Multiphoton processes in free-electron lasers

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Multiphoton processes in undulators with plane polarized magnetic field are considered. It is shown that the use of strong magnetic fields in the undulator, for beams with relatively low energy (5–15 MeV), makes it possible to increase substantially the frequencies of the amplified electromagnetic waves without noticeably decreasing the gain.

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1. INTRODUCTION

The operating principle of free-electron lasers is based on the interaction of a beam of relativistic electrons with the stationary periodic magnetic field of an undulator. The development of lasers of this type was reported in a number of papers.[1–4] One of the reasons interactions of the free-electron laser is a possibility of regulating the lasing frequency by varying the electron energy. This also raises hopes of advancing into the ultraviolet region.

At high frequencies, however, the gain obtained by perturbation theory in relatively weak fields of the undulator decreases with increasing frequency ω of the amplified wave like ω^{-2} (Refs. 5, 6). Therefore any method of increasing the gain in the infrared, optical, and ultraviolet bands is of interest. In particular, it makes sense to consider multiphoton processes, when the undulator field parameter $\zeta = eH_1 ƛ_0/mc^2 \gtrsim 1$ (H_1 is the amplitude of the undulator magnetic field, λ_0 is its period, and m is the electron mass).

The saturation of the intensity of the amplified electromagnetic wave in the case of a helical magnetic field of the undulator was considered by McIver and Fedorov.[7] In the present paper, the gain is calculated within the framework of a quantum-mechanical description of the behavior of the electrons in specified classical fields. The wave function of the electrons in a strong magnetic field is obtained without using perturbation theory; the field of the amplified wave is regarded as weak, and the probabilities of the induced radiation (absorption) are calculated in first-order perturbation theory with respect to this field. In the derivation of the equations it is assumed that the one-electron approximation criterion is satisfied, so that the considered effects are proportional to the first power of the electron density N_e in the beam.

2. BASIC EQUATIONS

We consider the motion of a relativistic electron situated in the spatially periodic magnetic field of an undulator and in the field of a traveling electromagnetic wave. We define the 4-potentials of the fields by the equations ($\hbar = c = 1$)

$$A_1(x) = (0, \mathbf{e}_x A_1 \cos q_0 z, 0, 0), \qquad (1)$$
$$A_2(x) = {}^1/_2 A_2 [e_2 e^{-ikx} + \text{c.c.}],$$

where $e_1 = (0, \mathbf{e}_x, 0, 0)$ and $e_2 = (0, \mathbf{e})$ are the unit vectors of the polarization of the fields of the undulator and of the wave, respectively (we are considering the case of planar polarization); A_1 and A_2 are the corresponding amplitudes; $k_1 = (0, 0, 0, q_0)$ and $k = (\omega, \mathbf{k})$ are the 4-momenta of the undulator field and of the wave respectively, $q_0 = 2\pi/\lambda_0$. We use in (1) the usual notation for the scalar product of 4-vectors: $kx = (kx) = \omega t - \mathbf{k}\cdot\mathbf{r}$.

As the basic equation, neglecting small spin corrections, we use the Klein-Gordon equation in fields $A_{1,2}(x)$. The dimensionless parameter $\zeta = eA_1/m$, which characterizes the intensity of the interaction of the electron with the undulator field, is assumed to be $\gtrsim 1$ and, accordingly, we take into account the field $A_1(x)$ in all orders of perturbation theory. We assume the field of the wave $A_2(x)$ to be weak enough and consider it in first order of perturbation theory.

The undulator field modulates the Ψ function of the electron in accordance with the equation

$$\left[-\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\mu} - 2ei\left(A_1^\mu \frac{\partial}{\partial x^\mu}\right) + (eA_1)^2 - m^2\right]\Psi = 0. \qquad (2)$$

The solution of Eq. (2) is given by a function of the type[8]

$$\Psi = e^{-ipx}F(k_1x) = (2\varepsilon V)^{-1/2}\exp\left[-i\tilde{p}x - i\frac{e(A_1p)}{(pk_1)}\sin k_1x - i\frac{(eA_1)^2}{8(pk_1)}\sin 2k_1x\right],$$
$$\tilde{p} = p + \frac{e^2A_1^2}{4(pk_1)}k_1, \qquad (3)$$

where $\tilde{p}$ is the average kinetic 4-momentum of the electron; $p = (\varepsilon, \mathbf{p})$ is the 4-momentum of the free electron (when the field is turned off). The function (3) corresponds to normalization of the wave function of the free particle to one particle in the volume V.

