Absorption of intense electromagnetic radiation in collisions of charged particles

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An effective collision frequency $\nu(E)$ for electrons and ions, which described the absorption of intense radiation with a frequency which exceeds the Langmuir frequency, is found. The amplitude of the electron oscillation velocity v_E in the radiation field is assumed to exceed the electron thermal velocity v_T. Under these conditions, when the parameter $Ze^2/\hbar v_E$ of the quantum-mechanical perturbation theory exceeds unity and the energy $\hbar\omega$ of the radiation quantum is less than the electron thermal energy $\varkappa T$, some simple asymptotic classical formulas are derived for the dependence of $\nu(E)$ on the radiation frequency and intensity. The role of quantum effects is studied in the case when at least one of the inequalities, $\hbar\omega > \varkappa T$ or $Ze^2/\hbar v_E < 1$, is fulfilled. Some qualitatively new quantum effects are deduced for linearly polarized radiation in the ranges $\hbar v_E > Ze^2 > \hbar v_T$ and $\hbar v_E > ze^2 > \hbar(\hbar\omega/m)^{1/2}$ for linearly polarized radiation and in the range $Ze^2 > \hbar v_E$, $\hbar\omega > \varkappa T$ for linearly and circularly polarized radiation. These effects, together with the classical dependences derived by us and with the extreme quantum dependences known previously (which hold for $\Theta v_T > Ze^2$ and $\Theta(\Theta\omega/m)^{1/2} > Ze^2$) complete the development of the asymptotic theory of absorption of high frequency radiation in a plasma.

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1. INTRODUCTION

The absorption of electromagnetic radiation in collisions of electrons and ions in a fully ionized plasma has been attracting the attention of many investigators. The theory of this phenomenon is to a certain degree fully developed in the case of low radiation intensity, when the velocity of the oscillations of the electron in the electric field $v_E = eE/m\omega$ ($-e$ is the charge and m is the mass of the electron, ω is the frequency of electromagnetic radiation) is small compared with the thermal velocity $v_T = (\varkappa T/m)^{1/2}$ ($\varkappa$ is Boltzmann's constant and T is the electron temperature) (see, e.g., Refs. 1 and 2). The situation is different when $v_E \gg v_T$. In fact, even though the investigation of absorption of intense radiation was started quite long ago,[3–8] this research is at present still incomplete. To explain this statement and justify the need for constructing the theory expounded below, of absorption of high-frequency intense electromagnetic radiation, let us discuss briefly the main results of the studies made in this direction and compare them with the results of the present investigation.

A kinetic equation was previously formulated,[3,4] describing the dissipative phenomena in a plasma under conditions when the strong electric field alters the electron trajectories enough to cause a substantial change in the electron-ion collisions. The use of such a kinetic equation has made it possible[6] (see also Ref. 7) to construct, under strong-field conditions $v_E \gg v_T$, a nonlinear theory of high-frequency plasma conductivity. In the field of a circularly polarized wave, this conductivity is independent of the time and is inversely proportional to v_E^3, which corresponds to the similar dependence of the frequency of the electron-ion collisions on the velocity v_E. For a field with another polarization, the nonlinear conductivity of the plasma depends on the time, and this corresponds to collision-induced generation of radiation harmonics. For that part of the conductivity which describes the heating of electrons by non-circularly-polarized radiation, the nonlinear dependence on the radiation field is characterized by the law $\Lambda(E)v_E^{-3}\ln(v_E/v_T)$.[6] We note that the theory of absorption of intense radiation based on the kinetic equation[3] is for the most part classical, and takes quantum effects into account only through some arbitrary functional dependence of the minimum impact distance that determines the form of the Coulomb logarithm (E).

Another inherently quantum trend of research is connected with the use of the inverse bremsstrahlung of an electron moving with a certain initial velocity v_0 in the field of a strong electromagnetic wave and in the field of an ion.[5,8–12] Rand[5] observed for the case $\mathbf{v}_0 \perp \mathbf{v}_E$ a nonlinear dependence of the inverse-bremsstrahlung cross section $\sigma \sim E^{-3}$, corresponding to the Rutherford law of scattering of charged particles. A similar problem was considered[8,9] in connection with the calculation of multiquantum cross sections of inverse bremsstrahlung absorption by an electron at an arbitrary relative orientation of $\mathbf{v}_0$ and $\mathbf{v}_E$. A calculation of the total absorption cross section with the aid of multiquantum cross sections was undertaken by Pert[10] and Elyutin,[11] who generalized Rand's results[5] to the case of parallel $\mathbf{v}_0$ and $\mathbf{v}_E$. Bunkin *et al.*[12] discussed the relation between the kinetic theory of absorption of intense radiation and the multiphoton quantum-mechanical approach, and traced the correspondence between the two.

When discussing the theory of absorption of intense radiation in a plasma, one must remember parametric instabilities.[13] Therefore an experimental investigation of this absorption (see, e.g., Ref. 14) calls, to prevent plasma heating, for the use of first, relatively short pulses (cf. Ref. 6) and second, for frequencies higher than the electron Langmuir frequency ($\omega \gg \omega_L$), when the conditions for the development of parametric instabilities are less favorable.[12–14] It is this last case of high frequencies which seems to us insufficiently investigated at present.

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The kinetic theory of Ref. 6 in the classical region

$$\hbar v_E < Ze^2, \quad \hbar\omega < \varkappa T$$

($\hbar$ is Planck's constant, Ze is the ion charge) describes the absorption of intense radiation in the high-frequency case ($\omega \gg \omega_L$). The general expressions obtained for the high-frequency case in that reference, however, were not given in detail, although tables of the collision frequencies were presented. In Ref. 15 are given numerical results only for a narrow frequency region $\omega \sim \omega_L$. A classical theory of high-frequency absorption was constructed in a number of papers.[16-18] Pert[16] obtained for the absorption coefficient of linearly polarized radiation an expression (33), which is similar to the classical expression[6]:

$$\sim v_E^{-3} \ln (v_E/v_T) \ln (r_{max}/r_{min})$$

(where r_{max} and r_{min} are the maximum and minimum impact distances), but the values of r_{max} and r_{min} were incorrectly determined in his paper. The equation obtained in Ref. 17 (see also Ref. 18) for the effective frequency of collisions of electrons with ions in an intense radiation field is likewise inaccurate, since it does not include principal terms that contain products of two large logarithms and are typical of the case, considered in Ref. 17, of absorption of linearly polarized radiation [cf. Refs. 5–7, 10–12, and 16; see also Eqs. (4.4) and (4.9) below].

