Critical scattering of polarized neutrons in ferromagnets subjected to a magnetic field above T_c

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An analysis is made of the physical information which can be obtained from the scattering of polarized neutrons in ferromagnets subjected to a magnetic field at temperatures above T_c. It is shown that an investigation of the polarization-dependent component of the scattering cross section makes it possible to: 1) investigate triple dynamic spin correlations in the paramagnetic phase; 2) verify predictions of the Polyakov-Kadanoff operator algebra and generalize it to the dynamic case; 3) select between two variants of the critical dynamics in the dipole range of temperatures. Moreover, an analysis is made of the question of verifying by nonpolarized neutron scattering the operator algebra predictions for quadruple correlations in the static case.

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1. INTRODUCTION

Okorokov *et al.*[1] investigated small-angle critical scattering of polarized neutrons in iron subjected to a magnetic field at temperatures above the Curie point. They found that the scattering cross section depends on the orientation of the initial polarization $\mathbf{P}_0$ relative to the magnetic field $\mathbf{H}$ and, when the momentum of the incident neutrons is not perpendicular to the field, it also depends on the sign of the projection of the transferred momentum perpendicular to the incident beam.

Our main purpose is to identify the physical information which can be obtained from experiments of this type. First of all, we shall show that such experiments can be used to study experimentally triple dynamic spin correlations in the paramagnetic phase, discussed earlier[2] and discovered by Okorokov *et al.*[3] Next, we shall demonstrate that by analysis of the angular and temperature dependences of the component of the cross section proportional to P_0 we can both check the predictions of the Polyakov principle of merging of correlations[4] (or, in other words, of the Polyakov–Kadanoff operator algebra[5]) generalized to triple dynamic vertices and select between two variants of the critical dynamics in the dipole temperature range. Finally, we shall show how scattering of neutrons in ferromagnetics subjected to a magnetic field can provide information on the behavior of quadruple vertices and check the validity of the correlation merging principle for these vertices.

We shall consider the critical scattering in cubic Heisenberg ferromagnets allowing for the exchange dipole interactions of atomic spins and we shall neglect completely the crystal anisotropy effects. The whole analysis of the behavior of the critical fluctuations in a magnetic field will be based on the hypothesis of dynamic similarity and will be of phenomenological nature.

In the second section we shall obtain general formulas for the scattering of polarized neutrons in a magnetic material subjected to a magnetic field and we shall consider the structure of these formulas. In the third section we shall analyze, in limiting cases, the investigated quantities and compare the theory with experiment. In the fourth (final) section we shall discuss the question how nonpolarized neutrons can be used to verify the principle of merging of correlations for four quadruple vertices in the static case.

2. MAGNETIC SCATTERING CROSS SECTION OF POLARIZED NEUTRONS

We can easily show that this cross section, representing magnetic scattering (by an angle θ) of polarized neutrons in a magnetic material subjected to a magnetic field, is related to the retarded spin Green function G as follows:

$$\frac{d\sigma}{d\Omega}=(r_0\gamma)^2\frac{T}{\pi}\int\frac{d\omega}{\omega}\frac{k'}{k}\operatorname{Im}\{G_{\alpha\beta}{}^{\perp}(\mathbf{q},\omega,\mathbf{H})(\delta_{\alpha\beta}+i\varepsilon_{\alpha\beta\gamma}P_{0\gamma})\},\tag{1}$$

where $\omega=E'-E$ is the transferred energy; $\mathbf{q}=\mathbf{k}'-\mathbf{k}$ is the transferred momentum; E and $\mathbf{k}$ is the energy and momentum of the incident neutrons; E' and $\mathbf{k}'$ are the corresponding properties of the scattered neutrons; r_0 is the classical electron radius; γ is the gyromagnetic ratio of a neutron; $G_{\alpha\beta}^{\perp}$ is the part of the retarded spin Green function $G_{\alpha\beta}$ perpendicular to the transferred momentum $\mathbf{q}$

$$G_{\alpha\beta}{}^{\perp}(\mathbf{q},\omega,\mathbf{H})=G_{\mu\nu}(\mathbf{q},\omega,\mathbf{H})(\delta_{\alpha\mu}-e_\alpha e_\mu)(\delta_{\beta\nu}-e_\beta e_\nu),\quad \mathbf{e}=\mathbf{q}q^{-1}.\tag{2}$$

Equation (1) is derived bearing in mind that, as frequently found in experimental situations, a typical energy transferred by scattering is low compared with temperature; in a more general case the factor T/ω should be replaced with $-N(-\omega)$, where N is the Planck function.

We shall first consider the case of low fields such that in the expansion $G_{\alpha\beta}(\mathbf{H})$ in powers of H we need consider only the linear approximation (the relevant criterion will be given below). Moreover, we shall assume that initially only the exchange interaction acts between spins in the investigated material. It is well known (see, for example, Ref. 6) that this exchange approximation can be used to describe critical fluctuations if the magnetic susceptibility of the material is low, i.e., when $4\pi\chi\ll 1$. Then, expanding $G_{\alpha\beta}(\mathbf{H})$ in powers of H, we find that

$$\begin{gathered}G_{\alpha\beta}(\mathbf{q},\omega,\mathbf{H})=G_{\alpha\beta}^{(1)}(\mathbf{q},\omega)+g\mu H_\gamma G_{\alpha\beta\gamma}^{(3)}(\mathbf{q},\omega;0,0;-\mathbf{q},-\omega),\\ G_{\alpha\beta}^{(1)}(\mathbf{q},\omega)=G_{\alpha\beta}(\mathbf{q},\omega,0)=G^{(1)}(\mathbf{q},\omega)\delta_{\alpha\beta},\quad G_{\alpha\beta\gamma}^{(3)}=-i\varepsilon_{\alpha\beta\gamma}G^{(3)},\end{gathered}\tag{3}$$

where $G^{(3)}$ is the triple dynamic spin Green function

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which vanishes at $\omega=0$. Its properties are discussed in detail in Refs. 2 and 8. Substituting this expansion in Eq. (1) and using Eq. (2), we obtain

$$\frac{d\sigma}{d\Omega}=2(r_0\gamma)^2\frac{T}{\pi}\int\frac{d\omega}{\omega}\frac{k'}{k}\{\mathrm{Im}\,G^{(1)}(q,\omega)+g\mu(\mathbf{He})(\mathbf{eP}_0)\,\mathrm{Im}\,G^{(3)}(q,\omega)\}. \quad (4)$$

As expected, if $H=0$, the scattering cross section is independent of P_0. The term in Eq. (4) proportional to P_0 can be described by an integral of the triple Green function with respect to ω. Thus, selection in the scattering cross section of a term linear in H and proportional to P_0 makes it possible to investigate triple dynamic spin correlations.