It should be noted that the function (3), in contrast to the solution of the relativistic equation for a traveling wave,[8] is an approximate solution of (2). When (3) was substituted in (2), we left out small terms $\sim eA_1p_\perp/p_\parallel^2$ and $q_0/p_\parallel$, where $p_\perp$ and $p_\parallel$ are the components of the electron momentum $\mathbf{p}$ perpendicular and parallel to the undulator axis. The solution (3) does not contain the reflected wave that arises when the particle enters the magnetic field of the undulator. In the case of collinear geometry, which will be considered from now on, the reflected wave can be neglected if the criterion $\zeta^2/\gamma^2 \ll 1$ is satisfied ($\gamma = \varepsilon/m$).

In the case of an helical undulator field the function $(eA_1)^2$ does not contain a spatial dependence, and allowance for the corresponding term in (2) leads only to a renormalization of the electron mass. In the case of a plane-polarized magnetic field this function contains a periodic dependence on the coordinate, which is in fact the cause of the multiphoton effects in strong fields.

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The processes considered are characterized by an S-matrix element given in perturbation theory in the first order of the interaction $\hat{V}=-2e^2(A_1A_2)$ by

$$S_{fi}=i\frac{e^2A_1A_2(e_1e_2)}{(\varepsilon\varepsilon')^{1/2}V}\int\exp[i(\tilde{p}'-\tilde{p})x-i\alpha\sin 2k_1x]\cos kx\cos k_1x\,d^4x, \quad (4)$$

where $p'=(\varepsilon',\mathbf{p}')$ is the 4-momentum of the final state of the electron; α denotes the dimensionless quantity

$$\alpha={}^1/_8e^2A_1^2[1/(pk_1)-1/(p'k_1)]. \quad (5)$$

We assume that the region of interaction of the electrons with the undulator field is infinite or, more accurately speaking, we assume satisfaction of the condition $\delta\varepsilon/\varepsilon \gg 1/N$, where N is the number of periods of the undulator and $\delta\varepsilon/\varepsilon$ is the relative energy spread in the initial beam of the electrons. In this case the integration in (4) is carried out formally with infinite limits, and the result, as usual, constitutes four δ functions that yield the system energy and momentum conservation laws.

We thus obtain from (4) the following expressions for the S-matrix elements that describe the processes with emission (S^e_{fi}) and absorption (S^a_{fi}) of a photon of the amplified wave of frequency ω:

$$S_{fi}^{e,a}=i\frac{e^2A_1A_2}{4(\varepsilon\varepsilon')^{1/2}V}(2\pi)^4\sum_n J_n(\alpha)\{\delta^{(4)}[\tilde{p}'\pm k-\tilde{p}-(2n-1)k_1] + \delta^{(4)}[\tilde{p}'\pm k-\tilde{p}-(2n+1)k_1]\}, \quad (6)$$

where the upper and lower signs correspond respectively to emission and absorption. In the derivation of (6) we used the Fourier expansion

$$\exp(-i\alpha\sin 2k_1x)=\sum_{n=-\infty}^{+\infty}J_n(\alpha)\exp(-in2k_1x),$$

where $J_n(\alpha)$ are Bessel functions.

From the conservation laws contained in the δ functions of Eq. (6)

$$\tilde{p}'\pm k-\tilde{p}-(2n-1)k_1=0, \quad \tilde{p}'\pm k-\tilde{p}-(2n+1)k_1=0, \quad (7)$$

and also from the conditions

$$(\tilde{p})^2=(\tilde{p}')^2=\tilde{m}^2\approx m^2(1+\zeta^2/2), \quad (k)^2=0,$$

follow the permissible values of n in the sums of (6). Thus, in the first sum for the process with emission (S^e_{fi}) we obtain $n\geqslant 1$, in the second we get $n\geqslant 0$; for the process with absorption (S^a_{fi}) we obtain $n\leqslant 0$ and $n\leqslant -1$ in the first and second sums respectively. Taking into account the permissible n, expressions (6) can be represented in the form

$$S_{fi}^e=i\frac{e^2A_1A_2}{4(\varepsilon\varepsilon')^{1/2}V}(2\pi)^4\sum_{n=0}^{\infty}[J_{n+1}(\alpha)+J_n(\alpha)]\delta^{(4)}[\tilde{p}'+k-\tilde{p}-(2n+1)k_1], \quad (8)$$

$$S_{fi}^a=i\frac{e^2A_1A_2}{4(\varepsilon\varepsilon')^{1/2}V}(2\pi)^4\sum_{n=0}^{\infty}(-1)^n[-J_{n+1}(\alpha) + J_n(\alpha)]\delta^{(4)}[\tilde{p}'-k-\tilde{p}-(2n+1)k_1].$$

