Properties of superconductors with a smeared transition temperature

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Percolation theory is used to calculate the critical current, the magnetic-field penetration depth, and the superconducting transition temperature in inhomogeneous superconductors whose parameters are random functions of the coordinates. The dependences of the percolation-theory parameters and the mean physical quantities on temperature and the degree of inhomogeneity are found for different types of inhomogeneity.

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INTRODUCTION

The thermal fluctuations smear out the phase transition in homogeneous superconductors only in a very narrow temperature region. Therefore, the smearing of the phase transition is often determined by the inhomogeneities of the sample. Only those superconductors with a greatly smeared transition temperature will be considered below. The coupling constant in inhomogeneous superconductors is a random function of the coordinates. The smearing of the transition near the upper critical field H_{c2} depends also on the inhomogeneities in the mean free path. The physical cause of such inhomogeneities may be dislocation, impurity, or crystallite pile-ups. In certain cases the cause is not clear; in others the inhomogeneities are artificially produced. Also greatly smeared are the superconducting phase transitions in heterogeneous systems in which the regions of the matrix separate out as a new phase with superconducting properties different from those of the matrix, as well as in systems of compressed superconducting balls covered with a thin dielectric layer or imbedded in a normal-metal matrix.

The details of the temperature dependence of the superconducting parameters vary from one type of system to another and strongly depend on the dimension of the superconducting regions and the magnitude of the local-transition-temperature spread, but the qualitative picture is the same in all the cases of a highly smeared transition. At a temperature close to the smeared-transition temperature, there exist in the superconductor superconducting regions whose energy is high compared to the temperature. The binding energy of these regions is exponentially small as compared to the energy of each region. The phase transition occurs at a temperature of the order of the binding energy. It is significant that the binding-energy spread is exponentially large. Therefore, we can consider the regions whose binding energy is higher than the temperature to be strongly coupled, and neglect the thermal fluctuations of the order parameter in them. We can neglect the couplings whose energy is less than the temperature. Strongly coupled regions form clusters whose dimensions increase with decreasing temperature. At some temperature one of the clusters becomes infinitely large. This is the transition temperature. As the temperature is decreased further, the number of regions belonging to the infinite cluster increases, and they form a cellular structure with characteristic dimension L. These quantities can be found from percolation theory. In the first part of the paper we express the superconductor's characteristics, such as the critical current, the magnetic-field penetration depth, and the superconducting transition temperature, averaged over the sample, in terms of the parameters of percolation theory. In the second part we express these parameters in terms of the macroscopic parameters of various systems with a highly smeared phase transition.

I. PERCOLATION MODEL OF A HIGHLY SMEARED SUPERCONDUCTING TRANSITION

Let us consider a model in which the modulus of the superconducting order parameter is large near the individual centers and exponentially small at points far away from them. The binding energy of such superconducting drops is exponentially small, the quantity in the exponential being, moreover, a random quantity. A similar exponentially weak random coupling arises in a system of superconducting drops separated by dielectric layers of varying thicknesses.

The thermal fluctuations have little effect on the properties of the individual superconducting drops; the phase φ and modulus Δ of the order parameter will therefore be considered to be constants inside a drop. The free energy of such a structure is equal to

$$F=\sum_i F_0(H,\Delta_i)+\sum_{i,j} V_{ij}\left[1-\cos\left(\varphi_i-\varphi_j-\frac{2e}{c}\int \mathbf{A}\,d\mathbf{r}\right)\right]. \quad (1)$$

Here the integral is evaluated over a path between the individual centers and F_0 is the free energy of a single drop. If V_{ij} is exponentially small, then the second term in the formula (1) is smaller than the first everywhere except at temperatures close to the transition temperature or to the H_{c2} of an individual drop. Outside this narrow region, the order parameter is the same in all the drops, and the free energy depends only on the phase difference, which is equivalent to saying that the first term in the formula (1) can be dropped. It is convenient to represent the quantity V_{ij} in the form

$$V_{ij}=Ve^{-\xi},$$

where ξ is a random quantity with a characteristic domain of variation much greater than unity. For example, for superconducting drops imbedded in a normal-metal matrix,

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$$V_{ij}=V\exp(-r_{ij}/\xi),$$

where ξ is the correlation length in the normal metal. In this case $\zeta_0=N^{-1/3}\xi^{-1}$, where N is the concentration of the drops.

