Anomalous magnetoresistance in semiconductors

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A theory of the anomalous magnetoresistance in semiconductors and metals is constructed. It is shown that when the interaction between the electrons is neglected the quantum corrections to the conductivity lead to the appearance of negative magnetoresistance. Allowance for the interaction between the electrons also gives rise to an anomalous magnetoresistance whose sign depends on the sign of the electron–electron interaction constant. It is shown that the various scattering mechanisms, such as spin–orbit scattering, intervalley transfers in many-valley semiconductors, and transitions between the light- and heavy-hole bands in semiconductors of the p-Ge type, have an effect on the magnitude and sign of the magnetoresistance. The effect of deformation on the anomalous magnetoresistance is discussed.

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I. INTRODUCTION

Negative anomalous magnetoresistance (NAM) has been experimentally observed in a broad class of semiconductors over a period of more than twenty years now. The explanation of this effect with the aid of Toyozawa's theory[1] meets with considerable difficulties. A new possibility of explaining NAM has opened up in connection with the study of the effect of a mangetic field on the quantum corrections to the kinetic coefficients.[2]

Another type of quantum correction to the conductivity arises when the interaction between the electrons is taken into account.[3-6] In the present paper we consider the anomalous magnetoresistances that arises in the cases of noninteracting and interacting particles. It is found that the effect changes significantly when we go from the two- to the three-dimensional case, and that it also depends on the specific band structure and the spin-relaxation mechanisms. The resulting magnetoresistance does not depend on the orientation of the magnetic field relative to the current, i.e., both longitudinal and transverse magnetoresistances exist. Some of these questions have already been discussed in the literature.[7-9]

The paper is organized as follows. In the second section we expound the theory of the anomalous magnetoresistance of noninteracting electrons for the two- and three-dimensional cases. In the two-dimensional case[2]

$$G(H)-G(0)=\frac{e^2}{2\pi^2\hbar}f_2\left(\frac{4DeH}{\hbar c}\tau_\varphi\right), \tag{1}$$

where $G(H)=1/R_\square(H)$ is the conductivity of a square film in a magnetic field H, D is the electron-diffusion coefficient, and τ_φ is the inelastic-collision-induced-relaxation time of the phase of the wave function. In the presence of magnetic impurities, we must add to the τ_φ^{-1} the reciprocal spin-flip-scattering time τ_s^{-1};

$$f_2(x)=\ln x+\psi(1/2+1/x)=\begin{cases}x^2/24, & x\ll 1,\\ \ln x & x\gg 1.\end{cases} \tag{2}$$

Here $\Psi(y)$ is the logarithmic derivative of the gamma function.

A remarkable property of the expression (1) is its prediction that the effect of the magnetic field is appreciable even in the region of classically weak fields, where

$$\omega_c\tau\sim 1/\mu\tau_\varphi\ll 1,$$

and the positive magnetoresistance is small. Here $\omega_c=eH/m^*c$ is the cyclotron frequency, m^* is the effective electron mass, μ is the Fermi energy, and τ is the electron relaxation time in momentum space. In the three-dimensional case[9]

$$\Delta\sigma(H)=\frac{e^2}{2\pi^2\hbar}\times f_3\left(\frac{4DeH}{\hbar c}\tau_\varphi\right)\left(\frac{eH}{\hbar c}\right)^{1/2}, \tag{3}$$

$$f_3(x)=\begin{cases}0.605, & x\gg 1,\\ x^{1/2}/48, & x\ll 1.\end{cases}$$

In conclusion of the second section, we discuss the relaxation time of the phase entering into (3) in the practically important—for semiconductors—case of quasi-elastic scattering of the electrons by the acoustic phonons.

In the third section we compute the magnetoresistance due to the interaction between the electrons. Allowance for the interaction leads to a magnetoresistance whose sign is determined by the sign of the interaction constant $g(T)$ at small total-momentum values: the magnetoresistance is negative in the case of attraction and positive in the case of repulsion. In the three-dimensional case

$$\sigma_{int}(H)-\sigma_{int}(0)=-g(T)\frac{e^2}{2\pi^2\hbar}\left(\frac{eH}{\hbar c}\right)^{1/2}\varphi_3\left(\frac{2DeH}{\pi cT}\right), \tag{4}$$

$$\varphi_3(x)=\begin{cases}1.90, & x\gg 1,\\ \frac{5\pi}{64}\zeta\left(\frac{5}{2}\right)x^{1/2}\approx 0.33x^{1/2}, & x\ll 1.\end{cases} \tag{4a}$$

In the two-dimensional case

$$\Delta G_{int}(H)=-g(T)\frac{e^2}{2\pi^2\hbar}\varphi_2\left(\frac{2DeH}{\pi cT}\right), \tag{5}$$

$$\varphi_2(x)=\begin{cases}\ln x, & x\gg 1,\\ \zeta(3)x^2/4\approx 0.30x^2, & x\ll 1,\end{cases} \tag{5a}$$

where $\zeta(y)$ is the Riemann zeta function. Figure 1 shows the plots of the functions $\varphi_2(\alpha)$ and $\varphi_3(\alpha)$.

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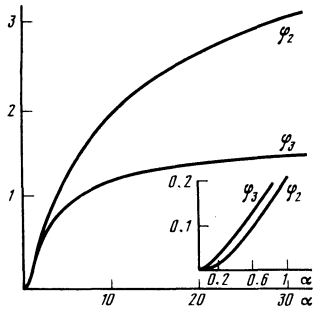

FIG. 1. Plots of the functions $\varphi_2(\alpha)$ and $\varphi_3(\alpha)$. The inset shows the plots of the functions φ_2 and φ_3 for small values of α.

Notice that in the three-dimensional case the absolute value of the magnetoconductivity for a noninteracting electron gas does not depend on the parameters of the material in strong magnetic fields, and has a universal value.

