Effect of external fields and impurities on the Josephson current in *SNINS* junctions

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The influence of the impurities on the stationary Josephson effect in *SNINS* systems is considered. It is shown that the expression for the current amplitude depends significantly on the ratio of the mean free path to the coherence range of the superconductor and also on the position of the dielectric barrier in the layer of the normal metal. The effect of the Andreev level shift in a magnetic field in a *SNINS* system [A. D. Zaikin and G. F. Zharkov, Sov. Phys. JETP **51**, 364 (1980)] on the current in the system is taken into account. A magnetic field enhances somewhat the current amplitude and leads to small current oscillations that depend on H and are related to the discrete levels of the system. A theory of the nonstationary Josephson effect in *SNINS* systems is developed. The discrete spectrum of the system affects the current-voltage characteristics and produces on them oscillations that are also dependent on the position of the dielectric layer. It is shown that for certain values of the voltage, such that $eV<2\Delta$, the Josephson-current component in an *SNINS* system diverges logarithmetically.

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1. INTRODUCTION

It is known that the spectrum of excitations localized in a normal metal between two superconductors (*SNS* system) is quantized.[1] If the normal-metal layer contains a certain irregularity (for example, a thin oxide film—*SNINS* system), the excitations are scattered by the film and this alters the form of the spectrum.[2] The Josephson effect is also strongly influenced thereby, since the expression for the stationary Josephson current in the *SNS* (or *SNINS*) system is determined in fact by the form of the excitation spectrum of this system. It was proposed in Ref. 2 that there are no impurities in the system. In the present paper, which is a further development of Ref. 2, we investigate the behavior of *SNINS* systems in the presence of external fields (impurity field, or magnetic and electric fields).

We start from the Eilenberger[3] quasiclassical equations for the matrix Green's function integrated over the energy:

$$\mathbf{v}_F\frac{\partial}{\partial\mathbf{r}}\hat{g}(\mathbf{v}_F,\mathbf{r})+\left[\omega\hat{\tau}_3+\hat{\Delta}-\frac{v_F}{2l}\langle\hat{g}_\omega(\mathbf{v}_F,\mathbf{r})\rangle,\ \hat{g}_\omega(\mathbf{v}_F,\mathbf{r})\right]=0, \qquad (1.1)$$

the use of which is quite effective for the solution of many problems of weak superconductivity (see, e.g., Ref. 4). Here

$$\hat{g}_\omega=\begin{pmatrix} g_\omega & f_\omega \\ f_\omega^+ & -g_\omega\end{pmatrix},\quad \hat{\tau}_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \hat{\Delta}=\begin{pmatrix}0 & \Delta\\ -\Delta^* & 0\end{pmatrix},$$

$g^2+f_\omega f_\omega^+=1$, v_F is the Fermi velocity, l is the mean free path, ω is the Matsubara frequency, and Δ is the order parameter. The square brackets in (1.1) stand for the commutators, and the angle brackets for averaging over the directions of the vector $\mathbf{v}_F$.

The current in the superconductor is determined from the equation ($\mathbf{p}_F=m\mathbf{v}_F$)

$$\mathbf{j}(\mathbf{r})=-\frac{iep_F}{\pi}T\sum_{\omega>0}\mathrm{Sp}\,\langle\mathbf{p}_F\hat{\tau}_3\hat{g}_\omega(\mathbf{v}_F,\mathbf{r})\rangle. \qquad (1.2)$$

As is customary, the presence of the dielectric interlayer is taken into account by introducing into the system a potential of the form $V_0\delta(x-x_0)$ (x_0 is the coordinate of the interlayer). This potential, however, does not enter at all in Eqs. (1.1) and must be taken into account with the aid of the boundary conditions for $\hat{g}_\omega$ at the point x_0. These conditions can be obtained from the Gor'kov equations[5] in which the potential $V_0\delta(x-x_0)$ is introduced, using the customary procedure of integrating these equations from $x_0-\delta$ to $x_0+\delta$ and letting δ tend to zero, after which taking the Fourier transforms of the exact Green's functions with respect to the coordinate difference and integrating with respect to energy. These conditions are of the form

$$\hat{g}_\omega(v_x,x_0+0)-\hat{g}_\omega(-v_x,x_0+0)=\hat{g}_\omega(v_x,x_0-0)-\hat{g}_\omega(-v_x,x_0-0)=i(v_x/V_0)[\hat{g}_\omega(v_x,x_0+0)-\hat{g}_\omega(v_x,x_0-0)]. \qquad (1.3)$$

Here and elsewhere the function $\hat{g}_\omega$ depends only on the coordinate x and on the Fermi-velocity component v_x, inasmuch as the superconducting system will be assumed homogeneous in the y and z directions. At $V_0=0$ it follows from (1.3) that $\hat{g}_\omega$ is continuous at the point x_0, and at $V_0=\infty$ we obtain the conditions for the impenetrability of the *IS* boundary[4]:

$$\hat{g}_\omega(v_x,x_0)=\hat{g}_\omega(-v_x,x_0). \qquad (1.4)$$

The order parameter of an *SNS* system is described by the model

$$\Delta(x)=\Delta\exp\{{}^1/_2 i\varphi\,\mathrm{sign}\,x\}[\Theta(x-d/2)+\Theta(-x-d/2)], \qquad (1.5)$$

which is customarily used under the condition $d\gg\xi_0$ [d is the thickness of the normal-metal layer, ξ_0 is the coherence length of the superconductor, $\Theta(x)$ is the Heaviside function, and φ is the phase difference between the order parameters of the superconducting edges].