1. Magnetic-field penetration depth

Let us first find the depth of penetration of a weak magnetic field into a highly inhomogeneous superconductor. In a weak magnetic field the expression (1) assumes the form

$$F=\sum_{ij}\frac{1}{2}V_{ij}\left(\varphi_i-\varphi_j-\frac{2e}{c}\int \mathbf{A}\,d\mathbf{r}\right)^2. \tag{2}$$

Minimizing this expression with respect to φ_j, we obtain for the free energy the expression

$$F=\frac{1}{4\pi\lambda^2}\int\left(\mathbf{A}-\frac{c}{2e}\nabla\varphi\right)^2 d^3r, \tag{3}$$

where λ is the depth of penetration of the magnetic field into the superconductor. The coefficient $c^2/16\pi e^2\lambda^2$ coincides with the mean conductivity of a system of drops joined by resistances equal to V_{ij}^{-1}.

The problem of the computation of the mean conductivity of highly inhomogeneous media has been solved by the methods of percolation theory.[1] According to this theory, the dominant contribution to the mean conductivity is made by critical resistances, equal in magnitude to

$$R_c=V_c^{-1}=V^{-1}e^{\zeta_c}, \tag{4}$$

which are found from the requirement that all the resistances less than R_c should form an infinite cluster near its percolation threshold. The resistances lower than R_c are not connected with each other, while the resistances much higher than R_c are shunted by resistances of the order of R_c, and do not contribute to the mean conductivity; ζ_c is determined from the condition

$$B_c=Nv_c, \tag{5}$$

where v_c is the volume of that region inside which two drops interact with V_{ij} greater than $Ve^{-\zeta_c}$; B_c is a number that depends weakly on the shape of the region; it is computed in Ref. 8. The quantity B_c is equal to 2.7 and 4.5 respectively in three- and two-dimensional systems.

For randomly arranged drops interacting according to the law (2), we have[1]

$$\zeta_c=\alpha N^{-1/d}\xi^{-1}, \tag{6}$$

where $\alpha=0.89$ for $d=3$ and $\alpha=0.95$ for $d=2$. For drops forming a regular lattice, and the interactions between which are distributed according to a law $p(\zeta)$, the values of ζ_c and, consequently, V_c are found from the equation

$$\int^{\zeta_c} p(\zeta)\,d\zeta=x_c, \tag{7}$$

where the numbers $x_c\sim 1$, and are given for different lattices in Table 5.1 in Shklovskiĭ and Éfros's book.[1]

The mean distance between the critical resistances is found from the condition $V_{ij}\sim V_c$, and is equal in order of magnitude to[1]

$$L=N^{-1/d}\zeta_c^{\nu}. \tag{8}$$

In the three-dimensional case $\nu=0.9$; for films $\nu=1.3$. The mean conductivity and, consequently, the penetration depth λ are expressed in terms of V_c and L by the formula

$$c^2/16\pi e^2\lambda^2=V_cL^{-1}. \tag{9}$$

Thus, the mangetic-field penetration depth is exponentially large, much larger than L. Therefore, we can use the local approximation to compute it, assuming that A in the formulas (3) and (4) does not depend on the coordinates, an assumption which was made earlier.

2. The critical current

The superconducting current flowing between two drops is equal to

$$I_{ij}=2eV_{ij}\sin(\varphi_i-\varphi_j). \tag{10}$$

It cannot exceed $2eV_{ij}$. Contacts with very small V_{ij} are unimportant, since they will be shunted. The dominant contribution to the critical current density will be made by contacts with $V_{ij}\sim V_c$. The critical current density is equal to

$$j_c=eV_cL^{-2}/c. \tag{11}$$

The same expression can be obtained by expanding (1) in a power series in A and finding the vector potential $\mathbf{A}$ from the condition that all the terms of the series should be of the same order of magnitude. Thus, the critical current is proportional to V_c, and exponentially small. From the formulas (9) and (11) we can eliminate V_c, and determine the critical length

$$L=c/16\pi e^2\lambda^2 j_c. \tag{12}$$

Like the formulas (9) and (11), this expression is valid up to a numerical factor of the order of unity.