When the interaction is taken into account, the magnetoconductivity in strong fields depends only on the value of the electron–electron interaction constant. Further, the high-field asymptotic form is attained in stronger fields in this case than in the case of noninteracting electrons, namely, in fields for which

$$\omega_c\tau \sim T/\mu \gg \hbar/\mu\tau_\varphi.$$

In conclusion of the third section, we discuss the so-called Maki-Thompson corrections to magnetoconductivity. Their sign does not depend on the sign of the interaction constant, and the field dependence is the same as the field dependence for noninteracting electrons.

The theory of the magnetoresistance of noninteracting particles is applicable when $\bar{\varepsilon} \gg \hbar/\tau \gg \hbar/\tau_\varphi$ ($\bar{\varepsilon}$ is the characteristic electron energy), and therefore it can easily be generalized to the case of nondegenerate statistics. At the same time, the theory of the magnetoresistance of interacting electrons is valid for $\bar{\varepsilon} \gg \hbar/\tau \gg T$, and is therefore applicable only to a degenerate electron gas. In the nondegenerate case the dominant contribution to the anomalous magnetoresistance in the region of magnetic fields $H \sim \hbar c/4De\tau_\varphi$ is apparently made by those considered effects which arise when the interaction is neglected.

If the diffusion coefficient is anosotropic, then ΔG and $\Delta\sigma$ are tensors, so that

$$\Delta G_{ik} = \frac{D_{ik}}{D_a^{(2)}} \frac{e^2}{2\pi^2\hbar} f_2\left(\frac{4D_c eH}{\hbar c}\tau_\varphi\right), \tag{1a}$$

$$\Delta\sigma_{ik} = \frac{D_{ik}}{D_a^{(3)}} \frac{e^2}{2\pi^2\hbar}\left(\frac{eH}{\hbar c}\frac{D_c}{D_a^{(3)}}\right)^{1/2} f_3\left(\frac{4D_c eH}{\hbar c}\tau_\varphi\right), \tag{3a}$$

where $D_c^2 = D_\perp(D_\parallel\cos^2\theta + D_\perp\sin^2\theta)$, θ is the angle between the axis of the ellipsoid and the magnetic field, D_{ik} is the diffusion coefficient tensor, and $D_a = [\det D_{ij}]^{1/d}$, so that, for example, $D_a^{(3)} = (D_\parallel D_\perp^2)^{1/3}$. The expressions for $\Delta G_{int}(H)$ and $\Delta\sigma_{int}(H)$ similarly change when the anisotropy is taken into account.

In a number of semiconductors, such as Ge, Si, and Te, the conduction bands consists of several nonequivalent valleys. The diffusion coefficient in each of the valleys is anisotropic. Therefore, in strong magnetic fields, in which the intervalley transfer processes can be neglected ($4eD_{\parallel,\perp}H\tau_v/c \gg 1$, τ_v is the intervalley transition time), the magnetoconductivity is given by the sum of the individual valleys' contributions each of which is given by the expressions (1a) and (3a). In this case the magnetoconductivity does not depend on the mutual orientation of the current and the field, but depends on the orientation of the magnetic field with respect to the crystallographic axes.[9,10]

In weak fields for which $4eD_{\parallel,\perp}H\tau_v/c \ll 1$, the magnetoconductivity does not depend on the orientation of the magnetic field with respect to the crystal axes, and is given by the expressions (1) and (3), where the diffusion coefficient D is connected with the total conductivity by the Einstein relation.

When the anisotropy in the coefficient of diffusion in a valley is sufficiently strong, there exists a region of intermediate magnetic fields

$$\frac{c}{4eD_{max}\tau_v} \ll H \ll \frac{c}{4eD_{min}\tau_v},$$

where the magnetoconductivity is anisotropic, but the degree of anisotropy depends on the magnetic field.

The fifth section is devoted to the study of the effect of the spin–orbit interaction on the anomalous magnetoresistance. It is shown in Ref. 7 that allowance for the spin–orbit interaction in electron scattering by an impurity leads to a change in the sign of the magnetoresistance, i.e., the magnetoresistance becomes positive in the case of noninteracting electrons, continuing to depend anomalously on the magnetic field (PMR). As the magnetic field intensity is increased, the magnetoresistance becomes negative again as soon as the condition

$$\omega_c\tau > \frac{1}{\mu\tau_{so}}, \tag{6}$$

where τ_{so} is the spin–orbit scattering time, is fulfilled.

Similar effects should be observed in ordinary scattering when allowance is made for the spin—orbit splitting of the band states, as obtained in, for example, the noncentrosymmetric cubic semiconductors (i.e., in compounds of the type $A^{III}B^{V}$). Since the spin–orbit scattering time in these compounds increases with increasing concentration,[11] the magnetoresistance changes its sign again when the condition (6) is fulfilled in the given magnetic field. Allowance for the complex structure of the valence band also leads to a change in the sign of the magnetoresistance in weak fields (PMR). The valence bands with $p = 0$ split up on being deformed, and as the deformation increases, the anomalous PMR decreases in magnitude, and when the splitting of the bands becomes of the order of the Fermi energy, the magnetoresistance changes its sign. The piezomagnetoresistance in p-Ge has been experimentally observed to have precisely this character.[12] Allowance for those spin–orbit effects in the magnetoresistance which are due to the interelectron interaction leads only to a change in the magnitude of the effect, leaving its sign unchanged.

In the sixth section we study the magnetoresistance in thin layers or in MDS structures.

II. THE MAGNETORESISTANCE OF NONINTERACTING ELECTRONS

As has been shown,[13] the main quantum contribution to the conductivity arises when the "fan" diagrams (Fig. 2) are taken into account. These diagrams describe the interference that arises during multiple back scattering. The amplitude of this interference [the "cooperon" $C(x, x')$] is given by the sum of the ladder diagrams for small total-momentum values. It is precisely this amplitude that is sensitive to weak magnetic fields,[2] a sensitivity which determines the anomalous magnetoresistance. The correction to the conductivity has the following form ($\hbar=1$):

$$\Delta\sigma(\omega, H)=-2e^2\pi^{-1}C(r, r; \omega)D, \tag{7}$$

where $C(r, r'; \omega)$ satisfies the equation[2]

$$\left\{-i\omega+D\left(-i\nabla-\frac{2e}{c}\mathbf{A}\right)^2+\frac{1}{\tau_\varphi}\right\}C(\mathbf{r}, \mathbf{r}'; \omega)=\delta(\mathbf{r}-\mathbf{r}'), \tag{8}$$

ω being the external-field frequency.