We described earlier[2] a method for investigating triple correlations in zero field. The basis of the method is as follows. The contribution to the scattering cross section calculated in the third order in respect of the magnetic interaction of a neutron with a medium is expressed in terms of an integral of a triple dynamic spin correlation function. In general, the relative magnitude of this contribution is governed by a small parameter $r_0/a \sim 10^{-5}$, where a is of the order of the interatomic distance, and only near T_c there is an increase in this contribution by approximately an order of magnitude. However, this part of the cross section includes components proportional to $\mathbf{k}\times\mathbf{k}'\cdot\mathbf{P}_0$ so that it can be investigated independently. The results of relevant experiments[3] are in qualitative agreement with the theoretical estimates,[2] but these experiments are difficult to perform because of the smallness of the effect.

On the other hand, the term in Eq. (3) proportional to H and P_0 may represent a few percent of the isotropic part of the cross section. Therefore, it is more realistic to investigate triple correlations using magnetic fields and modern apparatus. Nevertheless, we must bear in mind that information on triple correlations found by this method differs slightly from that obtained from measurements of the scattering asymmetry in zero field, because the two effects are expressed in different ways in terms of $G^{(3)}$.

We shall now analyze in greater detail the second term in Eq. (4). As a rule, small-angle critical scattering of neutrons is quasielastic and, therefore, the dependence of k' and ω is usually neglected. However, the formulas for the expansion of $G^{(3)}$ in terms of intermediate states readily show that $\mathrm{Im}G^{(3)}$ is an even function of ω, whereas $\mathrm{Re}G^{(3)}$ is an odd function. Therefore, in this quasielastic approximation the contribution to the cross section proportional to P_0 vanishes. This contribution is finite only if we continue expansion of $k'(\omega)$ right up to terms of the order of ω^3. As a result, we find that the

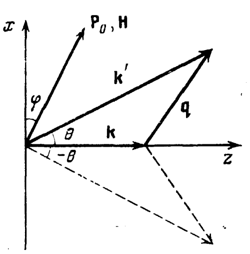

FIG. 1.

part of the cross section under investigation is (as shown in the next section) extremely sensitive to the spin dynamics of the scatterer. Obviously, the contribution to the scattering proportional to P_0 can be separated by reversing the direction of polarization. Let us assume that $\mathbf{P}_0$, $\mathbf{H}$, and $\mathbf{q}$ are in the same plane and $\mathbf{P}_0$ is directed parallel or antiparallel to $\mathbf{H}$. Then, in the coordinate system shown in Fig. 1 the angular factor in the second term of Eq. (1) has the form:

$$(\mathbf{eh})(\mathbf{eP}_0)=e_x^2h_xP_{0x}+e_z^2h_zP_{0z}+e_xe_z(h_xP_{0z}+h_zP_{0x}),\qquad \mathbf{h}=\mathbf{H}H^{-1}. \quad (5)$$

In this expression the signs of the last two terms are reversed when θ is replaced with $-\theta$. Therefore, if the angle between the magnetic field and the incident neutron momentum $\mathbf{k}$ differs from the right angle ($\varphi\neq0$), the scattering cross section depends on the sign of θ; in other words, in this case we have an asymmetry in the scattering plane. Therefore, in analyzing the polarization experiments it is convenient to separate the parts of the cross section which are symmetric and antisymmetric with respect to θ by introducing the quantities $P_S(\theta)$ and $P_A(\theta)$ both proportional to $\mathbf{P}_0$:

$$\begin{gathered}
P_S(\theta)=\frac{I(\theta,P_0)-I(\theta,-P_0)+I(-\theta,P_0)-I(-\theta,-P_0)}{I(\theta,P_0)+I(\theta,-P_0)+I(-\theta,P_0)+I(-\theta,-P_0)},\\
P_A(\theta)=\frac{I(\theta,P_0)-I(\theta,-P_0)-I(-\theta,P_0)+I(-\theta,-P_0)}{I(\theta,P_0)+I(\theta,-P_0)+I(-\theta,P_0)+I(-\theta,-P_0)},\\
2(r_0\gamma)^2TI(\theta,P_0)=d\sigma(\theta,P_0)/d\Omega.
\end{gathered} \quad (6)$$

The denominator in these expressions is independent of P_0 and it is governed by the symmetric (with respect to θ) part of the cross section. Substituting in Eq. (6) the expression (4) and separating in the integrand of Eq. (4) the part even in respect of ω when $\omega\ll E$ and $\theta\ll1$, we obtain

$$\begin{gathered}
P_S(\theta,\varphi)=P_{S1}(\theta)\cos2\varphi+P_{S2}(\theta)\sin^2\varphi,\qquad P_A(\theta,\varphi)=P_{A1}(\theta)\sin2\varphi;\\
P_{S1}(\theta)=P_0\frac{\theta^2g\mu H}{I_0(\theta)}\frac{1}{\pi}\int\frac{d\omega}{2E}\left\{\frac{(\omega/E)^2+2\theta^2}{[(\omega/2E)^2+\theta^2]^2}+\frac{\theta^2-(\omega/2E)^2}{(\omega/2E)^2+\theta^2}k^2\frac{\partial}{\partial q^2}\right\}\mathrm{Im}\,G^{(3)}(q,\omega),\\
P_{S2}(\theta)=P_0\frac{g\mu H}{I_0(\theta)}\frac{1}{\pi}\int\frac{d\omega}{2E}\left\{1+[\theta^2-(\omega/2E)^2]k^2\frac{\partial}{\partial q^2}\right\}\mathrm{Im}\,G^{(3)}(q,\omega),\\
P_{A1}(\theta)=P_0\frac{g\mu H}{I_0(\theta)}\frac{1}{\pi}\int\frac{d\omega}{2E}\frac{\theta}{(\omega/2E)^2+\theta^2}\mathrm{Im}\,G^{(3)}(q,\omega),\\
I_0(\theta)=\frac{1}{2}[I(\theta,P_0)+I(-\theta,P_0)]|_{P_0=0}=\frac{1}{\pi}\int\frac{d\omega}{\omega}\frac{k'}{k}\mathrm{Im}\,G^{(1)}(q,\omega).
\end{gathered} \quad (7)$$

First of all, we note that the dependence of P_S on the angle φ allows us to separate experimentally $P_{S1}(\theta)$ and $P_{S2}(\theta)$, which is necessary for the fullest analysis of the experimental results. It follows from Eqs. (4)–(6) that in the expression for P_{A1} there is an odd factor $e_z\propto\omega/E$, in the integral, even in the main order, whereas in the formulas for $P_{S1,2}$ odd factors appear only after expansion of the integrands in powers of ω/E. It is then found that the order of magnitude is given by $P_{A1}(\theta)\sim\theta^{-1}P_{S1}(\theta)$. Therefore, with the exception of very small angles and for almost all values of φ, the symmetric contribution to the cross section is small compared with the asymmetric part.

