Strong particle scattering in a random inhomogeneous magnetic field

S. I. Vaĭnshteĭn and L. L. Kichatinov

Institute of Terrestrial Magnetism, Ionosphere, and Radio Wave Propagation, Siberian Division, USSR Academy of Sciences
(Submitted 7 May 1981)
Zh. Eksp. Teor. Fiz. **81**, 1723–1730 (November 1981)

The motion of particles in randomly inhomogeneous fields is considered. The collisionless Boltzmann equation is averaged over the electromagnetic-field fluctuations by using an approximation developed in hydrodynamics theory. The approximation is presented in Sec. 2. The kinetic equation derived is also valid when the angle of particle scattering over the correlation length of the random magnetic field is not small. The diffusion approximation is considered by taking into acount particle scattering by a stochastic electric field. A very simple closed equation set is thus obtained.

PACS numbers: 41.70. + t, 05.20.Dd

1. INTRODUCTION

The kinetic theory of the propagation of charged particles in an electromagnetic field with random inhomogeneities is used in many problems of astrophysics, plasma theory, cosmic-ray physics, and others. Multiple scattering of the particles by an randomly inhomogeneous magnetic field was considered by Dolginov and Toptygin.[1] In their paper and in later works[2–5] the problem was solved by perturbation theory. The small parameter was the scattering angle with the particle traversing a distance equal to the correlation length l_c of the random magnetic field. This parameter depends on the particle energy. For a number of problems (for example, propagation of solar cosmic rays in interplanetary space) an important role is played by the energy region for which scattering by field inhomogeneities becomes strong and perturbation theory no longer holds. The purpose of the present paper is to derive a kinetic equation that describes the propagation of charged particles in an electromagnetic field with random inhomogeneities for the case of strong scattering.

The posed problem is similar to the theory of strong scattering of electromagnetic waves and to the problem of hydrodynamic turbulence, where there is no small parameter. The problem is solved by Orszag's scheme[6] which makes it possible to obtain a Kolmogorov spectrum is hydrodynamic turbulence theory, and offers other advantages.[7,8] This scheme is widely used also for magnetohydrodynamic turbulence.[9,10]

The employed approach, while quite simple, yields a kinetic equation for the average distribution function. In the weak-scattering limit this equation goes over automatically into the one obtained earlier. In addition, the Orszag scheme will be improved upon below, by obtaining an equation for the memory time τ of the system, rather than specifying this time.

2. DERIVATION OF EQUATION FOR THE AVERAGED DISTRIBUTION FUNCTION

We start from the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\mathbf{F}\frac{\partial f}{\partial \mathbf{p}}=0, \quad \mathbf{F}=e\mathbf{E}+\frac{e}{c}[\mathbf{v}\times\mathbf{H}], \tag{1}$$

where $\mathbf{v}=c^2\mathbf{p}/\varepsilon$ is the velocity of the particle with momentum $\mathbf{p}$ and energy ε. We assume that the mean values of $\mathbf{E}$ and $\mathbf{H}$ are zero:

$$\langle\mathbf{E}\rangle=0, \quad \langle\mathbf{H}\rangle=0.$$

The angle brackets denote averaging over the ensemble of the random field.

We shall assume that the fields $\mathbf{H}$ and $\mathbf{E}$ vary at frequency $\omega\ll v/l_c$. The distribution function f varies randomly in space and in time, and follows the variations of the random force. Interest attaches to the distribution function $\varphi=\langle f\rangle$ averaged over the ensemble of the random field. To obtain an equation for φ we must average (1):

$$\frac{\partial \varphi}{\partial t}+\mathbf{v}\frac{\partial \varphi}{\partial \mathbf{r}}=-\frac{\partial}{\partial \mathbf{p}}\langle\mathbf{F}f\rangle. \tag{2}$$

To make (2) closed, we change over first to the equation for the characteristic function—the Fourier transform of the function $f(\mathbf{r},\mathbf{p},t)$:

$$\frac{\partial f(\mathbf{k},\mathbf{p},t)}{\partial t}+i\mathbf{k}\mathbf{v}f(\mathbf{k}\ \mathbf{p},t)=-\frac{\partial}{\partial p_\alpha}\int d^3q F_\alpha(\mathbf{k}-\mathbf{q},\mathbf{p},t)f(\mathbf{q},\mathbf{p},t).$$

Here $\mathbf{F}(\mathbf{k},\mathbf{p},t)$ is the Fourier transform of the force $\mathbf{F}$.

