"Kinetic" instability of a strongly nonequilibrium system of spin waves and tunable radiation of a ferrite

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A new "kinetic" instability of a strongly excited state of parametric spin waves (PSW), in parallel pumping, is predicted theoretically and detected experimentally. This instability is caused by the vanishing of the attenuation of spin waves (SW) that are not related to the pumping. A negative contribution to the attenuation occurs because of four-magnon processes of spin-wave scattering in which the strongly nonequilibrium PSW participate. As a result of the development of kinetic instability, a narrow packet of secondary spin waves (SSW) originates, with minimum frequency $\omega_0 = \omega_{\mathbf{k}}$ at $k = 0$, $\vartheta_{\mathbf{k}} = 0$, and possessing minimum attenuation in the absence of PSW. The frequency of the secondary waves can be tuned over a wide range by variation of the magnetic field and is not related to the pumping frequency. The number of SSW is very large and may be comparable with the number of PSW. An abrupt change of the nonlinear susceptibility of the PSW beyond the threshold of kinetic instability, caused by flow of energy from PSW to SSW, is observed experimentally. Radiation of SSW at frequencies ω_0 and $2\omega_0$ is detected and investigated. The intensity of the latter attained 0.01% of the absorbed pumping power. The threshold of kinetic instability determined from the radiation always coincides with the threshold determined from the susceptibility.

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INTRODUCTION

So far, in the investigation of parametric excitation of spin waves (SW), primary attention has been given to the study of the properties of a narrow packet of parametric spin waves (PSW) with frequencies in the region of parametric instability. It has usually been assumed that the remaining SW are in a state close to thermodynamic equilibrium. But with increase of supercriticality, there occurs a considerable distortion of the equilibrium distribution of SW with frequencies far from parametric resonance, because of the influence of PSW. Thus at significant supercriticalities a number of new effects should occur, the experimental and theoretical study of which is only beginning. Among the phenomena of this sort, special interest attaches to the instability in a strongly nonequilibrium system of SW and to the major restructuring of the SW spectrum caused by them.

The present paper investigates, theoretically and experimentally, the instability of a system of PSW to generation of secondary SW, as a result of a four-magnon process: two PSW, fusing, give two secondary SW (SSW). Here the laws of conservation of energy and momentum must be satisfied:

$$\omega_{\mathbf{k}}+\omega_{\mathbf{k}'}=\omega_{\mathbf{k}_1}+\omega_{\mathbf{k}_2}, \quad \mathbf{k}+\mathbf{k}'=\mathbf{k}_1+\mathbf{k}_2. \tag{1}$$

Here $\mathbf{k}, \mathbf{k}'$ and $\omega_{\mathbf{k}}, \omega_{\mathbf{k}'}$ are the wave vectors and frequencies of the PSW, $\mathbf{k}_1, \mathbf{k}_2$ and $\omega_{\mathbf{k}_1}, \omega_{\mathbf{k}_2}$ are the wave vectors and frequencies of the SSW; the PSW frequencies are equal to half the pumping frequency,

$$\omega_{\mathbf{k}}=\omega_{\mathbf{k}'}=\omega_p/2,$$

but for the SW frequencies far from parametric resonance

$$\omega_{\mathbf{k}_1}\neq\omega_p/2, \quad \omega_{\mathbf{k}_2}\neq\omega_p/2.$$

In the nondecay region of the spectrum, this instability is of greatest interest, because all the other conceivable instabilities of the system of PSW are due to higher-order processes and therefore have a higher threshold.

A detailed theory of this SSW instability is presented in Sec. 1. Here we shall only illustrate its mechanism and estimate the threshold. As we shall show later, in the case when the attenuation decrements of secondary waves differ strongly, the presence of rapidly attenuating SSW may be neglected. The processes (1) lead, according to the kinetic equation, to a negative contribution to the attenuation of SSW:

$$\tilde{\gamma}_{\mathbf{k}_1}=\gamma_{\mathbf{k}_1}-2\pi\int |T_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}\mathbf{k}'}|^2 N_{\mathbf{k}} N_{\mathbf{k}'}\delta(\omega_{\mathbf{k}_1}+\omega_{\mathbf{k}_2}-\omega_p) \times\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}-\mathbf{k}')\,d^3\mathbf{k}_2\,d^3\mathbf{k}\,d^3\mathbf{k}', \tag{2}$$

where $\tilde{\gamma}_{\mathbf{k}_1}$ is the total attenuation of SSW, $\gamma_{\mathbf{k}_1}$ is their attenuation decrement in the absence of PSW, $N_{\mathbf{k}}$ is the number of PSW with wave vector $\mathbf{k}$, and $T_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}\mathbf{k}'}$ is the matrix element of the four-magnon interaction. If the number of PSW is sufficiently large, the attenuation diminishes noticeably for all waves in the nondecay part of the spectrum. The integral in (2) varies little with $\mathbf{k}_1$; therefore the attenuation becomes especially small for waves that have the smallest attenuation in the absence of PSW. These same waves are the first to become unstable, since for them the total attenuation becomes negative earliest, and the occupation numbers of these waves grow.

But which spin waves in a ferrite have the smallest attenuation? As is well known, the attenuation of SW caused by various processes, in which magnons and phonons take part, falls with increase of frequency. But for very long SW (that is, at small $\mathbf{k}_1$), additional channels of relaxation, due to the effect of the surface and to direct radiation of electromagnetic waves, are turned on. Thus the minimum attenuation, and consequently the maximum number of secondary SW, are attained at $\vartheta_{\mathbf{k}}=0$ ($\vartheta_{\mathbf{k}}$ is the angle between the wave vector and the magnetization) and at some small but nonzero value of $\mathbf{k}_1$. A reliable theoretical estimate

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of size-dependent mechanisms of relaxation is difficult, but numerous experimental investigations[1-4] show that for specimens of size ~1 mm the attenuation is smallest for $k_1 \sim 10^3$ to 10^4 cm^{-1}. The frequency of SW with such $\mathbf{k}$ is very close to the minimum $\omega_0 = \omega_{k\to 0, \vartheta\to 0}$, and the SSW may be considered to lie at the bottom of the spin-wave spectrum.