At high electromagnetic-radiation intensities, when $\hbar v_E > Ze^2$, quantum effects are significant. The quantum theory of absorption of intense high-frequency ($\omega \gg \omega_L$) radiation by a plasma was considered by many workers. Thus, in Refs. 19–21 there were obtained general quantum-mechanical formulas for the coefficient of radiation absorption due to pair collisions of particles. In these papers, however, only partial multiphoton absorption coefficients K_s, $s=1, 2, \ldots$ (s is the number of absorbed photons) were investigated in detail; the total absorption coefficient

$$K = \sum_{s=1}^{\infty} K_s$$

was not calculated.

The problem of the quantum-mechanical calculation of the total absorption coefficient for radiation in a plasma was solved in a number of papers[12,22-27] (see also Refs. 10 and 11). In these papers, the absorption theory was developed in a distinct quantum limit, when the absorption energy is independent of the minimum impact parameter defined in accordance with the classical mechanics.

The foregoing analysis of the theory of absorption of intense radiation reveals the purpose of the present investigation, consisting in a detailed study of absorption of high-frequency radiation ($\omega \gg \omega_L$) in a case of practical importance, that of radiation intensities and frequencies that can be treated by classical theory, when $\hbar\omega < \varkappa T$, $\hbar v_E < Ze^2$, as well as consideration of the quantum theory of absorption under the conditions (4.5), (4.12), and $\hbar\omega > \varkappa T$, when the absorption depends on the classical minimal impact parameter.

2. FUNDAMENTAL RELATIONS OF QUANTUM THEORY OF ABSORPTION

From the methodological point of view, the approach described below supplements the classical approach of Ref. 6, inasmuch as our analysis is based on the quantum equation for the single-particle density matrix $f_{\mathbf{p}',\mathbf{p}}(t)$:

$$\left\{ i\hbar\frac{\partial}{\partial t} + \varepsilon_{\mathbf{p}'}(t) - \varepsilon_{\mathbf{p}}(t) \right\} f_{\mathbf{p}',\mathbf{p}}(t) = \sum_{\mathbf{p}''} \{ \langle \mathbf{p}''|\hat{U}|\mathbf{p}\rangle f_{\mathbf{p}',\mathbf{p}''}(t) - \langle \mathbf{p}'|\hat{U}|\mathbf{p}''\rangle f_{\mathbf{p}'',\mathbf{p}}(t) \}. \tag{2.1}$$

Here

$$\varepsilon_{\mathbf{p}}(t) = \frac{1}{2m}\left[\mathbf{p} + \frac{e}{c}\mathbf{A}(t)\right]^2$$

are the eigenvalues of the single-particle operator of the Hamiltonian of the electron in an external electromagnetic field with a vector potential $\mathbf{A}(t)$, and $\hat{U}$ is the operator of the energy of the interaction of the electron with the ion. We assume the ions to be immobile, an assumption justified by the smallness of the ratio of the electron and ion masses, and also neglect the interaction between the electrons, which lead in particular to dynamic polarization of the plasma. The last assumption is justified because our purpose is to consider radiation frequencies much higher than the plasma frequency.

Bearing it in mind that the influence of the electric field of the radiation on the motion of the electron in the field of the ion is significant only under conditions of weak interaction of the electron with the ion, we can use the solution obtained for Eq. (2.1) by perturbation theory. In the zeroth approximation in the interaction of the electron with the ion, $f^{(0)}_{\mathbf{p}',\mathbf{p}}(t)$ is diagonal, with diagonal elements $n_{\mathbf{p}}$ that represent the number of electrons in the state $\mathbf{p}$. According to (2.1), $\partial n_{\mathbf{p}}/\partial t = 0$, i.e., the canonical-momentum distribution function does not depend explicitly on the time, and for the Maxwellian distribution considered below it is given by

$$n_{\mathbf{p}} = N_e (2\pi\hbar^2/m\varkappa T)^{3/2} \exp(-\mathbf{p}^2/2m\varkappa T),$$

where N_e is the electron density. The first-approximation correction is of the form

$$f^{(1)}_{\mathbf{p}',\mathbf{p}}(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} dt' \langle \mathbf{p}'|\hat{U}|\mathbf{p}\rangle [n_{\mathbf{p}} - n_{\mathbf{p}'}] \exp\left\{ \frac{i}{\hbar}\int_{t'}^{t} dt'' [\varepsilon_{\mathbf{p}'}(t'') - \varepsilon_{\mathbf{p}}(t'')] \right\}. \tag{2.2}$$

We have used here the initial condition customarily employed in scattering theory, but the interaction was adiabatically turned on in the infinite past.

Finally, we write down an equation that will be needed later on and determines the diagonal elements of the second-approximation correction

$$\frac{\partial f^{(2)}_{\mathbf{p},\mathbf{p}}(t)}{\partial t} = \frac{2}{\hbar^2}\int_{-\infty}^{t} dt' \sum_{\mathbf{p}'} [n_{\mathbf{p}'} - n_{\mathbf{p}}] \langle \mathbf{p}|\hat{U}|\mathbf{p}'\rangle \times \langle \mathbf{p}'|\hat{U}|\mathbf{p}\rangle \cos\left\{ \frac{1}{\hbar}\int_{t'}^{t} dt'' [\varepsilon_{\mathbf{p}'}(t'') - \varepsilon_{\mathbf{p}}(t'')] \right\}. \tag{2.3}$$

For a plasma interacting with a spatially inhomogeneous high-frequency field we have the following expression for the electric current:

$$\mathbf{J}(t)=-\frac{e}{m}\sum_{\mathbf{p}}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}(t)\right]f_{\mathbf{p},\mathbf{p}}(t). \qquad (2.4)$$

This expression can be used to determine the work performed by the electric field on the electrons colliding with an ion, averaged over the period of the radiation field,

$$Q_v=\frac{1}{T}\int_0^T dt\,\mathbf{E}(t)\mathbf{J}(t). \qquad (2.5)$$

Equations (2.4) and (2.5) jointly with (2.3) enable us to write down a general expression for the heat Q released per unit volume of the plasma under the influence of the radiation field as a result of the collisions of the electrons with the ions. We assume here that the ions are distributed, with a density N_i, uniformly over the volume V and that they scatter the electrons independently. Then, recognizing that $\mathbf{E}=-c^{-1}\partial\mathbf{A}/\partial t$, we have

$$Q=\frac{2eN_i}{mc\hbar^2}\int_0^T\frac{dt}{T}\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt''\sum_{\mathbf{p},\mathbf{p}'}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}(t)\right]\frac{\partial\mathbf{A}}{\partial t}(n_{\mathbf{p}'}-n_{\mathbf{p}})$$
$$\times\langle\mathbf{p}|\hat{U}|\mathbf{p}'\rangle\langle\mathbf{p}'|\hat{U}|\mathbf{p}\rangle\cos\left\{\frac{1}{\hbar}\int_{t''}^{t'}dt'''[\varepsilon_{\mathbf{p}'}(t''')-\varepsilon_{\mathbf{p}}(t''')]\right\}. \qquad (2.6)$$