On going to the probabilities of the processes, the δ functions of Eqs. (8) determine the three components of the momentum $\mathbf{p}'$ of the final state of the electron. The singularity that remains after integration with respect to $d\mathbf{p}'$, as is customary in problems of induced radiation in a given field, is eliminated by taking into account the real properties of the interacting objects, such as the spread of the initial electron beam with respect to direction or energy, the finite interaction region, the deviations of the undulator field from periodicity, etc. In our case the total probabilities of the processes should be averaged over the initial energy distribution of the electrons in the beam, given by the distribution function $f(\varepsilon)$. We assume that this function is normalized to unity by the condition

$$\int f(\varepsilon)\,d\varepsilon=1$$

and that the half-width of the function $\delta\varepsilon\ll\varepsilon$.

Taking all the foregoing into account, the probabilities, per unit time dw_e and dw_a of the processes with emission and absorption of a photon ω of the amplified wave are given respectively by the expressions (the upper and lower signs in this and all the succeeding equations correspond to emission and absorption, respectively)

$$dw_{e,a}=\frac{\pi}{8}(e^2A_1A_2)^2\sum_{n=0}^{\infty}[J_{n+1}(\alpha_{e,a})\pm J_n(\alpha_{e,a})]^2\frac{\delta(\varepsilon'_{e,a}\pm\omega-\varepsilon)}{\varepsilon\varepsilon'_{e,a}}f(\varepsilon)\,d\varepsilon, \quad (9)$$

where in accordance with the definition (5) and the conservation laws (7)

$$\alpha_{e,a}=\mp\frac{\alpha}{1\mp\omega/\varepsilon}; \quad \alpha=\frac{\zeta^2/2}{1+\zeta^2/2}\left(n+\frac{1}{2}\right). \quad (10)$$

From the conservation laws follow also formulas for the frequencies of the emitted ω_e and absorbed ω_a photons:

$$\omega_{e,a}=\frac{\omega}{1\pm\omega/\varepsilon}; \quad \omega=\frac{8\pi\gamma^2}{\lambda_0(1+\zeta^2/2)}\left(n+\frac{1}{2}\right). \quad (11)$$

In (9) these are no interference terms corresponding to different n. The results that follow indicate that for the parameters considered in the present paper the frequency difference $\delta\omega\equiv\omega(n+L)-\omega(n)$ is less than the spontaneous emission line width.

The rate of amplification of the wave is determined by the probability difference

$$dw_e-dw_a=\frac{\pi}{8}(e^2A_1A_2)^2\sum_{n=0}^{\infty}\left\{[J_{n+1}(\alpha_e)+J_n(\alpha_e)]^2\frac{\delta(\varepsilon_e'+\omega-\varepsilon)}{\varepsilon\varepsilon_e'} -[J_{n+1}(\alpha_a)-J_n(\alpha_a)]^2\frac{\delta(\varepsilon_a'-\omega-\varepsilon)}{\varepsilon\varepsilon_a'}\right\}f(\varepsilon)\,d\varepsilon. \quad (12)$$

Averaging over ε in (9) and (12) can be easily carried out by using the conditions $q_0\ll\omega\ll\varepsilon\approx|\mathbf{p}|$. In this approximation, the δ functions in (9) and (12) can be written in the form

$$\delta(\varepsilon_{e,a}\pm\omega-\varepsilon)=\frac{\delta(\varepsilon-\varepsilon_{e,a})}{|\partial\varepsilon'_{e,a}/\partial\varepsilon-1|}=\frac{\delta(\varepsilon-\varepsilon_{e,a})\varepsilon'_{e,a}}{2(2n+1)q_0}, \quad (13)$$

where $\varepsilon_{e,a}=\varepsilon_0\pm\Delta\varepsilon$ are the energies of the electrons that emit or absorb a photon of given frequency ω for a given period $\lambda_0=2\pi/q_0$ of the undulator;

$$\varepsilon_0=\tilde{m}[\omega/2q_0(2n+1)]^{1/2}; \quad \Delta\varepsilon=\omega/2. \quad (14)$$