An estimate of the threshold of instability is easily obtained from (2) on the assumption that the problem is isotropic, so that the matrix element T, the distribution $N_{\mathbf{k}}$, and the SW spectrum $\omega_{\mathbf{k}}$ are treated as independent of angles in k space. Then from (2)

$$\tilde{\gamma}_{k_1} = \gamma_{k_1} - \pi \frac{(TN)^2}{kv}, \tag{3}$$

where k and v are the wave vector and group velocity of the PSW, and where $N = \int N_{\mathbf{k}} d^3k$ is the total number of PSW. As a result, we get for the threshold number of PSW the estimate

$$N_c = \frac{1}{|T|} \left(\frac{\gamma_0 kv}{\pi} \right)^{1/2}, \tag{4}$$

where γ_0 is the minimum value of the attenuation of SW. This estimate was first obtained in a paper of one of the authors.[5]

It must be stated that the instability discussed here is not parametric. This is already evident from the fact that the phase relations between SW are not important, and only the occupation numbers enter in formula (2) for the threshold. Therefore the instability under consideration, with respect to production of waves of a second generation, has a general character. In particular, in systems with a nondecay spectrum its threshold is independent of the method of excitation of the waves of the first generation. The theory of the instability has been developed within the framework of the kinetic equation; therefore we have called it "kinetic."

The experimental part of our paper (Sec. 2) is devoted to study of kinetic instability and of some of its manifestations in a ferrite, yttrium-iron garnet (YIG), for parametric excitation of SW of the first generation by the method of parallel pumping at frequency 9.4 GHz. The experimental method is described in Sec. 2.1. We recorded the onset of kinetic instability of SW on the basis of the appearance of a second shear on the pumping pulse with increase of the pumping power (Sec. 2.2). The first shear corresponded to the threshold of parametric excitation of SW. The threshold for onset of kinetic instability of SW corresponded also to a kink on the dependence of the imaginary part of the susceptibility, in parallel pumping, on the supercriticality (Sec. 2.3).

In Sec. 3.1, we present a decisive proof of the existence of kinetic instability of PSW: experimental detection of electromagnetic radiation from a ferrite, at a frequency corresponding to the bottom of the spin-wave spectrum:

$$\omega_0 = \omega_{k\to 0,\, \theta\to 0} = g(H - 4\pi N_z M). \tag{5}$$

Here g is the gyromagnetic ratio for electron spin, H is the value of the external constant magnetic field, M is the magnetization of the ferrite, and N_z is the sum of the demagnetizing factors of shape and of crystallographic anisotropy of the specimen in the direction of the applied field. The power of the radiation was very small, since it was due to magnetic-dipole radiation from spin waves with wave vector $k \sim 10^3$ and 10^4 cm^{-1}, substantially exceeding the value of the wave vector of electromagnetic waves of the corresponding frequency, $k_{\rm rad} = \omega_0/c \sim 1$ cm^{-1}. Loss of momentum of SW occurred at random inhomogeneities of the specimen or at regular inhomogeneities of the internal magnetic field. In the latter case, the power of the radiation increased significantly.

Sections 2.2 and 3.2 study radiation at the double frequency $\omega_{\rm rad} = 2\omega_0$, which occurs as a result of fusion of two SSW to a photon:

$$\omega_{k_1} + \omega_{k_2} = \omega_{k_1} + \omega_{-k_1} = \omega_{rad}, \tag{6}$$

since

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_{rad}, \quad k_{rad} \ll k_1, k_2.$$

The last inequality is a consequence of the uniformity of the electromagnetic field of frequency $\omega_{\rm rad}$ over the volume of the specimen. The power of the radiation produced by the processes (6) can be calculated in the usual way[6]:

$$I = \frac{2}{3} \frac{\ddot{M}^2}{c^3} = \frac{8}{3} \omega_{rad} k_{rad}^3 \sum_{\mathbf{k}} |V_{\mathbf{k}}|^2 n_{\mathbf{k}}^2, \tag{7}$$

where $V_{\mathbf{k}} = (g\omega_M/4\omega_{\mathbf{k}}) \sin^2 \vartheta_{\mathbf{k}}$ is the matrix element of the radiation process, $n_{\mathbf{k}}$ is the number of SW near the bottom of the spin-wave spectrum, and $\omega_M = 4\pi g M$. Estimates made by us on the basis of (7) show that the total number of secondary SW in the region of the bottom of the spectrum should be comparable with or exceed the number of PSW. We note that for waves lying exactly at the bottom of the spectrum, $\vartheta_{\mathbf{k}}$ and therefore also $V_{\mathbf{k}}$ vanish. Therefore the radiation power (7) has an additional smallness, due to the angular size of the packet of secondary SW with respect to polar angles.

In contrast to the radiation at frequency ω_0, which is due to the presence of inhomogeneities, the radiation at frequency $2\omega_0$ is that allowed by the conservation laws (6) and therefore is more powerful. An additional enhancement of the radiation at the double frequency is achieved when the frequency $2\omega_0$ coincides with the frequency of a magnetostatic mode of oscillation. Here the power I increased because of the high quality factor of the magnetostatic oscillations and reached 10^{-4} W when the ferrite absorbed power ~10 W (under conditions of ferromagnetic resonance at frequency $2\omega_0$: $2\omega_0 = gH$). Such radiation was observed on spherical specimens in fields from 0.95 to 1.35 kOe; the corresponding range of the frequencies under study was $\omega_{\rm rad} = 2\pi \cdot (2.4$ to $2.5) \times 10^9$ sec^{-1}. The frequency of this radiation was independent of the pumping frequency, which varied over the range $\omega_p = 2\pi \cdot (9.2$ to $9.5) \cdot 10^9$ sec^{-1}. This radiation was apparently observed in Refs. 7, although the experimental variation of its frequency with field did not correspond to the lower limit of the spin-wave spectrum.

Estimates made by us on the basis of formula (7) and investigations of the behavior of the susceptibility show that already at small (~3 dB) excess over the threshold of kinetic instability, the number of SSW is comparable with the number of PSW and may exceed the number of the latter.

In general, it would be possible to assume another mechanism of accumulation of secondary SW in the region of the bottom of the spectrum, caused by multiple processes of scattering of magnons and by the resulting transfer over the spectrum, like the Kolmogorov spectra in hydrodynamics. But such an assumption cannot explain the accumulation of such a large number of SW because of the form of the dispersion law and of the matrix elements of the magnon interaction. On the other hand, it contradicts a number of experimental facts. First, for the same parameters for which radiation at frequency $2\omega_0$ occurs, there exists a significant additional absorption of pumping energy, which also has a threshold character. If long wave originated as a result of multiple transfer processes, the absorption of pumping energy would not be related to the accumulation of these waves and could not have a threshold character. Another argument against the transfer mechanism is the fast, i.e., occurring after a time of the order of the SW lifetime, reaction of the radiation and of the additional absorption when the pumping is turned on.