The matrix element of the energy operator $\hat{U}=-Ze^2/r$ of the Coulomb interaction of the electron with the ion is given by

$$\langle\mathbf{p}|\hat{U}|\mathbf{p}'\rangle=-\frac{(2\pi\hbar)^3}{V}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{4\pi Ze^2}{k^2}\delta(\mathbf{p}'-\mathbf{p}-\hbar\mathbf{k}). \qquad (2.7)$$

Using (2.7) as well as

$$\sum_{\mathbf{p}}n_{\mathbf{p}}=N_eV,\quad \sum_{\mathbf{p}}=\int\frac{d\mathbf{p}\,V}{(2\pi\hbar)^3},$$

we write down (2.6) in the form

$$Q=\frac{eN_i}{mc\hbar}\int_0^T\frac{dt}{T}\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt''\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\int\frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{4\pi Ze^2}{k^2}\right)^2\left(\mathbf{k}\frac{\partial\mathbf{A}}{\partial t}\right)$$
$$\times(n_{\mathbf{p}+\hbar\mathbf{k}/2}-n_{\mathbf{p}-\hbar\mathbf{k}/2})\cos\left\{\frac{1}{\hbar}\int_{t''}^{t'}dt'''[\varepsilon_{\mathbf{p}+\hbar\mathbf{k}/2}(t''')-\varepsilon_{\mathbf{p}-\hbar\mathbf{k}/2}(t''')]\right\}. \qquad (2.8)$$

In the case of a Maxwellian electron-momentum distribution we get from (2.8)

$$Q=-\frac{2N_eN_i}{\hbar T}\int_0^T dt\int_{-\infty}^{t}dt'\int_0 d\tau\int\frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{4\pi Ze^2}{k^2}\right)^2\frac{e\mathbf{k}}{mc}\frac{\partial\mathbf{A}}{\partial t}$$
$$\times\sin\left(\frac{\hbar k^2}{2m}\tau\right)\sin\left[\frac{e\mathbf{k}}{mc}\int_{t'-\tau}^{t'}dt''\,\mathbf{A}(t'')\right]\exp\left(-\frac{1}{2}k^2v_T^2\tau^2\right), \qquad (2.9)$$

where $v_T=(\varkappa T/m)^{1/2}$.

We consider hereafter the absorption of linearly polarized radiation $\mathbf{E}_l(t)=\mathbf{E}\cos\omega t$ and of circularly polarized radiation

$$\mathbf{E}_s(t)=(\mathbf{i}\cos\omega t-\mathbf{j}\sin\omega t)E/\sqrt{2},$$

where $\mathbf{i}\perp\mathbf{j}$ are unit vectors. In these cases we have from (2.9) (cf. Ref. 28)

$$Q_p=-\frac{4Z^2e^4N_eN_i}{\pi\hbar}\int_0^T\frac{dt}{T}\int_0^\infty d\tau\int\frac{d\mathbf{k}}{k^4}\sin\left(\frac{\hbar k^2}{2m}\tau\right.$$
$$\left.\times\exp\left(-\frac{1}{2}k^2v_T^2\tau^2\right)\frac{d}{d\tau}\cos\left\{2\varphi_p(\mathbf{k},\mathbf{v}_E)\sin\omega t\frac{\sin(\omega\tau/2)}{\omega}\right\}, \qquad (2.10)$$

where

$$\mathbf{v}_E=e\mathbf{E}/m\omega,\quad \varphi_l(\mathbf{k},\mathbf{v}_E)=\mathbf{k}\mathbf{v}_E,\quad \varphi_s(\mathbf{k},\mathbf{v}_E)=2^{-1/2}v_E(k_x^2+k_y^2)^{1/2}.$$

Equation (2.10) yields

$$O_p=N_emv_E^2\nu(E)/2, \qquad (2.11)$$

where $\nu(E)=16Z^2e^4N_im^{-2}v_E^{-3}R_p$ is the effective collision frequency,

$$R_p=-\frac{2mv_E}{\hbar}\int_0^T\frac{dt}{T}\int_0^\infty d\tau\int_0^{r_p^{-1}(t)}\frac{dk}{k^2}\sin\left(\frac{\hbar k^2}{2m}\tau\right)\exp\left(-\frac{1}{2}k^2v_T^2\tau^2\right)$$
$$\times\frac{d}{d\tau}\left\{S_p\left(\frac{2kv_E}{\omega}\sin\omega t\sin\frac{\omega\tau}{2}\right)\right\}, \qquad (2.12)$$

$$S_l(x)=x^{-1}\sin x,\quad S_s(x)=\int_0^{\pi/2}d\theta\sin\theta\cos(2^{-1/2}x\sin\theta);\quad p=l,s.$$

Let us explain the meaning of the limits of integration with respect to k in (2.12). First, since in the high-frequency case $\omega\gg\omega_L$ considered by us, the effect of dynamic polarizability of the plasma is insignificant, the lower limit is assumed equal to zero and not to the reciprocal Debye radius r_D, as is done in the theory of low-frequency absorption ($\omega\ll\omega_L$) (see, e.g., Ref. 1). Second, we introduce the upper limit of integration $r_p^{-1}(t)$, corresponding to restriction of the perturbation theory which was used in the derivation of (2.12) and which is not suitable when the distance between the electron and the ion becomes smaller than $r_p(t)$. According to Ref. 6 we have in the cases of interest to us

$$r_s=Ze^2(^3/_2\varkappa T+^1/_4mv_E^2)^{-1},$$
$$r_l(t)=Ze^2(^3/_2\varkappa T+^1/_2mv_E^2\sin^2\omega t)^{-1}.$$

We shall obtain below explicit expressions for R_p as a function[1] of ω and v_E.

3. ABSORPTION OF CIRCULARLY POLARIZED RADIATION

In this section we consider concrete consequences of Eq. (2.12) in the limit of a strong electric field $v_E\gg v_T$. We indicate first of all that if we assume $r_p^{-1}(t)=\infty$ in (2.12), then we get directly the results of the theory of the distinct quantum limit investigated in Refs. 25 and 26. On the contrary, there has not been a sufficient study of the theoretically more complicated manifestation of the classical restriction imposed on the perturbation theory by the finite value of $r_p^{-1}(t)$. It is precisely on an exposition of results pertaining to this case that we shall focus our attention below.