By varying the temperature or the drop concentration, we can vary ζ_c. By determining ζ_c with the aid of the formula (6), or with the aid of the formulas (5), (9), and (11), and comparing the formulas (8) and (12), we can experimentally determine the index ν, and compare it with the above-presented values, which were calculated on a computer.[3]

3. The superconducting transition temperature

We shall determine the superconducting transition temperature in the same way as the magnetic transition temperature for very dirty magnetic alloys was determined.[2] We shall assume that the drops whose interaction energy is higher than T have order parameters with the same phase, and that they are gathered in clusters. The transition temperature is found from the requirement that one cluster should become infinite:

$$T_c=V_c(T_c)=Ve^{-\zeta_c}. \tag{13}$$

In order to verify that the formula (13) does not contain ζ_c dependent pre-exponential factors, let us compute the correction to the penetration depth due to the thermal fluctuations. We shall estimate the quantity T_c as that temperature at which this correction assumes a value of the order of unity. For this purpose, let us expand the cosine in the expression for F up to terms of second order in $(\varphi_i - \varphi_j)^2$, and let us replace in the lowest approximation $(\varphi_i - \varphi_j)^4$ by $6(\varphi_i - \varphi_j)^2 \langle(\varphi_i - \varphi_j)^2\rangle_0$:

$$F=\sum_{ij}\frac{1}{2}V_{ij}\left[(\varphi_i-\varphi_j)^2-\frac{1}{4}(\varphi_i-\varphi_j)^2\langle(\varphi_i-\varphi_j)^2\rangle\right].$$

Here $\langle\ldots\rangle$ denotes averaging with the Hamiltonian

$$H=\sum V_{ij}(\varphi_i-\varphi_j)^2$$

over the thermal fluctuations: $\langle V_{ij}(\varphi_i - \varphi_j)^2\rangle = T$. In approximation

$$F=\sum\frac{1}{2}\left(V_{ij}-\frac{1}{2}T\right)(\varphi_i-\varphi_j)^2.$$

As a result, the V_{ij}'s greater than T are replaced by $V_{ij} - \frac{1}{2}T$, while the couplings smaller than T are shunted, and do not contribute to the penetration depth; thus, the V_c in the formula (9) for λ is replaced by $V_c - \frac{1}{2}T$ when $T \ll V_c$:

$$c^2/16\pi e^2\lambda^2=(V_c-{}^1/_2T)L^{-1}. \tag{14}$$

It can be seen from this formula that the correction to λ assumes a value of the order of unity when $T \sim T_c$ given by the expression (13).

Usually, V_c increases rapidly with decreasing temperature; therefore, the condition, $T \ll V_c$, of applicability of all the formulas obtained above is fulfilled everywhere except in a narrow region in the vicinity of T_c.

4. The magnetic moment

The critical current given by the formula (11) is a slowly varying function of the magnetic field. Therefore, strong coupling and hysteresis exist in the superconductor. The magnetic properties depend on the past history and the time the experiment is performed. The characteristic times are determined by the characteristic time of slippage of a flux quantum through the weakest contacts in the infinite cluster, and are equal to $t_1 \sim \exp(V_c/T)$. If the time of the experiment is shorter than t_1, then the magnetic field begins to penetrate the sample to a depth greater than λ when the current density on the surface attains its critical value, given by

$$H_c=4\pi\lambda j_c/c, \tag{15}$$

where λ is given by (9). Upon further increase, it gets distributed inside the sample according to the law

$$dH/dx=4\pi j_c/c. \tag{16}$$

It is distributed according to the same law when the field is switched off, and there remains in the superconductor a trapped macroscopic flux determined by the currents flowing through the entire sample.

If the time of the experiment is longer than t_1, then the magnetic field begins to penetrate into the superconductor when its intensity is equal to the value H_{c1} given by the condition for the minimum of the free energy:

$$H_{c1}=\lambda^{-2}\ln(\lambda/L). \tag{17}$$

When the field is switched off, not all the flux gets out within the time t_1. The part of the flux trapped by closely spaced drops with a large interaction energy remains in the superconductor significantly longer. This part of the flux remains trapped even at temperatures higher than T_c, when the sample contains small superconducting clusters, but no infinite cluster.