When the approximation linear in H is inapplicable, the tensor $G_{\alpha\beta}$ can be represented as follows:

$$\begin{gathered}
G_{\alpha\beta}(q,\omega,\mathbf{H})=\tilde{G}^{(1)}(q,\omega,H)(\delta_{\alpha\beta}-h_\alpha h_\beta)+\tilde{G}^{(2)}(q,\omega,H)h_\alpha h_\beta\\
-ie_{\alpha\beta\gamma}h_\gamma\tilde{G}^{(3)}(q,\omega,H),
\end{gathered} \quad (8)$$

where $\mathbf{h}=\mathbf{H}H^{-1}$. In the above expression the first and third terms describe fluctuations perpendicular to the fields, whereas the second term describes fluctuations parallel to the field.

We can show that the functions $\tilde{G}^{(i)}$ are independent of the direction of $\mathbf{H}$. Substituting the system (7) into Eqs. (2) and (1), we obtain

$$\frac{d\sigma}{d\Omega}=2(r_0\gamma)^2TI(\theta,H,P_0)=(r_0\gamma)^2\frac{T}{\pi}\int\frac{d\omega}{\omega}\frac{k'}{k}\{[1+(\mathbf{eh})^2]\operatorname{Im}\tilde{G}^{(1)}(q,\omega,H)$$
$$+[1-(\mathbf{eh})^2]\operatorname{Im}\tilde{G}^{(2)}(q,\omega,H)+2(\mathbf{eh})(\mathbf{eP}_0)\operatorname{Im}\tilde{G}^{(3)}(q,\omega,H)\}. \quad (9)$$

If H is small, then $\tilde{G}^{(3)}=g\mu HG^{(3)}$, and $\tilde{G}^{(1)}$ and $\tilde{G}^{(2)}$ differ from $G^{(1)}$ by a quantity of the order of H^2. The terms proportional to $(\mathbf{e}\cdot\mathbf{h})^2$ give rise to an asymmetry with respect to the angle of θ also in the case of nonpolarized neutrons. In low fields H this asymmetry is proportional to H^2, which allows us in principle to study four-spin correlations. It is also clear that the expressions for P_S in a strong field are obtained from the system (7) if $g\mu HG^{(3)}$ is replaced with $\tilde{G}^{(3)}$.

The formulas (4) and (9) are valid if only the anisotropic interaction is allowed for. However, in the direct vicinity of T_c the anisotropic and long-range dipole interactions of atomic spins play an important role. It is well known (see, for example, Ref. 8) that when these interactions are allowed for, $G_{\alpha\beta}$ is given by the following expression:

$$G_{\alpha\beta}=G_{1\alpha\beta}-\omega_0\frac{G_{1\alpha\nu}e_\nu e_\mu G_{1\mu\beta}}{1+\omega_0 e_\mu e_\nu G_{1\mu\nu}}, \quad (10)$$

where $\omega_0=4\pi(g\mu)^2v_0^{-1}$ is the characteristic energy of the dipole interaction; v_0 is the volume of a unit cell; G_1 is a function related to the inhomogeneous magnetic susceptibility of the material by

$$\omega_0 G_{1\alpha\beta}(\mathbf{q},\omega)=4\pi\chi_{\alpha\beta}(\mathbf{q},\omega). \quad (11)$$

The function $G_{1\alpha\beta}$ has the same symmetry as the spin Green function $G_{\alpha\beta}$ in the absence of dipole forces, and it can be described by the expansion given by Eq. (8). Substituting this expansion into Eqs. (10) and (1), we find that simple calculations yield

$$\left.\begin{aligned}
&\frac{d\sigma}{d\Omega}=\frac{d\sigma_0}{d\Omega}+\frac{d\sigma_P}{d\Omega};\quad \frac{d\sigma_0}{d\Omega}=(r_0\gamma)^2\frac{T}{\pi}\int\frac{d\omega}{\omega}\frac{k'}{k}\operatorname{Im}\Big\{(1+(\mathbf{eh})^2)\tilde{G}_1^{(1)}\\
&+(1-(\mathbf{eh})^2)\left[\tilde{G}_1^{(2)}-\omega_0\frac{(\mathbf{eh})^2(\tilde{G}_1^{(1)}-\tilde{G}_1^{(2)})^2+(\tilde{G}_1^{(3)})^2}{1+\omega_0[(1-(\mathbf{eh})^2)\tilde{G}_1^{(1)}+(\mathbf{eh})^2\tilde{G}_1^{(2)}]}\right]\Big\}\\
&\frac{d\sigma_P}{d\Omega}=2(r_0\gamma)^2\frac{T}{\pi}\int\frac{d\omega}{\omega}\frac{k'}{k}(\mathbf{eh})(\mathbf{eP}_0)\operatorname{Im}\Big\{\tilde{G}_1^{(3)}\Big[1\\
&-\omega_0\frac{(1-(\mathbf{eh})^2)(\tilde{G}_1^{(1)}-\tilde{G}_1^{(2)})}{1+\omega_0[(1-(\mathbf{eh})^2)\tilde{G}_1^{(1)}+(\mathbf{eh})^2\tilde{G}_1^{(2)}]}\Big]\Big\}
\end{aligned}\right\} \quad (12)$$

where $d\sigma/d\Omega$ is the differential scattering cross section nonpolarized neutrons.

In weak fields we have $\tilde{G}_1^{(1)}-\tilde{G}_1^{(2)}\propto H^2$, and $\tilde{G}_1^{(3)}\propto H$, i.e., in the formulas of the system (12) the terms proportional to ω_0 are small. Therefore, inclusion of the dipole forces in the case of H results only in the replacement in Eq. (4) of the functions $G^{(1)}$ and $G^{(3)}$ with $G_1^{(1)}$ and $G_1^{(3)}$, respectively. We can easily write down also the expressions for $P_{S,A}$ in this case. However, we shall not need them later.