We turn to the simple case of circular polarization, when r_s does not depend on the time, and $r_s=4Ze^2/mv_E^2$. Then (2.12) yields

$$R_s(v_E)=R_1(v_E)+R_2(v_E),$$

where

$$R_1(v_E)=-\frac{\pi}{2^{1/2}}\int_0^{r_s^{-1}}\frac{dk}{k}\int_0^{\pi/\omega}\frac{d\tau}{\pi}\omega\tau\frac{\sin(\hbar k^2\tau/2m)}{\hbar k^2\tau/2m}\exp\left(-\frac{k^2v_T^2\tau^2}{2}\right)$$
$$\times\frac{d}{d\tau}\left[\sin\left(\frac{2^{1/2}kv_E}{\omega}\sin\frac{\omega\tau}{2}\right)\Big/\sin\frac{\omega\tau}{2}\right], \qquad (3.1)$$

$$R_2(v_E)=-\frac{\pi}{2^{1/2}}\int_0^{r_s^{-1}}\frac{dk}{k}\int_{-\pi/\omega}^{\pi/\omega}\frac{d\tau}{\pi}\tau\frac{d}{d\tau}\left[\sin\left(\frac{2^{1/2}kv_E}{\omega}\sin\frac{\omega\tau}{2}\right)\Big/\sin\frac{\omega\tau}{2}\right]$$
$$\times\sum_{n=1}^{\infty}\frac{2m\omega}{\tau\hbar k^2}\sin\left[\frac{\hbar k^2}{2m\omega}(2\pi n+\omega\tau)\right]\exp\left[-\frac{k^2v_T^2}{2\omega^2}(2\pi n+\omega\tau)^2\right]. \qquad (3.2)$$

Since r_s is determined accurate to a numerical factor, it suffices to consider the quantity $R_s(v_E)$ with logarithmic accuracy. We recognize in this connection that when integrating with respect to τ we can use in expressions (3.1) and (3.2) the approximation[2)] $\sin(\omega\tau/2)=\omega\tau/2$. Next, in the limit of a strong field we can neglect the dependence of $R_1(v_E)$ on v_T, since the quantity

$$(kv_E\tau/2^{1/2})^{-1}\sin(kv_E\tau/2^{1/2})$$

causes the integrand of (3.1) to vanish rapidly at $kv_E\tau/2^{1/2}>1$. We then obtain with logarithmic accuracy

$$R_1(v_E)=2^{1/2}\int\limits_0^{\pi v_E/\sqrt{2}\omega r_s}\frac{dy}{y}\sin y\left[\operatorname{Ci}\left(\frac{\hbar y}{2^{1/2}mv_E r_s}\right)-\operatorname{Ci}\left(\frac{\hbar\omega y^2}{\pi m v_E^2}\right)\right], \quad (3.3)$$

$$\operatorname{Ci}(x)=-\int\limits_x^\infty\frac{dy}{y}\cos y.$$

We shall be interested in the case when the amplitude v_E/ω of the electron oscillations is large compared with r_s. We note that in the opposite limit ($r_s\gg v_E/\omega$) the influence of the electric field on the collisions can be disregarded.

According to (3.3), integration over the region $y>1$ leads to an insignificant contribution to R_1, and integration over the region $y<1$ yields a large logarithm only if the argument of at least one integral cosine is small compared with unity. Then, using the asymptotic form $\operatorname{Ci}(x)\approx\ln x$, we get for R_1

$$R_1(v_E)=\frac{\pi}{2^{1/2}}\ln\left(\frac{v_E}{\omega r_s}\right),\quad \frac{v_E}{\omega}\gg r_s,\quad Ze^2\gg\hbar v_E; \quad (3.4)$$

$$R_1(v_E)=\frac{\pi}{2^{1/2}}\ln\left(\frac{mv_E^2}{\hbar\omega}\right),\quad v_E\gg\left(\frac{\hbar\omega}{m}\right)^{1/2},\quad \hbar v_E\gg Ze^2. \quad (3.5)$$

We proceed now to $R_2(v_E)$. Noting that $2\pi n>\omega\tau$ for all $n\geq1$, we expand in (3.2) the function that depends on $2\pi n+\omega\tau$ in a series in $\omega\tau$, retaining the zeroth and first terms of the series. Then the integral with respect to τ of the zeroth term of the series is zero, and that of the first term of the series

$$\int\limits_{-\pi/\omega}^{\pi/\omega}\frac{d\tau}{2\pi}\omega\tau\frac{d}{d\tau}\left[\sin\left(\frac{2^{1/2}kv_E}{\omega}\sin\frac{\omega\tau}{2}\right)\Big/\sin\frac{\omega\tau}{2}\right]\approx-\eta\left(k-\frac{\omega}{2^{1/2}v_E}\right), \quad (3.6)$$

where $\eta(x)$ is equal to unity at $x\geq0$ and to zero at $x<0$. At the same time, in accordance with the Euler-Maclaurin formula, the summation in (3.2) reduces to the following (cf. Ref. 25):

$$\sum_{n=1}^{\infty}\frac{m\omega}{\pi\hbar k^2}\frac{d}{dn}\left\{\sin\left(\frac{\hbar k^2\pi n}{m\omega}\right)\exp\left[-\left(\frac{2^{1/2}\pi nkv_T}{\omega}\right)^2\right]\right\}$$
$$\approx\exp\left[-\left(\frac{2^{1/2}\pi kv_T}{\omega}\right)^2\right]\left\{\frac{1}{2}\cos\left(\frac{\hbar k^2\pi}{m\omega}\right)\right.$$
$$\left.+\left[-1-\left(\frac{2^{1/2}\pi kv_T}{\omega}\right)^2\right]\frac{\sin(\hbar k^2\pi/m\omega)}{\hbar k^2\pi/m\omega}\right\}. \quad (3.7)$$

It follows from (3.6) and (3.7) that the logarithmic contribution to $R_2(v_E)$ stems only from the integration over the region

$$\frac{2^{1/2}\omega}{\pi v_E}\leqslant k\leqslant\min\left\{r_s^{-1},\ \frac{\omega}{2^{1/2}\pi v_T},\ \left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\right\}, \quad (3.8)$$

when

$$R_2(v_E)\approx-\frac{\pi}{2^{1/2}}\int\frac{dk}{k}$$

From this we get

$$R_2(v_E)=-\frac{\pi}{2^{1/2}}\ln\frac{v_E}{(\hbar\omega/m)^{1/2}},\quad v_E\gg\left(\frac{\hbar\omega}{m}\right)^{1/2}\gg v_T,\quad \left(\frac{\hbar}{m\omega}\right)^{1/2}\gg r_s; \quad (3.9)$$

$$R_2(v_E)=-\frac{\pi}{2^{1/2}}\ln\frac{v_E}{v_T},\quad v_E\gg v_T\gg\left(\frac{\hbar\omega}{m}\right)^{1/2},\quad \frac{v_T}{\omega}\gg r_s; \quad (3.10)$$

$$R_2(v_E)=-\frac{\pi}{2^{1/2}}\ln\frac{v_E}{\omega r_s},\quad \frac{v_E}{\omega}\gg r_s\gg\max\left\{\frac{v_T}{\omega},\ \left(\frac{\hbar}{m\omega}\right)^{1/2}\right\}. \quad (3.11)$$

