Spontaneous radio emission of metals in a magnetic field in the presence of a temperature gradient

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We consider the spontaneous radio emission of a metal plate placed in a magnetic field perpendicular to its surface, under the condition that a temperature gradient exists in a direction parallel or antiparallel to the field. If the electric conduction is effected by electrons from several (at least two) bands independently of the sign of their charge, then self-excitation of helicoidal or (in metals without a Hall current) Alfvén waves sets in at easily obtainable magnetic field values and at sufficiently low temperatures of the order of several degrees. The nonlinear theory leads to the conclusion that the intensity of this emission at frequencies that can be of the order of 10^9 to 10^{10} sec^{-1} in the case of Alfvén waves the intensity of this emission can exceed 1 W/cm^2. In the case of helicoidal waves in bismuth, a power of the same order can be reached at frequencies 10^5 sec^{-1}.

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§1. INTRODUCTION

We investigate here a new effect: radio emission from a metallic sample located in a magnetic field (on the order of 10^3–$5\cdot10^4$ Oe) with a cyclotron frequency that exceeds the average carrier collision frequency at temperatures on the order of liquid-helium and at a small temperature gradient (of the order of 1–2 °K/cm). This emission takes place in a wide range of frequencies, 10^2–10^9 Hz and reaches intensities 1–10 W/cm^2. The described phenomenon is observed only in metals in which the electric conduction is effected by electrons from several bands, the carrier charges being of like or unlike sign. Detailed estimates of this effect for different metals will be given in §5. The spontaneous radio emission can take place also from semiconductors at nitrogen temperature. In this article, however, we confine ourselves to metals.

The emission is the result of self-excitation of circularly polarized helicoidal waves, and in metals such as bismuth also in Alfvén waves. Their spectrum and polarization are altered by the presence of a temperature gradient. The frequency ω of the Alfvén waves exceeds the average collision frequency ν, and in this sense these are high-frequency waves.

In earlier studies of thermomagnetic instability in solids (e.g., Ref. 1) the situation considered was such that spontaneous radio emission was impossible. In the case investigated the magnetic field was weak and the cyclotron frequency was lower than the average carrier collision density. In that case, however, only convective instability is produced without feedback, and cannot lead to emission (Ref. 2, Chap. 6).

We consider for simplicity a metal plate whose thickness d is much less than that of the remaining dimensions, so that edge effects are immaterial. The temperature gradient and the magnetic field are perpendicular to its plane.

§2. LINEAR KINETIC THEORY IN THE CASE OF TWO BANDS

The alternating field of the electromagnetic wave is of the form

$$E'\propto B'\propto \exp(ik'z-i\omega't-k''z+\omega''t)$$

(the z axis is directed along ∇T). It was shown in a preceding paper[3] that the alternating current proportional to ∇T is strong enough to change substantially the dispersion relation of the waves only if carriers from at least two bands participate in the conduction, and these carriers should be dragged by phonons. This dragging can be described by a "thermodynamic force" $[-\chi(\varepsilon)\nabla T]$ added to the usual forces $e\mathbf{E}$ and $e\mathbf{v}\times\mathbf{B}/c$ [here $\chi(\varepsilon)$ is a quantity that can be called the dragging coefficient[3]]. A calculation similar to the earlier one[3] but with allowance for the presence of a constant magnetic field B [in the approximation of the collision frequency $\nu(\varepsilon_F)$] leads to the following expression for the alternating current $\mathbf{j}'$ in the presence of carriers in bands a and b: *

$$\mathbf{j}'=\sigma\mathbf{E}'+\sigma_1[\mathbf{E}'\mathbf{B}]+\eta[\mathbf{B}\nabla T]+\eta_1[\mathbf{B}'[\mathbf{B}'\nabla T]], \tag{1}$$

where

$$\sigma=\sigma_a+\sigma_b,\qquad \sigma_1=\sigma_{1a}+\sigma_{1b},$$

$$\eta=cC_{ab}[\chi_a(\varepsilon_F)-\chi_b(\varepsilon_F)]\left[\frac{1}{m_b}\frac{\nu_b-i\omega}{(\nu_b-i\omega)^2+\Omega_b^2}-\frac{1}{m_a}\frac{\nu_a-i\omega}{(\nu_a-i\omega)^2+\Omega_a^2}\right],$$