We shall need below the system memory time τ, i.e., the correlation time for expressions of the type $\langle\mathbf{F}f\rangle$. The physical meaning of the memory of a system is that the correlations, i.e., the initial values, are forgotten as a result of scattering of the particles in the random fields. It is clear at the same time that any initial correlation $\langle\mathbf{F}f\rangle$ vanishes after a time l_c/v even in the absence of an interaction of the particles with the field, merely on account of the convective term $(\mathbf{v}\cdot\nabla)f$ in (1).

We change over therefore to the auxiliary function

$$g(\mathbf{k},\mathbf{p},t)=e^{i\mathbf{k}\mathbf{v}t}f(\mathbf{k},\mathbf{p},t),$$

the equation for which is

$$\frac{\partial g(\mathbf{k},\mathbf{p},t)}{\partial t}=-e^{i\mathbf{k}\mathbf{v}t}\int d^3q F_\alpha(\mathbf{k}-\mathbf{q},\mathbf{p},t)\frac{\partial}{\partial p_\alpha}g(\mathbf{q},\mathbf{p},t)e^{-i\mathbf{q}\mathbf{v}t}, \tag{3}$$

$$i\omega g(\mathbf{k},\mathbf{p},\omega)=\int d^3q\,d\omega_1 F_\alpha(\mathbf{k}-\mathbf{q},\mathbf{p},\omega-\omega_1+\mathbf{k}\mathbf{v})\frac{\partial}{\partial p_\alpha}g(\mathbf{q},\mathbf{p},\omega_1-\mathbf{q}\mathbf{v}). \tag{4}$$

From Eq. (4) for g, which does not contain the Fourier transform of the convective term, it is seen that the

 © 1982 American Institute of Physics

memory time of the correlation $\langle \mathbf{F}g\rangle$ is connected only with the interaction with the fields, and that τ has the meaning of the free path time of the particle. To obtain equations for $\langle \mathbf{F}g\rangle$ we multiply (1) by $\mathbf{F}$ and add to the expression $g\partial \mathbf{F}/\partial t$, obtaining thereby in the left-hand side $\partial g\mathbf{F}/\partial t$, and average. We use the scheme of Ref. 6, where no small parameter corresponding to weak scattering is employed.

This scheme was developed in the hydrodynamic theory of turbulence. It is being effectively used of late, since it leads to correct results. Thus, in the stationary case we obtain a Kolmogorov spectrum, the spectral function in dynamics satisfies the non-negativity property, and the general equations satisfy the fundamental properties of angular-momentum conservation. Following Ref. 6, we replace the resultant semi-invariant by a τ-relaxing term in the right-hand side: $-\langle \mathbf{F}g\rangle/\tau$. In the $\mathbf{k}, \omega$ space this means that

$$\int d^3q d\omega_1 \langle F_\mu(\mathbf{k}',\mathbf{p},\omega')F_\alpha(\mathbf{k}-\mathbf{q},\mathbf{p},\omega-\omega_1+\mathbf{kv})\frac{\partial}{\partial p_\alpha}g(\mathbf{q},\mathbf{p},\omega_1-\mathbf{qv})\rangle$$
$$-\int d^3q d\omega_1 \langle F_\mu(\mathbf{k}',\mathbf{p},\omega')F_\alpha(\mathbf{k}-\mathbf{q},\mathbf{p},\omega-\omega_1+\mathbf{kv})\rangle\frac{\partial}{\partial p_\alpha}\langle g(\mathbf{q},\mathbf{p},\omega_1-\mathbf{qv})\rangle$$
$$=-\frac{1}{\tau}\langle F_\mu(\mathbf{k}',\mathbf{p},\omega')g(\mathbf{k},\mathbf{p},\omega)\rangle.$$