Equations (3.4), (3.5), and (3.9)–(3.11) permit a complete description of the absorption of a circularly polarized wave in all those cases when the Landau logarithmic approximation is sufficient for the determination of the effective collision frequency. In the limit when the de Broglie wavelength $\hbar/mv_E$ of the electron is large compared with the classical impact parameter r_s, Eqs. (3.5), (3.9), and (3.10) lead to the results of the quantum theory of absorption[25,26] (see also Refs. 10 and 11):

$$R_s(v_E)=\frac{\pi}{2^{1/2}}\ln\frac{mv_E^2}{\hbar\omega},\quad v_E\gg\left(\frac{\hbar\omega}{m}\right)^{1/2}\gg v_T,\quad \hbar v_E\gg Ze^2; \quad (3.12)$$

$$R_s(v_E)=\frac{\pi}{2^{1/2}}\left(\ln\frac{mv_E^2}{\hbar\omega}+\ln\frac{\varkappa T}{\hbar\omega}\right),\quad v_E\gg v_T\gg\left(\frac{\hbar\omega}{m}\right)^{1/2},\quad \hbar v_E\gg Ze^2. \quad (3.13)$$

We note here that in the paper of Seely and Harris[23] no logarithmic dependence of the effective collision frequency on the field was revealed, thus contradicting the results of Shima and Yatom,[25] of Karapetyan,[26] as well as our own results.

In the opposite limit, when r_s is less than $\hbar/mv_E$, i.e., $Ze^2\ll\hbar v_E$, Eqs. (3.4) and (3.9)–(3.11) yield

$$R_s(v_E)=\frac{\pi}{2^{1/2}}\ln\frac{v_T}{\omega r_s},\quad v_E\gg v_T\gg\left(\frac{\hbar\omega}{m}\right)^{1/2},\quad \frac{v_T}{\omega}\gg r_s\gg\frac{\hbar}{mv_E}; \quad (3.14)$$

$$R_s(v_E)=\frac{\pi}{2^{1/2}}\ln\frac{(\hbar\omega/m)^{1/2}}{\omega r_s},\quad \left(\frac{\hbar\omega}{m}\right)^{1/2}\gg v_T,\quad \left(\frac{\hbar}{m\omega}\right)^{1/2}\gg r_s\gg\frac{\hbar}{mv_E}. \quad (3.15)$$

Equation (3.14) generalizes the asymptotic result[6] to include the region of high frequencies ($\omega\gg\omega_L$) in such a way that the maximum impact parameter is not the Debye radius but v_T/ω, the distance traversed by the thermal electron during one period of the field.

Our other result (3.15) goes over, with decreasing radiation frequency, at $\omega<\omega_L$ into the expression

$$R_s(v_E)=\frac{\pi}{2^{1/2}}\ln\left\{\frac{(\hbar/m\omega_L)^{1/2}}{r_s}\right\},$$

which differs from that previously known[6] in that under the conditions $\hbar\omega_L\gg\varkappa T$ the maximum impact parameter is determined by the characteristic velocity $(\hbar\omega_L/m)^{1/2}$, and not by the thermal velocity of the electrons. One other result for $R_s(v_E)$, which will be useful later, follows from (3.4) and (3.11) under the conditions

$$v_E/\omega\gg r_s\gg\max\left[v_T/\omega,\ (\hbar/m\omega)^{1/2}\right],$$

when we have with logarithmic accuracy

$$R_s(v_E)\approx0. \quad (3.16)$$

Considering the absorption of circularly polarized radiation, we present asymptotic results, which will be needed later (Sec. 4), for $R_s(v_E)$ in a weak electric field, when in the essential region of integration with respect to k and τ we can assume that

$$(2^{1/2}kv_E/\omega)\sin(\omega\tau/2)\ll1$$

[see (3.1) and (3.2)]. We then have

$$R_s(v_E)=\frac{(2\pi)^{1/2}}{12}\left(\frac{v_E}{v_T}\right)^3\left(\frac{2\varkappa T}{\hbar\omega}\right)\mathrm{sh}\left(\frac{\hbar\omega}{2\varkappa T}\right)\times\int_0^{r_s^{-1}}\frac{dk}{k}\exp\left(-\frac{\omega^2}{2k^2v_T^2}-\frac{\hbar^2k^2}{8m^2v_T^2}\right). \quad (3.17)$$

In analogy with the case of a strong field ($v_E\gg v_T$), when we were interested in the absorption of the radiation at $v_E/\omega\gg r_s$, we investigate expression (3.17) under the conditions

$$\max\,[v_T/\omega,\ (\hbar/m\omega)^{1/2}]\gg r_s.$$

We emphasize that under conditions not discussed by us, when the minimum impact parameter turns out to be not small,

$$r_s\gg\max\,[v_T/\omega,\ (\hbar/m\omega)^{1/2},\ v_E/\omega],$$

the assumption of the small change of the momentum of the scattered particles does not hold. This leads automatically to suppression of the absorption.[1,2]

We consider first the case $\varkappa T\gg\hbar\omega$. If the de Broglie wavelength of the electron in the weak field $\hbar/mv_T$ is less than r_s, then Eq. (3.17) yields the known result of the kinetic theory of rapidly alternating processes[1,3,4]:

$$R_s(v_E)=\frac{(2\pi)^{1/2}}{12}\left(\frac{v_E}{v_T}\right)^3\ln\left(\frac{2^{1/2}v_T}{\omega r_s}\right),\quad v_T\gg v_E,\quad \frac{v_T}{\omega}\gg r_s\gg\frac{\hbar}{mv_T}. \quad (3.18)$$

Also in the limit $\varkappa T\gg\hbar\omega$, but $\hbar/mv_T\gg r_s$, when the upper limit of the integral with respect to k in (3.17) can be regarded as infinite, we have

$$R_s(v_E)=\frac{(2\pi)^{1/2}}{12}\left(\frac{v_E}{v_T}\right)^3\ln\frac{4\varkappa T}{\hbar\omega},\quad v_T\gg v_E,\quad \frac{v_T}{\omega}\gg\frac{\hbar}{mv_T}\gg r_s. \quad (3.19)$$

In the opposite case $\hbar\omega\gg\varkappa T$, the condition under which r_s can be neglected takes the form $(\hbar/m\omega)^{1/2}\gg r_s$, according to which we obtain from (3.17) (see Ref. 25)

$$R_s(v_E)=\frac{\pi}{6\sqrt{2}}\left(\frac{mv_E^2}{\hbar\omega}\right)^{3/2}. \quad (3.20)$$

Bearing in mind the assumption we made when writing down (3.17)

$$(2^{1/2}kv_E/\omega)\sin(\omega\tau/2)\ll 1,$$

Eq. (3.20) should be used if $v_E\ll(\hbar\omega/m)^{1/2}$.