$$\eta_1=C_{ab}[\chi_a(\varepsilon_F)-\chi_b(\varepsilon_F)]\left[\frac{e_b}{m_b^2}\frac{1}{(\nu_b-i\omega)^2+\Omega_b^2}-\frac{e_a}{m_a^2}\frac{1}{(\nu_a-i\omega)^2+\Omega_a^2}\right], \tag{2}$$

$$C_{ab}=\frac{e^2}{c^2}\frac{n_an_b}{n_am_b\nu_b+n_bm_a\nu_a},\quad \Omega=\frac{eB}{mc}.$$

This expression calls for two stipulations. First, we have assumed that $\mathbf{k}\cdot\mathbf{v}\ll\omega$. This is permissible for helicoidal waves, for which $\mathbf{k}\cdot\mathbf{v}\ll\nu$ and $\omega\ll\nu$, and we neglect both these quantities. For Alfvén waves the frequency $\omega\gg\mathbf{k}\cdot\mathbf{v}$. We shall be interested hereafter only in situations in which $(\nu-i\omega)^2$ can be neglected compared with Ω^2, so that σ_1 and η_1 can be regarded as independent of frequency. Maxwell's equations with allowance for (1) leads to the dispersion relation

$$\varepsilon_0\omega^2-c^2k^2+4\pi i\omega\sigma+\mathscr{P}\cdot4\pi\omega\sigma_1B_z-4\pi ic\eta k_z\nabla T+\mathscr{P}\cdot4\pi c\eta_1k_zB_z\nabla T=0. \tag{3}$$

Here $\mathscr{P}=\pm1$ for right—and left—circular polarization of the waves. The subscript z denotes the vector component, which can be either positive or negative.

*$[\mathbf{E}'\mathbf{B}]\equiv\mathbf{E}'\times\mathbf{B}$, etc.

§3. INSTABILITY OF THERMOMAGNETIC WAVES IN A BOUNDED MEDIUM

If a right-circularly polarized wave is incident from the inside on a crystal boundary in the positive direction, then the reflected wave is left-polarized, and the real part k' of its wave vector turns out to be negative. For helicoidal and Alfvén waves "feedback" is present in the entire instability region, i.e., the reflected wave also increases with time. Therefore the multiple reflections from the crystal boundaries cause continuous amplification of the wave. This amplification, as we shall show later, is only slightly weakened by the radiation at the boundaries. The feedback couples the right-hand wave, which propagates in the positive direction (we shall call it "forward") and has a complex wave vector $k_{r+}=k'_{r+}+ik''_{r+}$ with a backward left-hand wave with wave vector $k_{l-}=k'_{l-}+ik''_{l-}$. These two waves form therefore one mode of the natural oscillations of the crystal. The second mode is formed by a right-hand wave propagating in the backward direction with wave vector k_{r-} to a left-hand wave in the forward direction with a wave vector k_{l+}. The spatial growth (or attenuation) rates $k''_{r\pm}$ and $k''_{l\pm}$ of the waves of these two modes are different. Therefore the field can be single-valued only for each mode separately. For the first mode ($r+$ and $l-$) it is of the form

$$q_{r+}q_{l-}\exp[i(k_{r+}-k_{l-})d]=1. \tag{4}$$

Here

$$q_{r+}=-(ck_{r+}-\omega)/(ck_{l-}-\omega)$$

is the reflection coefficient of the $r+$ wave at the right-hand boundary and is connected with the change of the polarization;

$$q_{l-}=-(ck_{l-}+\omega)/(ck_{r+}+\omega)$$

is the coefficient of the inverse transformation upon reflection from the left boundary.