II. SPECIFIC SUPERCONDUCTING SYSTEMS

As noted in the Introduction, the model of superconducting drops with weak and strong random couplings can be realized in different physical systems. Here we can express the random quantity V and its parameters V_c and L in terms of the temperature and the microscopic parameters, and then find with the aid of the above-obtained formulas the mean macroscopic characteristics.

1. Widely-spaced superconducting drops in a normal-metal matrix

We consider a system consisting of drops of a superconductor with a high transition temperature T_{cd} imbedded in a matrix with a lower transition temperature T_{cm} in the temperature range $T_{cm} < T < T_{cd}$. Here we assume that the drops have sufficiently large dimensions, so that the proximity effect does not destroy the superconductivity in an individual drop. For this purpose it is sufficient at any rate that the drop dimension be greater than the correlation length ξ in the drop. If the transmittance of the boundary between the drops and the matrix is low, or if the temperature is close to T_{cm}, then the proximity effect is weak, and the condition on the drop dimension is less rigid. The case of small-sized drops will be considered later.

Let us write down the Ginzburg–Landau equation for the order parameter at temperatures slightly higher than T_{cm}:

$$-\xi^2\nabla^2\Delta+\Delta+\Delta^2\Delta/\Delta_m^2=0,$$
$$\xi^2=\frac{D}{(T-T_{cm})}\frac{\pi}{8},\quad \Delta_m^2=\frac{T-T_c}{B}, \tag{18}$$

and D is the diffusion coefficient. If Δ inside a drop is greater than Δ_m, then the solution to (18) in the vicinity of the drop surface decreases over distances $\sim\xi$ to Δ_m; at large distances Eq. (18) reduces to a linear equation, and has the solution

$$\Delta_j=\Delta_0\frac{\xi}{r}\exp\left(-\frac{r}{\xi}+i\varphi_j\right). \tag{19}$$

In the case in which Δ_d inside a drop is smaller than Δ_m, or the transmittance of the boundary between the drops and the matrix is low, Δ_0 in the formula (19) should be replaced by the Δ value inside the drop multiplied by the transmittance; in the opposite case, i.e., when $\Delta_m < \Delta_d$, Δ_0 in this formula coincides with Δ_m.

The parameter characterizing the order between two drops located at a distance much greater than ξ from each other is equal to $\Delta=\Delta_i+\Delta_j$. The current flowing between two such drops is

$$I=e\nu D\frac{\pi}{4}\int \mathrm{Im}\,\Delta\nabla\Delta^* dS=e\nu D\,2\pi^2|\Delta_0|^2\frac{\xi^2}{R_{ij}}\exp(-R_{ij}/\xi)\sin(\varphi_i-\varphi_j). \quad (20)$$

Comparing this expression with the formula (10), we obtain

$$V_{ij}=\nu D|\Delta_0|^2\,2\pi^2\frac{\xi^2}{R_{ij}}\exp\left(-\frac{R_{ij}}{\xi}\right).$$

Comparing the expression (20) with the formula (5), we obtain the parameters of the percolation model:

$$V=\nu D2\pi^2\xi^2N^{1/3}|\Delta_0|^2, \quad (21)$$

with ξ_c and L given respectively by (6) and (8).

The above formulas are applicable in the case in which the temperature is not close to the transition temperature of the matrix; in this case instead of the formula (18) for ξ we should use a more general expression for this quantity; in the particular case of $T_{cm}\ll T$ we can use the expression[4]

$$\xi^2=D/2\pi T. \quad (22)$$

The percolation picture is applicable in those cases in which ξ is small compared to the distance between the drops. Therefore, it is not applicable in the region of temperatures very close to the transition temperature of the matrix, where ξ is large. The standard Ginzberg-Landau equation with averaged parameters is applicable in this region.