4. ABSORPTION OF LINEARLY POLARIZED RADIATION

We proceed now to consider the results pertaining to the theoretically more complicated case of linear polarization of the field. We shall use hereafter the following representation of R_l, obtained with the aid of (2.12):

$$R_l=\frac{2^{1/2}}{\pi}\int_0^{\pi/2\omega}\frac{\omega\,dt}{\sin\omega t}R_s(2^{1/2}v_E\sin\omega t). \quad (4.1)$$

Bearing in mind that the classical minimum impact parameter $r_l(t)$ is determined accurate to a numerical factor of the order of unity, we confine ourselves in the discussion of R_l to only the terms of highest order, which contain products of two large logarithms. The results corresponding to this calculation accuracy follow directly from (4.1) if we use for $R_s(2^{1/2}v_E\sin\omega t)$ the asymptotic equations obtained in Sec. 3. The limits of the applicability of the approximate equations (3.12)–(3.16), (3.18)–(3.20) for $R_s(v_E)$ then determine the corresponding limits of the regions of integration with respect to t in (4.1).

In analogy with the weak-field absorption theory restricted by the Landau logarithmic approximation, we assume first that the distance traversed by a thermal electron during one period of the field is large compared with $Ze^2/\varkappa T$:

$$v_T/\omega\gg Ze^2/\varkappa T. \quad (4.2)$$

We consider first of all the classical limit of Eq. (4.1), which is realized under the conditions

$$Ze^2\gg\hbar v_E, \quad (4.3)$$

when only the classical asymptotic forms of $R_s(v_E)$ should be used in the calculation of the integral (4.1). In addition, the condition (4.2) makes the asymptotic form (3.14) convenient for arbitrary $2^{1/2}v_E\sin\omega t>v_T$, while Eq. (3.18) can be used at $v_T>2^{1/2}v_E\sin\omega t$. We then have

$$R_l\approx\ln\frac{v_E}{v_T}\ln\frac{mv_T^2v_E}{Ze^2\omega}. \quad (4.4)$$

We emphasize that the appearance of doubly logarithmic expressions corresponds in accordance with Ref. 6 to a possible vanishing of the velocity of the electron oscillations in a linearly polarized electric field. In the limit of radiation frequencies lower than the electron plasma frequency ($\omega\ll\omega_L$), when the maximum impact parameter is the Debye radius r_D, expression (4.4) goes over into (cf. Ref. 6)

$$R_l=\ln\frac{v_E}{v_T}\ln\frac{\varkappa Tv_E}{Ze^2\omega_L}.$$

Comparing the classical (non-quantum) Eq. (4.4) with the results of others, we note first of all that our Eq. (4.4) differs qualitatively from the result of Ref. 17 [see Eq. (4.8) of Ref. 17 as well as Eq. (45.9) of Ref. 18], in which the products of the two large logarithms were completely lost. The doubly logarithmic expressions

$$\ln(v_E/v_T)\ln(r_{max}/r_{min}),$$

which determine the absorption coefficient were obtained by Pert.[16] In contrast to our result (4.4), Pert proposes to use

$$r_{max}=v_E/\omega,\quad r_{min}=\varkappa T/eE\quad \text{at}\quad Ze^2/\varkappa T\gg\varkappa T/eE,$$
$$r_{max}=\varkappa T/eE,\quad r_{min}=Ze^2/\varkappa T\quad \text{at}\quad \varkappa T/eE\gg Ze^2/\varkappa T.$$

These prescriptions by Pert are due to an inconsistent treatment of the motion of the electron in a strong electric field when scattered through small angles in the field of an ion.

We now turn to the quantum consequences of Eq. (4.1). In the limit $\hbar v_T>Ze^2$ the quantum theory of absorption in a fully ionized plasma was developed in a number of papers (see Refs. 10–12, 22, 24–27). At the same time, no attention was paid to the peculiar quantum region that is realized only in the case of a strong field when the following inequalities are satisfied:

$$\hbar v_E\gg Ze^2\gg\hbar v_T. \quad (4.5)$$

These conditions correspond to a plasma with a temperature lower than several dozen electron volts, and at large Z lower than hundreds of electron volts, whereas in the theory of absorption of a weak electromagnetic field the classical (non-quantum) expression for the Coulomb logarithm holds[1,2] [see Eq. (3.18)]. We note here that simultaneous satisfaction of the inequality (4.2) and the right-hand inequality (4.5) yields $\varkappa T \gg \hbar\omega$. It then follows from (4.1), in accord with the results (3.13), (3.14), and (3.18), that

$$R_l = \ln\frac{v_E}{v_T}\ln\frac{mv_E v_T^2}{Ze^2\omega} - \frac{1}{2}\ln^2\frac{\hbar v_E}{Ze^2}$$
$$= \frac{1}{2}\ln\frac{v_E}{v_T}\left(\ln\frac{mv_E v_T}{\hbar\omega} + \ln\frac{\varkappa T}{\hbar\omega}\right) - \frac{1}{2}\ln^2\frac{Ze^2}{\hbar v_T}. \quad (4.6)$$

In the region of higher electron temperatures, when in addition to condition $\varkappa T \gg \hbar\omega$ the inequality $\hbar v_T \gg Ze^2$ is also satisfied, in accordance with (3.13) and (3.19), we obtain from (4.1) the known results[25,26]:

$$R_l \approx \frac{1}{2}\ln\frac{v_E}{v_T}\left(\ln\frac{mv_E v_T}{\hbar\omega} + \ln\frac{\varkappa T}{\hbar\omega}\right). \quad (4.7)$$

At the boundaries of the region (4.5), expression (4.6) goes over respectively into (4.4) and (4.7). From Fig. 1, which shows the dependence R_l on $Ze^2/\hbar$, it is seen that in the region (4.5) the absorption is lower than those values obtained when Eqs. (4.4) and (4.7) are continued into this region.