For the waves considered we have $|k''|\ll|k'|, \omega^+=\omega^-$, $\omega''\ll\omega'$; in addition, $k'_{l-}\approx k'_{r+}$ with an error much smaller than k''/k'. Finally $c|k'|\gg\omega'$. It follows from these inequalities that

$$q_{r+}=|q_{r+}|e^{i\varphi_{r+}}\approx\frac{k_{r+}'}{|k_{l-}'|}\left[1+\frac{1}{2}\left(\frac{k_{r+}''}{k_{r+}'}+\frac{k_{l-}''}{|k_{l-}'|}\right)^2\right]$$
$$\times\exp\left[i\left(\frac{k_{r+}''}{k_{r+}'}+\frac{k_{l-}''}{|k_{l-}'|}\right)\right]$$

and analogously for q_{l-}, with $\varphi\ll1$, $1-|q|\ll1$. It follows therefore that

$$(k_{r+}'-k_{l-}')d\approx2\pi p,\quad p=1,2,3,\ldots,$$

i.e., the presence of radiation has very little effect on the spectrum of the wave vectors. The condition that the field of the mode $r+,l-$ be of the same sign takes then the form

$$|q_{r+}||q_{l-}|\exp[(k_{l-}''-k_{r+}'')d]=1.$$

This condition is very close to the global instability condition.[2] This means that the "coordinate enhancement" $\exp|k''_+|d$ of the forward wave be almost completely cancelled by the "coordinate attenuation" $\exp(-|k''_-|d)$ of the backward wave, so that on the average the coordinate amplification is practically nonexistent. In the presence of instability, however, the self-excited field continuous to grow with time like $\exp\omega''t$ until a stationary value is established (see §4).

Following to Akhiezer and Polovin,[4] we obtain the instability condition $\omega''>0$ by substituting in (4) the value of k'' obtained from (3). For helicoidal waves, in the case of electron and hole bands, the instability condition for the $r+,l-$ mode is of the form

$$\omega''=\frac{2\pi|\eta\eta_1|\nabla T^2}{|\sigma_1|}\left(1+\frac{\omega_0\eta_1}{2\pi|\eta\sigma_1|}\right)-\frac{\pi p\omega_0|\eta|\nabla T}{|\sigma_1|B}>0,$$

whence

$$\nabla T>\nabla T_{cr}=\frac{p\omega_0}{2|\eta_1|B}\left(1+\frac{\omega_0|\eta_1|}{2\pi|\eta\sigma_1|}\right)^{-1}$$

or

$$B_{min}<B<B_{cr}\approx2|\eta_1|B^2\nabla T/p\omega_0,$$

where $\omega_0=c/d, B_{\min}=mcv/e$. In this case

$$\omega'<\omega'_{max}=\frac{2\pi|\eta\eta_1|B\nabla T^2}{\sigma}\left(1+\frac{\omega_0|\eta_1|}{2\pi|\eta\sigma_1|}\right),$$

$$k'=\pi p/d<k'_{max}=\sigma\omega'_{max}/c|\eta|\nabla T.$$

In the case of two bands of the same type, the coefficient η_1 decreases in the ratio ν^2/Ω^2 and the instability takes place at

$$\nabla T>\nabla T_{cr}=\frac{p\omega_0}{2B}\left[\frac{1}{2\pi|\sigma_1\eta|}\left(\frac{\pi\sigma}{|\eta_1|}+\frac{\omega_0^2}{8\pi|\sigma_1\eta|}\right)\right]^{1/2}-\frac{p\omega_0^2}{8\pi|\sigma_1\eta|B}$$

or

$$B_{min}<B<B_{cr}=\frac{2}{p}B^2\nabla T\left(\frac{2|\eta\eta_1\sigma_1|}{\omega_0^2\sigma}\right)^{1/2}.$$

Then

$$\omega'<\omega'_{max}=\left\{\frac{\omega_0|\eta_1|B\nabla T}{\sigma(4\pi|\sigma_1|B)^{1/2}}+\left[\frac{|\eta_1|B\nabla T^2}{\sigma}\left(2\pi|\eta|+\frac{\omega_0^2|\eta_1|}{4\pi\sigma|\sigma_1|}\right)\right]^{1/2}\right\}^2,$$

$$k'=\pi p/d<k'_{max}=c^{-1}(4\pi|\sigma_1|B\omega'_{max})^{1/2}.$$

(We have taken it into account here that σ, σ_1, η, and η_1 are proportional to $1/B^2$.) These inequalities can take place in metals as well as in semimetals.