3. ESTIMATES OF THE POLARIZATION EFFECTS IN THE SCATTERING BY CRITICAL FLUCTUATIONS

We do not know the explicit form of the functions G occurring in Eqs. (4), (7), (9), and (12). However, we know their structure which follows from the theory of dynamic similarity, and also the explicit form (apart from numerical coefficients) in limiting cases. This makes it possible to obtain estimates for the quantities $P_{S,A}$ and to find their dependences on the parameters in those cases when one can use asymptotic formulas for the functions G.

We shall first consider the case of weak magnetic fields. It is well known (see, for example, Ref. 5) that in the static theory of similarity the criterion of low fields is the condition[1)]

$$g\mu H\ll T_c(\varkappa a)^{5/2}. \quad (13)$$

Here, H is the internal field in the sample of $\varkappa$ is the reciprocal radius of critical fluctuations. We recall that $\varkappa=a^{-1}\tau^\nu$, where $\tau=(T-T_c)T_c^{-1}$, a is a distance of the order of the interatomic spacing, and $\nu\approx\frac{2}{3}$. Clearly, this criterion applies also in the dynamic theory. The only exception is the hydrodynamic exchange region, which we shall consider later.

Obviously, the function $G^{(3)}$ occurring in Eqs. (4) and (7) can be written in the form[2)]

$$G^{(3)}(q,\omega)=(G^{(1)}(q,\omega))^2G^{(1)}(0,0)\Gamma_3(q,\omega), \quad (14)$$

where Γ_3 is a three-spin vertex. In accordance with the theory of similarity, the scaling size of Γ_3 is governed by the factor $\varkappa^{3/2}$ (see Refs. 2 and 8).

Polyakov[4] formulated the principle of merging of correlations in the static similarity theory (this principle is also known as the Polyakov–Kadanoff of operator algebra), according to which if one of the momenta q at any vertex is large compared with the others, the dependence on this momentum can be separated in the form of a factor which has the scaling size $q^{1/\nu-1}q^{1/2}$. If we assume that this principle applies also in the dynamic case for Γ_3 in the critical region, where $q\gg\varkappa$, we have

$$\Gamma_3(q,\omega)=T_c(qa)^{1/2}(\varkappa a)\gamma_3(\omega/\Omega(q)). \quad (15)$$

Here, $\Omega(q)$ is the characteristic energy of critical fluctuations with the momentum q. In the static limit, we have $\Gamma_3=0$ and, therefore, $\gamma_3(0)=0$. The static Green function $G(0,0)$ is proportional to $\varkappa^{-2}$. It therefore follows from Eqs. (15), (14), and (7) that throughout the critical range of angles $k\theta\gg\varkappa$ the temperature dependences of the quantities $P_{S,A}$ are governed by the factor $\varkappa^{-1}\propto\tau^{-2/3}$. This conclusion is based only on the principle of merging of correlations and it is valid both in the exchange range of temperatures as well as in the dipole range if the condition of low fields (13) is satisfied.

We shall now obtain more detailed estimates of the quantities $P_{A,S}$ in the critical region. We shall first consider the exchange range of temperatures, where $4\pi\chi\ll1$. In this range the dipole forces are weak and, as is well known (see, for example, Ref. 9), the energy of critical fluctuations is $\Omega_e(q)=T_c(qa)^{5/2}$. In estimating $P_{S,A}$ we have to know the asymptotic behavior of the functions $G^{(1)}$ and γ_3 at high and low values of ω. In accordance with the theory of dynamic similarity, we shall assume that if $\omega \ll \Omega_e$, we have

$$G^{(1)}(q,\omega)=\frac{z}{T_c(qa)^2}\left(1+ig_0\frac{\omega}{\Omega_e(q)}\right),\quad \gamma_3\left(\frac{\omega}{\Omega_e(q)}\right)=\gamma_0\frac{\omega}{\Omega_e(q)}, \tag{16}$$

where $z>0$, $g_0>0$, and γ_0 is a constant of the order of unity. In the opposite limiting case of $\omega \gg \Omega_e(q)$, we find that $G^{(1)}$ and γ_3 are described by

$$G^{(1)}(q,\omega)=\frac{(qa)^2}{T_c}\left(\frac{iT_c}{\omega}\right)^{4/5}\left[g_1+ig_2\frac{\Omega_e(q)}{\omega}\right], \tag{17a}$$

$$\gamma_3\left(\frac{\omega}{\Omega_e(q)}\right)=i\left(\frac{\omega}{i\Omega_e(q)}\right)^{1/5}\left[\gamma_1+i\gamma_2\frac{\Omega_e(q)}{\omega}\right], \tag{17b}$$

where $g_1>0$, g_2, γ_1, and γ_2 are constants of the order of unity. The first term in Eq. (17a) was obtained earlier by one of the present authors[10] and the first term in Eq. (17b) is a consequence of the usual requirement that in the main order when $\omega \gg \Omega_e(q)$ the value of Γ_3 should be independent of q. However, it then follows from Eq. (14) that $G^{(3)} \propto \omega^{-3}$ and, consequently, this quantity is odd in respect of ω and real. Therefore, in the brackets in Eqs. (17a) and (17b) we should include also the next higher corrections to $G^{(1)}(\omega/\Omega_e)$ and $\gamma_3(\omega/\Omega_e)$.

We shall now consider the kinematics of scattering. If $\omega \ll E$, then

$$q^2 \approx k^2[\theta^2+(\omega/2E)^2],$$

i.e., q^2 consists of two terms: elastic and inelastic. Then, the condition $\omega \sim \Omega_e(q)$ may be satisfied only for some values of $k\theta$. We shall first assume that $k\theta=0$. Then the equality $\omega=\Omega_e(q)$ applies both at $\omega=0$ and for characteristic inelastic values of ω_i and q_i given by

$$\omega_i=T_c\left(\frac{2E}{T_cka}\right)^{5/3}=T_c(q_ia)^{5/2},\quad q_i=\frac{1}{a}\left(\frac{2E}{T_cka}\right)^{2/3}. \tag{18}$$

If $k\theta \neq 0$ and $q_i \ll k$, it then follows that for $k\theta \ll q_i$ there are two ranges of the values of ω where $\omega \sim \Omega_e(q)$. In the first range the scattering is quasielastic, i.e., $\omega \ll 2E\theta$ and $\omega \sim \Omega_e(q)$. In the second range, the scattering is strongly inelastic, i.e., $q \approx k\omega/2E$ and the condition $\omega \sim \Omega_e(q)$ can be written in the form $\omega \sim \omega_i=\Omega_e(q)$.