2. White noise in the coupling constant

When the size of the regions with an increased value of the coupling constant is small, the superconductivity in each of them is suppressed on account of the proximity effect; nevertheless, it can exist in clusters of such regions. Such a structure is a particular case of white noise in the coupling constant[9]:

$$1/g=\langle 1/g\rangle+g_1,\quad \langle g_1(r)g_1(r')\rangle=\gamma\delta(r-r'). \quad (23)$$

In the case of small regions with temperature T_{cd} the quantity γ is given by the formula

$$\gamma=[(T_{cd}-T_{cm})/T_{cm}]^2v_d^2n, \quad (24)$$

where v_d is the volume of a region and n is the concentration of the regions.

The order parameter in clusters of these regions changes over distances $\xi(T)\gg\xi_0$; therefore, we can write for it the Ginzburg-Landau equation with a noise addend in the term with τ:

$$[-\nabla^2/2m+(\tau+g_1)+B\Delta^2]\Delta=0, \quad (25)$$

where $m=4T/\pi^2D$, $\tau=(T-T_{c0})/T_{c0}$, and T_{c0} is the transition temperature of a homogeneous superconductor with the mean coupling constant.

The temperature region $T<T_{c0}$ has been studied by Ovchinnikov and one of the present authors.[5] They have shown that the inhomogeneities smear the superconducting phase transition over a region of the order of

$$\delta\tau=\gamma^2T^2/D^3; \quad (26)$$

also smeared out in this region are the specific-heat jump, the temperature dependence of the order parameter, and the penetration depth.

Below we shall consider the temperature region $T>T_{c0}$, with $\tau\gg\delta\tau$. In this temperature region, the last term in Eq. (25) is small, and can be taken into consideration with the aid of perturbation theory. Let us, for a given realization of g_1, expand Δ in a series in the eigenfunctions of the linear equation

$$(-\nabla^2/2m+E_n+g_1)\psi_n=0. \quad (27)$$

Only one term, $\Delta=\Delta_0\psi_0$, in this expansion is large. Multiplying Eq. (25) from the left by ψ_0, and integrating over the coordinates, we obtain

$$\Delta_0^2=(E-\tau)/B\int\psi_0^4\,d^3r. \quad (28)$$

When we average over the various g_1 realizations, the dominant contribution is made by that realization for which $g_1\sim\psi_0^2$. Therefore, ψ_0 is found by solving the nonlinear Schrödinger equation

$$[-\nabla^2/2m+E-\psi_0^2]\psi_0=0$$

and normalizing the solution. For E close to τ, the function ψ_0 depends weakly on E. Therefore, the averaging over the realizations amounts to multiplication by the density of states, which, to within an additive number, is equal to[9]

$$\rho(E)=\rho_0\gamma^{-(d+1)/2}E^{d(5-d)/4}\exp\left[-37\frac{E^{2-d/2}}{\gamma}(2m)^{-d/2}\right]. \quad (29)$$

As a result, we obtain

$$\int\Delta^2\,d^3r=\int_\tau^\infty\rho(E)(E-\tau)\,dE/B\int\psi_0^4\,d^3r. \quad (30)$$

Here it is supposed that τ_{c0} in the formulas (29) and (30) has been renormalized on account of the very-small-scaled fluctuations. As shown in Ref. 10, these fluctuations increase T_{c0} by an amount greater than the smearing width $\delta\tau$.

The superconductivity-related correction to the specific heat is equal to

$$C=T^{-1}\frac{\partial}{\partial\tau}\nu\int\Delta^2\,d^3r=\frac{T^{-1}\nu}{B}\rho_0\gamma^{-(d-1)/2}\tau^{(6d-d^2)/4-2}\times m^{d/2}\exp\left[-37\frac{\tau^{2-d/2}}{\gamma}(2m)^{-d/2}\right]. \quad (31)$$

Here τ is reckoned from the renormalized transition temperature T_{c0}, which is defined as the temperature corresponding to the middle of the specific-heat jump.