For Alfvén waves, in contrast to helicoidal, only the $r+,l-$ mode is stable at

$$\omega''=\frac{|\eta_1|B\nabla T}{p\omega_0(\pi\Gamma)^{1/2}}\left(2\pi|\eta|\nabla T+\frac{\omega_0}{(\pi\Gamma)^{1/2}}\right)-\frac{\sigma_v}{2\Gamma}>0,$$

$$\sigma=\sum_{a,b}\frac{nmc^2}{B^2}(\nu-i\omega)=\sigma_v-i\omega\Gamma,$$

$$\sigma_v=\nu_a\Gamma_a+\nu_b\Gamma_b,\quad\Gamma=\Gamma_a+\Gamma_b.$$

If one of the following inequalities is satisfied

$$\nabla T>\nabla T_{cr}\approx\frac{p}{4}\left(\frac{\omega_0\sigma_v}{\eta\eta_1B(\pi\Gamma)^{1/2}}\right)^{1/2}$$

or

$$B_{min}<B<B_{cr}\approx\left(\frac{4|\eta\eta_1|B^3\nabla T^2}{p\omega_0\sigma_v}\right)^{1/2}$$

is satisfied, then

$$\omega'<\omega'_{max}=\frac{|\eta_1|B\nabla T}{\sigma_v}\left(2\pi|\eta|\nabla T+\frac{\omega_0}{(\pi\Gamma)^{1/2}}\right),$$

$$k'=\pi p/d<k'_{max}=(4\pi\Gamma)^{1/2}\omega'_{max}/c.$$

Finally at $\Omega^2 \ll \nu^2$, as already mentioned, there is no feedback in a bounded medium. Since, however $|k''_{l-}| < |k''_{r+}|$, the backward wave is less attenuated than the forward one, so that the medium can serve as an amplifier for waves propagating in the ∇T direction at $\mathscr{P}\sigma_1 B_z > 0$ and in the $-\nabla T$ direction at $\mathscr{P}\sigma_1 B_z < 0$.

We consider now radiation from the crystal into vacuum. The attenuation of the field of the $r+$ wave upon reflection from the right-hand boundary is

$$s=(ck_{r+}+ck_{l-}-2\omega)/(ck_{l-}-\omega).$$

After a time $\Delta t = 2d/u_{gr}$ (where u_{gr} is the group velocity of the wave packet), during which the packet propagates from one boundary to the other and back, the relative field attenuation is $2s$. Since $|k''| \ll |k'|$, $\omega'' \ll \omega'$ and, for the considered waves, $c|k'| \gg \omega'$, we have

$$s\approx(2\omega'+c|k_{l-}'|-ck_{r+}')/c|k_{l-}'|.$$

It is easily seen from (3) that $k'_{r+}+k'_{l-} \ll |k'_{l-}|$. From the condition for the "quantization of the wave vectors k' it follows therefore that $k'_{r+}+k'_{l-} \approx \varphi/d$, and therefore

$$s\approx\frac{2\omega'+\varphi\omega_0}{c|k_{l-}'|}\ll 1.$$

It is convenient to introduce the decrement ω_d, which is equal to the mean value of the relative field attenuation per unit time:

$$\frac{1}{E'}\frac{\Delta E'}{\Delta t}=-\omega_d=-s\frac{u_{gr}}{d}.$$

In this case $\omega_d \ll u_{gr}/d$ and in real situations for Alfvén waves, and even more strongly for helicoidal waves, the inequality $\omega_d \ll \omega''$ is satisfied. The radiation therefore does not stop the growth of the field, which continues with an effective growth rate $\gamma = \omega'' - \omega_d \approx \omega''$.

The electric field of the wave that goes out to the vacuum is

$$E_e=E'(ck_{l-}-ck_{r+})/(ck_{l-}-\omega)\approx E'(|k_{l-}'|+|k_{r+}'|)/|k_{l-}'|\approx 2E',$$

so that the density of the radiated energy is

$$S_e=c\bar{E}_e^2/4\pi=4\omega'S/ck', \tag{5}$$

where S is the flux density incident on the boundary from the inside.