Using (1.1)–(1.3) and (1.5), we have at a low impurity density ($\Omega^2=\omega^2+\Delta^2$)

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$$j=\frac{2ep_F^2\Delta^2}{\pi}\sin\varphi\cdot T\sum_{\omega>0}\int_0^1\alpha d\alpha\left\{\Delta^2\left[\cos\varphi+\frac{V_0^2}{v_F^2\alpha^2}\operatorname{ch}\frac{4\omega x_0}{v_F\alpha}\right]\right.$$
$$\left.+\left(1+\frac{V_0^2}{v_F^2\alpha^2}\right)\left[(\omega^2+\Omega^2)\operatorname{ch}\frac{2\omega d}{v_F\alpha}+2\omega\Omega\operatorname{sh}\frac{2\omega d}{v_F\alpha}\right]\right\}^{-1}. \quad (1.6)$$

This result can of course be obtained also (albeit after more cumbersome calculations) with the aid of the expressions obtained in Ref. 2 for the exact Green's functions of a *SNINS* system. We note that Eq. (1.6) describes the stationary Josephson effect in impurity-free superconducting systems with weak couplings of various types at arbitrary values of V_0. In fact, let initially $d=0$. Then the current density takes the form

$$j=\frac{ep_F^2\Delta^2\sin\varphi}{4\pi}\int_0^1\alpha d\alpha\frac{D(\alpha)}{R(\alpha)}\operatorname{th}\left\{\frac{\Delta}{2T}R(\alpha)\right\}, \quad (1.7)$$

$$R(\alpha)=\{1-D(\alpha)\sin^2(\varphi/2)\}^{1/2},\quad D(\alpha)=\{1+(V_0/v_F\alpha)^2\}^{-1}.$$

At large V_0, Eq. (1.7) leads directly to the Ambegaokar-Baratov equation.[6] Equation (1.7) is suitable also for the description of such a state in short superconducting bridges. When the current density (1.7) is multiplied by the area of the opening, the results of Refs. 4 ($V_0=0$) and 7 ($V_0\neq 0$) are obtained directly.

Let now $d\neq 0$ (and in fact $d\gg\xi_0$). Then at $V_0=0$ Eq. (1.6) yields the well-known equation for the current density in an *SNS* junction.[8] On the other hand, if the resistance of the insulating interlayer is large: $R\gg 8\pi^2/e^2p_F^2$ [$R=2\pi^2V_0^2/e^2\mu^2$ (Ref. 9), and μ is the chemical potential], then we arrive at the result of Ref. 2 for *SNINS* junctions. These two limiting cases are precisely those of greatest interest. The case $V_0=0$ has been sufficiently well studied. We confine ourselves therefore below to the influence of external fields and impurities on the Josephson effect in *SNINS* junctions at large values of R (i.e., the tunneling through the insulating interlayer is weak). In Sec. 2 we obtain an expression for the stationary Josephson current in the field of nonmagnetic impurities. In the presence of such impurities, the discrete excitation spectrum of the *SNINS* junction becomes smeared out. This (as in the case of *SNS* junctions[10]) decreases the amplitude of the Josephson current. In Sec. 3 we investigate the influence of an external magnetic field on the Josephson effect in a *SNINS* system. It was shown in Ref. 2 that the magnetic field in an *INS* systems shifts the Andreev levels by an amount proportion to this field. In an *SNS* junction, the presence of a magnetic field leads to instability of the discrete spectrum.[11,12] It is clear in this connection that the Josephson behavior of *SNS*[13] and of *SNINS* junctions will be significantly different. In Sec. 4 is developed a microscopic theory of the tunnel current in the presence of an electric field in the system (the nonstationary Josephson effect).

2. STATIONARY JOSEPHSON EFFECT IN A *SNINS* JUNCTION WITH IMPURITIES

To take into account the presence of impurities in a system one can use the usual averaging technique.[5] In matrix formalism, the corresponding equation for the Green's function takes the following graphic form:

(2.1)

A thin line denotes here the matrix Green's function of a pure *SNS* junction,[8] a thick line corresponds to the analogous function for the *SNINS* junction, averaged over the disposition of the impurities, and the wavy line is used to introduce the vertex $V_0\hat{\tau}_3\delta(x-x_0)$. To solve Eq. (2.1) we use the condition that the parameter V_0^{-1} is small. It suffices for us to find an equation for the renormalized quantity $\tilde{\omega}$ in the zeroth order in this parameter (i.e., for the two isolated half-spaces $x<x_0$ and $x>x_0$). Taking (1.4) into account, we obtain after standard calculations (for simplicity, we present the result at low temperatures $T\ll T_c$)

$$\tilde{\omega}_\pm=\omega+\frac{v_F}{2l}\int_0^1 d\alpha\operatorname{th}\frac{\tilde{\omega}_\pm(d\mp 2x_0)}{v_F\alpha},\quad x\gtrless x_0. \quad (2.2)$$

Strictly speaking, we must also take into account the renormalization of Δ. We confine ourselves first to the case of a relatively small number of impurities, $l>\xi_0$. Under this condition, it is easy to show[10] that the renormalization of Δ can be neglected. Using the conditions (1.13) and Eq. (1.2) for the current, we arrive at

$$j=\frac{8\pi\sin\varphi}{eR}T\sum_{\omega>0}\int_0^1\alpha^3 d\alpha\left[\operatorname{ch}\frac{\tilde{\omega}_+(d-2x_0)}{v_F\alpha}\operatorname{ch}\frac{\tilde{\omega}_-(d+2x_0)}{v_F\alpha}\right]^{-1}. \quad (2.3)$$

It is seen that at $l>d$ the presence of impurities has little effect on the value of the current. In the opposite case, $l\ll d$, we get from (2.2) and (2.3)

$$j=\frac{16\pi}{eR}\sin\varphi\cdot T\int_0^1\alpha^3 d\alpha\left[\exp\left(\frac{d}{l\alpha}\right)\operatorname{sh}\frac{2d}{\xi_T\alpha}\right]^{-1},\quad \xi_T=\frac{v_F}{\pi T}. \quad (2.4)$$