1. THEORY OF KINETIC INSTABILITY OF SPIN WAVES

The behavior of waves far from the region of parametric instability is described by the kinetic equation[1,8]

$$\frac{\partial n_{\mathbf{k}}}{\partial t}-\mathrm{St}\{n\}=0. \tag{1.1}$$

The collision term receives contributions from all possible processes of scattering of SW; but in the present paper, we are interested in the case in which the PSW are in a nondisintegration region of the spectrum, and long SW can interact with parametric only in processes of four-magnon scattering (5). In all other processes, the occupation numbers of the quasiparticles vary little, and their influence leads only to a linear relaxation of SW, if the number of the latter is not too large. Thus the collision term may be written in the form

$$\begin{aligned}-\mathrm{St}\{n\}=2\gamma_{\mathbf{k}}^{(0)}(n_{\mathbf{k}}-n_{\mathbf{k}}^{0})+4\pi\int\{|T_{\mathbf{k}1,23}|^2[n_{\mathbf{k}}n_1(n_2+n_3)-n_2n_3(n_{\mathbf{k}}+n_1)]\\ +\mathrm{Re}\,T_{\mathbf{k}1,23}T^{*}_{\mathbf{k}\bar{2},\bar{1}3}\Sigma_1^{*}[\Sigma_2(n_{\mathbf{k}}-n_3)+\Sigma_3(n_{\mathbf{k}}-n_2)]\}\delta(\omega_{\mathbf{k}}+\omega_{\mathbf{k}_1}\\ -\omega_{\mathbf{k}_2}-\omega_{\mathbf{k}_3})\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,d^3\mathbf{k}_1\,d^3\mathbf{k}_2\,d^3\mathbf{k}_3.\end{aligned} \tag{1.1a}$$

Here $\gamma_{\mathbf{k}}^{(0)}$ is the frequency of relaxation of SW because of all processes except (1): $\gamma_{\mathbf{k}}^{(0)}=\gamma_{\mathbf{k}}-\gamma_{\mathbf{k}}^{(4)}$, where $\gamma_{\mathbf{k}}^{(4)}$ is the frequency of relaxation of SW caused by four-magnon scattering, and where $\gamma_{\mathbf{k}}$ is the total relaxation frequency. Equation (1.1a) differs from the standard equation by the presence of anomalous correlators of parametric waves $\Sigma_{\mathbf{k}}=\langle a_{\mathbf{k}}a_{-\mathbf{k}}\rangle$.

In the absence of parametric pumping, the solution of the kinetic equation is the thermodynamic equilibrium distribution $n_{\mathbf{k}}=n_{\mathbf{k}}^0$. In the presence of PSW, this distribution may prove unstable. To calculate the threshold and the increment of this instability, we linearize the kinetic equation (1.1), (1.1a):

$$n_{\mathbf{k}}=n_{\mathbf{k}}^0+\tilde{n}_{\mathbf{k}},$$

$$\begin{aligned}\left(\frac{\partial}{\partial t}+2\gamma_{\mathbf{k}}\right)\tilde{n}_{\mathbf{k}}+4\pi\int\{|T_{\mathbf{k}1,23}|^2[N_1N_2(\tilde{n}_{\mathbf{k}}-\tilde{n}_3)+N_1N_3(\tilde{n}_{\mathbf{k}}-\tilde{n}_2)-N_2N_3(\tilde{n}_{\mathbf{k}}+\tilde{n}_1)]\\ +\mathrm{Re}\,T_{\mathbf{k}1,23}T^{*}_{\mathbf{k}\bar{2},\bar{1}3}[\Sigma_1^{*}\Sigma_2(\tilde{n}_{\mathbf{k}}-\tilde{n}_3)+\Sigma_1^{*}\Sigma_3(\tilde{n}_{\mathbf{k}}-\tilde{n}_2)]\}\delta(\omega_{\mathbf{k}}+\omega_{\mathbf{k}_1}\\ -\omega_{\mathbf{k}_2}-\omega_{\mathbf{k}_3})\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,d^3k_1\,d^3k_2\,d^3k_3.\end{aligned} \tag{1.2}$$

Here $\gamma_{\mathbf{k}}$ is the attenuation decrement of SW in the absence of PSW, and $N_{\mathbf{k}}$ are the occupation numbers of the PSW. Since the distribution of PSW with respect to frequency $\omega_{\mathbf{k}}$ is very narrow,[9] it is convenient to transform in (1.2) from the variable $\mathbf{k}$ to the variables $\omega_{\mathbf{k}}$ and Ω, where Ω is a solid angle in k space on the surface $\omega_{\mathbf{k}}$ =const. If

$$N_k=N_\Omega J_\Omega\delta(\omega_{\mathbf{k}}-\omega_p/2),$$

then the general solution of the linear integral equation (1.2) can be represented in the form of a sum of special solutions of the form

$$\tilde{n}_{\mathbf{k}}=\tilde{n}_\Omega'J_\Omega'\delta(\omega_{\mathbf{k}}-\omega)+\tilde{n}_\Omega''J_\Omega''\delta(\omega_{\mathbf{k}}-\bar{\omega}), \tag{1.3}$$

where

$$\bar{\omega}=\omega_p-\omega,$$

where

$$J_\Omega=\left|\frac{\partial(\omega_{\mathbf{k}},\vartheta_{\mathbf{k}},\varphi_{\mathbf{k}})}{\partial(k_x,k_y,k_z)}\right|=\frac{v_\Omega}{k_\Omega^2},\quad J_\Omega'=J_\Omega(\omega_{\mathbf{k}}=\omega),\quad J_\Omega''=J_\Omega(\omega_{\mathbf{k}}=\bar{\omega}) \tag{1.4}$$

are the Jacobians of the transformation to the new variables, and where k_Ω is the length of the wave vector at the point $\omega_{\mathbf{k}},\Omega$.