The coherence of the electronic wave functions is destroyed over a time period of the order of the inelastic-collision-induced-relaxation time of the phase of the electron wave function, which is the cause of the appearance of the τ_φ^{-1} term in Eq. (8). In metals at low temperatures each inelastic-scattering event changes the energy by an amount of the order of T. Under these conditions, τ_φ coincides with τ_ε, the energy relaxation time.

It must be noted that the time τ_ε significantly depends on the mean free path. In the three-dimensional case the interelectron-collision-induced energy relaxation time $\tau_\varepsilon^{-1}\sim\varepsilon^{3/2}\mu^{-2}\tau^{-3/2}$ (Refs. 5 and 16); in the quasi-two-dimensional case $\tau_\varepsilon^{-1}\sim\varepsilon/\mu\tau p_F a$ (a is the sample thickness).[6] In semiconductors the collisions with the acoustic phonons are quasielastic, and therefore τ_ε and τ_φ differ greatly from each other. In fact, the time τ_φ is the period of time over which the phase changes by an amount of the order of unity, i.e., for which

$$\Delta\varphi\sim\tau_\varphi\delta\varepsilon\sim 1. \tag{9}$$

In an electron-phonon collision, because of the quasielastic nature of the collision, the energy of the electron changes by an amount

$$\Delta\varepsilon\sim s\bar{\varepsilon}/v,$$

where $\bar{\varepsilon}$ is the characteristic electron energy, equal

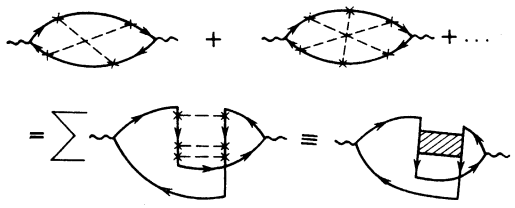

FIG. 2. The dominant quantum correction to the conductivity of noninteracting electrons.

to T in the nondegenerate case and to μ in the case of a degenerate electron gas, v is the characteristic electron velocity, and s is the velocity of sound. Then the change in the energy over the time period τ_φ is, in order of magnitude, equal to

$$\delta\varepsilon\sim\varepsilon\frac{s}{v}\left(\frac{\tau_\varphi}{\tau_{ph}}\right)^{1/2}, \tag{10}$$

where τ_{ph}^{-1} is the electron-phonon collision rate. Substituting (10) into (9), we obtain

$$\frac{1}{\tau_\varphi}\sim\left[\frac{1}{\tau_{ph}}\left(\frac{s\varepsilon}{v}\right)^2\right]^{1/3}\sim\frac{(\varepsilon\tau_\varepsilon)^{2/3}}{\tau_\varepsilon}, \quad \tau_\varepsilon^{-1}=\tau_{ph}^{-1}\left(\frac{s}{v}\right)^2, \tag{11}$$

and therefore $\tau_{ph}\ll\tau_\varphi\ll\tau_\varepsilon$.

It should be noted that the expression (11) is valid if

$$\tau_{ph}\Delta\varepsilon\sim\tau_{ph}\frac{s}{v}\varepsilon\ll 1. \tag{11a}$$

In the opposite case the phase changes by an amount of the order of unity over a time period of the order of τ_{ph}, i.e.,

$$\tau_\varphi=\tau_{ph} \text{ for } \tau_{ph}\Delta\varepsilon\gtrsim 1. \tag{11b}$$

The criterion (11a) differs significantly from the standard quasi-elasticity condition $\Delta\varepsilon\ll\bar{\varepsilon}, T$.

The solution to Eq. (8) can be written in the form

$$C(\mathbf{r}, \mathbf{r}'; \omega)=\sum_{n,\alpha}\frac{\psi_{n,\alpha}(\mathbf{r})\psi_{n,\alpha}(\mathbf{r}')}{-i\omega+DQ_z^2+4DeHc^{-1}(n+1/2)+1/\tau_\varphi}, \tag{12}$$

where the $\Psi_{n,\alpha}(\mathbf{r})$ are the normalized wave functions of a particle with charge $2e$ and mass $(2D)^{-1}$ in a magnetic field. The sample will be quasi-two-dimensional if its thickness $a<(D\tau_\varphi)^{1/2}=L_\varphi$ (Refs. 14 and 15). In this case the term DQ_z^2 in (12) should be dropped. The substitution of (12) into (7) leads to the expressions (1) for the quasi-two-dimensional, and (3) for the three-dimensional, case. The function $f_3(x)$ then has the form[9]

$$f_3(x)=\sum_{n=0}^{\infty}\{2[(n+x+1)^{1/2}-(n-x)^{1/2}]-(n+1/2+x)^{-1/2}\}. \tag{13}$$

III. THE MAGNETORESISTANCE FOR AN INTERACTING ELECTRON GAS

The quantum corrections to the conductivity that are due to the effects of the interelectron interaction have been discussed by a number of authors.[2-6] There are two types of corrections: 1. The corrections that arise when the interaction in the particle-hole channel, i.e., the interaction involving small momentum transfers, is taken into consideration. These corrections begin to depend essentially on the magnetic field only in the region of classically strong magnetic fields, i.e., in the region where $\omega_c\tau\sim 1$. 2. The corrections due to the interaction in the particle-particle channel (the Cooper channel). These corrections, as we shall now show, depend essentially on the magnetic field even in the region of classically weak mangetic fields, namely, in the region where $\omega_c\tau\sim T/\mu\ll 1$. Therefore, we can, in investigating the magnetoresistance, limit ourselves to the consideration of only the interaction in the Cooper channel. If