§4. NONLINEAR KINETIC THEORY

The electromagnetic field of the self-exciting wave, which we shall assume to be monochromatic, produces an electric current that is determined from the solution of the nonlinear kinetic equation

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+e\left(\mathbf{E}+\frac{1}{c}[\mathbf{vB}]\right)\frac{\partial}{\partial \mathbf{p}}\right\}f(\mathbf{p},\mathbf{r},t)+\hat{I}(f)=0, \tag{6}$$

where $E = E_0 + E'$ (E_0 is the thermoelectric field), $B = B_0 + B'$ (we shall leave out hereafter the subscript of B), and $\hat{I}(f)$ is the collision integral. Since it is unwieldy and there are no detailed data on the kinetic coefficients in the two-band model of §2, we assume the following approximations:

a) a quadratic isotropic carrier spectrum;

b) the average-collision-frequency approximation:

$$\hat{I}(f)=\nu(\varepsilon)(f-f_0(\varepsilon)),$$

where $f_0(\varepsilon)$ is the equilibrium distribution function—in this case we assume $\nu = \nu(\varepsilon_F)$ to be the same in all the approximations;

c) we shall not label the distribution function with the band numbers a and b. On the basis of the arguments in §2, $\mathbf{v}\partial f/\partial \mathbf{r}$ can be neglected.

We shall find it convenient to express the electric field $\mathbf{E}'$ of the wave in terms of the magnetic field $\mathbf{B}'$. Since the wave is transverse, it follows that

$$\mathbf{E}'=\omega[\mathbf{B}'\mathbf{k}_1]/ck,$$

where $\mathbf{k}_1 = \mathbf{k}/k$. The waves are circularly polarized, and for a right-hand wave propagating in the $k_z > 0$ direction we put

$$B_x'=B'\cos(kz-\omega t),\quad B_y'=B'\sin(kz-\omega t),$$

so that $B'^2 = B_x'^2 + B_y'^2 = \text{const}$. We are interested in the condition under which a stationary amplitude sets in and we therefore assume $\omega = \omega'$; next, on the basis of the arguments in §3, we can put $k = k'$.

The wave equation is of the form

$$\left(\varepsilon_0\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial z^2}\right)\mathbf{B}'=4\pi c\,\mathrm{rot}\,\mathbf{j}',\quad \mathbf{j}'=\mathbf{j}_1+\mathbf{j}_2+\mathbf{j}_3.$$

The current $\mathbf{j}_1$ is linear, $\mathbf{j}_2$ is quadratic, and $\mathbf{j}_3$ is cubic in the alternating electric and magnetic fields.

To find the stationary amplitude of the self-exciting wave it suffices to solve (6) up to third-order approximation. The current $\mathbf{j}_2$ is inessential here, since it does not depend on the time, and we need only the currents $\mathbf{j}_1$ and $\mathbf{j}_2$. If $\nabla T = \nabla T_{cr} + \delta\nabla T$, where $\delta\nabla T \ll \nabla T_{cr}$, Maxwell's equations can be written in the form

$$\begin{aligned}&\left(\varepsilon_0\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial z^2}\right)\mathbf{B}'-4\pi c\,\mathrm{rot}\,\mathbf{j}_1(\nabla T_{cr})\\&=4\pi c[\mathrm{rot}\,\mathbf{j}_1(\delta\nabla T)+\mathrm{rot}\,\mathbf{j}_3(\nabla T_{cr})].\end{aligned} \tag{7}$$

The distribution function can be represented in the form of the series

$$f(\mathbf{v},z,t)=f_0(\varepsilon)+f_T(\mathbf{v})+\sum_{r=1}^{3}f_r(\mathbf{v},z,t). \tag{8}$$

Here $f_T(v)$ is the time-independent deviation from an equilibrium distribution in the presence of ∇T. When account is taken of the relation $eE_0 = \alpha\nabla T$ we have

$$f_T(\mathbf{v})=\frac{\chi(\varepsilon)-\alpha}{\nu(\varepsilon)}\frac{\partial f_0}{\partial\varepsilon}(\mathbf{v}\nabla T)=\mathbf{v}\mathbf{g}_T(\varepsilon). \tag{9}$$