The true transition temperature, which corresponds to the disappearance of the long-range order, is given by the formula (13); it lies above T_{c0}. In the region between T_{c0} and T_c, the critical current and the penetration depth are given by the above-obtained formulas (11), (12), (8), (5), and (6). These formulas contain the drop concentration N. The drops exist in exponentially sparse regions, where the inequality $-g_1>\tau$ is fulfilled in a fairly large volume, so that Eq. (27) has an eigenvalue $E_n>\tau$. It is precisely these regions that contribute to the density of states $\rho(E)$, (29); therefore, the number of drops

$$N=\int_{\tau}^{\infty}\rho(E)\,dE=\rho_0\gamma^{-(d-1)/2}\,\tau^{(6d-d^2)/4-2}\,m^{d/2}\exp\left[-37\frac{\tau^{2-d/2}}{\gamma}(2m)^{-d/2}\right]. \quad (32)$$

Inside each drop, Δ_m is given by the formula (28). The important values of $E-\tau\sim\delta\tau$; therefore,

$$\Delta_0\sim\delta\tau^{1/2}. \quad (33)$$

Outside a drop Δ decreases exponentially, and, just as in the case of large drops, it is given by the formula (19), where

$$\xi^2=D\pi/8(T-T_{c0}). \quad (34)$$

The interaction V between the drops will be determined by the same formula, (21), as for large drops, the quantities Δ_0 and ξ in which are given by the expressions (33) and (34).

Because of the exponential decrease of the number of drops with increasing T, the dependence of the critical current and the penetration depth is very strong.

From the formulas (21), (6), and (32) we obtain an expression for the transition temperature shift:

$$T_c-T_{c0}=T\delta\tau\frac{1}{170}\ln^2\frac{\alpha}{\ln\nu\gamma T}. \quad (35)$$

In a real sample, there is a spread in the dimensions of the superconducting regions, and a relatively small number of them may each have a dimension greater than ξ. Each such region forms a superconducting drop, and the concentration of these drops may be higher than the drop concentration produced by white noise and given by the expression (32). In this case the structure coincides with the structure considered in the preceding subsection.

3. The smearing of H_{c2}

The percolation picture is applicable not only in the vicinity of T_c, but also in the vicinity of H_{c2}. Let the metallic matrix contain widely-space impregnations of a material with H_{c2} higher than the H_{c2} of the matrix, and let each of them be in the superconducting state.

For sufficiently large distances, Δ is given by the solution to the linear Ginzburg–Landau equation, and has, in the Landau gauge, the form

$$\Delta(r)=\Delta_0\left(\frac{r_\perp}{\xi}\right)^m\exp\left(-\frac{r_\perp^2}{4\xi^2}-\frac{z}{\xi_\parallel}+im\phi+i\varphi\right). \quad (36)$$

Here $r_\perp$, z, and ϕ are cylindrical coordinates and

$$\xi_\parallel^2=\xi^2H_{c2}/(H-H_{c2}); \quad (37)$$

m is the number of vortices in a drop. If H is higher than the H_c of a drop, then the magnetic field penetrates the drop, and m is the number of flux quanta passing through the cross section of the drop; Δ_0 is given by the solution to the one-dimensional nonlinear equation. In a drop with semitransparent walls, Δ_0 coincides with the value of Δ inside the drop multiplied by the transmittance.

Let us find an expression, analogous to (20), for the current flowing between two drops at a distance $R\gg\xi$ apart. Their order parameter at a distance much greater than ξ is

$$\Delta=\Delta_i+\Delta_j.$$

Here the Δ_i are given by the formula

$$\Delta_i=\Delta(r-R_i)\exp(i\varphi_i+2i[\mathbf{H}\times\mathbf{R}_i]\mathbf{r}), \quad (38)$$

where $\mathbf{R}_i$ is the coordinate of the center of the drop. Computing the total current flowing between the two drops, we obtain

$$I_{ij}=e\nu D\frac{\pi}{4}\int \operatorname{Im}\Delta^*\nabla\Delta\,d\mathbf{S}=e\nu D|\Delta(R_i-R_j)|^2\xi_\parallel\sin(\varphi_i-\varphi_j). \quad (39)$$