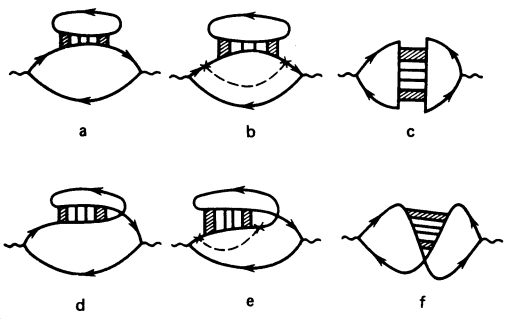

FIG. 3. The corrections to the conductivity that arise when the interaction between the electrons is taken into consideration.

the bare interaction does not depend on the resultant momentum, then, as shown in the Appendix, only the diagrams shown in Fig. 3 are important. It can in the process be verified that the contributions corresponding to the diagrams 3b and 3e are respectively equal to minus one half the contribution made by the diagrams 3a and 3d, and that the contribution of the diagram 3a differs in sign from, and is two times greater than, the contribution of 3d. Therefore, the sum of the diagrams 3a, 3b, 3d, and 3e is equal to one fourth the contribution of 3a. All these contributions to the anomalous magnetoconductivity stem from the magnetic-field-dependent interelectron-interaction-induced corrections to the density of states.[6] The diagram 3c and 3f describe the so-called Maki-Thompson corrections, which are well known in the theory of fluctuational superconductivity,[16] and whose magnetic-field dependence in the two-dimensional case was investigated earlier.[8]

The effective interaction $\Gamma(\omega_n)$ is determined by the equation depicted in Fig. 4:

$$\left\{g^{-1}(T)-\psi\left(\frac{1}{2}+\frac{\omega_n+D(-i\nabla-2e\mathbf{A}/c)^2}{4\pi T}\right)+\psi\left(\frac{1}{2}\right)\right\}\Gamma(\omega_n,\mathbf{r},\mathbf{r}')=\frac{2}{\nu}\delta(\mathbf{r}-\mathbf{r}'), \tag{14}$$

where $\nu=m^*p_F/\pi^2$ is the density of electron states at the Fermi level,

$$g^{-1}(T)=\frac{1}{\lambda}+\ln\frac{\gamma\eta}{\pi T}, \tag{14a}$$

λ is the dimensionless bare interaction constant, η is the cutoff parameter, equal to μ in the case of mutual repulsion of the electrons and to ω_D in the case of phonon-mediated attraction, ω_D is the Debye frequency, and $\ln\gamma=C=0.577$. If Γ is treated as a constant, then only the diagrams 3a, 3b, 3d, and 3e need to be taken into consideration. The contribution of the diagrams 3c and 3f can in this case be neglected.

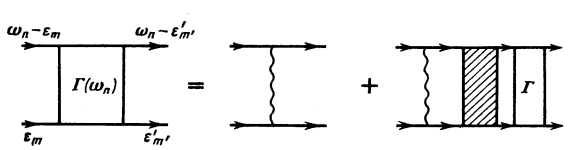

FIG. 4. Equation determining the electron–electron vertex part in the case of low total momentum and low total energy ω_n.

Computing the contribution of the diagram 3a with the aid of the rules of the temperature technique after performing the analytic continuation in the Matsubara-frequency space, we obtain the following expression for the magnetic-field-induced correction to the conductivity in the three-dimensional case:

$$\Delta\sigma_{int}(H)=\frac{e^2D}{\pi^2}\operatorname{Im}\int d\omega g(T)\frac{\partial}{\partial\omega}\left(\omega\operatorname{cth}\frac{\omega}{2T}\right)\times\int_{-\infty}^{\infty}dQ_z\left\{2eH\sum_{n=0}^{\infty}\frac{1}{[-i\omega+4DeHc^{-1}(n+1/2)+DQ_z^2]^2}-\frac{1}{2}\int_0^{\infty}\frac{Q_\perp dQ_\perp}{[-i\omega+D(Q_z^2+Q_\perp^2)]^2}\right\}. \tag{15}$$

Formally, each of the terms in (15) when integrated over ω diverges at the upper limit. In the $\Delta\sigma_{int}$ difference, the diverging parts cancel each other. Using the relation

$$\alpha\sum_{n=0}^{\infty}\int_0^{\infty}\frac{dy}{[-iz+\alpha(n+1/2)+y^2]^2}=-\frac{i\alpha(\pi)^{1/2}}{4}\frac{\partial}{\partial z}\int\frac{dt}{t^{1/2}}\frac{e^{izt}}{\operatorname{sh}(\alpha t/2)} \tag{16}$$

we can represent the expression (15) in the form

$$\Delta\sigma_{int}(H)=-\frac{e^2}{2\pi^2}g(T)\left(\frac{eH}{c}\right)^{1/2}\varphi_3\left(\frac{2eHD}{\pi cT}\right), \tag{17}$$

$$\varphi_3(x)=\left(\frac{\pi}{2x}\right)^{1/2}\int_0^{\infty}\frac{t^{1/2}dt}{\operatorname{sh}^2 t}\left[1-\frac{xt}{\operatorname{sh}xt}\right]. \tag{18}$$

The asymptotic expressions obtainable from (17) and (18) for $\Delta\sigma_{int}(H)$ have the form (4a). As can be seen from Eq. (13), the electron–electron interaction depends on the magnetic field. In weak fields, this dependence can be neglected, since the corrections that arise when it is taken into consideration are small compared to (17), being smaller by a factor of the order of the parameter $g(T)\ll 1$. In strong magnetic fields, allowance for the dependence $\Gamma(H)$ in (14) leads to a situation in which the interaction constant

$$g^{-1}(H)=\frac{1}{\lambda}+\ln\frac{c\eta}{DeH} \tag{19}$$

enters into (17). It can be seen from the expressions (17), (18), and (19) that, as the magnetic-field intensity increases, the conductivity decreases in the case of mutual repulsion of the electrons and increases in the case of mutual attraction. Therefore, the study of this effect can yield information about the sign and magnitude of the effective electron–electron interaction constant for small total momenta.