At temperatures $v_F/d\ll T\ll T_c$ and at arbitrary values of l we obtain from (2.2) and (2.3)

$$j=\frac{8v_F}{eRd}\frac{T\sin\varphi}{T+2\pi v_F/l}\exp\left\{-\frac{2d}{\xi_T}-\frac{d}{l}\right\}. \quad (2.5)$$

In the case $l\ll d$, Eq. (2.4) for the current is exponentially small even at $T=0$:

$$j=\frac{8v_F}{eRd}\frac{l}{d}\sin\varphi\exp\left\{-\frac{d}{l}\right\}. \quad (2.6)$$

The results (2.4)–(2.6) are valid at $d/2-|x_0|\gg\xi_T$. At $|x_0|\sim d/2-\xi_T$ the tunnel current begins to decrease somewhat and at $x_0=\pm d/2$ (*SINS* junction) the corresponding expressions turn out to be half as large as expressions (2.4)–(2.6). This difference between *SINS* and *SNINS* systems (in the absence of impurities) was already pointed out in Ref. 2. The reason for this effect is that at low temperature the main contribution to the current is made by excitations with energy $\sim v_F/d\ll\Delta$. In *SINS* junctions these excitations are present only in one half-space, whereas in the *SNINS* system (at $d/2-|x_0|\gg\xi_T$) they are present on both sides of the insulating barrier.

Thus, the physical cause of the effect of impurities on the Josephson current in *SNINS* junctions (just as in *SNS* junctions[10]) is the smearing of the discrete spectrum of the levels as a result of scattering of the electronic excitations by impurities localized in the normal-metal layers. The actual difference from the case of *SNS* systems (under the analogous condition $l > \xi_0$) lies in the presence of two systems of levels, which leads to different contributions to the renormalization of the frequency on the opposite sides of the insulating interlayer.

We consider now the inverse case $l \ll \xi_0$. Under this condition the Eilenberger functions take the form

$$g_\omega(\mathbf{v}_F, \mathbf{r}) = G_\omega(\mathbf{r}) + \frac{\mathbf{v}_F}{v_F}\mathbf{G}_\omega(r), \quad f_\omega(\mathbf{v}_F, \mathbf{r}) = F_\omega(\mathbf{r}) + \frac{\mathbf{v}_F}{v_F}\mathbf{F}_\omega(\mathbf{r}), \tag{2.7}$$

and the properties of the system are described by the Usadel equations[14]:

$$2\Delta G_\omega(\mathbf{r}) - 2\omega F_\omega(\mathbf{r}) + D\frac{\partial}{\partial \mathbf{r}}\left[G_\omega(\mathbf{r})\frac{\partial}{\partial \mathbf{r}}F_\omega(\mathbf{r}) + \frac{1}{2}\frac{F_\omega(\mathbf{r})}{G_\omega(\mathbf{r})}\frac{\partial}{\partial \mathbf{r}}|F_\omega(\mathbf{r})|^2\right] = 0,$$
$$G_\omega(r) = \{1 - |F_\omega(\mathbf{r})|^2\}^{1/2}, \quad D = {}^1/_3 v_F l. \tag{2.8}$$

From the Eilenberger equations (1.1) it is also easy to obtain[14]

$$2\omega \mathbf{F}_\omega(\mathbf{r}) + v_F\frac{\partial}{\partial \mathbf{r}}F_\omega(\mathbf{r}) = 2\Delta \mathbf{G}_\omega(r) + \frac{v_F}{l}[\mathbf{G}_\omega(\mathbf{r})F_\omega(\mathbf{r}) - \mathbf{F}_\omega(\mathbf{r})G_\omega(\mathbf{r})]. \tag{2.9}$$

We now integrate the boundary conditions (1.3) over the directions of the vector $\mathbf{v}_F$. Then, taking (2.7) into account, we have

$$\frac{iv_F}{2V_0}(F_\omega(x_0+0) - F_\omega(x_0-0)) = F_{1\omega}(x_0+0) = F_{1\omega}(x_0-0) \tag{2.10}$$

with analogous relations for the G-functions. We have taken it into account here that in our problem only the x components of the vectors $\mathbf{F}$ and $\mathbf{G}$ differ from zero:

$$\mathbf{F}_\omega(\mathbf{r}) = \{F_{1\omega}(x), 0, 0\}, \quad \mathbf{G}_\omega(\mathbf{r}) = \{G_{1\omega}(x), 0, 0\}. \tag{2.11}$$

Using relations (2.9)–(2.11) under the condition $l \ll \xi_0$, we arrive at the boundary conditions for Eqs. (2.8):

$$\left.\frac{\partial F_\omega}{\partial x}\right|_{x=x_0+0} = \left.\frac{\partial F_\omega}{\partial x}\right|_{x=x_0-0} = \frac{iv_F}{2V_0 l}[G_\omega(x_0+0)F_\omega(x_0-0) - G_\omega(x_0-0)F_\omega(x_0+0)], \tag{2.12}$$

at $V_0 = 0$ there follows from (2.12) the usual condition of the continuity of the Usadel functions at the point x_0, and at $V_0 = \infty$ we obtain the condition $\partial F_\omega/\partial x = 0$ on the boundary with an impenetrable insulator[14].