Comparing this expression with the formula (10), we obtain the drop–drop coupling constant:

$$V_{ij}=V\exp(-\zeta_{ij}),\qquad V=\nu D(r_\perp/\xi)^{2m}\xi_\parallel|\Delta_0|^2, \quad (40)$$

$$\zeta_{ij}=(r_\perp/\xi)^2+z/\xi_\parallel. \quad (41)$$

The volume v_c, which is limited by the condition $\zeta_{ij}>\zeta_c$, is equal to $v_c=\pi\xi^2\xi_\parallel\zeta_c^2$. We find ζ_c from the condition (5):

$$\zeta_c=(B_c\pi N\xi^2\xi_\parallel)^{-1/3}. \quad (42)$$

In the region v_c the characteristic $r_\perp\sim\xi\zeta_c$, while $z\sim\zeta_c\xi_\parallel$; therefore,

$$V=\nu D\zeta_c^{2m}\xi_\parallel|\Delta_0|^2. \quad (43)$$

This result applies in the two-dimensional case as well; we only need to replace $\xi_\parallel$ in the formulas (39) and (43) by the film thickness d and, in the formula (42), drop $\xi_\parallel$ and replace B_c by its two-dimensional value, N being then the surface concentration. In the three-dimensional case the H_{c2}-smearing region is given by the formula (13) for T_c after substituting the formulas (42) and (43) in it, and the H_{c2} shift (δH) is given by the formula

$$\frac{\delta H}{H}=\frac{8\pi D}{T-T_c}\left[B_cN\xi^2\ln^2\frac{\nu D\xi|\Delta_0|^2}{T}\right]^2.$$

In the region $H_{c2}<H<H_{c2}+\delta H$ the critical current, given by the formula (11), decreases exponentially with increasing H.

In the case of a film with a sufficiently high drop concentration satisfying the inequality

$$N\xi^2>\left(\pi B_c\ln^2\frac{\nu Dd|\Delta_0|^2}{T}\right)^{-1} \quad (44)$$

the H_{c2}-smearing region is of the order of H_{c2}. The critical current in this region is given by the formula (11), and depends smoothly on H. If the inequality (44) is not fulfilled, then the percolation mechanism of superconductivity in films does not occur.

4. White noise in the mean free path

The mean-free-path fluctuations in zero magnetic field have no effect on the smearing of the phase transition. But H_{c2} depends on the mean free path, which is therefore smeared by these fluctuations. If the dimension of a region with a short mean free path is large compared to ξ, then this structure is equivalent to the structure considered in Subsection 3. If the dimension of these regions is smaller than ξ, then the mean free paths can be considered to be distributed according to the Gauss law:

$$\langle l(r_1)l(r_2)\rangle-\langle l\rangle^2=\langle l\rangle^2\gamma_1\delta(r_1-r_2). \quad (45)$$

These fluctuations give rise to fluctuations in the coefficient in the derivative term of the Ginzberg-Landau equations. For fields close to H_{c2},

$$-D(\nabla-ie\mathbf{A})^2\Delta+(-|\tau|+g)\Delta+B\Delta^3=0,\quad g=g_1+\tau(l/\langle l\rangle-1), \quad (46)$$

where g_1 is white noise in the coupling constant; g has also a Gaussian distribution with a variance $\gamma+\tau^2\gamma_1$.

As before, the drop concentration coincides with the number, (32), of states with energy smaller than $|\tau|$. As will be shown elsewhere, the density of states in a magnetic field is equal to

$$\rho(E)=\left(\frac{H_{c2}}{H-H_{c2}}\right)^{-5/2}\frac{E^3\xi^3}{(\gamma+\tau^2\gamma_1)^2}\exp\left[-\frac{32}{3}\pi\left(\frac{H-H_{c2}}{H_{c2}}\right)^{3/2}\frac{E^2\xi^3}{\gamma+\tau^2\gamma_1}\right]. \quad (47)$$

Therefore, the drop concentration is equal to

$$N=\int_{|\tau|}^{\infty}\rho(E)\,dE=\left(\frac{H-H_{c2}}{H_{c2}}\right)^2\frac{\tau^2}{\gamma+\tau^2\gamma_1}\exp\left[-\frac{32}{3}\pi\left(\frac{H-H_{c2}}{H_{c2}}\right)^{3/2}\frac{\tau^2\xi^3}{\gamma+\tau^2\gamma_1}\right]. \quad (48)$$

For a given g realization, $\Delta^2 B$ in each drop is $\sim|\tau|-E$; therefore, the mean order parameter in each drop is given by the expression

$$\Delta_0^2=\frac{1}{B}\frac{\gamma+\tau^2\gamma_1}{\tau\xi^3}\left(\frac{H_{c2}}{H-H_{c2}}\right)^{3/2}. \quad (49)$$

The properties of an ensemble of such drops are equivalent to the large-drop structure considered in the preceding subsection, where the quantities N and Δ_0 in the formulas (42) and (43), for ζ_c and V are given by by the formulas (48) and (49).