In the other limiting case when $k\theta \gg q_i$ the transferred energy is always small compared with $\Omega_e(q)$, so that $G^{(1)}$ and Γ_3 are described by the formulas (16). Then, the main contribution to the integrals (7) is clearly due to the range $\omega \sim 2E\theta \ll \Omega_e(q)$ and $I_0(\theta)$ and $P_{S,A}(\theta)$ are described by

$$I_0(\theta) \sim \frac{1}{T_c(ka\theta)^2}\left(\frac{q_i}{k\theta}\right)^{3/2}, \tag{19}$$

$$P_{S1} \sim P_{S2} \sim \theta P_{A1} \sim -P_0\frac{g\mu H}{E}\left(\frac{q_i}{k\theta}\right)^2\frac{q_i}{\varkappa}. \tag{20}$$

We can see because of the strong inelasticity the scattering intensity $I_0(\theta)$ decreases more rapidly on increase in the angle than in the quasielastic approximation when we have $I_0(\theta) \propto \theta^{-2}$; this result was first obtained by one of the present authors.[11] The negative sign in the expression for $P_{S,A}$ is a consequence of matching to the hydrodynamic region, which we shall discuss below.

Next, if $\varkappa \ll k\theta \ll q_i$, the integrals with respect to ω in Eq. (7) split into three parts: 1) $0<\omega<\Omega_e(k\theta)$; 2) $\Omega_e(k\theta)<\omega<\omega_i$; 3) $\omega>\omega_i$. Then, in estimating first and third parts we have to use the formulas in Eq. (16), and in estimating the second part we have to use the system (17). It is thus found that the main contribution to the investigated quantities is made by the quasielastic range where $\omega \ll 2E\theta$. Therefore, the scattering intensity has the usual quasielastic Ornstein-Zernike form and

$$P_{S1} \sim P_{S2} \sim \theta P_{A1} \sim -P_0\frac{g\mu H}{E}\frac{k\theta}{\varkappa}. \tag{21}$$

Comparing Eqs. (20) and (21), we can see that the quantities $P_{S1,2}$ have an absolute maximum at $k\theta \sim q_i$.

We shall now analyze the hydrodynamic range $k\theta \ll \varkappa$. For this angular range we have to allow for a new physical phenomenon, which is the homogeneous precession of the magnetization. Therefore, even when the condition (13) is satisfied, the field is weak only if it is low compared with the precession damping. If it is not low, then the Green functions longitudinal and transverse to the field are described by the following simple formulas;

$$\left.\begin{aligned} &G_{+-}(q,\omega,H)=G_{-+}(q,\omega,-H)=\tilde{G}^{(1)}-\tilde{G}^{(3)}=G_0(\varkappa)\frac{-g\mu H+i\Gamma_q}{\omega-g\mu H+i\Gamma_q},\\ &G_{\parallel}(q,\omega,H)=\tilde{G}^{(2)}(q,\omega,H)=G_0(\varkappa)\frac{i\Gamma_q}{\omega+i\Gamma_q}\\ &\Gamma_q=Dq^2+\Gamma_0,\quad G_0(\varkappa)=z/T_c(\varkappa a)^2. \end{aligned}\right\} \tag{22}$$

Here, G_0 is the static Green function, $D \sim T_c(\varkappa a)^{1/2}$ is the spin diffusion coefficient, and Γ_0 is the homogeneous damping which is of the form[6,12]

$$\Gamma_0 \sim \frac{\omega_0^2}{T_c(\varkappa a)^{3/2}} \sim \Omega_e(\varkappa)(4\pi\chi)^2 \sim \Omega_e(\varkappa)\left(\frac{q_0}{\varkappa}\right)^4, \tag{23}$$

where $q_0=a^{-1}(\omega_0/T_c)^{1/2}$ is a characteristic dipole momentum (see below). If the condition (13) is satisfied, then G_0, $\varkappa$, D, and Γ_0 are all independent of H.

Using Eq. (22), we can readily determine the functions $\tilde{G}^{(i)}$ occurring in Eqs. (8) and (9) and use them to calculate both $I_0(\theta)$ and $P_{S,A}$:

$$\left.\begin{aligned} &I_0(\theta,\varphi)=G_0(\varkappa)\left\{1-\frac{E\theta}{2E\theta+\Gamma_0}\frac{(g\mu H)^2\cos 2\varphi}{(2E\theta+\Gamma_0)^2+(q\mu H)^2}\right\};\\ &P_{S,A}=\tilde{P}_{S,A}I_0^{-1}G_0,\quad \tilde{P}_{S2}=-P_0 g\mu H/2E,\\ &\tilde{P}_{S1}=-P_0\frac{g\mu H}{2E}(2E\theta)^2\frac{(2g\mu H)^2+3(2E\theta+\Gamma_0)^2+\Gamma_0^2-(2E\theta)^2}{[(2E\theta+\Gamma_0)^2+(g\mu H)^2]^2},\\ &\tilde{P}_{A1}=-P_0\frac{g\mu H\cdot 2E\theta}{(2E\theta+\Gamma_0)^2+(g\mu H)^2}. \end{aligned}\right\} \tag{24}$$

We can see that, firstly, the scattering intensity depends—in spite of the condition $k\theta \ll \varkappa$—on θ and also on φ. Both I_0 and $P_{S,A}$ depend only on Γ_0 and are independent of D. This is due to the fact that in the case under consideration only the poles $\omega=iD(k\theta)^2$ and $\omega=\pm 2E\theta$ are effective. We can also see that all $P_{S,A}$ are negative. It then follows from the matching conditions that these quantities are negative also for other values of θ. Figure 2 shows schematically the dependence of $|P_{S,A}|$ on $k\theta$.

In a strong field when the condition (13) is reversed, all the formulas still apply if we replace $\varkappa$ with $\varkappa_H=a^{-1}(g\mu HT_c^{-1})^{2/5}$, naturally provided the condition $\varkappa_H \ll q_i$ is satisfied.