As a result, we get from (1.2)

$$\begin{aligned}\left(\frac{\partial}{\partial t}+2\gamma_\Omega'\right)\tilde{n}_\Omega'+4\pi\frac{k_\Omega'^2}{v_\Omega'}\int\{|T_{\mathbf{k}1,23}|^2[2N_{\Omega_1}N_{\Omega_2}(\tilde{n}_\Omega'-\tilde{n}_{\Omega_3}')-N_{\Omega_2}N_{\Omega_3}(\tilde{n}_\Omega'+\tilde{n}_{\Omega_1}'')]\\ +2\,\mathrm{Re}\,T_{\mathbf{k}1,23}T^{*}_{\mathbf{k}\bar{2},\bar{1}3}\Sigma_{\Omega_1}^{*}\Sigma_{\Omega_2}(\tilde{n}_\Omega-\tilde{n}_{\Omega_3}')\}\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,d^2\Omega_1\,d^2\Omega_2\,d^2\Omega_3,\\ \left(\frac{\partial}{\partial t}+2\gamma_\Omega''\right)\tilde{n}_\Omega''+4\pi\frac{k_\Omega''^2}{v_\Omega''}\int\{|T_{\mathbf{k}1,23}|^2[2N_{\Omega_1}N_{\Omega_2}(\tilde{n}_\Omega''-\tilde{n}_{\Omega_3}'')-N_{\Omega_2}N_{\Omega_3}(\tilde{n}_\Omega''+\tilde{n}_{\Omega_1}')]\\ +2\,\mathrm{Re}\,T_{\mathbf{k}1,23}T^{*}_{\mathbf{k}\bar{2},\bar{1}3}\Sigma_{\Omega_1}^{*}\Sigma_{\Omega_2}(\tilde{n}_\Omega''-n_{\Omega_3}'')\}\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,d^2\Omega_1\,d^2\Omega_2\,d^2\Omega_3.\end{aligned} \tag{1.5}$$

Here $\gamma_\Omega',\gamma_\Omega''$ are the values of the attenuation on the surface $\omega_{\mathbf{k}}=\omega$, $\omega_{\mathbf{k}}=\bar{\omega}$ in the direction Ω; $\mathbf{k},\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3$ are functions of the frequencies and of the solid angles.

The functions $\tilde{n}_\Omega'e^{2\nu t}$ and $\tilde{n}_\Omega''e^{2\nu t}$ will be solutions of this equation. It is not difficult to show that

$$\int(\nu+\gamma_\Omega')\tilde{n}_\Omega'\,d^2\Omega=\int(\nu+\gamma_\Omega'')\tilde{n}_\Omega''\,d^2\Omega. \tag{1.6}$$

It is impossible to obtain an analytical solution of equation (1.5) in the general case; we shall therefore consider several simple but comprehensive model cases.

1.1 Isotropic model. The simplest case is that in which the matrix elements $T_{\mathbf{k}1,2,3}$, the attenuation γ_Ω, and the PSW distribution N_Ω are isotropic. Then

$$\begin{aligned}&N_\Omega=N/4\pi,\quad \gamma_\Omega'=\gamma',\quad \gamma_\Omega''=\gamma'',\\ &\omega_{\mathbf{k}}=\omega_k,\quad \tilde{n}_\Omega'=\tilde{n}'/4\pi,\quad \tilde{n}_\Omega''=\tilde{n}''/4\pi.\end{aligned} \tag{1.7}$$

As a result, we get from (1.5)

$$\begin{aligned}&(\nu+\gamma')\tilde{n}'-\pi\frac{(TN)^2}{kv}(\tilde{n}'+\tilde{n}'')=0,\\ &(\nu+\gamma'')\tilde{n}''-\pi\frac{(TN)^2}{kv}(\tilde{n}'+\tilde{n}'')=0.\end{aligned} \tag{1.8}$$

The threshold of instability is reached when the number of PSW is N_c:

$$\pi\frac{(TN_c)^2}{kv}=\frac{\gamma'\gamma''}{\gamma'+\gamma''}. \tag{1.9}$$

As a rule, the minimum of the right side is attained if one of the attenuations, for example γ'', is minimal, i.e., for $k'\to 0$. Usually $\gamma''\gg\gamma'$, and the threshold formula simplifies to the form

$$\pi\frac{(TN_c)^2}{kv}=\gamma'=\gamma_0, \tag{1.10}$$

where γ_0 is the minimum SW attenuation. On determining the number of PSW from S theory,[5] we get an estimate for the threshold supercriticality, i.e., the excess of the pumping amplitude h_c over the threshold of parametric instability h_1 at which the instability discussed here sets in:

$$p_c=(h_c/h_1)^2=\frac{1}{\pi}\left(\frac{S}{T}\right)^2\frac{\gamma_0k_0v}{\gamma^2}. \tag{1.11}$$

Here S is the matrix element of interaction of PSW pairs (the ratio of T to S is of order unity[9]), and γ is the attenuation of PSW.

1.2. It makes sense to carry out further investigation of the original equations (1.5) when $\gamma'_\Omega\ll\gamma''_\Omega$. As is evident from the integral relation (1.6), the number of secondary SW with large attenuation is small, and therefore they may be neglected in the linearized kinetic equation for $\tilde n_\Omega\equiv\tilde n'_\Omega$. Then instead of (1.5) we get

$$\left\{\frac{\partial}{\partial t}+2\gamma_\Omega-4\pi\frac{k_\Omega^2}{v_\Omega}\int|T_{\mathbf{k}1,23}|^2N_2N_3\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,d^2\Omega_1\,d^2\Omega_2\,d^2\Omega_3\right\}\tilde n_\Omega$$
$$=8\pi\frac{k_\Omega^2}{v_\Omega}\int(|T_{\mathbf{k}1,23}|^2N_1N_2+\operatorname{Re}T_{\mathbf{k}1,23}T_{\mathbf{k}\bar 2,\bar 1 3}\Sigma_1{}^*\Sigma_2)(\tilde n_3-\tilde n_\Omega)\delta(\mathbf{k}+\mathbf{k}_1$$
$$-\mathbf{k}_2-\mathbf{k}_3)\,d^2\Omega_1\,d^2\Omega_2\,d^2\Omega_3. \tag{1.12}$$

We point out that the integral of the right side of this expression is zero. This is a consequence of the fact that in four-magnon processes of scattering of secondary waves by parametric, as a result of which PSW and secondary SW appear, the number of secondary SW is conserved. As a result, (1.12) leads to the important integral relation

$$\int\left\{\frac{\partial}{\partial t}+2\gamma_\Omega-4\pi\frac{k_\Omega^2}{v_\Omega}\int|T_{\mathbf{k}1,23}|^2N_2N_3\delta(\mathbf{k}+\mathbf{k}_1\right.$$
$$\left.-\mathbf{k}_2-\mathbf{k}_3)\,d^2\Omega_1\,d^2\Omega_2\,d^2\Omega_3\right\}\tilde n_\Omega\,d^2\Omega=0. \tag{1.13}$$