5. Superconducting drops in a dielectric shell

The structure consisting of superconducting drops in a dielectric can easily be prepared by slightly oxidizing the surface of each drop. If the thickness of the oxide layer is a random quantity, then percolation theory is applicable here.

The Josephson current flowing through the contact between two balls is equal to[6]

$$I_{ij}=\frac{\Delta(T)}{2eR_{ij}^n}\,\mathrm{th}\,\frac{\Delta(T)}{2T}\sin(\varphi_i-\varphi_j). \quad (50)$$

Here R_{ij}^n is the resistance of the contact in the normal state. Comparing the formula (50) with (10), we obtain

$$V_{ij}=\frac{\Delta(T)}{4e^2R_{ij}^n}\,\mathrm{th}\,\frac{\Delta(T)}{2T}. \quad (51)$$

The resistance between the drops is exponentially high:

$$R_{ij}=R_0e^{\zeta}, \quad (52)$$

where $\zeta\sim l/a$ (l is the thickness of the dielectric layer) is distributed with probability $p(\zeta)$. The parameter V_c entering into the percolation theory is equal to

$$V_c=V_0e^{-\zeta_c},\quad V_0=\frac{\Delta(T)}{4e^2R_0}\,\mathrm{th}\,\frac{\Delta(T)}{2T}, \quad (53)$$

and ζ_c is given by the relation (7). The form of the function $p(\zeta)$ and the parameter x_c are not known; therefore, ζ_c cannot be expressed in terms of the microscopic quantities, but in this case there is an independent method of determining ζ_c and V_c in terms of the conductivity of the sample in the normal state. As has been shown in percolation theory, the conductivity of the structure consisting of resistances distributed according to the formula (52) is given by the expression

$$\sigma=e^{-\zeta_c}/R_0L, \quad (54)$$

where L is expressed in terms of ζ_c by the formula (8). From (54) we can determine ζ_c and V_c. With logarithmic accuracy,

$$\zeta_c=-\ln\sigma R_0N^{-1/3},\quad V_c=\frac{\Delta(T)}{4e^2}L\sigma\,\mathrm{th}\,\frac{\Delta(T)}{2T}. \quad (55)$$

The superconducting transition temperature T_c is determined from (55) and Eq. (12). At temperatures below T_c the critical current and the penetration depth are given by the formulas (9) and (10).

Other physical situations are possible for superconducting drops in a dielectric matrix: a) there may not be any spread in the oxide-layer thicknesses at all; b) an infinite cluster may be formed by drops between which there is no dielectric layer at all.[7] In these cases the transition temperature, the penetration depth, and the critical current are expressed in terms of the resistance in the normal state by the same formulas, but the percolation dimension L in them has a different value: in the case a) $L\sim N^{-1/3}$; in the case b) L is given by the classical percolation theory.

Above we neglected the Coulomb energy[11]; this is admissible when the drop size is not very small: $V_c\gg e^2/L$.

CONCLUSION

In this paper we have not studied the temperature dependence of the resistance. We have considered only the temperature region below T_c; T_c was defined as that temperature at which the resistance vanishes or becomes exponentially small. This temperature is higher than the superconducting transition temperature of the matrix, and its dependence on the number of drops is given by the formula (13). At temperatures lower than T_c, the critical current and the penetration depth depend exponentially on the temperature. The explicit form of this dependence in each specific case is obtained by substituting the formulas of Sec. II into (11) and (9). From these quantities we can construct a combination that has no exponential smallness with respect to the formula (12). This quantity has the meaning of a percolation length, and it can be compared with the formula (8), which expresses it in terms of the microscopic quantities.

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