The Maki-Thompson corrections corresponding to the diagrams 3c and 3f lead to the following expressions for the magnetoresistance:

$$\Delta\sigma^{MT}(H)=-\beta(T)\Delta\sigma(H),\quad \Delta G^{MT}(H)=-\beta(T)\Delta G(H),$$

where $\Delta G(H)$ and $\Delta\sigma(H)$ are given by the expressions (1), (3), and (13). The quantity $\beta(T)$ does not depend on the dimensionality, and is computed in Ref. 8. The sign of the Maki-Thompson correction does not depend on the sign of the interaction constant $g(T)$. The function $\beta(T)$ has the following asymptotic forms[8]:

$$\beta(T)=\begin{cases}\frac{\pi^2}{4}|g(T)| & \text{for } -g(T)\gg 1,\\ \frac{\pi^2}{6}g^2(T) & \text{for } |g(T)|\ll 1\end{cases}$$

and has been tabulated by one of the present authors.[8]

Notice that these corrections are small when $DeH/cT \gg 1$.

IV. THE MAGNETORESISTANCE IN MANY-VALLEY SEMICONDUCTORS

In the absence of umklapp processes, the contributions of the various valleys in many-valley semiconductors are additive. In this case the number of particles in each valley is conserved, and the distribution of the particle-density fluctuation in each valley is described by the diffusion equation. In the presence of umklapp processes, only the total density is conserved, and only the fluctuations in this quantity are described by the diffusion equation. The symmetric fluctuations in the valley-occupation numbers, on the other hand, relax over time periods of the order of the intervalley transition times τ_v, which can be much longer than the intravalley relaxation times. A similar situation obtains for the phase fluctuations, which are described by the cooperon: only the symmetric—in the indices of the equivalent valleys—cooperon is preserved by the diffusion pole $(-i\omega+DQ^2)^{-1}$; the remaining components die down over time periods of the order of τ_v when $\tau_v \gg \tau$. Thus, of the n independent cooperons only one remains at times much greater than τ_v (the quantity D in it is equal to the mass diffusion coefficient).

As a result, if $\tau_v \ll \tau_\varphi$, then the expression for the magnetoconductivity of noninteracting electrons has the form

$$\Delta\sigma(H)=\frac{e^2}{2\pi^2}\left(\frac{eH}{c}\right)^{1/2} f_3\left(\frac{4DeH}{c}\tau_\varphi\right), \tag{20}$$

when $4D_{\parallel,\perp}eH\tau_v/c \ll 1$. In the opposite limiting case, i.e., for $4D_{\parallel,\perp}eH\tau_v/c \gg 1$,

$$\Delta\sigma_{\alpha\beta}(H)=\sum_i \Delta\sigma_{\alpha\beta}^{(i)}(H), \tag{20a}$$

where $\Delta\sigma_{\alpha\beta}^{(i)}(H)$ is given by the expression (3a), and to each term in (20a) corresponds an angle between the direction of the magnetic field and the axes of the ellipsoid. It should be noted that the entire anisotropy of the magnetoconductivity is due to the anisotropy of the effective masses (cf. Ref. 10).

As shown above, the interaction gives rise to two contributions to the magnetoconductivty: the diagrams 3a, 3b, 3d, and 3e. In the many-valley case we should distinguish between the electron–electron interaction constants corresponding to the coupling of both the diagonal—with respect to the valley induces—and the off-diagonal elements of the density matrix. In the case of the Coulomb interaction the off-diagonal constants are small compared to the diagonal constants, and, if we neglect them, then the diagrams 3a and 3b have an extra factor of $2n_1$ (because of the presence of the electronic loop) in comparison with the diagrams 3d and 3e.

The effect of the anisotropy, which always obtains in many-valley semiconductors, of the coefficient of diffusion in one valley is to make only the consideration of the interaction between the electrons pertaining to the equivalent (i.e., the identically oriented) ellipsoids necessary.[1)] If the number of equivalent valleys is n_1 (for Ge, $n_1=1$ and for Si, $n_1=2$), and if the umklapp processes can be neglected, then the diagrams 3a and 3b have only $2n_1$ as an extra factor, and therefore for $T\tau_v \ll 1$ and $4DeH\tau_v/c \ll 1$

$$\Delta\sigma_{inl}(H)=-g(T)\frac{e^2}{2\pi^2}\left(\frac{eH}{c}\right)^{1/2}\varphi_3\left(\frac{2DeH}{\pi cT}\right). \tag{21}$$

For $4D_{\parallel,\perp}eH\tau_v/c \gg 1$ or $T\tau_v \gg 1$

$$\Delta\hat{\sigma}_{inl}(H)=(2n_1^2-n_1)\sum_i \Delta\hat{\sigma}_{inl}^{(i)}(H), \tag{21a}$$

where the summation is over the nonequivalent ellipsoids and

$$\Delta\sigma_{inl}^{\alpha\beta}=-g(T)\frac{e^2}{2\pi^2}\frac{D_{\alpha\beta}}{D_a^{(3)}}\left(\frac{eH}{c}\frac{D_c}{D_a^{(3)}}\right)^{1/2}\varphi_3\left(\frac{2D_c eH}{\pi cT}\right).$$

These assertions are valid with regard to the contribution of the Maki-Thompson diagrams (Figs. 3c and 3f).

The off-diagonal constants cannot be taken into consideration generally: each type of many-valley semiconductor requires a special treatment.