Alternately, using (4),

$$\langle F_\mu(k_1,\mathbf{p})g(k,\mathbf{p})\rangle=-\frac{G_{\mu\alpha}(-k_1)}{1/\tau-i\omega}\frac{\partial}{\partial p_\alpha}\langle g(\mathbf{k}+\mathbf{k}_1,\mathbf{p},\omega+\omega_1-\mathbf{k}_1\mathbf{v})\rangle,$$

where $k=\{\mathbf{k},\omega\}$ is a 4-vector. We have used here the property of a process that is homogeneous and stationary in the statistical sense:

$$\langle F_\alpha(k_1)F_\mu(k)\rangle=\delta(k+k_1)G_{\alpha\mu}(k).$$

When account is taken of the equality

$$f(\mathbf{k},\ \mathbf{p},\ \omega)=g(\mathbf{k},\ \mathbf{p},\ \omega-\mathbf{kv})$$

this corresponds to

$$\langle F_\mu(k-k_1)f(k,\mathbf{p})\rangle-\frac{G_{\mu\alpha}(k_1-k)}{1/\tau-i\omega+i\mathbf{kv}}\frac{\partial\varphi(k_1,\mathbf{p})}{\partial p_\alpha},$$
$$\langle F_\mu(\mathbf{r},\mathbf{p},t)f(\mathbf{r},\mathbf{p},t)\rangle=\int d^4k d^4k_1\langle F_\mu(k-k_1)f(k_1,\mathbf{p})\rangle e^{i\mathbf{kr}-i\omega t}. \tag{5}$$

We note that in the scheme of Ref. 6 the problem was to calculate the second moments. They are expressed in terms of the third. In the next step, the third are expressed in terms of the fourth, in which case the semi-invariant of fourth order is replaced by the τ-relaxing term. In our case the problem consists of calculating the first moment $\langle f\rangle=\varphi$. It is expressed in terms of the second (2), and the closure is effected also in the next step—in the equation for the second moment.

Recognizing that the spatial and temporal scales of variation of the average distribution function φ are large compared with the corresponding scales of the random field, we reduce (2) with the aid of (5) to the form

$$\frac{\partial\varphi}{\partial t}+\mathbf{v}\frac{\partial\varphi}{\partial\mathbf{r}}=\frac{\partial}{\partial p_\mu}D_{\mu\alpha}\frac{\partial\varphi}{\partial p_\alpha},$$
$$D_{\mu\alpha}=\int d^4k\frac{G_{\mu\alpha}(k)}{1/\tau-i\omega+i\mathbf{kv}}=\int_0^\infty dt' B_{\mu\alpha}(\mathbf{v}t',t')\exp\left(-\frac{t'}{\tau}\right). \tag{6}$$

Here $B_{\mu\alpha}$ is the correlation tensor of the force:

$$B_{\mu\alpha}(\mathbf{r},t)=\int d^4k e^{i\mathbf{kr}-i\omega t}G_{\mu\alpha}(k).$$

Expression (6) is the kinetic equation for the averaged distribution function φ, whose right-hand side takes into account the scattering of the particles by the random inhomogeneities of the field. We shall show that in the case of weak scattering the previously employed scheme of closing (2) is equivalent to a perturbation theory in terms of the small parameter $l_c/R \ll 1$ (R is the average Larmor radius of the particle, $R=cp/e\langle H^2\rangle^{1/2}$), developed in Refs. 1–3. The free-path time τ is in this case much longer than $\tau_c=l_c/v$.

Equation (6) takes then the form

$$\frac{\partial\varphi}{\partial t}+\mathbf{v}\frac{\partial\varphi}{\partial\mathbf{r}}=\frac{\partial}{\partial p_\mu}\int_0^\infty dt' B_{\mu\alpha}(\mathbf{v}t',t')\frac{\partial\varphi(\mathbf{r},\mathbf{p},t)}{\partial p_\alpha},$$

which coincides with the kinetic equation obtained for the case of weak scattering.[1-3] We note that it does not contain the parameter τ.