After substituting (13) in (12) and integrating with respect to $d\varepsilon$, we obtain for the difference between the total probabilities, per unit time, of emission and absorption of a photon of frequency ω,

$$\Delta w\equiv w_e-w_a=\frac{\pi}{8}(e^2A_1A_2)^2\frac{[J_{n_0+1}(\alpha)-J_{n_0}(\alpha)]^2}{2n_0+1}\frac{f(\varepsilon_e)-f(\varepsilon_a)}{2\varepsilon_0q_0}. \quad (15)$$

we note that in the derivation of (15) we used the inequality $|df/d\varepsilon| \gg f(\varepsilon)/\varepsilon$, which is the consequence of the condition formulated above on the degree of non-monochromaticity of the beam[1)]: $\delta\varepsilon/\varepsilon \ll 1$.

The difference between the values of the function $f(\varepsilon)$ at the points ε_e and ε_a, using formulas (14), is

$$f(\varepsilon_e)-f(\varepsilon_a)\approx 2\Delta\varepsilon\frac{df}{d\varepsilon}\approx\frac{\omega}{\gamma^2 m^2}\left(\frac{\varepsilon}{\delta\varepsilon}\right)^2,\quad \delta\varepsilon\gg\omega \tag{16}$$

(the derivative $df/d\varepsilon$ is calculated at the point $\varepsilon=\varepsilon_0$). Substituting (16) in (15) and using Eq. (11) for ω, we obtain

$$\Delta w=\frac{\pi(eA_2)^2\zeta^2}{8\gamma m(1+\zeta^2/2)}\left(\frac{\varepsilon}{\delta\varepsilon}\right)^2[J_{n_0+1}(\alpha)-J_{n_0}(\alpha)]^2. \tag{17}$$

The difference Δw of the total probabilities of the emission and absorption per unit time of a photon with frequency ω determines the gain G at this frequency:

$$G=\frac{\Delta w\,\omega N_e}{E_2^2/8\pi}, \tag{18}$$

where N_e is the electron density in the beam; E_2 is the amplitude of the electric field intensity in the amplified wave.

3. THE GAIN

Substituting (17) in the definition (18) of G, we obtain for the gain per pass the expression

$$G=\frac{\pi^2e^2N_eL\zeta^2}{\gamma m\omega(1+\zeta^2/2)}\left(\frac{\varepsilon}{\delta\varepsilon}\right)^2[J_{n_0+1}(\alpha)-J_{n_0}(\alpha)]^2, \tag{19}$$

where L is the longitudinal dimension of the interaction region (in our formulation, L coincides with the undulator length). It is easy to verify that formulas (11), (17) and (19) go over in the limit as $\zeta\to 0$ into the known perturbation-theory ($n=0$) formulas.[6,7]

With the aid of (11), given the frequency ω of the amplified wave and the electron energy (γ), we obtain the corresponding index n_0 of the Bessel function

$$n_0+\frac{1}{2}=\frac{\omega\lambda_0(1+\zeta^2/2)}{8\pi\gamma^2}. \tag{20}$$

The energy spread of the electrons in the initial beam determines the effective width of the summation δn in (19):

$$\delta n=2n_0\frac{\delta\varepsilon}{\varepsilon}\approx\frac{\omega\lambda_0(1+\zeta^2/2)}{4\pi\gamma^2}\frac{\delta\varepsilon}{\varepsilon}. \tag{21}$$

Bearing in mind the condition $n_0>1$, we obtain from (10) the argument of the Bessel functions:

$$\alpha=\frac{\zeta^2/2}{1+\zeta^2/2}n. \tag{22}$$

The parameter ζ at a given value of γ should satisfy the inequalities $\gamma\gg\zeta\gg 1$. More accurately, this parameter should be such as to satisfy the condition

$$|\alpha-n|\lesssim n^{1/3}. \tag{23}$$

If the condition (23) is not satisfied, then the Bessel function J_n ($\alpha<n$) are exponentially small and consequently the gain is small. To choose the parameters of the problem it is convenient to formulate the condition (23) in the form

$$(\omega\lambda_0)^{2/3}\lesssim(8\pi)^{2/3}\gamma^{4/3}(1+\zeta^2/2)^{1/3};\quad \gamma\gg\zeta \tag{24}$$

[in the derivation of (24) we used Eqs. (20) and (22)].