In reporting our results that describe the absorption of strong linearly polarized radiation, we now forgo the assumption (4.2) and postulate in its place satisfaction of the inequalities

$$Ze^2/\varkappa T \gg v_T/\omega \gg Ze^2/mv_E^2. \quad (4.8)$$

The classical limit of (4.1) corresponding to the conditions (4.8) is realized in the limit of (4.3) and at $\varkappa T \gg \hbar\omega$. Under these conditions, remembering that in accordance with the remark made at the end of Sec. 3 the contribution of the region of large momentum transfers is negligible, so that integration over values of T for which

$$r_l(t) > \max\left(v_T/\omega,\ \sqrt{2}\,\omega^{-1}v_E\sin\omega t\right),$$

leads to an inessential contribution to R_l, and using the asymptotic expressions (3.14) and (3.16), we obtain

$$R_l \approx \frac{1}{4}\ln^2\frac{mv_T v_E^2}{Ze^2\omega}. \quad (4.9)$$

In Fig. 1 the region where formula (4.9) is realized correspond to the outer-most right-hand section of the curve. Obviously, at $Ze^2/\hbar \sim v_T(mv_E^2/\hbar\omega)$ the logarithm in Eq. (4.9) becomes small, corresponding to the absence of a doubly logarithmic dependence on Fig. 1 at large $Ze^2/\hbar$.

One more case corresponding to the inequalities (4.8) takes place if the condition (4.5) is satisfied in place of (4.3). Such conditions can be realized when

$$v_E \gg v_T\varkappa T/\hbar\omega \gg (\hbar\omega/m)^{1/2}.$$

R_l is then determined in analogy with (4.9), but the interval of integration with respect to t is broken up into two:

$$(Ze^2\omega/mv_T v_E^2)^{1/2} \leqslant \omega t \leqslant Ze^2/\hbar v_E, \quad Ze^2/\hbar v_E \leqslant \omega t \leqslant \pi/2,$$

in which it is necessary to use respectively the asymptotic relations (3.14) and (3.13). We then have

$$R_l = \frac{1}{4}\ln^2\frac{mv_T v_E^2}{Ze^2\omega} - \frac{1}{2}\ln^2\frac{\hbar v_E}{Ze^2}. \quad (4.10)$$

Figure 2 shows the set of the doubly logarithmic relations that are realized under the conditions

$$v_E \gg v_T\varkappa T/\hbar\omega \gg (\hbar\omega/m)^{1/2}.$$

The left-hand horizontal section, just as in Fig. 1, corresponds to Eq. (4.7). In analogy with Fig. 1, this curve with that described by Eq. (4.6). However, at

$$Ze^2/\hbar \sim v_T\varkappa T/\hbar\omega$$

Eq. (4.6) goes over into (4.10), which subsequently merges with (4.9) in accordance with Fig. 2.

We have discussed so far absorption under conditions when the energy $\hbar\omega$ of the emission photon is much lower than the energy $\varkappa T$ of the thermal motion of the electrons. It is clear at the same time that the quantum effects in absorption should manifest themselves most pronouncedly in the opposite limit, $\hbar\omega \gg \varkappa T$. We proceed now to an examination of the features of the absorption in this limit. We assume everywhere hereafter that the amplitude v_E of the electron oscillation velocity in the radiation field is large not only compared with thermal velocity of the electrons, but also exceeds the velocity $(\hbar\omega/m)^{1/2}$ determined by the energy of the emission photon: $v_E \gg (\hbar\omega/m)^{1/2}$. We note first of all that at sufficiently high frequencies, corresponding to photon energies that are large compared with the ionization energy

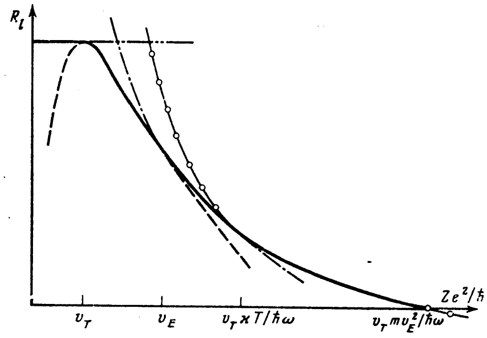

FIG. 1. Dependence of the function R_l on $Ze^2/\hbar$ under the conditions $v_T\varkappa T/\hbar\omega \gg v_E \gg v_T$.

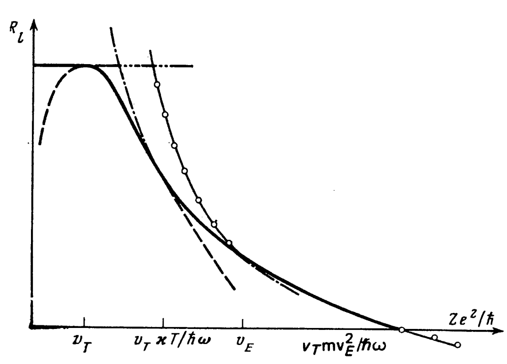

FIG. 2. Dependence of the function R_l on $Ze^2/\hbar$ under the conditions $v_E \gg v_T\varkappa T/\hbar\omega \gg (\hbar\omega/m)^{1/2}$.

$\hbar\omega \gg Z^2e^4m\hbar^{-2}$, if allowance is made for the insignificant contribution made to R_l by the integration with respect to t, for which $(\hbar\omega/m)^{1/2} \geqslant 2^{1/2}v_E \sin\omega t$ [see (3.20) and the remark at the end of Sec. 3], then Eq. (4.1) yields the known quantum-mechanical result[25,26] (see also Ref. 10):

$$R_l \approx \frac{1}{2}\ln^2\frac{v_E}{(\hbar\omega/m)^{1/2}}. \qquad (4.11)$$

The appearance of effects due to the limits of applicability of perturbation theory at small momentum transfers it is possible at lower frequencies when [cf. (4.5)]

$$\hbar v_E \gg Ze^2 \gg \hbar(\hbar\omega/m)^{1/2}. \qquad (4.12)$$

Under these conditions, using the asymptotic forms (3.12) at

$$Ze^2/\hbar v_E \leqslant \varphi \leqslant \pi/2, \quad \varphi = \omega t,$$

(3.15) at

$$[Ze^2\omega/mv_E^2(\hbar\omega/m)^{1/2}]^{1/2} \leqslant \varphi \leqslant Ze^2/\hbar v_E,$$

and (3.6) at

$$(Ze^2\omega/mv_E^3)^{1/3} \leqslant \varphi \leqslant [Ze^2\omega/mv_E^2(\hbar\omega/m)^{1/2}]^{1/2}$$

and neglecting the contribution made to R_l by the region

$$0 \leqslant \varphi \leqslant (Ze^2\omega/mv_E^3)^{1/3},$$

we get from (4.1)

$$R_l = \frac{1}{2}\ln^2\frac{v_E}{(\hbar\omega/m)^{1/2}} - \frac{1}{4}\ln^2\frac{Ze^2}{\hbar(\hbar\omega/m)^{1/2}} = \frac{1}{4}\ln^2\frac{mv_E^2(\hbar\omega/m)^{1/2}}{Ze^2\omega} - \frac{1}{2}\ln^2\frac{Ze^2}{\hbar v_E}. \qquad (4.13)$$

On the boundaries of the region (4.12), Eq. (4.13) goes over at $Ze^2 \sim \hbar(\hbar\omega/m)^{1/2}$ into the result (4.11), and at $Ze^2 \sim \hbar v_E$ it goes over into still another new result:

$$R_l = \frac{1}{4}\ln^2\frac{mv_E^2(\hbar\omega/m)^{1/2}}{Ze^2\omega}. \qquad (4.14)$$

Equation (4.14) is valid at

$$Ze^2 \gg \hbar v_E, \quad (\hbar/m\omega)^{1/2} \gg Ze^2/mv_E^2$$

and in analogy with (4.13) it follows direction from (4.1). The only difference between the calculations and the case of (4.13) is that at $Ze^2 \gg \hbar v_E$ the asymptotic form (3.15) is suitable for all

$$\varphi \geqslant [Ze^2\omega/mv_E^2(\hbar\omega/m)^{1/2}]^{1/2}.$$

We emphasize that there is no dependence on the thermal motion of the electrons in Eqs. (4.11), (4.13), and (4.14); this is due to satisfaction of the condition $\hbar\omega \gg \varkappa T$.