It is convenient to use an expression for the tensor $D_{\mu\alpha}$ in terms of the spectral tensor $G_{\mu\alpha}(k)$, since the latter can be expressed in terms of the spectral tensor of the electric field $T_{\mu\alpha}(k)$. Indeed, the Fourier transform of the force is

$$F_\mu(k)=eE_\mu(k)+\frac{e}{c}\varepsilon_{\mu\alpha\beta}v_\alpha H_\beta(k).$$

The magnetic field $H_\mu(k)$ can be expressed in terms of the electric field using the electrodynamics equation

$$\frac{\omega}{c}H_\alpha(k)=\varepsilon_{\alpha\beta\gamma}k_\beta E_\gamma(k).$$

We then obtain

$$F_\mu(k)=\frac{e}{\omega}([\omega-\mathbf{kv}]\delta_{\mu\alpha}+k_\mu v_\alpha)E_\alpha(k).$$

Therefore

$$G_{\mu\alpha}(k)=\frac{e^2}{\omega^2}([\omega-\mathbf{kv}]\delta_{\mu\gamma}+k_\mu v_\gamma)([\omega-\mathbf{kv}]\delta_{\alpha\beta}+k_\alpha v_\beta)T_{\gamma\beta}(k). \tag{7}$$

In the limiting case of a stationary magnetic field $\omega\to 0$, $E\sim\omega$, $T_{\alpha\beta}\sim\omega^2$, and $G_{\mu\alpha}$ is expressed in terms of the spectral tensor of the magnetic field $F_{\mu\alpha}(k)$:

$$G_{\mu\alpha}(k)=\frac{e^2}{c^2}\varepsilon_{\mu\nu\sigma}\varepsilon_{\alpha\beta\gamma}F_{\gamma\sigma}(\mathbf{k})v_\nu v_\beta\delta(\omega).$$

3. DIFFUSION APPROXIMATION

One of the most frequently employed methods of simplifying the kinetic equation is the diffusion approximation. In the energy range for which the mean free path Λ prior to scattering is much less than the characteristic scale of the problem, the distribution function is weakly anisotropic in momentum space and it suffices to use the first two terms of the expansion of the distribution function φ in spherical harmonics:

$$\varphi(\mathbf{r},\mathbf{p},t)=\frac{1}{4\pi}\left(N(\mathbf{r},p,t)+3\frac{\mathbf{pj}(\mathbf{r},p,t)}{pv}\right), \tag{8}$$

where N and $\mathbf{j}$ are respectively the particle density and the particle-current density.

We shall assume hereafter that the spectral tensor of the electric field is symmetrical with respect to the reversal of the sign of its arguments **k** and ω:

$$T_{\alpha\beta}(\mathbf{k},\omega)=T_{\alpha\beta}(-\mathbf{k},-\omega).$$

Averaging (6) over the solid angle of the vector **p**, with allowance for (7) and (8), and using the relation

$$\left\langle\frac{\partial}{\partial p_\alpha}B_\alpha(\mathbf{p})\right\rangle_\Omega=\frac{1}{p^2}\frac{\partial}{\partial p}p\langle p_\alpha B_\alpha(\mathbf{p})\rangle_\Omega$$

($\langle\ldots\rangle_\Omega$ signifies averaging over the solid angle of the vector **p**, and **B(p)** is a certain vector function of the particle momentum), we obtain

$$\frac{\partial N}{\partial t}+\operatorname{div}\mathbf{j}=\frac{1}{p^2}\frac{\partial}{\partial p}D_p\frac{\partial N}{\partial p},\qquad (9)$$

$$D_p=\left\langle e^2\int d^4k\,\frac{p_\alpha p_\beta T_{\alpha\beta}(k)}{1/\tau-i\omega+i\mathbf{kv}}\right\rangle_\Omega.$$