To estimate G, we use the known asymptotic representation of the Bessel functions for $n\gg 1$ and $|\alpha-n|\lesssim n^{1/3}$ (Ref. 9):

$$J_n(\alpha)=\frac{1}{3}\left[\frac{2(\alpha-n)}{\alpha}\right]^{1/2}\left\{J_{-1/3}\left(\frac{[2(\alpha-n)]^{3/2}}{3\alpha^{1/2}}\right)+J_{1/3}\left(\frac{[2(\alpha-n)]^{3/2}}{3\alpha^{1/2}}\right)\right\}.$$

Using this representation, we obtain for the difference of the Bessel functions at $\alpha\approx n$

$$|J_n(\alpha)-J_{n-1}(\alpha)|=\frac{2^{2/3}}{3^{1/3}\Gamma(1/3)n^{2/3}}\approx\frac{0.42}{n^{2/3}}. \tag{25}$$

Substituting (25) in (19) and also using formula (20), we obtain for the gain the expression

$$G=\frac{0.2(8\pi)^{4/3}\pi^2e^2N_eL\gamma^{5/3}\zeta^2}{m\omega^{7/3}\lambda_0^{4/3}(1+\zeta^2/2)^{7/3}}\left(\frac{\varepsilon}{\delta\varepsilon}\right)^2. \tag{26}$$

Expression (26) is valid if $n_0\gg 1$. On the other hand if $n_0\sim 1$, then the gain should be calculated from a formula that follows directly from (19):

$$G=\frac{\pi^2e^2N_eL\zeta^2}{\gamma m\omega(1+\zeta^2/2)}[J_{n_0+1}(\alpha)-J_{n_0}(\alpha)]^2\left(\frac{\varepsilon}{\delta\varepsilon}\right)^2. \tag{27}$$

Equations (26) and (27) were obtained in an approximation in which the undulator is assumed infinite, or more accurately, for an undulator and beam with relative energy-distribution width satisfying the inequality $\delta\varepsilon/\varepsilon\gg 1/N$. If $\delta\varepsilon/\varepsilon\ll 1/N$, then the derivative $df/d\varepsilon\sim(\delta\varepsilon)^{-2}$ in the equations for the probability difference (17) and for the gains (26) and (27) should be replaced by[6,10]

$$\frac{4\pi N^2}{(\delta\varepsilon)^2}\left(\frac{\delta\varepsilon}{\varepsilon}\right)^2\frac{d}{du}\frac{\sin^2u}{u^2}.$$

In this case the gain should be calculated from the equations

$$G=\frac{0.8(8\pi)^{4/3}\pi^3e^2N_eL^3\gamma^{5/3}\zeta^2}{m\omega^{7/3}\lambda_0^{10/3}(1+\zeta^2/2)^{7/3}}\frac{d}{du}\frac{\sin^2u}{u^2},\quad n_0\gg 1, \tag{28}$$

$$G=\frac{4\pi^3N_eL^3\zeta^2}{\gamma m\omega\lambda_0^2(1+\zeta^2/2)}[J_{n_0+1}(\alpha)-J_{n_0}(\alpha)]^2\frac{d}{du}\frac{\sin^2u}{u^2},\quad n_0\sim 1. \tag{29}$$

4. ESTIMATES AND CONCLUSIONS

We shall distinguish between two limiting cases: relatively low-energy beams with $\gamma\lesssim 30$, and high-energy beams with $\gamma\sim 10^3$. We consider first the case with relatively small γ. The conditions (24), for a given parameter, determine the permissible value of ζ, and also the limiting values of the electromagnetic-wave frequencies that can be effectively amplified for the given beam (γ) and the given undulator (ζ, λ_0). Table I lists the cal-

TABLE I.

γ	ζ^2	$\omega\lambda_0$, eV·cm	λ_0, cm	ω, eV	ω_p, 10^{-3} eV
10	10	0.12	3 6	0.04 0.02	
	20	0.17	3 6	0.06 0.03	0.8 0.4
	30	0.20	3 6	0.07 0.03	
20	10	0.48	3 6	0.16 0.08	
	20	0.68	3 6	0.23 0.11	3.4 1.7
	30	0.80	3 6	0.27 0.13	
30	10	1.1	3 6	0.36 0.18	
	20	1.5	3 6	0.51 0.25	7.5 3.8
	30	1.8	3 6	0.6 0.3	

TABLE II.