Next, f_1, f_2, and f_3 are functions linear, quadratic, and cubic in the alternating fields. The system of equations for them is

$$\frac{\partial f_1}{\partial t}+[\mathbf{v}\Omega]\frac{\partial f_1}{\partial \mathbf{v}}+\nu f_1\equiv\hat{D}f_1=-\frac{e}{m}\mathbf{E}'\frac{\partial f_0}{\partial \mathbf{v}}-[\mathbf{v}\Omega']\frac{\partial f_T}{\partial \mathbf{v}}, \tag{10a}$$

$$\hat{D}f_2=-\frac{e}{m}\mathbf{E}'\frac{\partial f_1}{\partial \mathbf{v}}-[\mathbf{v}\Omega']\frac{\partial f_1}{\partial \mathbf{v}}, \tag{10b}$$

$$\hat{D}f_3=-\frac{e}{m}\mathbf{E}'\frac{\partial f_2}{\partial \mathbf{v}}-[\mathbf{v}\Omega']\frac{\partial f_2}{\partial \mathbf{v}}. \tag{10c}$$

To solve them it is convenient to set

$$f_r(\mathbf{v}, z, t)=f_{r1}(\mathbf{v})\cos r\psi+f_{r2}(\mathbf{v})\sin r\psi,$$
$$\psi=kz-\omega t,\quad r=1, 2, 3.$$

We have

$$f_1=\mathbf{v}\mathbf{g}(\varepsilon,\ \psi)=\mathbf{v}\mathbf{g}_1(\varepsilon)\cos\psi+\mathbf{v}\mathbf{h}_1(\varepsilon)\sin\psi, \tag{11}$$
$$f_2=\Phi^{(2)}(\varepsilon)+\mathbf{v}\mathbf{g}_2(\varepsilon)+v_l v_m h_{lm}^{(2)}(\varepsilon, 2\psi), \tag{12}$$
$$f_3=\Phi^{(3)}(\varepsilon, \psi, 3\psi)+\mathbf{v}\mathbf{g}_3(\varepsilon, \psi, 3\psi)$$
$$+v_l v_m h_{lm}^{(3)}(\varepsilon, \psi, 3\psi)+v_l v_m v_n h_{lmn}(\varepsilon, \psi, 3\psi), \tag{13}$$

where $\Phi^{(2)}$ and $\Phi^{(3)}$ are scalars, $\mathbf{g}_r$ are vectors, and $h_{lm}^{2,3}$ and h_{lmn} are tensors of second and third rank. The arguments 2ψ and 3ψ are due to the fact that

$$\cos^2\psi={}^1/_2(1+\cos 2\psi),\quad \sin\psi\sin 2\psi={}^1/_2(\cos\psi-\cos 3\psi),$$

and analogously for the other trigonometric functions.

The solution of (10a) is compactly expressed in the complex form:

$$g_{1x}+ih_{1y}=A_z(\varepsilon)(\Omega_z\Omega_+'+i\omega\Omega_-')/\Omega^2,$$
$$g_{1y}-ih_{1x}=vA_z(\varepsilon)(\Omega^2\Omega_+'+2i\omega\Omega_z\Omega_-')/\Omega^4,$$
$$\Omega_\pm'=\Omega_x'\pm i\Omega_y',\quad \mathbf{A}(\varepsilon)=\frac{\partial f_0}{\partial\varepsilon}\left(\frac{m\omega}{k}\mathbf{k}_1+\frac{\chi(\varepsilon)-\alpha}{v}\nabla T\right).$$

The components of the current $\mathbf{j}_1(\delta\nabla T)$ are[1]:

$$j_{1x}=\delta\nabla T(\eta_1 B_z B_x'+B_y'\operatorname{Im}\eta^*)\cos\psi,$$
$$j_{1y}=\delta\nabla T(\eta_1 B_z B_y'+B_x'\operatorname{Im}\eta^*)\sin\psi.$$

For the parameters of the function f_2, which enter in (10c), we have

$$\Phi^{(2)}(\varepsilon)=\frac{\omega}{k}\frac{\Omega_1^4}{\Omega^4}A_z(\varepsilon),\quad \mathbf{g}_2(\varepsilon)=-\frac{\Omega_1^4}{\Omega^4}\mathbf{A}(\varepsilon),$$
$$\operatorname{Sp}h^{(2)}=\frac{m\omega}{k}\frac{\Omega_1^4}{\Omega^4}\frac{\partial A_z(\varepsilon)}{\partial\varepsilon},$$
$$h_{xy}^{(2)}+h_{yx}^{(2)}=\frac{\omega+\Omega_z}{4\Omega^6}\frac{m\omega}{k}(3v\Omega_1^4\cos 2\psi+2\Omega^2\Omega_2^3\sin 2\psi)\frac{\partial A_z(\varepsilon)}{\partial\varepsilon},$$
$$h_{xx}^{(2)}-h_{yy}^{(2)}=\frac{1}{2k}\frac{\partial}{\partial z}(h_{xy}^{(2)}+h_{yx}^{(2)}),$$

where

$$\Omega_1^4=\Omega^2\Omega_x'^2+2\omega\Omega_z\Omega_x'\Omega_y',\quad \Omega_2^3=\Omega_z\Omega_x'^2+\omega\Omega_x'\Omega_y'.$$