A similar averaging with weight p_α yields

$$\chi_{\mu\alpha}j_\alpha=-\frac{pv}{3}\frac{\partial N}{\partial r_\alpha},\qquad (10)$$

where

$$\chi_{\mu\alpha}=\frac{3e^2}{pv}\left\langle\int d^4k\,\frac{\omega^{-2}(k_\mu v_\gamma-\mathbf{kv}\delta_{\mu\gamma})(k_\alpha v_\beta-\mathbf{kv}\delta_{\alpha\beta})T_{\gamma\beta}(k)}{1/\tau-i\omega+i\mathbf{kv}}\right\rangle_\Omega.\qquad (11)$$

From (10) we obtain the following expression for the particle flux density

$$j_\mu=\varkappa_{\mu\alpha}\frac{\partial N}{\partial r_\alpha},\quad \varkappa_{\mu\alpha}=\frac{pv}{3}(\chi^{-1})_{\mu\alpha}.\qquad (12)$$

With the aid of (11) we reduce Eq. (9) to the form

$$\frac{\partial N}{\partial t}=\frac{\partial}{\partial r_\mu}\varkappa_{\mu\alpha}\frac{\partial N}{\partial r_\alpha}+\frac{1}{p^2}\frac{\partial}{\partial p}D_p\frac{\partial N}{\partial p}.\qquad (13)$$

The right-hand side of this equation describes the particle diffusion in phase space. The first term of the right-hand side of (13) corresponds to spatial diffusion of the particles, due to the scattering of the particles by random spatial inhomogeneities of the magnetic field. The term

$$\frac{1}{p^2}\frac{\partial}{\partial p}D_p\frac{\partial N}{\partial p}$$

describes the diffusion of particles in momentum space, corresponding to the acceleration particles by a stochastic electric field.

In the important particular case of a magnetic field frozen in a certain conducting medium that moves with velocity $\mathbf{u}(\mathbf{r},t)$ ($\langle\mathbf{u}\rangle=0$), the electric field $\mathbf{E}=-c^{-1}\mathbf{u}\times\mathbf{H}$. Assuming that

$$\langle u_\mu u_\alpha H_\beta H_\gamma\rangle=\langle u_\mu u_\alpha\rangle\langle H_\beta H_\gamma\rangle,$$

we obtain the coefficient

$$D_p=\frac{e^2}{c^2}\varepsilon_{\alpha\mu\nu}\varepsilon_{\beta\gamma\sigma}\int d^4k\,d^4k_1\left\langle\frac{p_\mu p_\gamma\Gamma_{\alpha\beta}(k-k_1)F_{\gamma\sigma}(k)}{1/\tau-i\omega+i\mathbf{kv}}\right\rangle_\Omega,$$

where $\Gamma_{\alpha\beta}$ is the spectral tensor of the velocity **u**.

In this case the particles are accelerated as they are multiply scattered by the chaotically moving random inhomogeneities of the magnetic field; this corresponds to the known Fermi acceleration mechanism.[11] In the particular case of an isotropic magnetic field

$$\langle H_\alpha(\mathbf{r},t)H_\beta(\mathbf{r}+\rho,t)\rangle={}^1/_3\langle H^2(\mathbf{r})\rangle\left\{\psi(\rho)\delta_{\alpha\beta}+\psi_1(\rho)\frac{\rho_\alpha\rho_\beta}{\rho^2}\right\},$$

$$\psi(\rho)=\exp[-(\rho/l_c)^2],$$

(ψ and ψ_1 are then connected by the condition that the field **H** be solenoidal) Eqs. (11) and (12) yield the following expression for the diffusion tensor:

$$\varkappa_{\mu\alpha}=\delta_{\mu\alpha}\frac{vc^2p^2}{\pi^{1/2}l_c\langle H^2\rangle e^2}\frac{\exp(-l_c^2/4v^2\tau^2)}{1-\operatorname{erf}(l_c/2v\tau)},$$

where $\operatorname{erf}(x)$ is the error function. In the case of strong scattering, $l_c/v\tau\sim1$, this equation yields for the diffusion coefficient a value larger than given by perturbation theory. We note that an experiment on the diffusion of solar cosmic rays[12] yields a similar deviation from the perturbation-theory predictions.