In the case of interest to us, that of large values of the parameter V_0, the problem is solved by successive approximations in V_0^{-1}. In the zeroth approximation, the Usadel equations in a normal-metal layer can be reduced to an equation of the physical-pendulum type:

$$\frac{d^2\alpha_\omega^\pm}{dx^2} = \frac{2\omega}{D}\sin\alpha_\omega^\pm, \quad x \gtrless x_0. \tag{2.13}$$

Here $F_\omega^{(0)} = \sin\alpha_\omega^\pm \exp\{\pm i\varphi/2\}$. We assume that the F-function on the *NS* boundary is equal to its value in the interior of the superconductor in the absence of current. This condition is valid at $T \ll T_c$ (and also for bridge structures at higher temperatures). This is easily verified by considering two half-spaces occupied by a normal metal and a superconductor. We assume that the order parameter changes jumpwise on the *NS* boundary from zero to its equilibrium value Δ in the interior of the superconductor. Then the Usadel equations also reduce to the physical-pendulum equation

$$\frac{d^2\alpha_\omega}{dx^2} = \frac{2\Omega}{D}\sin(\alpha_\omega - \delta), \quad \delta = \arcsin\frac{\Delta}{\Omega}. \tag{2.14}$$

Let the regions occupied by the normal metal and by the superconductor be located on the left and on the right, respectively. The solutions of Eqs. (2.13)–(2.14) can be easily obtained. In the normal metal

$$\alpha_\omega = 4\,\mathrm{arctg}\exp\{(2\omega/D)^{1/2}x + C_1\}, \tag{2.15}$$

and in the superconductor

$$\alpha_\omega = \delta - 4\,\mathrm{arctg}\exp\{-(2\Omega/D)^{1/2}x + C_2\}, \tag{2.16}$$

C_1 and C_2 are arbitrary constants. Matching expressions (2.15) and (2.16) as well as their derivatives on the *NS* boundary, we find that at $\omega \ll \Delta$ the Usadel function on the *NS* boundary is equal to its equilibrium value Δ/Ω. In the temperature region $T \ll T_c$ the main contribution to the current is determined precisely by this region of Matsubara frequencies.

Thus, the boundary conditions for Eq. (2.13) take the form

$$\alpha_\omega^\pm\left(\pm\frac{d}{2}\right) = \arcsin\frac{\Delta}{\Omega}, \quad \left.\frac{d\alpha_\omega^\pm}{dx}\right|_{x=x_0} = 0. \tag{2.17}$$

The solutions of Eq. (2.13) with analogous boundary conditions were already investigated in detail by us when we obtained the stationary states of a Josephson junction in an inhomogeneous magnetic field.[15] For the case of interest to us the $\alpha_\omega^\pm(x)$ are determined from the equations

$$\left(\frac{2\omega}{D}\right)^{1/2}(x - x_0) = \pm K\left(\cos\frac{\alpha_\omega^\pm(x)}{2}\right) \mp F\left(\chi^\pm, \cos\frac{\alpha_\omega^\pm(x)}{2}\right),$$
$$\chi^\pm = \arccos\left[(\cos\alpha_\omega^\pm(x_0) - \cos\alpha_\omega^\pm(x))^{1/2}/2^{1/2}\cos\alpha_\omega^\pm(x)\right], \quad x \gtrless x_0, \tag{2.18}$$

where

$$F(\chi, z) = \int_0^\chi \frac{dy}{(1 - z^2\sin^2 y)^{1/2}}, \quad K(z) = F\left(\frac{\pi}{2}, z\right)$$

are respectively incomplete and complete elliptic integrals of the first kind.

Let now $F_\omega^\pm = F_\omega^{(0)\pm} + F_\omega^{(1)\pm}$, where

$$F_\omega^{(1)\pm} = \beta_\omega^\pm \cos\alpha_\omega^\pm \exp(\pm i\varphi/2) \tag{2.19}$$

is the first approximation for the Usadel function in terms of the parameter V_0^{-1}. The functions $\beta_\omega^\pm$ satisfy in the normal-metal layer the equation

$$\frac{d^2\beta_\omega^\pm}{dx^2}=\frac{2\omega}{D}\beta_\omega^\pm\cos\alpha_\omega^\pm,\quad |x|\leqslant\frac{d}{2}. \tag{2.20}$$

In the superconducting regions the equations for $\beta_\omega^\pm$ assume the very simple form

$$\frac{d^2\beta_\omega^\pm}{dx^2}-\frac{2\Omega}{D}\beta_\omega^\pm=0,\quad x\gtrless\pm\frac{d}{2}. \tag{2.21}$$

At infinity, the functions $\beta_\omega^\pm$ should tend to zero. Using this and matching $F_\omega^{(1)\pm}$ and their derivatives at the points $\mp d/2$, we obtain

$$\frac{d\beta_\omega^\pm}{dx}\Big|_{x=\pm d/2}+\left[2\Delta\left(\frac{\Omega-\omega}{2\omega\Omega}\right)^{1/2}\pm\left(\frac{2\Omega}{D}\right)^{1/2}\right]\beta_\omega^\pm\left(\pm\frac{d}{2}\right)=0. \tag{2.22}$$

The general solution of (2.20) is of the form

$$\beta_\omega^\pm(x)=A_\pm h_\pm(x)+B_\pm h_\pm(x)\int^x\frac{dx}{h_\pm^2(x)},\quad h_\pm(x)=\frac{d\alpha_\omega^\pm}{dx}. \tag{2.23}$$

By substituting (2.23) in the boundary conditions (2.12) and (2.22) we can determine the arbitrary constants $A_\pm$ and $B_\pm$.