We have assumed so far that the condition $\varkappa \ll q_i$ is obeyed. In the opposite limiting case of $\varkappa > q_i$, the in-

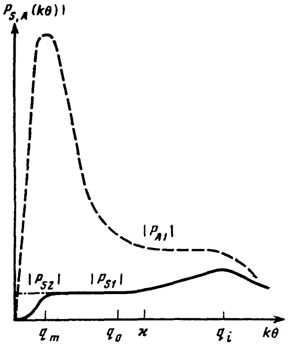

FIG. 2. Schematic representation of the dependences of the asymmetry effects on $k\theta$ in the exchange temperature range $\varkappa \gg q_0$, $q_m \sim q_0^4(\varkappa q_i)^{-3/2}$, $q_i = a^{-1}(2E/T_c ka)^{2/3}$.

elastic part of the expression for Dq^2 becomes dominant, and quantities I_0 and $P_{S,A}$ are large only well inside the hydrodynamic range $k\theta \ll \varkappa$ and they depend strongly on D, Γ_0, θ, and $g\mu H$. The condition $\varkappa \gg q_i$ is generally difficult to attain experimentally. However, the relevant experiments would be very interesting because they would permit us not only to determine D and Γ_0, but also to verify the hydrodynamic formulas (22). The expressions for I_0 and $P_{S,A}$ can be derived directly from Eqs. (22) and (9), but they are cumbersome and are therefore given in the Appendix. Here, we shall simply mention that an interesting feature of the behavior of $P_{S,A}(\theta, H)$ in the range $\Gamma_0 \ll g\mu H \ll T_c(\varkappa a)^{5/2}$ is the fact that it is governed entirely by the spin diffusion coefficient and by the magnetic field. Therefore, an experimental investigation of $P_{S,A}(\theta, H)$ under these conditions allows us to check directly the existence of a diffusion mode in a weak field. One should point out that the inequality $q_i \gg \varkappa$ can be rewritten in the form of the condition of smallness of the characteristic fluctuation lifetime $\Omega_e^{-1}(\varkappa)$ introduced in Ref. 11, compared with the time taken by a neutron to travel a distance $\varkappa^{-1}$. Therefore, the smallness of the scattering cross section in the $k\theta \ll \varkappa$ case is then a consequence of the fact that during the time taken by a neutron to travel a distance of the order of $\varkappa^{-1}$ the action of the fluctuating field is strongly averaged out.

We shall now discuss the dipole temperature range where $4\pi\chi \gg 1$. In this range the dipole momentum q_0 introduced in Ref. 6 plays the dominant part and this momentum is defined by $\omega_0 G_0(q_0) = 4\pi\chi(q_0) = 1$. Its order of magnitude is $q_0 \sim a^{-1}(\omega_0/T_c)^{1/2}$; clearly, in the dipole range we have $\varkappa \ll q_0$. We shall consider the most interesting, from the practical point of view, case when $q_0 \ll q_i$; then, if $k\theta > q_0$, the formulas obtained above for the exchange temperature range are still valid.

The main difference between the dipole and the exchange ranges is the change in the nature of the characteristic energy of critical fluctuations if $q < q_0$. A method developed in Ref. 6 gives the following expression for this energy:

$$\Omega_d(\varkappa) = {}^1/_2 G_{01}^{-1}(\varkappa)\{\alpha(q_0 a)^{1/2} + [\alpha^2(q_0 a)^2 + 4\beta T_c G_{01}(\varkappa)(q_0 a)^3]^{1/2}\}, \quad (25)$$

where α and β are numbers of the order of unity, and $\varkappa$ should be replaced with q if $q \gg \varkappa$. In the limit $T \to T_c$, we have

$$\Omega_d(\varkappa) \sim T_c \beta^{1/2}(q_0 a)^{3/2}\varkappa a. \quad (26)$$

This form of $\Omega_d(\varkappa)$ is obtained in Ref. 6 including diagrams "with rescattering." As pointed out in Ref. 6, these diagrams may be numerically small which means that $\beta \ll \alpha^2$. Then, in a wide range of temperatures where $4\pi\chi \gg 1$, we can ignore the term β in Eq. 25 and $\Omega_d(\varkappa)$ has its "usual" form, i.e., its temperature dependence is governed by the factor χ^{-1}. We cannot exclude the possibility that the onset of transition from conventional to the hard dynamic range described by Eq. (26) was observed in Ref. 13 in an investigation of the critical dynamics in EuS. One should mention that in the case of ferromagnets with a large atomic spin S the value of β should be considerably less than for $S=\frac{1}{2}$ or 1, because an increase in S suppresses strongly the processes of "rescattering" associated with odd vertices.

We will consider weak fields once again. In the critical dipole region we can easily demonstrate that for both variants of the dynamics the inelastic part of q^2 is small. Then, using Eqs. (14) and (15) and the dynamic similarity for $G_1^{(1)}(q, \omega)$ and $\Gamma_3(q, \omega)$, we obtain

$$P_{S1,2} \sim \theta P_{A1} \sim -P_0 \frac{g\mu H}{E} \frac{\Omega_d(k\theta)}{\Omega_e(k\theta)} \frac{k\theta}{\varkappa}, \quad \varkappa \ll k\theta \ll q_0. \quad (27)$$

Here and later we shall retain the negative sign in the expressions for $P_{S,A}$ because it corresponds to the sign obtained for these quantities in the exchange hydrodynamic range. The denominator of Eq. (27) contains a factor $\Omega_e(k\theta) = T_c(ka\theta)^{5/2}$. [In the hydrodynamic range $k\theta \ll \varkappa$ if the inelasticity is weak, i.e., if $2E\theta \gg \Omega_d(\varkappa)$, it is necessary to modify Eq. (27) by replacing $k\theta$ with $\varkappa$.] Finally, it should be noted that Eq. (21) is obtained from Eq. (27) by the replacement of $\Omega_d(k\theta)$ with $\Omega_e(k\theta)$. Thus, in a very wide range of values of parameters the quantities $P_{S,A}$ are proportional to the ratio of the characteristic dynamic similarity energy to the exchange energy of critical fluctuations. Therefore, an investigation of the angular and temperature dependences of $P_{S,A}$ will make it possible to study the whole range of transition from the exchange to the dipole dynamics. We do not know even one experimental method which can achieve this.