In the situation of interest to us (a cubic ferromagnet), minimum attenuation is attained at $\vartheta_k=0$ and at a small wave vector k_0. The SW spectrum has the form

$$\omega_{\mathbf{k}}=\omega_{k,\vartheta}=\{[\omega_0+\omega_{ex}(ak)^2+\omega_M\sin^2\vartheta][\omega_0+\omega_{ex}(ak)^2]\}^{1/2}, \tag{1.14}$$

therefore on the surface of constant frequency $\omega_{k,\vartheta}=\omega_{k_0,0}$ the permissible values of the angle ϑ are small:

$$\vartheta^2\leqslant 2\omega_{ex}(ak_0)^2/\omega_M\ll 1. \tag{1.15}$$

The expression in square brackets in (1.13) varies little with the angle ϑ over the narrow range of angles where $\tilde n_\Omega\neq 0$, and the identity (1.13) takes the form

$$\left[\frac{\partial}{\partial t}+2\gamma_0-4\pi\int|T_{0,\mathbf{k}'+\mathbf{k}'',\mathbf{k}',\mathbf{k}''}|^2N_{\mathbf{k}'}N_{\mathbf{k}''}\right.$$
$$\left.\times\delta(\omega_0+\omega_{\mathbf{k}'+\mathbf{k}''}-\omega_p)\,d^3\mathbf{k}'\,d^3\mathbf{k}''\right]\tilde n=0. \tag{1.16}$$

Here

$$\tilde n=\int\tilde n_\Omega\,d^2\Omega.$$

In (1.16), it was convenient to transform the integral part to the variables $\mathbf{k}'=\mathbf{k}_2$, $\mathbf{k}''=\mathbf{k}_3$; γ_0 is the minimum value of $\gamma_{\mathbf{k}}$.

For accurate calculation of the threshold of instability according to formula (1.16), it is necessary to know an expression for the matrix element $T_{0,\mathbf{k}'+\mathbf{k}'',\mathbf{k}',\mathbf{k}''}$ and the the PSW distribution $N_{\mathbf{k}}$. In the experimental situation of interest to us, the dipole-dipole contribution to the matrix element T is comparable with or exceeds the exchange. For the calculation, it is necessary to make a u, v transformation and to take into account the contribution of three-magnon processes to four-magnon in the second order of perturbation theory.

The expression thus obtained for T is very cumbersome, and we shall not give it. By analyzing it, one can show that in the situations studied experimentally, the matrix element $T_{0,\mathbf{k}'+\mathbf{k}'',\mathbf{k}',\mathbf{k}''}$ varies by no more than a factor two for all possible values of $\mathbf{k}'$ and $\mathbf{k}''$. For example, for $\omega_p=2\pi\cdot 9.4\cdot 10^9$ sec^{-1} and $\omega_M=2\pi\cdot 4.9\cdot 10^9$ sec^{-1} (yttrium-iron garnet at $T=300$ K) and for $\mathbf{k}'$, $\mathbf{k}''\perp\mathbf{M}$,

$$T_{0,\mathbf{k}'+\mathbf{k}'',\mathbf{k}',\mathbf{k}''}=2\pi g^2\cdot\begin{cases}0.65 & \text{when } H=H_1=0.9\text{ kOe}\\ 0.69 & \text{when } H=H_2=1.3\text{ kOe}\end{cases}, \tag{1.17}$$

where H_1 and H_2 are the minimum and maximum values of the external field H at which radiation at frequency $2\omega_0$ is observed.

For calculation of the threshold of kinetic instability it is also necessary to know the distribution of PSW with respect to angles, i.e., the function $N_{\vartheta,\varphi}$. We first use the simple expression for $N_{\vartheta,\varphi}$ obtained in S theory at supercriticalities less than the threshold for generation of the second group of pairs[9]:

$$N_{\vartheta,\varphi}=\frac{N}{2\pi}\delta(\vartheta-\pi/2), \tag{1.18}$$

$$N=\frac{\gamma}{|S|}(p-1)^{1/2}. \tag{1.19}$$

Then in accordance with (1.16) we get when

$$(TN_c)^2=2\pi\gamma_0\omega_M\left(\frac{H_c-H}{2\pi M}\right)^{1/2}A^{1/2}\quad\text{when}\quad A\geqslant 0. \tag{1.20}$$

Here the following notation has been introduced:

$$A=4(\Omega_p^2+1)^{1/2}-3-6\Omega_0-[(\Omega_p-\Omega_0)^2+{}^1/_4]^{1/2},$$
$$\Omega_p=\omega_p/\omega_M,\quad \Omega_0=\omega_0/\omega_M. \tag{1.21}$$

It is not difficult to show that the quantity A and with it the threshold number N_c of (1.20) vanish at a certain field $H=H_2$:

$$H_2=4\pi M\{N_z+[12(\Omega_p^2+1)^{1/2}-2\Omega_p-9-[(2+4\Omega_p$$
$$-6(\Omega_p^2+1)^{1/2})^2+20\Omega_p-7]^{1/2}/16]\}. \tag{1.22}$$

At fields larger than H_2, the processes (1) involving participation of PSW at the equator are suppressed, and the threshold (1.20) becomes infinite. Vanishing of the threshold (1.20) at the boundary of permissibility of the processes (1) is due to the infinitely large density of states that occurs because of a singularity in the distribution of PSW with respect to ϑ in (1.18). In fact, the PSW distribution N_ϑ has a finite width, which increases with the supercriticality[10]:

$$\Delta\vartheta \approx [(\gamma/kv)(p-1)]^{1/2}. \tag{1.23}$$

In order to take this into account, we expand the integrand in (1.16) in the small deviation of ϑ from $\pi/2$. As a result we get

$$T^2 \int \frac{N_x N_{x'}\, dx\, dx'}{[A+B(x^2+x'^2)+2Cxx']^{1/2}} = 2\pi\gamma_0\omega_M\left(\frac{H_c-H}{2\pi M}\right)^{1/2}. \tag{1.24}$$