The tensor components of $h_{lm}^{(2)}$ and h_{lmn} with z labels are equal to zero. The current j_{3x} receives contributions from the vector $\mathbf{g}_3$ and from the components of the tensors $h_{xxx}, h_{xyy}+h_{yxy}+h_{yyx}$; j_{3y} is obtained by substituting $x\to y$. The current $\mathbf{j}_3=\mathbf{j}_3(\omega)+\mathbf{j}_3(3\omega)$. A simple expression based on (10c) and (13) yields

$$j_{3x}(\omega)=-\frac{B'^2}{B^2}\sum_{a,b}\left(\frac{\omega+\Omega_z}{\Omega_z}\right)^2\left[\frac{10}{3}\left(\frac{\omega}{kv_F}\right)^2-1\right]\left[\left(\frac{\omega}{ck_z}\sigma_1-\eta_1\nabla T_{cr}\right)B_zB_x'\right.$$
$$\left.+\left(\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right)B_y'\right]\cos\psi,$$
$$j_{3y}(\omega)=-\frac{B'^2}{B^2}\sum_{a,b}\left(\frac{\omega+\Omega_z}{\Omega_z}\right)^2\left[\frac{10}{3}\left(\frac{\omega}{kv_F}\right)^2-1\right]\left[\left(\frac{\omega}{ck_z}\sigma_1-\eta_1\nabla T_{cr}\right)B_zB_y'\right.$$
$$\left.+\left(\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right)B_x'\right]\sin\psi. \tag{14}$$

According to Bogolyubov and Mitropol'skiĭ,[5] a stationary amplitude sets in when the "resonant" perturbation having the wave frequency and equal to $\mathbf{j}_1(\omega, \delta\nabla T)+\mathbf{j}_3(\omega, \nabla T_{cr})$ in (7) vanishes. This condition leads to two equations that determine the stationary amplitude B':

$$\delta\nabla T(\eta_1 B_zB_x'+B_y'\operatorname{Im}\eta^*)=\frac{B'^2}{B^2}\sum_{a,b}\left(\frac{\omega+\Omega_z}{\Omega_z}\right)^2\left[\frac{10}{3}\left(\frac{\omega}{kv_F}\right)^2-1\right]$$
$$\times\left[\left(\frac{\omega}{ck_z}\sigma_1-\eta_1\nabla T_{cr}\right)B_zB_x'+\left(\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right)B_y'\right],$$
$$\delta\nabla T(\eta_1 B_zB_y'+B_x'\operatorname{Im}\eta^*)=\frac{B'^2}{B^2}\sum_{a,b}\left(\frac{\omega+\Omega_z}{\Omega_z}\right)^2\left[\frac{10}{3}\left(\frac{\omega}{kv_F}\right)^2-1\right]$$
$$\times\left[\left(\frac{\omega}{ck_z}\sigma_1-\eta_1\nabla T_{cr}\right)B_zB_y'+\left(\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right)B_x'\right].$$

Their solution is

$$\frac{B'^2}{B^2}=\delta\nabla T(|\eta_1|B+|\operatorname{Im}\eta^*|)\left\{\sum_{a,b}\left(\frac{\omega+\Omega_z}{\Omega_z}\right)^2\left|\left(\frac{10}{3}\left(\frac{\omega}{kv_F}\right)^2-1\right)\right|\right.$$
$$\left.\times\left[\left|\frac{\omega\sigma_1}{ck_z}-\eta_1\nabla T_{cr}\right|B+\left|\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right|\right]\right\}^{-1}. \tag{15}$$

It can be shown that the presence of forward and backward waves in the nonlinear approximation makes no noticeable contribution to the resonant perturbation. For helicoidal waves we can neglect in (15) the ratio ω/Ω, and for Alfvén waves $\sigma_1=0$.