In the hydrodynamic range the inelastic part of q is large if $2E\theta \ll \Omega_d(\varkappa)$. Consequently, in the case of hard

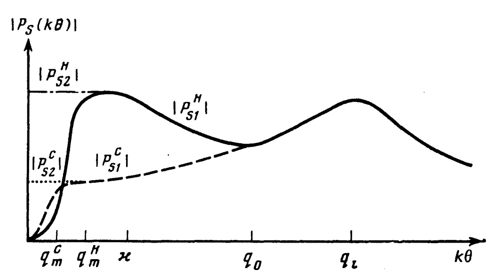

FIG. 3. Schematic representation of the dependences of the asymmetry effects on $k\theta$ for the hard $|P_S^H|$ and conventional $|P_S^C|$ variants of the dipole dynamics $q_m^C \sim \varkappa^2 q_0^{1/2} q_i^{-3/2}$, $q_m^H \sim \varkappa q_0^{3/2} q_i^{-3/2}$. The values of $q_m^{H,C} = k\theta_m^{H,C}$ correspond to the maxima of the functions $|P_A^{H,C}(\theta)| \sim \theta^{-1}|P_S^{H,C}(\theta)|$.

dynamics we have

$$P_{S1}\sim\theta P_{A1}\sim -P_0\frac{g\mu H}{E}\frac{(E\theta)^2}{\Omega_d(\varkappa)\Omega_e(\varkappa)}$$
$$\sim -P_0\frac{g\mu H}{E}\left(\frac{q_i^2}{q_0\varkappa}\right)^{1/2}\left(\frac{k\theta}{\varkappa}\right)^2,\quad k\theta\ll\varkappa\left(\frac{q_0}{q_i}\right)^{1/2};$$
$$P_{S2}\sim -P_0\frac{g\mu H}{E}\left(\frac{q_0}{\varkappa}\right)^{1/2},\quad k\theta\ll\varkappa, \tag{28a}$$

whereas in the conventional case, we obtain

$$P_{S1}\sim\theta P_{A1}\sim -P_0\frac{g\mu H}{E}\frac{(E\theta)^2}{\Omega_d(\varkappa)\Omega_e(\varkappa)}\sim -P_0\frac{g\mu H}{E}\left(\frac{q_i}{\varkappa}\right)^2\frac{k\theta}{(q_0\varkappa)^{1/2}},$$
$$k\theta\ll\left(\frac{q_0}{q_i}\right)^{1/2}\frac{\varkappa^2}{q_i}, \tag{28b}$$
$$P_{S2}\sim -P_0\frac{g\mu H}{E}\left(\frac{q_0}{\varkappa}\right)^{1/2},\quad k\theta\ll\varkappa.$$

Figure 1 shows qualitatively the angular dependences of the quantities $|P_{S1,2}|$ for both cases. The right-hand maximum at $k\theta\sim q_i$ lies in the exchange range, and its temperature dependence $\tau^{-2/3}$ is governed by the principle of merging of correlations. The minimum at $k\theta\sim q_0$ and the left-hand maximum at $k\theta\lesssim\varkappa$ occur only in the hard dynamics case. The amplitude of this maximum depends on temperature[3)] as τ^{-1}. We have not included a graphical illustration of the behavior of P_{A1} because this behavior differs little in the two cases under discussion. However, the maximum of $|P_{A1}|$ lies in the hydrodynamic range $k\theta\ll\varkappa$ and the magnitude of the effect of the maximum is of the order of $g\mu H/\Omega_e(\varkappa)$ in both cases. On the other hand, the dependences of the angles $\theta_m(\tau)$ at which $|P_A(\theta)|$ reaches its maximum differ greatly in the hard variant $\theta_m^H(\tau)\propto\tau^{2/3}$ and in the conventional case $\theta_m^c(\tau)\propto\tau^{4/3}$. The dependences $P_{A1}(\tau)$ is the hydrodynamic range are also different for these variants. For example, to the right of the maximum we have $P_A\propto\tau^{-1}$ in the hard dynamics case and $P_{A1}\propto\tau^{1/3}$ in the conventional case.

We shall not estimate $P_{S,A}$ in strong fields because in this case, in addition to the replacement of $\varkappa$ with $\varkappa_H$, we have to allow for the second term in brackets in Eq. (12) in the expression for $d\sigma_P/d\Omega$, which is difficult to do. We shall simply note that if $\varkappa\ll\varkappa_H$, all the quantities should be independent of τ.

We shall conclude this section by considering briefly the experimental situation. Okorokov *et al.*[1] observed the asymmetry effects discussed above. Their dependences on temperature, scattering angle, and field are in qualitative agreement with the conclusions reached by us above. It is worth noting a tendency for a minimum to appear on the $P_S(\theta)$ curve at $k\theta\sim q_0$ in the dipole temperature range, although because of the small number of results and the small difference between q_0 and q_i, a definite conclusion is difficult to draw. Moreover, the fairly poor angular resolution in the experiments of Okorokov *et al.*[1] prevented the authors from attaining the region $\theta\sim\theta_m(\tau)$ and the observed position of the maximum of the effect $P_A(\theta)$ was clearly associated with the instrumental angular width. For this reason there is insufficient justification for a comparison of the experimental data with the theoretical predictions for the hydrodynamic range. In particular, in spite of the fact that near an experimental maximum the value of $P_A(\tau)$ is proportional to τ^{-1}, it is not yet possible to draw a final conclusion from this observation in favor of the hard dipole dynamics.

We can thus regard the experiments of Okorokov *et al.*[1] as preliminary and demonstrating the basic potentialities of a new experimental method for investigating critical dynamics, triple dynamic correlations, and critical phenomena in a magnetic field.

4. POSSIBILITY OF VERIFICATION OF THE PRINCIPLE OF MERGING OF CORRELATIONS BY APPLYING A STATISTICAL THEORY

We shall consider the exchange range of temperatures and assume that $k\theta\ll q_i$. It then follows from the above discussion that the scattering is quasielastic and the cross section can be expressed in terms of the static Green functions. Applying Eq. (9) to nonpolarized neutrons, we find that

$$d\sigma/d\Omega=(r_0\gamma)^2T\{(1+\cos^2\varphi)\tilde{G}^{(1)}(k\theta,H)+\sin^2\varphi\tilde{G}^{(2)}(k\theta,H)\}, \tag{29}$$

where $\tilde{G}^{(1)}$ and $\tilde{G}^{(2)}$ are static Green functions perpendicular and parallel to the field, and the angle φ is defined in Fig. 1. In weak fields [Eq. (13)], the functions $\tilde{G}^{(1,2)}$ can be expanded as series in terms of H^2, and in the lowest order we have

$$\tilde{G}^{(1,2)}(k\theta,H)=G_0(k\theta)+G_0^2(k\theta)G_0^2(0)(g\mu H)^2\Gamma_{1,2}(k\theta,0), \tag{30}$$

where Γ_1 is the vertex part with pairwise coincident tensor components ($\Gamma_1=\Gamma_{\xi\rho\rho\xi}$; the ρ axis is along the field), whereas in the case of Γ_2 the tensor components are identical ($\Gamma_2=\Gamma_{\rho\rho\rho\rho}$).