4. DERIVATION OF EQUATION FOR τ

So far we have analyzed only the general properties of τ, which was defined as the free path time. At high energies, the property $vk\gg\tau^{-1}$ was used. The scheme of Ref. 6 does not make it possible to determine this quantity, which is in its way a parameter of the theory. We obtain below an equation for the particular case of an isotropic time-independent random process (**E**=0, **H** is a random function of the coordinates) in the diffusion approximation. To derive this equation we turn to the next approximation in accordance with the scheme of Ref. 6. In other words, the equation for the second moment is written in exact form, while in the equation for the third moments the semi-invariant of fourth order is replaced by a τ-relaxing term. The expression for the second moment, which enters in (2), and obtained in the new approximation, should coincide with expression (5). This condition yields an equation for τ.

In the case considered

$$\mathbf{F}(\mathbf{k},\mathbf{p})\frac{\partial}{\partial\mathbf{p}}=-\hat{\mathbf{L}}\mathbf{H}(\mathbf{k}),\quad \hat{\mathbf{L}}=\frac{e}{c}\left[\mathbf{v}\times\frac{\partial}{\partial\mathbf{p}}\right],$$

therefore we have in place of (5)

$$\langle H_\alpha(\mathbf{k}-\mathbf{q})g(\mathbf{q},\mathbf{p},t)\rangle=\int_0^t dt'\exp\left(-\frac{t-t'}{\tau}-i\mathbf{qv}t'\right)F_{\alpha\mu}(\mathbf{q})\hat{L}_\mu\langle g(\mathbf{k},\mathbf{p},t')\rangle\times\exp(-i\mathbf{kv}t').\qquad (14)$$

The new approximation leads to the following equation for the second moment:

$$\frac{\partial}{\partial t}\langle H_\alpha(\mathbf{k}-\mathbf{q})g(\mathbf{q},\mathbf{p},t)\rangle=e^{-i\mathbf{qv}t}\hat{L}_\mu\int d^3q_1e^{-i\mathbf{q}_1\mathbf{v}t}\int_0^t dt'\exp\Big(-\frac{t-t'}{\tau}-i\mathbf{q}_1\mathbf{v}t'\Big)\hat{L}_\gamma\{\exp(-i\mathbf{qv}t')F_{\mu\gamma}(\mathbf{q}_1-\mathbf{q})\langle H_\alpha(\mathbf{k}-\mathbf{q})g(\mathbf{q},\mathbf{p},t')\rangle$$
$$+\exp(-i(\mathbf{q}_1-\mathbf{q})\mathbf{v}t')F_{\alpha\gamma}(\mathbf{q})\langle H_\mu(\mathbf{q}-\mathbf{q}_1)g(\mathbf{k}-\mathbf{q}+\mathbf{q}_1,\mathbf{p},t')\rangle\}+e^{-i\mathbf{qv}t}\hat{L}_\mu F_{\alpha\mu}(\mathbf{q})\langle g(\mathbf{k},\mathbf{p},t)\rangle e^{i\mathbf{kv}t}.\qquad (15)$$

Substituting the expression (14) for $\langle H_\alpha g\rangle$ in (15), and changing to the r-representation, we obtain, taking (8) into account,

$$\int_0 dt_1\psi(vt_1)\left\{\frac{1}{\tau}e^{-t_1/\tau}-\frac{2\Omega^2}{3}\int_{t_1}^\infty dt_2e^{-t_2/\tau}\psi(vt_2)\right\}\varepsilon_{\alpha\beta\gamma}v_\beta j_\gamma=0,\quad \Omega=\frac{v}{R}.$$

This equation should be satisfied at any direction of the vector **v**, therefore

$$\int_0^\infty dt_1\psi(vt_1)\left\{\frac{1}{\tau}e^{-t_1/\tau}-\frac{2\Omega^2}{3}\int_{t_1}^\infty dt_2e^{-t_2/\tau}\psi(vt_2)\right\}=0.\qquad (16)$$

This is in fact the equation for τ.