Here $x=\cos\vartheta$, $x'=\cos\vartheta'$,

$$B+C=\frac{3}{2}+\frac{1}{[4(\Omega_p-\Omega_0)^2+1]^{1/2}}-\frac{2}{(\Omega_p^2+1)^{1/2}}$$
$$B-C=4\Omega_0-2\frac{[(\Omega_p^2+1)^{1/2}-1]^2}{(\Omega_p^2+1)^{1/2}}, \tag{1.25}$$

and the integration extends over the region where the radicand is positive. The minimum of the threshold is attained at $H=H_2$, when $A=0$. Assigning to N_x the model value

$$N_x=\frac{N}{(2\pi)^{1/2}\Delta}e^{-x^2/2\Delta^2},\quad \Delta^2\ll 1,$$

we get from (1.24) an expression for the threshold in this field:

$$(TN_c)^2 \approx 2(2\pi)^{1/2}\Delta\gamma_0\omega_M\left(\frac{H_c-H}{2\pi M}\right)^{1/2} B^{1/2}\left(1+\frac{3}{16}\frac{C^2}{B^2}\right). \tag{1.26}$$

Here we have used the fact that in the experimental situation, $|C|< B$. It is evident that N_c depends strongly on the angular width Δ of the PSW packet.

In the limit of large supercriticalities, one can obtain from S theory a model expression for N_ϑ:

$$N_\vartheta = {}^4/_3 N \sin^2\vartheta. \tag{1.27}$$

Substituting it in (1.24), we get approximately

$$(TN_c)^2 \approx \frac{8}{3}\gamma_0\omega_M\left(\frac{H_c-H}{2\pi M}\right)^{1/2} B^{1/2}, \tag{1.28}$$

which corresponds to an effective width $\Delta\approx 0.5$ of the packet (1.27).

A comparison of the expressions obtained for N_c with experiment will be made in the next section.

2. EXPERIMENTAL STUDY OF KINETIC INSTABILITY

2.1 Experimental method

A block diagram of the experimental setup is shown in Fig. 1. The signal from the pumping generator, via a series of waveguide elements, entered the measurement section 1, containing the ferrite and located in a constant magnetic field H. The measurement section was a volume resonator, made from a section of standard three-centimeter waveguide, with the H_{011} type of oscillation. The ferrite specimen was placed near the rear end wall of the resonator and was surrounded by a coupling loop, connected to the coaxial-cable input. In order to introduce minimum disturbance of the fields in the resonator, the plane of the loop was parallel to the lines of force of the constant and microwave magnetic fields.

The frequency of the pumping generator was 9.4 GHz,

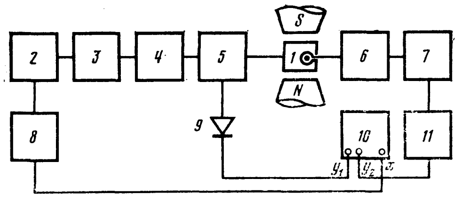

FIG. 1. Block diagram of the experimental setup. 1, measurement section containing ferrite; 2, pumping generator; 3, valve; 4, precision attenuator; 5, bidirectional coupler; 6, low-frequency filter; 7, transistor microwave amplifier; 8, 8, trigger pulse generator; 9, microwave detector; 10, two-beam oscillograph; 11, measuring receiver.

the pulse duration 2 to 400 μsec, the repetition frequency of the pulses 5 to 50 Hz. The necessary level of the pumping signal was established by means of the precision attenuator 4. The low-pass filter 6 served to protect the transistor amplifier 7 from the powerful pumping signal. On the oscillograph 10, depending on the tuning of the measuring receiver 11, it was possible to observe the form of the pulse radiated by the ferrite at frequency ω_0 or $2\omega_0$, and also the form of the pumping pulse reflected from the resonator.

2.2 Transition processes

As has already been mentioned in the Introduction, the value of the supercriticality, in relation to the threshold power p_c for parametric instability, at which kinetic instability occurred for SW lying at the bottom of the spin-wave spectrum was fixed on the basis of a shear on the pulse reflected from the measurement section. This pulse, after detection of it by the detector 9, could be observed on the screen of the oscillograph 10. Simultaneously with the shear there occurs radiation at frequency $2\omega_0$, fixed in the same experiment. This supports our assumption of fast buildup of PSW at the bottom of the spectrum, without multiple scattering processes and transfer along the spectrum. Figure 2 shows the form of the pulse of radiation at frequency $2\omega_0$ for small excesses over the threshold for excitation of secondary SW; here the duration of the pumping pulse is 2 μsec. It is seen that during the time of action of the pump the radiated power, beginning with a certain instant, increases rapidly. In accordance with what was said above, this instant coincides with the appearance of a shear on the pumping pulse. After the shutting off of the pumping pulse, the power of the radiation drops exponentially, with relaxation time 1 μsec. Since the power of the radiation at frequency $2\omega_0$ is proportional to the square of the number of SSW, it is possible, by plotting the trailing edge of the pulse

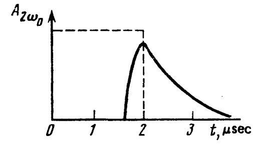

FIG. 2. Form of the pulse radiated by a ferrite (YIG) sphere of diameter 2.26 mm, at frequency $2\omega_0$, H|| [111], H = 1.25 kOe, p/p_c = 1.5 dB. Dotted: form of the pumping pulse, of duration 2 μsec; linear detector.

of radiation, averaged over the background of self-oscillations (Fig. 2), on a logarithmic scale, to estimate the lifetime and relaxation frequency γ_0 of these spin waves.

It was found that the relaxation frequency γ_0 of SW with the minimum frequency $\omega_{\mathbf{k}}=\omega_0$ is less than the relaxation frequency of PSW at $\vartheta=\pi/2$, determined from the parallel-pumping threshold. For example, when $H_c-H=300$ Oe and $\mathbf{H}\|[111]$, $\gamma_0\approx 0.5\cdot 10^6$ sec^{-1} and $\gamma_{\mathbf{k}}\approx 3\cdot 10^6$ sec^{-1}, so that $\gamma_{\mathbf{k}}/\gamma_0\approx 6$. When $\mathbf{H}\|[100]$, $\gamma_0\approx 1.5\cdot 10^6$ sec^{-1} and $\gamma_{\mathbf{k}}/\gamma_0\approx 2$.

2.3 Dependence of the threshold of kinetic instability on the magnetic field

Figure 3 shows the dependence of the threshold for generation of SSW (and for radiation at frequency $2\omega_0$) on the value H of a magnetic field directed along the easy axis [111] of the ferrite. It is seen that the experimentally allowed region of excitation of SSW is enclosed within the bounds $H_1\lesssim H\lesssim H_2$, where $H_1=0.9$ kOe and $H_2=1.27$ kOe. Outside these bounds, the threshold rises abruptly.