V. EFFECT OF THE SPIN–ORBIT INTERACTION ON THE MAGNETORESISTANCE

The spin-orbit interaction has a strong effect on the magnetoresistance, and can even change its sign. In noncentrosymmetric cubic crystals the effective Hamiltonian of the electrons in the conduction band has the form[18]

$$\hat{\mathcal{H}}=p^2/2m^*+\sigma\Omega(\mathbf{p}), \tag{22}$$

$$\Omega_x(\mathbf{p})=\delta\cdot p_x(p_y^2-p_z^2). \tag{22a}$$

If $\Omega(\mathbf{p})\tau \ll 1$, then there occur over a period of time of the order of $\Omega^{-1} \gg \tau$ a large number of elastic collisions, which, because of the spin-orbit interaction, lead to spin relaxation. The effective time of this relaxation is equal to[11]

$$\frac{1}{\tau_{so}}=\tau\int\frac{d\theta_p}{4\pi}\sum_i \Omega_i^2(\mathbf{p}). \tag{23}$$

As a result, the cooperon assumes the following form[7,19]:

$$C_{\alpha\beta\beta\alpha}=\frac{3}{2}\frac{1}{-i\omega+DQ^2+1/\tau_\varphi+2/\tau_{so}}-\frac{1}{2}\frac{1}{-i\omega+DQ^2+1/\tau_\varphi}. \tag{24}$$

The expression (24) can be interpreted in the following simple manner[19]: the cooperon can be represented in the form of a sum of correlators of the singlet Δ_0 and triplet Δ wave functions of two particles:

$$C_{\alpha\beta\beta\alpha}=\langle\Delta_{\alpha\beta}\Delta_{\beta\alpha}\rangle=\frac{3}{2}\langle\Delta\Delta\rangle-\frac{1}{2}\langle\Delta_0\Delta_0\rangle. \tag{25}$$

In the absence of spin–orbit interaction, each of the correlators has the same form as the second term in (24). The spin–orbit interaction gives rise to spin relaxation, and, in consequence, the correlator $\langle\Delta\cdot\Delta\rangle$

acquires $1/\tau_{so}$ in the denominator. Therefore, for $\tau_{so} \gg \tau_\varphi \gg \tau$ the correction to the conductivity of noninteracting electrons is negative, as is the magnetoresistance, while for $\tau_\varphi \gg \tau_{so}$ the increase of τ_φ causes this negative correction to decrease, and ultimately makes it negative.

If $\tau\Omega \gtrsim 1$, then $\tau_{so} \sim \tau$. Under these conditions, it is not necessary at all to write down the first term in the formula (24), and we need to consider only the second term with its sign and coefficient. We should then expect a positive correction to the conductivity and a PMR in the entire temperature range.

A similar effect occurs in the computation of $\Delta\sigma_{int}(H)$. But since it is the square of the Cooper pole that enters into the expression (15), allowance for the spin–orbit scattering does not change the sign of $\Delta\sigma_{int}(H)$, but decreases its magnitude by a factor of four. In the two-dimensional case, the magnetoresistance is similarly affected by the spin–orbit scattering of the impurities. This effect is considered in Ref. 7.

In the cubic crystals the valence band with $p=0$ is fourfold degenerate, and the Hamiltonian of the holes in the spherical approximation can be written in the form

$$\mathcal{H} = \frac{p^2}{4}\left(\frac{3}{m_h} - \frac{1}{m_l}\right) + (\mathbf{Jp})^2\left(\frac{1}{m_l} - \frac{1}{m_h}\right), \tag{26}$$

where m_h and m_l are respectively the masses of the heavy and light holes $\mathbf{J}$ is the operator corresponding to the $^3/_2$ spin. The cooperon can be expanded in terms of the states characterized by the total spin of the pair of particles:

$$C = \sum_{I=0}^{2J} (-1)^{I+1}\frac{2I+1}{2J+1}\langle\Delta^{(I)}\Delta^{(I)}\rangle = \frac{7}{4}\langle\Delta^{(3)}\Delta^{(3)}\rangle - \frac{5}{4}\langle\Delta^{(2)}\Delta^{(2)}\rangle + \frac{3}{4}\langle\Delta^{(1)}\Delta^{(1)}\rangle - \frac{1}{4}\langle\Delta^{(0)}\Delta^{(0)}\rangle, \tag{27}$$

where I is the total spin of the pair of particles and $\Delta^{(I)}$ is the wave function of a pair of particles with total spin I. If the light- and heavy-hole masses coincide, then each of the correlators has the form of a diffusion pole. If, on the other hand, $m_h \neq m_l$, then the scattering on the impurities causes transitions between the various branches of the spectrum. As a result, all the higher spin multipoles rapidly relax, and only the last term in the expression (27) remains. Thus, the magnetoresistance fo noninteracting electrons in the p-type cubic crystals should be positive, and its magnitude should be four times smaller than the magnetoresistance in a simple band. Therefore, under conditions of a uniaxial deformation that splits the fourfold degenerate band into two twofold degenerate one, the magnetoresistance becomes negative and increases in magnitude by a factor of four when the deformation-induced band splitting goes above μ.

It should be noted that allowance for the cubic anisotropy does not change the result obtained, since the coefficient $\frac{1}{4}$ in the last term in (27) is due only to the multiplicity of the hold states with $p=0$.

As a result of the transitions between the various spectral branches, $\Delta\sigma_{int}(H)$ decreases in magnitude by a factor of 16 as compared to the expression (3) for the case of a simple band without changing sign. Therefore, $\Delta\sigma_{int}(H)$ is much greater in a deformed crystal—in limit of large deformations, 16 times greater—than in an undeformed crystal.

Let us emphasize that it is the mass hole-ddiffusion coefficient, which is connected with the electrical conductivity by the Einstein relation

$$D = \sigma/e^2\nu,$$

where ν is the total density of hole states at the Fermi level, that enters into the expressions (3) and (4).

VI. THE MAGNETORESISTANCE IN THE TWO-DIMENSIONAL CASE

We have thus far been discussing the magnetoresistance in the three-dimensional case. In order that the sample might be considered to be three-dimensional, its dimensions should be greater than $L\varphi = (D\tau_\varphi)^{1/2}$ for the effects occuring in the absence of interaction[14] and $L_T = (D/T)^{1/2}$ for the effects due to the interaction between the electrons.[4] If, on the other hand, its thickness is less than these characteristic lengths, then the sample behaves like a two-dimensional object, even though a may be macroscopic.

The computations of the magnetoconductivities $G(H)$ and $G_{int}(H)$ of a square film differ from the corresponding computations in the three-dimensional case in that there is no integration over Q_z. As a result, we obtain for $G(H)$ the expressions (1) and (2) and for $G_{int}(H)$ the expression

$$G_{int}(H) - G_{int}(0) = -\frac{e^2}{2\pi^2}g(T)\varphi_2\left(\frac{2eHD}{\pi cT}\right), \tag{28}$$

$$\varphi_2(\alpha) = \int_0^\infty \frac{tdt}{\mathrm{sh}^2 t}\left[1 - \frac{\alpha t}{\mathrm{sh}\,\alpha t}\right]. \tag{28a}$$

The asymptotic expressions for $\varphi_2(\alpha)$ are given by the formulas (5a).