The current density in the system is obtained from the equation

$$j=-2ieN(0)\pi TD\sum_{\omega>0}(F_\omega^*\nabla F_\omega-F_\omega\nabla F_\omega^*). \tag{2.24}$$

The presented relations enable us to calculate the Josephson current at arbitrary temperatures. The final result for the current density is expressed in terms of elliptic functions and is too unwieldy to present here. However, under the condition

$$T\gg D/d^2 \tag{2.25}$$

the analysis becomes much simpler. Let initially $x_0=-d/2$ (*SINS* junction). The quantity $\alpha_\omega^+(x_0)$ is easily calculated from Eqs. (2.18) with allowance for (2.25) (see Ref. 15):

$$\alpha_\omega^+(x_0)=\frac{8\Delta}{\Omega+\omega+[2\Omega(\Omega+\omega)]^{1/2}}\exp\left\{-d\left(\frac{2\omega}{D}\right)^{1/2}\right\}. \tag{2.26}$$

If (2.25) is satisfied, Eq. (2.20) has a solution that satisfies the boundary condition (2.12) and takes the form (we assume here $\sin\alpha_\omega^-=\Delta/\Omega$)

$$\beta_\omega^+=-\frac{iv_F}{2V_0l}\left(\frac{D}{2\omega}\right)^{1/2}\left[\frac{\omega}{\Omega}\frac{d\alpha_\omega^+}{dx}+\frac{\Delta}{\Omega}e^{-i\varphi}\exp\left\{-x\left(\frac{2\omega}{D}\right)^{1/2}\right\}\right], \tag{2.27}$$

and this expression is valid at distances not too close to the point $x=d/2$ [under the condition $d/2-x\gg(d/\omega)^{1/2}$]. From (2.24) and (2.27) we obtain directly the result for the Josephson-current density in a dirty *SINS* junction:

$$j=\frac{32\pi^3d(2\pi TD)^{1/2}}{e^3RR_Np_F^2l^2}\frac{\Delta^2\sin\varphi}{\pi^2T^2+\Delta^2}\frac{T}{W}\exp\left\{-d\left(\frac{2\pi T}{D}\right)^{1/2}\right\}, \tag{2.28}$$
$$W=(\pi^2T^2+\Delta^2)^{1/2}+\pi T+[2(\pi^2T^2+\Delta^2)+2\pi T(\pi^2T^2+\Delta^2)^{1/2}]^{1/2}.$$

Here $R_N=d/DN(0)e^2$ is the resistance per unit area of the normal-metal layer. At $T\ll T_c$ the general equation (2.28) goes over into

$$j=32\pi^3(2-\sqrt{2})\frac{T}{\Delta}\frac{d(\pi TD)^{1/2}\sin\varphi}{e^3RR_Np_F^2l^2}\exp\left\{-d\left(\frac{2\pi T}{D}\right)^{1/2}\right\}. \tag{2.29}$$

Let now $d/2-|x_0|\gg(D/T)^{1/2}$ (*SNINS* junction). The calculations yield in this case

$$j=\frac{32\pi^2d(2\pi TD)^{1/2}\Delta^2\sin\varphi}{e^3RR_Np_F^2l^2W^2}\exp\left\{-d\left(\frac{2\pi T}{D}\right)^{1/2}\right\}. \tag{2.30}$$

At $T\ll T_c$ we obtain

$$j=32\pi^2(3\sqrt{2}-4)\frac{d(\pi TD)^{1/2}\sin\varphi}{e^3RR_Np_F^2l^2}\exp\left\{-d\left(\frac{2\pi T}{D}\right)^{1/2}\right\}. \tag{2.31}$$

We see that in the considered case $l\ll\xi_0$ the Josephson current (just as in the case of a low impurity density) depends on the location of the insulating interlayer inside the normal-metal layer. Thus, the current density in a *SNINS* junction turns out to be considerably higher than the analogous value for the *SINS* junction [cf. (2.29) and (2.31)]. It is seen also that the expressions for the current in the cases $l\gg\xi_0$ and $l\ll\xi_0$ are substantially different. The role of the coherence length in a normal-metal layer under the conditions $l\ll\xi_0$ (just as in the case of *SNS* junctions[16]) is assumed by the quantity $(D/2\pi T)^{1/2}$, whereas for a small number of impurities ($l\gg\xi_0$) the analogous quantity is $(2\pi T/v_F+1/l)^{-1}$.

3. EFFECT OF EXTERNAL MAGNETIC FIELD ON THE JOSEPHSON CURRENT

Let now an external magnetic field H directed along the z axis (parallel to the *NS* boundaries) be present in the normal-metal layer of an impurity-free *SNINS* junction. It is easy (for details see Ref. 2) to obtain a dispersion equation for the energy of the excitations under these conditions. It takes the form ($E\ll\Delta$):

$$\cos\left[\frac{2Ed}{v_x}-\gamma_++\gamma_-\right]+\frac{\cos[4Ex_0/v_x+\gamma_++\gamma_-]}{1+v_x^2/V_0^2}+\frac{\cos\tilde\varphi}{1+V_0^2/v_x^2}=0, \tag{3.1}$$

$$\gamma_\pm=\frac{ev_yH}{v_x}\left(\frac{d}{2}\mp x_0\right)^2,\quad \tilde\varphi=\varphi-2eHy(d+2\lambda).$$

Here v_y is the electron-excitation velocity component in the y direction, and λ is the depth of penetration of the magnetic field in the superconductor. Strictly speaking, Eq. (3.1) is contradictory, since an eigenvalue of the energy E should not depend on the coordinates. This is in fact that cause of the instability of the spectrum in *SNS* junctions.[11,12] However, if the transparency of the insulator is low, then it follows from (3.1) in the principal approximation

$$\cos\left[\frac{2Ed}{v_x}-\gamma_++\gamma_-\right]+\cos\left[\frac{4Ex_0}{v_x}+\gamma_++\gamma_-\right]=0. \tag{3.2}$$

It is seen that in this case there is no smearing, and only a shift of the levels by an amount proportional to the magnetic field H and to v_y (Ref. 2) and the result is valid for arbitrary H.