The regularities of the absorption at $\hbar\omega \gg \varkappa T$ are illustrated in Fig. 3. The left-hand horizontal section of the curve corresponds to (4.11), and then goes over into the curve described by Eq. (4.13) and finally, at still larger values of $Ze^2/\hbar$, it goes over into a curve described by Eq. (4.14), which is the boundary of the region of existence of the doubly logarithmic approximation.

All the results obtained by us on the absorption of high frequency ($\omega > \omega_L$) radiation, both linearly and circularly polarized, are easily generalize to the case of low frequencies ($\omega < \omega_L$). To this end it suffices to replace

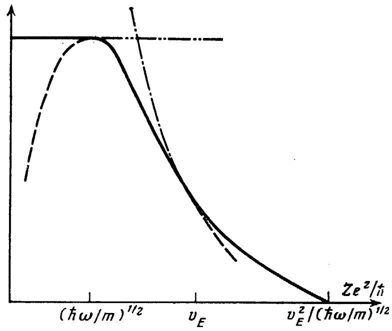

FIG. 3. Dependence of the function R_l on $Ze^2/\hbar$ under the conditions $v_E \gg (\hbar\omega/m)^{1/2} \gg v_T$.

ω by ω_L in all the equations of Secs. 3 and 4. We note also that the results of the theory of absorption of low-frequency radiation follows directly from the general formula (2.12) if the lower limit of the integral with respect to k in the latter is assumed to be equal to the smaller of the quanities r_D^{-1} and $(m\omega_L/\hbar)^{1/2}$. Introduction into (2.12) of the minimal limit of integration with respect to k corresponds to allowance for the effect of dynamic polarizability of the plasma at $\omega < \omega_L$.

The transition of the formulas for R_p that are valid at $\omega > \omega_L$ into the corresponding formulas in the region $\omega < \omega_L$ indicates that there is no increase in the absorption at $\omega \sim \omega_L$. This conclusion agrees with the results of a numerical investigation of $\nu(E)$,[15] where it is shown that allowance for the dynamic polarizability of the plasma increases $\nu(E)$ at $\omega \sim \omega_L$ only by numerical factor close to unity. We indicate in this connection that the recently predicted[22] anomalous increase of the absorption of intense radiation at $\omega \sim \omega_L$ does not correspond to reality. The error of that paper lies in the neglect of the spatial dispersion of the dielectric constant in the short-wave region $k \sim r_D^{-1}$.

5. CONCLUSIONS

Summarizing all the foregoing, we can state first of all that the results obtained even prior to our paper in the theory of absorption of linearly polarized high-frequency ($\omega \gg \omega_L$) strong electromagnetic field $v_E \gg v_T$, $(\hbar\omega/m)^{1/2}$ in a fully ionized plasma, under conditions when the dissipation is determined by the collisions, were correct in the very strongly pronounced quantum region [see Eqs. (4.7) and (4.11), which correspond to the results of Shima and Yatom[25] and of Karapetyan[26]]. In the classical region, in place of the differing results of Pert[16] and Klimontovich and Puchkov,[7] who described the absorption of linearly polarized radiation, we obtained Eq. (4.4). The reason why Pert's result[16] is incorrect is that he introduced in his theory arbitrary procedures which do not follow directly from this theory.[16] The result of Klimontovich and Puchkov[17] is wrong because of the cumbersome and physically unnecessary allowance for the dynamic polarizability, which prevented them from observing our correct results (4.4), which actually follows also from the general formulas of the theory of Ref. 17. Another new result pertaining to the case of linear polarization and also corresponding to classical mechanics of collitions (4.9) stems from the qualitative difference between the conditions for the interaction of charged particles in a strong field from those customarily realized. Namely, the result (4.9) pertains to the situation wherein the distance v_T/ω traversed by the thermal electron during one period of the field oscillation is large compared with the minimum, from the point of view of perturbation theory, impact parameter Ze^2/mv_E^2, although v_T/ω is simultaneously small compared with the usual minimal impact parameter $Ze^2/\varkappa T$. Finally, even in the classical region we obtained for circularly polarized radiation the result (3.14), which generalizes the result of Ref. 6 to include high frequencies.

For linearly polarized radiation, we obtained new results in a region where the collisions of the charged particles obey quantum mechanics when, the condition $Ze^2 > \hbar v_E$ is violated despite the satisfaction of the usual condition for the applicability of classical mechanics $Ze^2 > \hbar v_T$. Our formulas (4.6) and (4.10) have demonstrated the qualitative peculiarity of the new quantum-mechanical rules that are obtained in this case. The quantum effects manifested themselves most strongly in the limit when the energy $\hbar\omega$ of the emission photon is larger than the energy $\varkappa T$ of the thermal motion of the electrons, when the absorbed energy does not depend on $\varkappa T$ at all. Under these conditions, we obtained also the new equations (4.13) and (4.14) for linearly polarized radiation and (3.15) for circular polarization. In particular, the result (3.15) supplements the known essentially quantum results[25,26] [see also (3.12) and (3.13)] with allowance for the dependence of R_s on the classical minimum impact parameter.

Our asymptotic analysis was carried out in the case of circularly polarized radiation with logarithmic accuracy, and in the case of linearly polarized radiation with accuracy limited by allowance for the products of large logarithms. It seems to us that this accuracy is sufficient at the present experimental state of the art. At the same time, the relative simplicity of the theory of high-frequency absorption offers promise of obtaining results with higher accuracy.

[1)]The use of a minimal impact parameter $r_l(t)$ that depends only on the time is justified by the smallness of the time of interaction of the electron with the ion compared with the period of the radiation field.

[2)]The discarded expansion terms $\sin(\omega\tau/2)$ in the series in $\omega\tau/2$ add a numerical factor of the order of unity in the equations for R_p under the logarithm sign.

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