In the two-dimensional case the magnetoresistance in classically weak fields due only to the magnetic-field component perpendicular to the plane of the sample. In those cases in which the film is thick, i.e., in the absence of dimensional quantization (the quasi-two-dimensional case), all the effects of the spin-orbit interaction and the many-valley character of the semiconductor are similar to the effects in the three-dimensional case.

In the truly two-dimensional case, which is realized in, for example, metal–dielectric–semiconductor (MDS) structures, the effects of the spin–orbit interaction must be reconsidered. Let us first consider the spin–orbit scattering on the impurities. Let the z axis be directed along the normal to the surface. Then the scattering amplitude contains only the term

$$f_{\alpha\beta}^{so} = ib[\mathbf{pp}']_z\sigma_{\alpha\beta}^z. \tag{29}$$

Computing the cooperon $C_{\alpha\beta\gamma\delta}$ by a method similar to the one used in Ref. 7, we obtain

$$C_{\alpha\beta\gamma\delta} = A\delta_{\alpha\beta}\delta_{\gamma\delta} + B\sigma_{\alpha\beta}^z\sigma_{\gamma\delta}^z;$$

$$A = \frac{1}{2}\left[\frac{1}{-i\omega + DQ^2 + 2/\tau_{so}^z} + \frac{1}{-i\omega + DQ^2}\right],$$

$$B=\frac{1}{2}\left[\frac{1}{-i\omega+DQ^2+2/\tau_{so}^z}-\frac{1}{-i\omega+DQ^2}\right].$$

The expression for $G(H)$ contains the quantity

$$C_{\alpha\beta\beta\alpha}=2(A+B),$$

which does not become infinite as $\omega\to 0$, $Q\to 0$. Therefore, for $DeH/c\ll 1/\tau_{so}^z$, $\tau_\varphi\gg\tau_{so}^z$ the magnetoresistance for noninteracting electrons is equal to zero.

The situation changes when we consider $G_{int}(H)$. The contributions of the diagrams a)–c) in Fig. 3 are proportional to

$$C_{\alpha\beta\delta\uparrow}C_{\beta\alpha\uparrow\delta}=4(A^2+B^2).$$

Therefore, allowance for the spin–orbit scattering decreases these contributions by a factor of two. On the other hand, the corrections due to the diagrams c), d), and f) in Fig. 3 are proportional to

$$-C_{\alpha\beta\delta\uparrow}C_{\beta\uparrow\delta\alpha}=-2(A+B)^2,$$

and as a result, when τ_{so}^z is taken into account, the corrections due to the diagrams 3c, 3d, and 3f do not contribute to the magnetoconductivity in sufficiently weak fields.

When the spin–orbit scattering is neglected, the contributions of the diagrams 3c and 3d are two times smaller than the contributions of the diagrams 3a and 3b, and have the opposite sign. As a result, the spin–orbit scattering does not affect the magnitude of $G_{int}(H)$.

This phenomenon can be explained as follows. The electron–impurity interaction, (29), does not change the direction of the electron spin. Therefore, the cooperons with spin components parallel and anti-parallel to the normal to the sample surface can be considered separately. It turns out that the pole at $\omega=0$ and $Q=0$ is preserved only in the cooperon with anti-parallel spins. This means that only the contribution of the diagrams 3a–3c to the magnetoconductivity $G_{int}(H)$ remains.

The D'yakonov–Perel' spin-relaxation mechanism affects the magnetoconductivity differently. Let, for simplicity, the fourfold axis [100] be perpendicular to the plane of the film. Then we can only have $\tau_{so}^x=\tau_{so}^y$ (the situation does not qualitatively change if the z axis does not coincide with the [100] axis). In this case[7]

$$C_{\alpha\beta\beta\alpha}=\left\{\frac{1}{-i\omega+DQ^2+2/\tau_{so}^z}+\frac{1}{2(-i\omega+DQ^2+4/\tau_{so}^z)}-\frac{1}{2(-i\omega+DQ^2)}\right\}.$$

Therefore, because of this spin-relaxation mechanism, the magnetoresistance in the absence of interaction changes its sign and becomes positive, being equal in magnitude to one half the expression (1). At the same time $\Delta G_{int}(H)$ simply decreases by a factor of four in comparison with (5).

In inversion layers of the p type, the magnetoresistance also depends logarithmically on H, and two situations are possible for the coefficient of the logarithm. If the band splitting due to the dimensional quantization in the inversion layer is greater than the Fermi energy, then the magnitude and sign of the coefficient are the same as for the case of a simple band or in deformed three-dimensional semiconductors of the p type. But if the dimensional splitting is small, then the situation is exactly the same as in three-dimensional p-type samples when the deformation is slight.

TABLE I.

	Three-dimensional case		Two-dimensional case	
	c_3	c_3^{int}	c_2	c_2^{int}
1. Simple band without–orbit effects	1	1	1	1
2. Impurity spin–orbit scattering (weak fields)	$-1/2$	$1/4$	Quasi-two-dimensional case	
			$-1/2$	$1/4$
			Two-dimensional case	
			0	1
3. n-GaAs	$-1/2$	$1/4$	$-1/2$	$1/4$
4. p-Ge	$-1/4$	$1/16$	$-1/4$	$1/16$
5. p-Ge deformed	1	1	1	1

VII. CONCLUSION

Thus, the sign and magnitude of the magnetoconductivity and the characters of the temperature and field dependences are determined by many factors: the band structure of the semiconductor, the spin- and intervalley-relaxation mechanisms, and the effective dimensionality of the sample. The total magnetoconductivity for the three-dimensional case is a sum of two contributions

$$\sigma(H)-\sigma(0)=[c_3-c_3^{int}\beta(T)]\Delta\sigma(H)+c_3^{int}\Delta\sigma_{int}(H). \qquad (30)$$

The coefficients c_2 and c_2^{int} for the two-dimensional case can be similarly defined. The expressions for $\Delta\sigma(H)$ and $\Delta\sigma_{int}(H)$, as well as for $\Delta G(H)$ and $\Delta G_{int}(H)$ for the two-dimensional case, are given by the formulas (1), (1a), (3), (3a), (4), and (5).