The substantial differences in the behavior of the spectra of the *SNS* and *SNINS* junctions in an external magnetic field should determine also the different influence of this field on the stationary Josephson effects in these junctions. In the case of the *SNINS* system it is quite easy to take the magnetic field into account. The shift of the levels (3.2) leads to a renormalization of the Matsubara frequency. Replacing E in (3.2) by $-i\omega$, we obtain directly

$$\tilde{\omega}_{\pm}=\omega\mp\frac{1}{4}ieHv_F(d\mp 2x_0)\sin\theta\cos\chi,\quad x\gtrless x_0. \tag{3.3}$$

Here $v_x=v_F\cos\theta$ and $v_y=v_F\sin\theta\cos\chi$. Thus, the magnetic field "shifts" the frequency along the imaginary axis [in contrast to impurities, whose presence leads to the frequency shift (2.2) along the real axis].

The remaining calculations are quite similar to those of Sec. 2. We obtain ultimately ($T\ll T_c$)

$$j=\frac{2\sin\tilde{\varphi}}{eR}T\sum_{\omega}\int_0^{2\pi}d\chi\int_0^1\cos^3\theta d(\cos\theta)\left[\operatorname{ch}\frac{\tilde{\omega}_+(d-2x_0)}{v_F\cos\theta}\operatorname{ch}\frac{\tilde{\omega}_-(d+2x_0)}{v_F\cos\theta}\right]. \tag{3.4}$$

At $T=0$, expression (3.4) for a symmetrical *SNINS* junction ($x_0=0$) goes over into

$$j=\frac{2v_F\sin\varphi}{\pi eRd}\int_0^{2\pi}d\chi\int_0^1\cos^4\theta d(\cos\theta)\frac{\arcsin Q(H)}{Q(H)},$$
$$Q(H)=\sin\left\{\frac{eHd^2}{2}\operatorname{tg}\theta\cos\chi\right\}. \tag{3.5}$$

The dependence of the critical current density on the magnetic field is shown in Fig. 1. It is seen that the magnetic field leads to a certain increase of j_c and also to oscillations of this quantity. We emphasize that we are dealing here precisely with the current density. On the other hand, the total current through a junction with dimensions of the order of (or larger than) the Josephson penetration depth will obviously be substantially decreased by the interference effect.

For a *SINS* junction, (3.4) ($T=0$) yields

$$j=\frac{\pi}{5}\frac{v_F}{eRd}\sin\varphi,$$

i.e., in this case the magnetic field does not influence the amplitude of the Josephson current. At $T\gg v_F/d$ the current receives contributions only from excitations whose velocity component v_x differs little from v_F ($\theta\lesssim v_F/Td)^{1/2}$). For these excitations, the level shift is small and consequently the presence of a magnetic field in the normal-metal layer has no effect whatever on the critical density of the tunnel current at these temperatures, and this holds for all x_0.

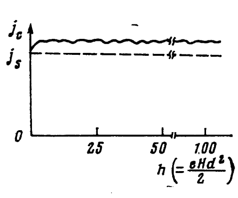


FIG. 1.

We see that the Josephson behavior of a *SNINS* system in an external magnetic field differs not at all trivially and quite substantially from the analogous behavior of *SNS* junctions. In the case of the latter, the dependence of the Josephson current on the magnetic field is quite complicated.[13] For *SNINS* junctions the amplitude of the tunnel current depends on the magnetic field only at low temperatures, $T\lesssim v_F/d$, and this dependence is somewhat different for different values of x_0. The phase dependence of such a current remains sinusoidal at all temperatures.

4. NONSTATIONARY JOSEPHSON EFFECT IN AN *SNINS* JUNCTION

When a potential difference is produced between the superconducting edges, the tunnel junction begins to depend on the time. Since the resistance of the insulating barrier is large, the entire voltage drop is precisely across this barrier. Let the potential difference between the superconductors be $V(t)$. The distribution of the electric potential in space can then be written in the form

$$\Phi(t)={}^1/_2V(t)\operatorname{sign}(x-x_0). \tag{4.1}$$

We assume hereafter that there is no vector potential in the system.

The Gor'kov equations in the Keldysh method[17] take for this system the form[18]

$$\left(\left\{i\hat{\tau}_3\frac{\partial}{\partial t}+\frac{1}{2m}\frac{\partial^2}{\partial\mathbf{r}^2}+\hat{\Delta}-e\Phi-V_0\delta(x-x_0)+\mu\right)\check{1}-\check{\Sigma}\right\}$$
$$\times\check{G}(r,r';t,t')=\check{1}\delta(t-t')\delta(\mathbf{r}-\mathbf{r}'). \tag{4.2}$$

Here

$$\check{G}=\begin{pmatrix}\hat{G}^R & \hat{G}\\ 0 & \hat{G}^A\end{pmatrix},\quad \check{\Sigma}=\begin{pmatrix}\hat{\Sigma}^R & \hat{\Sigma}\\ 0 & \hat{\Sigma}^A\end{pmatrix}.$$

With the aid of (4.2) we can obtain for the energy-integrated Green's functions quasiclassical equations that go over into (1.1) in the stationary case. Such equations were first obtained by Eliashberg.[19] In our case they take the form

$$\mathbf{v}_F\frac{\partial\check{G}}{\partial\mathbf{R}}+\hat{\tau}_3\frac{\partial\check{G}}{\partial t}+\frac{\partial\check{G}}{\partial t'}\hat{\tau}_3+[(ie\Phi(t)-i\hat{\Delta}(t))\check{1},\check{G}]$$
$$+i\int_{-\infty}^{\infty}dt_1\{\check{\Sigma}_{tt_1}\check{G}_{t_1t'}-\check{G}_{tt_1}\check{\Sigma}_{t_1t'}\}=0, \tag{4.3}$$

where

$$\check{G}=\check{G}(\mathbf{v}_F,\mathbf{R};t,t'),\quad \check{\Sigma}=\frac{v_F}{2l}\langle\check{G}(\mathbf{v}_F,\mathbf{R};t,t')\rangle+\check{\Sigma}_{ph}.$$