§ 5. ESTIMATE OF THE INTENSITY OF THE SPONTANEOUS RADIO EMISSION

For Alfvén waves, according to (15), the stationary amplitude of the electric field of the wave is

$$E'=\frac{\omega}{ck}B[\delta\nabla T(|\eta_1|B+|\operatorname{Im}\eta^*|)]^{1/2}\left\{\frac{10}{3}\sum_{a,b}\left(\frac{\omega+\Omega_z}{\Omega_z}\frac{\omega}{kv_F}\right)^2\right.$$
$$\left.\times\left[\left|\frac{\omega}{ck_z}\sigma_1-\eta_1\nabla T_{cr}\right|B+\left|\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right|\right]\right\}^{-1/2}.$$

Therefore, by virtue of (5), the flux density of the radiated energy is

$$S_e=\frac{c}{\pi}\left(\frac{\omega}{ck}\right)^2B^2\delta\nabla T(|\eta_1|B+|\operatorname{Im}\eta^*|)\left\{\frac{10}{3}\sum_{a,b}\left(\frac{\omega+\Omega_z}{\Omega_z}\frac{\omega}{kv_F}\right)^2\right.$$
$$\left.\times\left[\left|\frac{\omega}{ck_z}\sigma_1-\eta_1\nabla T_{cr}\right|B+\left|\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right|\right]\right\}^{-1}. \tag{16}$$

For a bismuth plate of thickness $d=0.5$ cm, placed in a magnetic field $B=10^3$ Oe at a temperature $T\approx 5$ K, assuming in accord with Kopylov[6] that $\chi\approx 10-10^2$, we obtain the estimates $\eta\approx 3\cdot 10^{23}$, $\sigma_v\approx 10^{12}$, $\eta_1\approx 10^{24}$, $\nabla T_{cr}=5\cdot 10^{-16}$ erg/cm (1.75°K/cm). Putting $\delta\nabla T=0.1\nabla T_{cr}$, we obtain from (16) a flux $S_e\approx 1-10$ W/cm². This value does not contradict the second law of thermodynamics, since the heat flux under these conditions exceeds substantially the radiated energy.[7]

For helicoidal waves,

$$S_e=\frac{c}{\pi}\left(\frac{\omega}{ck}\right)^2B^2\delta\nabla T(|\eta_1|B+|\operatorname{Im}\eta^*|)$$
$$\times\left[\sum_{a,b}\left(\left|\frac{\omega}{ck_z}\sigma_1-\eta_1\nabla T_{cr}\right|B+\left|\frac{\omega}{ck_z}\operatorname{Im}\sigma^*-\nabla T_{cr}\operatorname{Im}\eta^*\right|\right)\right]^{-1}.$$

For metals such as Al and Cu, at $B=5\times 10^4$ Oe, $\chi\approx 1$, and $\delta\nabla T=0.1\nabla T_{cr}=7\cdot 10^{-17}$ erg/cm (0.25°K/cm) we have $S_e\approx 5\cdot 10^{-3}$ W/cm², and in the case of doped Bi ($n_+\neq n_-$) at $B=10^3-10^4$ Oe, under the same temperature condition, a value $S_e\approx 1$ W/cm² can be obtained.

The radiation frequencies are determined from the equations of §3. For Alfvén waves these frequencies are of the order of 10^9-10^{10} sec⁻¹. For helicoidal waves $\omega\sim 10^2-10^3$ sec^{-1} in metals and $\omega\sim 10^5-10^6$ sec^{-1} in bismuth. Since self-excitation of either type of wave takes place, the actual radiation is determined by the highest field intensity.

The radiation intensity depends on the magnetic field intensity B and by the supercriticality of the temperature gradient $\delta\nabla T$. Besides the intensity, these quantities determine the "quantized" levels of the wave vector k, i.e., the quantum number p, and with them also the radiation frequency ω. We have therefore confined ourselves to order-of-magnitude estimates.

In conclusion, we thank V.I. Perel' for a discussion of general instability theory, which was of help to us.

[1)] In these and succeeding expressions for the currents we neglect terms of order $\nu/\Omega \ll 1$.

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