In the particular case $\psi(r)=e^{-r/a}$ we obtain for τ the expression

$$\tau=a/v[(1+2a^2\Omega^2/3v^2)^{1/2}-1]. \quad (17)$$

The diffusion coefficient corresponding to this choice of the function ψ is of the form

$$\varkappa=\frac{vc^2p^2}{2e^2a\langle H^2\rangle}\left(1+\frac{2e^2a\langle H^2\rangle}{3c^2p^2}\right)^{1/2}.$$

Here τ is eliminated with the aid of (17).

At $\Omega\tau_c\ll 1$ (weak scattering) $\tau=3v/\Omega^2a\sim\Lambda/v$. At $\Omega\tau_c\sim 1$ (strong scattering) $\tau\sim a/v\sim\Lambda/v$. Thus in both weak and strong scattering τ does indeed coincide with the free-path time. This agrees with the assumptions made above concerning the value of τ.

If we consider the limit $\Omega\tau_c\gg 1$, then it follows from (17) that $\tau=1/\Omega$. This time corresponds to isotropization of the distribution function in momentum space. Indeed, in this case the motion of the particle is close to Larmor rotation and, when averaged over scales larger than l_c, all the momentum directions become equally probable within the Larmor-rotation time $1/\Omega$. In this case, however, the scheme of Ref. 6 yields an incorrect value for the diffusion coefficient. The point is that the problem has now one more memory time, namely the time of passage of the particle along a force line through an inhomogeneity is much longer than $1/\Omega$. It is this which is responsible for the spatial diffusion. The presence of at least two memory times does not make it possible to use the Orszag scheme directly. It is necessary to use here the drift approximation and generalize the scheme of Ref. 6 so as to take the two memory times into account. The solution of such a problem is beyond the scope of the present analysis.

5. CONCLUSION

Orszag's scheme[6] is widely used in the theory of hydrodynamic, magnetohydrodynamic, and two-dimensional turbulence, where the beneficial aspects of this approach were demonstrated. In the present article, this method was developed to obtain an averaged kinetic equation. This made it possible to describe strong scattering of particles, which up to now was treated within the framework of certain model representations. The transition ot the high-energy limit, when the scattering is weak, leads to the known kinetic equation obtained by perturbation theory. The advantage of this approach is the simplicity of the deviation and of the obtained closed system of equations: The kinetic equation (6) and the equation for τ (16). At the same time, the employed scheme does not make it possible to go to the low-energy limit.

[1]A. Z. Dolginov and I. N. Toptygin, Zh. Eksp. Teor. Fiz. **51**, 1771 (1966) [Sov. Phys. JETP **24**, 1195 (1967)].
[2]A. Z. Dolginov and I. N. Toptygin, Icarus **8**, 54 (1968).
[3]L. I. Dorman and M. E. Katz, Space Sci. Rev. **20**, 529 (1977).
[4]L. I. Dorman, M. E. Kats, Yu. I. Fedorov, and B. A. Shakhov, Zh. Eksp. Teor. Fiz. **79**, 1267 (1980) [Sov. Phys. JETP **52**, 640 (1980)].
[5]A. M. Bykov and I. N. Toptygin, Izv. AN SSSR, ser. fiz. **43**, 2552 (1980).
[6]S. A. Orszag, Fluid Mech. **41**, 363 (1970).
[7]R. H. Kraichnan and D. Montgomery, Rept. Progr. Phys. **43**, 547 (1980).
[8]A. P. Mirabel' and A. S. Monin, Uspekhi mekhaniki **2**, 47 (1979).
[9]U. Frisch, A. Pouguet, T. Leorat, and A. Marure, J. Fluid Mech. **68**, 769 (1975).
[10]S. I. Vaĭnshteĭn, Magnitnaya gidrodinamika **2**, 3 (1980).
[11]E. Fermi, Phys. Rev. 75, 1169 (1949).
[12]I. Ilenchik, Yu. Dubinskiĭ, and L. I. Miroshnichenko, Physica Solariterrestris, No. 12, 33 (1979).

Translated by J. G. Adashko