The existence of an upper bound H_2 to the field, i.e., of a minimum of the PSW wave vector, finds a natural explanation within the framework of the instability model proposed above and is simply related to the law of conservation of energy and of momentum. On substituting in formula (1.22) the numerical values $\Omega_p=\omega_p/\omega_M\approx 1.92$, $4\pi M=1750$ G, $N_z=\frac{1}{3}-H_a/4\pi M=0.30$ (for $\mathbf{H}\|[111]$), we get $H_2=1250$ Oe, which agrees excellently with the experimental value of this quantity.

The increase of the instability threshold in fields less than H_1 is due to the fact that for sufficiently short PSW, three-magnon interaction processes-fusions and disintegration of PSW—are permitted. As a result, a sharply nonlinear attenuation of PSW occurs, and attainment of the threshold level N_c of PSW requires a large pumping power. Under the conditions of the experiment, the field H^* for which processes of fusion of two PSW with $\vartheta=\pi/2$ are permitted is 1.0 kOe. Disintegrations of PSW with $\vartheta=\pi/2$ are permitted for $H<0.7$ kOe; with $\vartheta=0$, for $H<1.14$ kOe. The experimental value of the field H_1, as is seen from Fig. 3, is located in this interval.

2.4 The threshold number of PSW and its anisotropy

Figure 4 shows the variation of the threshold supercriticality p_c with the orientation of the external field $\mathbf{H}$

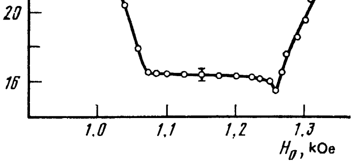

FIG. 3. Variation of the threshold supercriticality for onset of secondary instability of spin waves with a constant magnetic field H. Specimen: YIG sphere, diameter 1.5 mm, H|| [111].

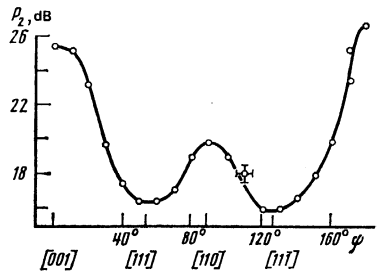

FIG. 4. Variation of threshold supercriticality with angle between direction of constant magnetic field and crystallographic axis [001], for YIG sphere: diameter 2.9 mm, $H=1.26$ kOe.

with respect to crystallographic axes [in the (110) plane]. The threshold is highest for $\psi=0$, $\mathbf{H}\|[100]$. The minimum value of the threshold ($p_c\approx 16$ dB) corresponds to $\mathbf{H}\|[111]$. A local maximum occurs in direction [110].

There are at present no reliable direct methods of measuring the number of PSW. We shall therefore estimate an "experimental" value of N_c from data on the susceptibility χ," assuming that the attenuation $\gamma_{\mathbf{k}}$ of PSW is independent of the number N of PSW and of the direction of $\mathbf{k}$. Then

$$N_c=\frac{2M}{g}\frac{\gamma}{\omega_M}\left(\frac{\omega_p}{\omega_M}\right)^2 4\pi\chi''(p_c)p_c.$$

Using the experimental values $4\pi\chi''(p_c)=0.2\pm 0.05$, $p_c\approx 40$ (16 dB), we get, for $\mathbf{H}\|[111]$ and $\gamma/\omega_M\approx 10^4$, $(N_c)_{\rm exp}\approx 3\cdot 10^{-3}$(M/g). The corresponding data for $\mathbf{H}\|[100]$ give $(N_c)_{\rm exp}\approx 2\cdot 10^{-2}$(M/g). A theoretical value of N_c can be obtained from formula (1.28) by substituting in it $T=0.7\times 2\pi g^2$ from (1.17), $H_c-H=300$ Oe, $B(H_2)\approx 0.65$, $\gamma_0\approx 0.5\cdot 10^6$ sec^{-1} ($\mathbf{H}\|[111]$) and $\gamma_0\approx 1.5\cdot 10^6$ sec^{-1} ($\mathbf{H}\|[100]$). As a result, $(N_c)_{\rm th}$ is found to be 10^{-2}(M/g) for $\mathbf{H}\|[111]$ and $2\cdot 10^{-2}$(M/g) for $\mathbf{H}\|[100]$.

In view of the nature of the assumptions made, the qualitative agreement of the experimental data and of the estimates presented must be considered quite satisfactory. The quantitative discrepancy may be due to a number of reasons: inaccurate determination of N_c from the susceptibility data because of the dependence of $\gamma_{\mathbf{k}}$ on ϑ, the unknown distribution of the PSW with respect to ϑ, self-oscillations of the PSW, the presence in addition to PSW of "heated" SW with frequencies of the order of $\omega_p/2$, etc.

2.5 The nonlinear susceptibility

The variation of the imaginary part of the nonlinear susceptibility with supercriticality is shown in Fig. 5. It is seen that the curve $\chi''(p)$ has a break at $p=p_c$, caused by instability of secondary SW. The absolute value of χ'' depends little on the angle ψ between H and the crystal axes.

For different fields H, as a result of the action of different nonlinear mechanisms, the form of the $\chi''(p)$ vs p curve is considerably different.[9] For fields close to H_c, where the phase mechanism of limitation of amplitude acts, the value of χ'' decreases monotonically at large supercriticalities. For large fields, becasue of the influence of nonlinear three-magnon mechanisms, an increase of χ'' occurs, as is seen from Fig. 5. A

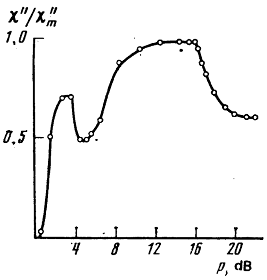

FIG. 5. Variation of the relative value of the imaginary part of the nonlinear susceptibility, in parallel pumping, with supercriticality, for a YIG sphere: diameter 2.26 mm; $H = 1.21$ kOe; H‖[111]; $p/p_c = 16$ dB; duration of pumping pulse 2 μsec.