The Green's function G satisfies the normalization condition[18]

$$\int_{-\infty}^{\infty} dt_1 \check{G}_{tt_1}\check{G}_{t_1t'} = \check{1}\cdot\delta(t-t').$$

In addition, in perfect analogy with the stationary case, we get from (4.2) boundary conditions for G at the point x_0 (we leave out the time arguments here):

$$\begin{aligned}&\check{G}(v_x, x_0+0)-\check{G}(-v_x, x_0+0)\\&=\check{G}(v_x, x_0-0)-\check{G}(-v_x, x_0-0)\\&=i(v_x/V_0)\{\check{G}(v_x, x_0+0)-\check{G}(v_x, x_0-0)\}.\end{aligned} \tag{4.4}$$

In (4.4), just as in (1.3), we made allowance for the fact that $\check{G}$ depends in our problem on x and v_x.

Equations (4.3) have already been used to solve a number of problems in the theory of weak superconductivity (for example, to describe the nonstationary Josephson effect in short microbridges[20,21]). The solutions of Eqs. (4.3) in the superconducting regions are sought in the form

$$\check{G}_\pm(v_x, x)=\check{A}_\pm+\exp\left\{-\check{K}_\pm\frac{(x\mp d/2)}{v_x}\right\}\check{B}_\pm(v_x)\exp\left\{\check{K}_\pm\frac{(x\mp d/2)}{v_x}\right\}, x\gtrless\frac{d}{2}. \tag{4.5}$$

The matrices $\check{A}_\pm$ are known[20]:

$$\check{A}_\pm=\begin{pmatrix}\hat{A}^R_\pm & \hat{A}_\pm\\ 0 & \hat{A}^A_\pm\end{pmatrix},\quad \hat{A}^{R(A)}_\pm=\hat{S}_\pm(t)\int\hat{g}^{R(A)}(\varepsilon)e^{-i\varepsilon(t-t')}\frac{d\varepsilon}{2\pi}\hat{S}^+_\pm(t),$$
$$\hat{g}^{R(A)}=g^{R(A)}(\varepsilon)\hat{\tau}_3+f^{R(A)}(\varepsilon)i\hat{\tau}_2,\quad g^{R(A)}(\varepsilon)=\frac{\varepsilon}{[(\varepsilon\pm i0)^2-\Delta^2]^{1/2}}, \tag{4.6}$$
$$f^{R(A)}(\varepsilon)=\frac{\Delta}{[(\varepsilon\pm i0)^2-\Delta^2]^{1/2}},$$
$$\hat{S}^\pm(t)=\begin{pmatrix}\exp\{\pm i\varphi(t)/4\} & 0\\ 0 & \exp\{\mp i\varphi(t)/4\}\end{pmatrix},$$
$$\varphi(t)=\varphi+2e\int^t V(t_1)dt_1,\quad \hat{\tau}_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}.$$

It can be shown[21] that the matrices A and B are connected by the relations

$$\check{B}_\pm(v_x)\check{A}_\pm=\pm\check{B}_\pm(v_x)\,\mathrm{sign}\, v_x. \tag{4.7}$$

The following condition also obtains

$$\begin{aligned}&\hat{A}_\pm=\hat{A}_\pm{}^R\hat{n}_\pm-\hat{n}_\pm\hat{A}^A,\\&\hat{n}_\pm(t,t')=\hat{S}_\pm(t)\int\mathrm{th}\frac{\varepsilon}{2T}e^{-i\varepsilon(t-t')}\frac{d\varepsilon}{2\pi}\hat{S}_\pm{}^+(t).\end{aligned} \tag{4.8}$$

The solution of Eqs. (4.3) at $|x|\lesssim d/2$ (i.e., at $\Delta=0$) is trivial. It yields (in the absence of impurities)

$$\begin{aligned}&\check{G}(v_x, x_0\pm0)=\hat{C}_\pm(v_x)(\check{A}_\pm+\check{B}_\pm(v_x))\hat{C}_\pm{}^+(v_x),\\&\hat{C}_\pm(v_x;t,t')=\hat{S}_\pm(t)\int\hat{C}_\pm(v_x,\varepsilon)e^{-i\varepsilon(t-t')}\frac{d\varepsilon}{2\pi}\hat{S}_\pm{}^+(t'),\end{aligned} \tag{4.9}$$
$$\hat{C}_\pm(v_x,\varepsilon)=\begin{pmatrix}\exp\left\{\mp i\frac{\varepsilon(d\mp2x_0)}{v_x}\right\} & 0\\ 0 & \exp\left\{\pm i\frac{\varepsilon(d\mp2x_0)}{v_x}\right\}\end{pmatrix}.$$

It remains now to obtain the matrices $\check{B}_\pm(v_x)$. Since the resistance of the insulating barrier is high, we can represent these matrices in the form

$$\check{B}_\pm=\check{B}_\pm^{(0)}+\frac{v_x}{V_0}\check{B}_\pm^{(1)}+\frac{v_x{}^2}{V_0{}^2}\check{B}_\pm^{(2)}\ldots, \tag{4.10}$$

and to find the current, as usual, it is necessary to calculate the term $\sim V_0^{02}$ of this expansion. Expressing the current in terms of the matrix $\hat{B}$, we obtain

$$j(t)=-\frac{\pi}{eR}\int_{-1}^{1}\alpha^3 d\alpha\,\mathrm{Sp}\,\hat{\tau}_3\hat{B}^{(2)}(v_F\alpha;t,t). \tag{4.11}$$