In Table I we have collected the values of the coefficients c_d and c_d^{int} for the various cases in the limit of weak magnetic fields. According to the established tradition of experiment, the data are represented in graphs as dependences of $[\rho(H)-\rho(0)]/\rho(0)$ on the magnetic field. It should be noted that, for the purpose of a comparison with the present theory, it is convenient to plot $\sigma(H)-\sigma(0)$ vs. $H^{1/2}$ in the three-dimensional $G(H)-G(0)$ vs. $\ln H$ in the two-dimensional case, since it is precisely in such plots that the asymptotic dependences are represented by straight lines with universal slopes. In the anisotropic case, as well as in the case of the many-valley semiconductors these slopes depend on the ratio of the components of the diffusion tensor in one ellipsoid, as well as on the angle between the direction of the magnetic field and the crystal axes.

The theory expounded in the present paper has two characteristic magnetic-field scales:

$$H_0=\hbar c/4eD\tau_\varphi,\quad H_{int}=\pi cT/2eD.$$

In the case $T\tau_\varphi\gg 1$, $H_0\ll H_{int}$. Therefore, the field dependences of the magnetoconductivity are given by the formula (30), in which $\Delta\sigma_{int}(H)$ assumes its asy-

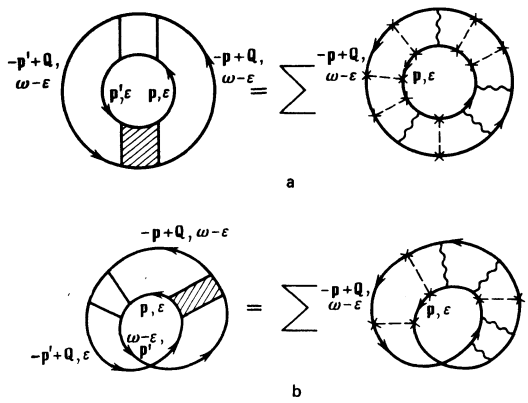

FIG. 5.

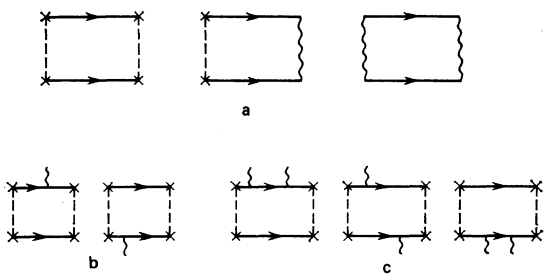

FIG. 6.

mptotic form at $H \gg H_{int}$, while $\Delta\sigma(H)$ does so at $H \gg H_0$. The graphs of such a function have two points of inflection at $H \sim H_0$ and $H \sim H_{int}$. Moreover, the various intervalley- and spin-relaxation mechanisms produce new magnetic-field scales, and therefore the dependence $\sigma(H)$ may have several points of inflection. A detailed study of these dependences can provide information about the relaxation-time scales. It is apparently convenient to set up such experiments in the longitudinal geometry, in which the classical magnetoresistance is equal to zero. If T and τ_φ^{-1} do not differ greatly from each other, then the magnetoconducitivty curve may have only one point of inflection. Such a situation is possible in the case of quasielastic scattering. When the momentum relaxation occurs on account of the quasielastic scattering by the acoustic phonons, the diffusion coefficient depends on the temperature, and this leads to an additional temperature dependence of the quantities H_0 and H_{int}.

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APPENDIX

All the diagrams that should be considered in the study of the effect of the electron-electron interaction on the conductivity can be obtained by inserting in all possible ways two current vertices in the diagrams shown in Figs. 5a and 5b.

The contribution of each of the diagrams in Fig. 4 can be written in the form

$$\int R(Q)\,(dQ) = \int (dQ) \sum_{\omega} \sum_{\varepsilon_i} \prod_{i} \zeta_i(Q_i, \varepsilon_i),$$

where the ζ_i, are the elements of the diagram in Fig. 5 depicted in Fig. 6a. It turns out that the sum of the diagrams in which the sign of the energy does not change at the current vertices vanishes if the bare interaction does not depend on the total momentum Q.

Indeed, the sum of the diagrams in Fig. 6b is equal in this case to $\partial\zeta_i/\partial Q_\alpha$, while the sum of the diagrams in Fig. 6c can be represented in the form $\partial^2\zeta_i/\partial Q_\alpha \partial Q_\beta$. Therefore, the total contribution to the conductivity from the diagrams in which the electron energy does not change its sign at a current vertex has the form

$$\int (dQ) \sum_{\omega} \sum_{\varepsilon_i} \left\{ \prod_{i \neq j} \zeta_i \frac{\partial^2 \zeta_j}{\partial Q_\alpha \partial Q_\beta} + \prod_{i \neq j,k} \zeta_i \frac{\partial \zeta_j}{\partial Q_\alpha} \frac{\partial \zeta_k}{\partial Q_\beta} \right\} = \int (dQ) \frac{\partial^2 R(Q)}{\partial Q_\alpha \partial Q_\beta} = 0.$$

Thus, we need to consider only those diagrams in which the retarded Green function is changed into the advanced function, and vice versa, at a current vertex. If we also take into account the fact that the cooperon is made up of a retarded and an advanced Green function, then only the diagrams in Figs. 3a–3c and Figs. 3d–3f, which arise from the diagrams 5a and 5b upon the insertion in them of the current vertices, remain.

[1)] The point is that a cooperon made up of electron wave functions belonging to nonequivalent valleys dies away over a time period of the order of $\hbar m/\mu \Delta m$ because of the effective-mass anisotropy.

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