fact that the state density has a Coulomb gap and decreases in the vicinity of the Fermi level like φ^2. Since $\varphi \approx e^2/\varkappa r_\omega$, and the electric conductivity contains a product of two state densities, it follows that the expression for the electric conductivity, under the conditions $r_\omega \gg e^2/\varkappa\Delta$ and $\hbar\omega \ll e^2/\varkappa r_\omega$ should contain four less powers of the logarithm than expression (1.5). From this we get

$$\sigma(\omega) \propto \omega/\ln\frac{2I_0}{\hbar\omega}, \qquad (3.3)$$

i.e., the dependence turns out to be weaker than linear.

This result pertains only to the three-dimensional case. In the one-dimensional case the Coulomb gap has only a logarithmic character and its width is exponentially small. It seems to us therefore that formula (2.19) (with perhaps a slight modification of the logarithmic factor) should be applicable at arbitrarily low frequencies.

4. CONDUCTIVITY AT FINITE TEMPERATURE

We have assumed so far $T = 0$. We consider now how the results are altered at finite temperatures. We assume first that $e^2/\varkappa r_\omega \ll \hbar\omega$. This case seems to us the most important. Indeed, for hydrogenlike localized states $I_0 \approx e^2/\varkappa d$ and, using the definition (1.2), we readily see that the strong opposite inequality $e^2/\varkappa r_\omega \ll \hbar\omega$ is impossible.

Formula (1.5) is applicable at $kT \ll \hbar\omega$. In the region $\hbar\omega \ll kT \ll e^2/\varkappa r_\omega$ the populations of the states E_1^+ and E_1^- become close to each other, so that the stimulated emission almost offsets the transitions to the upper level. This is manifest in the fact that the factor $1 - \exp(-\hbar\omega/kT)$ in (2.11) becomes of the order of $\hbar\omega/kT \ll 1$. At the same time, the width of the region of integration with respect to E_1^-, defined by the inequalities (2.12), is equal to $e^2/\varkappa r_\omega \gg kT$ and does not become smeared out by the temperature. Thus, the following formula is valid in the entire region $kT, \hbar\omega \ll e^2/\varkappa r_\omega$,

$$\sigma(\omega) = \frac{\pi^2}{3}[1 - e^{-\hbar\omega/kT}]ag^2\frac{e^4}{\varkappa}\omega r_\omega^3. \qquad (4.1)$$

In the region $kT \gg e^2/\varkappa r_\omega$, $\hbar\omega$ the states E_1^+ and E_1^- can be higher or lower than the Fermi level by an amount $\sim kT$. In this region

$$\exp\frac{\Omega - E_1^- + \mu}{kT} = \frac{1}{4}\,\text{ch}^{-2}\frac{E_1^- - \mu}{2kT}. \qquad (4.2)$$

Substituting (4.2) in (2.11) we obtain

$$\sigma(\omega) = \frac{1}{3}\pi^2 e^3 ag^2\hbar\omega^2 r_\omega^4, \qquad (4.3)$$

which coincides with Mott's formula (1.3) with $\alpha = \pi 2/3$. Finally, in the region $\hbar\omega \gg e^2/\varkappa r_\omega$, kT formula (2.17) is valid when the term in the square bracket is neglected, after which formula (4.3) is obtained.

As already mentioned in the introduction, at finite temperatures there exists also the so called relaxation mechanism of conduction. It consists in the fact that the electron goes from level to level by absorbing and emitting phonons with characteristic frequency

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$$\tau^{-1} = \nu_{ph} e^{-2r/a}, \qquad (4.4)$$

where ν_{ph} is a frequency of the order of the phonon frequencies (usually 10^{12} Hz). The electric field alters the equilibrium occupation numbers of the sites and causes a relaxation to the equilibrium state determined by the instantaneous value of the field. Retardation and the associated energy dissipation take place.

The main contribution to the conductivity is made by pairs of sites with energy difference $\sim kT$ and with transition frequency τ^{-1} of the order of the field frequency ω. It follows therefore that the characteristic distance in the pair is of the order of

$$r_\omega' = \frac{a}{2} \ln \frac{\nu_{ph}}{\omega}. \qquad (4.5)$$

The first to consider absorption of this type were Pollak and Geballe[7] for the case of the impurity band of a weakly doped semiconductor. For the model considered above with a large scatter of levels of non-Coulomb character, which is frequently used to describe amorphous semiconductors, the formula for the conductivity connected with the relaxation absorption was derived by Austin and Mott.[9] It is of the form

$$\sigma(\omega) = \frac{\pi^4}{24} a e^2 g^2 kT \omega r_\omega'^4. \qquad (4.6)$$

(We have presented here the correct, from our point of view, numerical coefficient obtained in Refs. 5 and 10.)

In the derivation of (4.6) no account was taken of the quantum splitting of the levels, so that this equation is valid so long as the distance kT between the levels exceeds the quantum splitting, which equals $2I(r'_\omega)$. At $\omega = \omega_1 \approx (kT/2I_0)^2$ this condition is violated. In the region $\omega_1 \ll \omega \ll T/\hbar$ the decisive role is played by pairs of length $r_T = a \ln(2I_0/kT)$, for which $2I(r_T) = kT$ and the electric conductivity is of the form[10]

$$\sigma = \frac{\pi^5}{288} e^2 g^2 a I_0^{-2} \nu_{ph} (kT)^3 r_T^4 \qquad (4.7)$$

and is independent of frequency.

One of us[5] has shown that at low temperatures $kT < e^2/\varkappa r'_\omega$ it is necessary to take into account the Coulomb correlation of the occupation numbers inside the pair. Just as in the resonance-absorption problem discussed above, this results in the pair having only one electron

and contributing to the relaxation absorption only when its levels are lower than the Fermi level by an amount $e^2/\varkappa r'_\omega$, and not by an amount $\sim kT$ as was assumed in Refs. 9 and 10. As a result, at $\omega < \omega_1$ the following formula is valid[5]

$$\sigma(\omega) = \frac{\pi^2 e^4}{6\varkappa} a g^2 \omega r_\omega'^3, \qquad (4.8)$$

and differs from (4.6) by a factor $e^2/\varkappa r'_\omega kT$ and by a numerical coefficient. Formula (4.7) should analogously be multiplied by $e^2/\varkappa r_T kT$. We then have in order of magnitude

$$\sigma \approx (e^4/\varkappa) a g^2 \nu_{ph} I_0^{-2} (kT)^2 r_T^3. \qquad (4.9)$$

We consider now briefly the expected form of the experimentally observable electric conductivity at a given temperature, assuming that $e^2/\varkappa r_\omega$, $e^2/\varkappa r'_\omega$, $e^2/\varkappa r_T \gg \hbar\omega, kT$. The static conductivity goes over into the relaxation conductivity (4.8), followed by the plateau (4.9). The conductivity connected with the resonance transitions can become larger than the relaxation conductivity at $\hbar\omega \ll kT$ or at $\hbar\omega \approx kT$. In both cases it is described by formula (4.1).

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