γ	ζ^2	ω, eV	G, %	H_1, kG
10	10 20 30	0.04 0.06 0.07	37 9 4	11 15.7 19
20	2.5 16	0.1 0.2	36 2	5.5 14
30	1.6 6	0.2 0.3	16 4	4.4 8.6

culated limiting frequencies ω of the amplified waves for different beam and undulator parameters. The last column of the table indicates the frequencies ω_p obtained by perturbation theory ($\zeta \ll 1$, $n=0$) with the same values of γ and λ_0 (the upper figure is for $\lambda_0=3$ cm, the lower for $\lambda_0=6$ cm).

Table II gives the values of the gain for different values of γ and ζ and of the frequencies ω of the amplified waves [G is calculated from Eq. (28) with the parameters $I=10$ A, $d=0.5$ cm, $\lambda_0=3$ cm, and $L=3$ m]. The last column of this table gives the maximum amplitudes of the undulator magnetic field intensity H_1.

For comparison we present the result of a calculation of the gain G_p by perturbation theory ($\zeta \ll 1, n=0$). If the beam and undulator parameters are $\gamma=50$, $I=10$ A, $d=0.5$ cm, $\omega=0.2$ eV, $\zeta^2=0.1$, $\lambda_0=3$ cm, and $L=3$ m, the gain turns out to be $G_p=14\%$.

From Table I and from the presented estimates of the gain it is seen that the use of strong plane-polarized magnetic fields makes it possible to increase substantially the frequencies of the amplified electromagnetic waves, using beams of relatively low energy, without a noticeable decrease of the gain.

We turn now to the case of large $\gamma \sim 10^3$. Table III lists for different undulator parameters and for different n_0 the frequencies of the electromagnetic waves that can be amplified with the aid of an electron beam in a storage ring with $\gamma=7\times10^2$ ($\varepsilon=350$ MeV). The fourth column of the table gives the maximum amplitudes of the undulator magnetic field intensity. In the last column are given the corresponding gains [the numerical values of G were obtained from Eq. (29) at $I=100$ A, $d=0.2$ cm, $\lambda_0=3$ cm, and $L=3$ m].

For comparison, Table IV gives the gains $G_{\rm hel}$ for the case of an undulator with a helical magnetic field. $G_{\rm hel}$ was calculated from an equation given by Pellegrini,[11] with beam and undulator parameters $\gamma=7\times10^2$, $I=100$ A, $d=0.2$ cm, $\lambda_0=3$ cm, and $L=3$ m.

From a comparison of the data of Tables III and IV it is seen that an undulator with planar polarization of the magnetic field, in the case of large γ and $\zeta>1$, offers no advantages in comparison with a helical undulator.

TABLE III.

n_0	ω, eV	ζ^2	H_1, kG	G, %
1	10 5 3	22.5 47 80	16.4 24 31	3 6 10
2	10 5 3	39 80 134	22 31 40.6	2 3 5

TABLE IV.

ω, eV	ζ^2	$G_{\rm hel}$, %
10 5 3	3.1 7.2 12.7	20 47 82

Thus, the use of undulators with planar polarization of the magnetic field is advantageous in the case of electron beams with relatively small γ (~ 10–30), inasmuch as multiphoton processes can increase in this case the frequency of the amplified wave at least to the infrared band. It must be borne in mind here that the equations obtained in this paper are valid only for maximum undulator field values that satisfy the criterion $\zeta^2/\gamma^2 \ll 1$.

Although the expressions for the gain G were obtained formally within the framework of perturbation theory in the interaction $\hat{V}=-2e^2(\mathbf{A}_1\cdot\mathbf{A}_2)$, the range of validity of these equations should be regarded as wider.[7] Using the results of McIver and Fedorov,[7] we obtain an estimate for the field intensity E^2 at which saturation sets in ($\gamma=20$, $L=3$ m, $\lambda_0=3$ cm, $\hbar\omega=0.1$ eV, $H_1=10$ kG), namely, $E_2 \sim 10^2$ V/cm.

The analysis proposed in this paper is valid when the role of the undulator is played by a traveling electromagnetic pump wave with planar polarization.[12] In this case, when the electron interacts with the wave it can capture many photons and emit one photon of the amplified wave. The corresponding gain differs from Eq. (19) only by a numerical factor.

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[1)]The effective width of the summation over n in Eq. (12) is determined by the energy spread $\delta\varepsilon$ in the beam. For realistic values $\delta\varepsilon \lesssim 1\%$ we have $\delta n<1$ [see (21)], and we shall therefore omit hereafter the sign of summation over n.

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