Bremsstrahlung of gravitons

M. I. Krivoruchenko and B. V. Martem'yanov

Institute of Theoretical and Experimental Physics
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An expression is obtained for the cross section for bremsstrahlung of a graviton resulting from the scattering of two charged particles. It is pointed out that the Weizsäcker-Williams method is not valid for processes with the participation of gravitions.

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1. In this paper, we study the bremsstrahlung of gravitons resulting from the scattering of two charged scalar particles, one of which is assumed to be much heavier than the other. In the Born approximation, we obtain the differential cross section of this process in the rest frame of the heavy particle. The calcuations show that at both nonrelativistic and ultrarelativistic energies of the incident particle the bremsstrahlung of gravitons is Gm^2/α times weaker in order of magntiude than the bremsstrahlung of photons (G is the Newtonian gravitational constant, α is the fine-structure constant, and m is the mass of the particle), which agrees with the assertions made in Refs. 1–3 concerning the weak nature of the interaction between gravitons and matter. This conclusion is of interest in connection with the establishment of thermal equilibrium between graviton radiation and the remaining matter in the early stage in the evolution of the Universe.

We also consider the applicability of the Weizsäcker-Williams method to processes with the participation of gravitons and show that this method cannot be extended in the general case to such processes.

2. We calculated the amplitude for graviton bremsstrahlung by the standard Feynman rules for processes with the participation of gravitons.[4-6] In the calculation of the cross section, an important simplification was achieved by assuming one of the scalar particles to be heavy. The results of the calculations of the cross section are as follows:

$$d\sigma=8G\alpha^2Z^2\frac{p'}{p}\frac{d\omega}{\omega}\Big[\frac{14}{3}-\frac{11}{3}\frac{\varepsilon\varepsilon'}{pp'}L_q+(-3\varepsilon\varepsilon'-\omega\varepsilon)\frac{L}{\omega p} -(-3\varepsilon\varepsilon'+\omega\varepsilon')\frac{L'}{\omega p'}-\frac{1}{2}(\varepsilon\varepsilon'-m^2)\frac{LL'}{pp'} +\frac{1}{2}(2\varepsilon^2-m^2)\frac{\varepsilon\varepsilon'}{\omega p^2p'}LL_q-\frac{1}{2}(2\varepsilon'^2-m^2)\frac{\varepsilon\varepsilon'}{\omega pp'^2}L'L_q\Big]; \quad (1)$$

$$L=\ln\frac{\varepsilon+p}{\varepsilon-p},\quad L'=\ln\frac{\varepsilon'+p'}{\varepsilon'-p'},\quad L_q=\ln\frac{(p+p')^2-\omega^2}{(p-p')^2-\omega^2},$$

where ε, p and ε', p' are, respectively, the energy and momentum of the incident and the scattered light particle, and ω is the graviton frequency.

At nonrelativistic energies, (1) agrees with the result of Ref. 7. Note that at low frequencies $\omega \ll m$ the main contribution to the cross section is made by small-angle scattering, and since scalar and spinor particles undergo the same small-angle scattering, the expression (1) also describes the scattering of spinor particles with the emission of soft gravitons ($\omega \ll m$).

3. It is well known that at high energies of the colliding particles the Weizsäcker-Williams method[8] is applicable to the bremsstrahlung of photons, and it is therefore of interest to consider whether this method also applies to the bremsstrahlung of gravitons. We shall show that in the general case it cannot be extended to processes with the participation of gravitons in the final state.

The Weizsäcker-Williams theorem was proved for processes in which the radiation of the massive particle can be ignored. In electrodynamics, the amplitude of the process corresponding to radiation of the heavy particle is proportional to $1/M$ as $M \to \infty$, and in this case its contribution to the total amplitude can be ignored. Note that this assertion does not depend on the gauge of the photon, since the amplitudes corresponding to radiation by light and massive particles are separately gauge invariant.

For processes with the emission of a graviton, there are two important differences from electrodynamics. First, the amplitude of graviton radiation by the heavy particle ($M \to \infty$) in an arbitrary gauge has the same order in $1/M$ as the amplitude for radiation by the light particle. Second, the amplitudes corresponding to graviton radiation by the light and heavy particles are not separately gauge invariant; their contributions to the total amplitude depend on the gauge of the graviton polarization tensor. In particular, one can choose a gauge in which the heavy particle does not radiate at all. However, the graviton polarization tensor, and with it the electromagnetic current of the light particle then depend on the momentum of the heavy particle, which violates the assumptions of the Weizsacker-Williams theorem. In the invariant derivation of the theorem,[9] one change will be that the number of structure functions in the electromagnetic tensor is increased from two to four, and the simple proportionality between the cross sections of photoproduction and bremsstrahlung will in general cease to hold. Thus, the theorem is no longer valid.

4. It can be seen from the main result (1) that the cross section of graviton bremsstrahlung, like the cross section for graviton production in particle annihilation,[10] does not exhibit a growth of ε^2/m^2 as $\varepsilon \to \infty$, in contrast to the assertions of Matzner.[11]

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Although we made all our calculations for the case of electromagnetic interaction of the scattered particles, only trivial modifications are required to make the result valid in the case of QCD interaction in a hot quark-gluon plasma. It may therefore be concluded that at this stage of cosmological evolution gravitational radiation cannot be strong.

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