Thus, an investigation of the dependence of the cross section on the angle φ in a weak field can be used to investigate the tensor structure of the static vertex Γ. Next, if $k\theta\gg\varkappa$, it follows from the principle of merging that Γ is described by

$$\Gamma_{1,2}=T_c(ka\theta)^{1/2}(\varkappa a)^{1/2}\Lambda_{1,2}, \tag{31}$$

where $\Lambda_{1,2}$ are constants of the order of unity. Using the formulas (16) and (22) for the functions G_0, we can represent the expression for the cross section in the form

$$\frac{d\sigma}{d\Omega}=(r_0\gamma)^2\left\{2G_0(k\theta)+\left(\frac{z^2}{ka\theta}\right)^2\left(\frac{g\mu H}{T_c(\varkappa a)^{5/2}}\right)^2[\Lambda_1(1+\cos^2\varphi)+\Lambda_2\sin^2\varphi]\right\}. \tag{32}$$

We can thus see that investigations of the temperature and angular dependences of the field correction to the cross section makes it possible to verify experimentally the principle of merging in the static case. In strong fields, we have to replace $\varkappa$ in Eq. (32) with $\varkappa_H$. Then, the second term is of the order of $(\varkappa_H/k\theta)^{3/2}$; clearly, this dependence is correct $k\theta\gg\varkappa_H$.

We shall conclude by thanking A. I. Okorokov, A. G. Gukasov, V. V. Runov, and V. E. Mikhailov for many interesting discussions and an opportunity to see the results of their experiments well before publication. Moreover, we are grateful to E. M. Pavlenko for the great help in the preparation of the manuscript.

APPENDIX

We shall give the expressions for $I_0(\theta, \varphi)$ and $P_{A1}(\theta)$ in the $\varkappa \gg q_i$ case for the hydrodynamic temperature range $k\theta \ll \varkappa$ assuming that the field is weak so that $g\mu H \ll \Omega_c(\varkappa)$. The formulas for $\tilde{P}_{S1,2}$ are very cumbersome and the values of these quantities are small; for these reasons we shall not give the formulas. Using Eq. (22), we find from Eqs. (8), (9), and (24) that

$$\tilde{P}_{A1}=-P_0 g\mu H 2E\theta\left\{\frac{1}{(2E\theta+\Gamma_0)^2+(g\mu H)^2}-\frac{\Phi(\theta)+\Phi(-\theta)}{2\Delta(2(\Delta_0+\Delta))^{1/2}}\right\}, \quad (A.1)$$

$$\Delta=[\Delta_0^2+(g\mu H)^2]^{1/2}, \quad \Delta_0=E^2/Dk^2+D(k\theta)^2+\Gamma_0, \quad (A.2)$$

$$\Phi(\theta)=\frac{2^{1/2}(\Delta+\Delta_0)^{1/2}+D^{1/2}k\theta+ED^{-1/2}k^{-1}}{\Delta+(D^{1/2}k\theta+ED^{-1/2}k^{-1})[2^{1/2}(\Delta+\Delta_0)^{1/2}+D^{1/2}k\theta+ED^{-1/2}k^{-1}]}. \quad (A.3)$$

The first term in Eq. (A.1) is governed by the poles $\omega=\pm 2iE\theta$ and it is identical with Eq. (24) for P_{A1}. The second term is due to inelastic poles of the Green functions G_{+-} and G_{-+}. In the range of very small angles we have $\tilde{P}_{A1} \propto \theta$. However, if $\theta \gg 2E/Dk^2$ and $2E\theta$ is greater than Γ_0, $g\mu H$, and E^2/Dk^2, we find that

$$\tilde{P}_{A1}\sim-\frac{P_0}{\theta}\frac{g\mu H}{D(k\theta)^2}\left(\frac{2E}{Dk^2}\right)^2.$$

In the case when $g\mu H$ is large, we have

$$\tilde{P}_{A1}\sim-P_0\left(\frac{E}{g\mu H}\right)^{3/2}\left(\frac{E}{Dk^2}\right)^{1/2}.$$

Under the same conditions, $I_0(\theta, \varphi)$ is described by

$$I_0(\theta,\varphi)=TG_0\frac{E}{D^{1/2}k}\left[\frac{(2(\Delta+\Delta_0))^{1/2}}{\Delta}-\left(\frac{1}{2}\frac{(2(\Delta+\Delta_0))^{1/2}}{\Delta}-\frac{1}{\Delta_0^{1/2}}\right)\cos^2\varphi\right]$$
$$-E\theta\cos 2\varphi\left\{\frac{(g\mu H)^2}{[(2E\theta+\Gamma_0)^2+(g\mu H)^2][2E\theta+\Gamma_0]}-\frac{1}{2\Delta}[F(H,\theta)-F(0,\theta)-F(H,-\theta)+F(0,-\theta)]\right\}, \quad (A.4)$$

$$F(H,\theta)=\frac{\Delta_0+{}^1/_2(2(\Delta+\Delta_0))^{1/2}(D^{1/2}k\theta+ED^{-1/2}k^{-1})}{\Delta+(D^{1/2}k\theta+ED^{-1/2}k^{-1})[(2(\Delta+\Delta_0))^{1/2}+D^{1/2}k\theta+ED^{-1/2}k^{-1}]}. \quad (A.5)$$

In the range $\varkappa \gg q_i$ (and $k\theta \ll \varkappa$) it is quite difficult to carry out experiments because the scattering cross section is fairly small. However, the asymmetry can then reach a few tenths of percent and it should be possible to measure it easily.

1) Here and later we shall always assume that the Fisher exponent η vanishes.

2) If allowance is made for the dipole forces. Eq. (14) is valid also for the function $G_1^{(3)}$ which occurs in Eqs. (4) and (7), provided we replace $G^{(1)}$ in this formula with $G_1^{(1)}$.

3) If we introduce a critical index of the dynamic similarity using the formula $\Omega_d(\varkappa) \sim T_c \varkappa^z q_0^{3/2-z}$ and consider it as a free parameter, we can show that the minimum of the function $|P_{S1}|$ occurs only if $z < 3/2$.

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