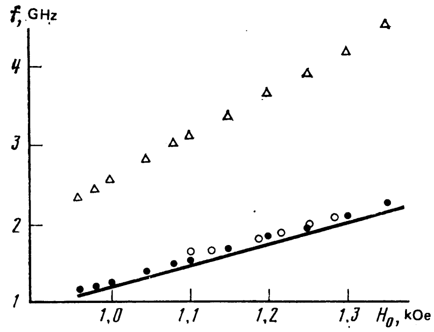

FIG. 6. Variation of the frequency of the radiation from a ferrite with constant magnetic field, for a YIG sphere: diameter 1.5 mm; H‖[111]. Solid line, experimentally determined lower limit of the spin-wave spectrum for $k \to 0$. ○, radiation at frequency ω_0; △, radiation at the double frequency $2\omega_0$; ●, radiation frequency $2\omega_0$, divided by two.

break in the $\chi''(p)$ curve near the point $p = p_c$ always occurs, but in the first case it leads to an increase of the susceptibility, and in the second to a decrease.

It is seen from Fig. 5 that when the threshold of kinetic instability is exceeded by 3 to 4 dB, the imaginary part of the nonlinear susceptibility changes by a factor of about two. This means that even at small excesses over this threshold, the number of SSW becomes comparable with the number PSW. A nonlinear theory, describing the behavior of the susceptibility and other beyond-threshold phenomena, will be published by us later.

3. ELECTROMAGNETIC RADIATION

3.1 Direct radiation of SSW at frequency ω_0

In order to prove directly the buildup of SSW with the minimum frequency, we did an experiment aimed at detection of radiation by the ferrite at frequency ω_0. Since, as was indicated in the Introduction, direct processes of radiation of SW are forbidden by the law of conservation of momentum, direct radiation of SSW is possible only in a spatially inhomogeneous specimen or in an inhomogeneous magnetic field.

The power of the radiation of SW from a spherical specimen at frequency ω_0, caused by loss of momentum of the waves at random inhomogeneities, registered at the limit of sensitivity of the experimental apparatus, which was 10^{-13} W. Therefore in the experiment with a sphere, we succeeded in measuring only the frequency of this radiation; measurements of the amplitudes and of the spectrum proved impossible. The power of the radiation at frequency ω_0 could be raised by using, instead of a spherical specimen, a normally magnetized disk of finite thickness. Here the internal magnetic field, and consequently also the spin-wave frequency, is somewhat smaller at the center than at the boundary.[1] Therefore SW excited in the volume of the specimen may have, at the boundary, wave vectors close to $k_{\rm rad}$ and may be capable of radiating electromagnetic waves. This process is the inverse of the well-known process of excitation of traveling SW. The power of the radiation from the disk attained values of 10^{-10} W; but this radiation had a whole series of peculiarities, caused by the inhomogeneity of the internal field, discussion of which here does not seem expedient. We remark only that the frequency band of the radiation from the disk amounted to several hundred MHz, which is larger than for the sphere by more than an order of magnitude (see below).

The variation of the frequency of the radiation for a spherical specimen with the constant magnetic field is shown in Fig. 6. Here also (solid line) is shown the experimentally determined lower limit of the spin-wave spectrum. The good agreement of the radiated frequencies with this limit is evident.

The necessity for experimental determination of the lower limit of the SW spectrum is due to the difficulty of exact determination of the demagnetizing field $N_z M$ in formula (4), dependent as it is on the nonspherical shape of the specimen, the crystallographic anisotropy, the temperature, etc. For this purpose it is sufficient to determine just one point on the $\omega_{\bf k} - H$ plane, since on this plane the lower limit of the spin-wave spectrum must be a straight line with a slope equal to the gyromagnetic ratio g. To determine this point, the method of parametric excitation of spin waves was used, under conditions of coincidence of the fundamental resonance with the subsidiary.[11] The loop surrounding the ferrite (see Fig. 1) received a signal from the external microwave generator that excited parametric instability in the ferrite. The frequency $\omega_{\rm gen}$ of this generator corresponded to ferromagnetic resonance of the specimen. With increase of the frequency $\omega_{\rm gen}$, the instability threshold increased; it reached a maximum at $\omega_{\rm gen}/2 = \omega_0$. Thus it was possible, by finding the maximum of the instability threshold under conditions of coincidence of the fundamental resonance with the subsidiary, to determine experimentally the value of ω_0.

Here it is necessary to make an important comment. It is clear that in the course of the experiment it was possible to determine the position both of the lower and also of the upper (by measurement of the instability threshold in parallel pumping) limits of the spin-waves for $k \to 0$. It was found that the distance between these limits in frequency, i.e., the width of the spin-wave spectrum at small k, exceeded the theoretical value by at least 50 MHz.

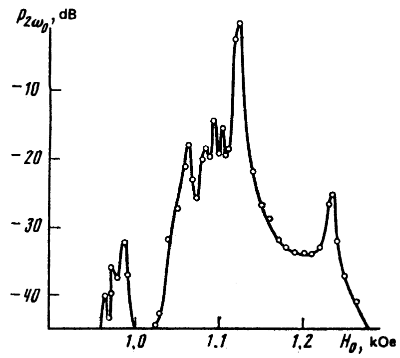

FIG. 7. Variation of the amplitude of radiation from a ferrite at frequency $2\omega_0$ with constant magnetic field, for a YIG sphere: diameter 1.5 mm; H ∥ [111]. Zero dB on the vertical axis corresponds to (40 ± 5) dB/W in absorption by the ferrite of power ~10 W.

3.2 Radiation at the double frequency

The variation of the power of the radiation from a spherical specimen at double the frequency of the lower limit of the spin-wave spectrum with the constant magnetic field is shown in Fig. 7. The radiation power is considerably larger in this case than at frequency ω_0 and reaches hundreds of mW. The power absorbed in the ferrite was (10 ± 3) W.

The maximum of the radiation at frequency $2\omega_0$, at field H = 1125 Oe, corresponds to the case of ferromagnetic resonance at this frequency. The other, less strong maxima at fields 1230 Oe, 1093 Oe, etc. correspond to resonances at frequency $2\omega_0$ of higher magnetostatic types of precession, which lead to enhancement of the radiation because of the high quality factor of these oscillations.

An investigation of the spectral composition of the radiation at frequency $2\omega_0$ was made on a setup whose block diagram differs from that shown in Fig. 1 only in the fact that the measuring receiver in it was replaced by a spectral analyzer. The width of the spectrum for supercriticalities $p > p_c$ depended on the value of the constant magnetic field and lay between the limits 5 and 15 MHz. To interpret the spectral properties of the radiation, it is necessary to apply a nonlinear theory of kinetic instability.

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