Using (4.4)–(4.11) we arrive at the final result

$$j(t)=\mathrm{Im}\int\int d\omega d\omega'\{W(\omega)W^*(\omega')e^{i(\omega-\omega')t}j_q(\omega')+e^{i\varphi}W(\omega)W(\omega')e^{i(\omega+\omega')t}j_p(\omega')\},$$
$$\mathrm{Re}\, j_q(\omega)=\frac{i}{2eR}\int_0^1\alpha^3 d\alpha\int_{-\infty}^{\infty}d\varepsilon\{T_+(\varepsilon)[X^{RR}Y^{RR}-X^{AA}Y^{AA}]+T_-(\varepsilon)[X^{RA}Y^{RA}-X^{AR}Y^{AR}]\},$$
$$\mathrm{Im}\, j_q(\omega)=\frac{1}{2eR}\int_0^1\alpha^3 d\alpha\int_{-\infty}^{\infty}d\varepsilon T_-(\varepsilon)\{X^{RR}Y^{RR}-X^{AR}Y^{AR}+X^{AA}Y^{AA}-X^{RA}Y^{RA}\}, \tag{4.12}$$
$$\mathrm{Re}\, j_p(\omega)=\frac{i}{eR}\int_0^1\alpha^3 d\alpha\int_{-\infty}^{\infty}d\varepsilon\{T_+(\varepsilon)[f^R(\varepsilon-\omega)f^R(\varepsilon)Y^{RR}-f^A(\varepsilon-\omega)f^A(\varepsilon)Y^{AA}]+T_-(\varepsilon)[f^A(\varepsilon-\omega)f^R(\varepsilon)Y^{RA}-f^R(\varepsilon-\omega)f^A(\varepsilon)Y^{AR}]\},$$
$$\mathrm{Im}\, j_p(\omega)=\frac{1}{eR}\int_0^1\alpha^3 d\alpha\int_{-\infty}^{\infty}d\varepsilon T_-(\varepsilon)\{f^R(\varepsilon-\omega)[f^R(\varepsilon)Y^{RR}-f^A(\varepsilon)Y^{AR}]+f^A(\varepsilon-\omega)[f^A(\varepsilon)Y^{AA}-f^R(\varepsilon)Y^{RA}]\},$$
$$X^{ij}=[1+g^i(\varepsilon)g^j(\varepsilon-\omega)]\cos\beta_+-[1-g^i(\varepsilon)g^j(\varepsilon-\omega)]\cos\beta_-+i[g^i(\varepsilon)+g^j(\varepsilon-\omega)]\sin\beta_+-i[g^i(\varepsilon)-g^j(\varepsilon-\omega)]\sin\beta_-,$$
$$Y^{ij}=\{[1+g^i(\varepsilon)g^j(\varepsilon-\omega)]\cos\beta_++[1-g^i(\varepsilon)g^j(\varepsilon-\omega)]\cos\beta_-+i[g^i(\varepsilon)+g^j(\varepsilon-\omega)]\sin\beta_++i[g^i(\varepsilon)-g^j(\varepsilon-\omega)]\sin\beta_-\}^{-1},\quad i(j)=R, A,$$
$$T_\pm(\varepsilon)=\mathrm{th}\frac{\varepsilon}{2T}\pm\mathrm{th}\frac{\varepsilon-\omega}{2T},$$
$$\beta_\pm=\frac{\varepsilon(d-2x_0)}{v_F\alpha}\pm\frac{(\varepsilon-\omega)(d+2x_0)}{v_F\alpha}.$$

In the equation (4.2) for the current we used the usual representation[22]:

$$\exp\left\{ie\int^t V(t_1)dt_1\right\}=\int_{-\infty}^{\infty}W(\omega)e^{i\omega t}d\omega.$$

At $V=0$, Eqs. (4.12) lead to the result of Ref. 2 for the stationary Josephson current in a *SNINS* system in the absence of impurities. In the case $d=x_0=0$ the result (4.12) coincides with the known expression for the current in an *SIS* junction, first obtained by the tunnel-Hamiltonian method.[22,23]

We proceed now to investigate the expressions obtained for wide *SNINS* junctions. Let first the voltage V on the barrier be constant in time. Then

$$W(\omega)=\delta(\omega-eV).$$

In this case j_q does not depend on the time, and $j_p(t)$ takes the form

$$j_p(t)=j_1\sin\varphi(t)+j_2\cos\varphi(t).$$

At $T=0$ we obtain, after the necessary manipulations,

$$j_0=\operatorname{Im} j_q(eV)=j_2=0,\quad e|V|\leqslant\Delta;\quad j_1=j^r+j_1^a,$$

$$j_1^{(r)}\left(x_0=\pm\frac{d}{2}\right)=\begin{cases}\dfrac{\pi v_F}{10eRd}\left(1+\dfrac{\Delta}{(\Delta^2-e^2V^2)^{1/2}}\right), & \dfrac{v_F}{d}\ll e|V|,\quad \Delta-e|V|\gg\dfrac{v_F}{d}\\ \dfrac{2}{9eR}\left(\dfrac{2\pi v_F\Delta}{d}\right)^{1/2}\sum\limits_{k=0}^{\infty}\dfrac{(-1)^k}{(2k+1)^{1/2}}, & |\Delta-e|V||\ll\dfrac{v_F}{d}\end{cases};$$

$$j_1^r(x_0=0)=\frac{4v_F}{eRd}\int_0^1\alpha^4d\alpha\frac{\operatorname{arctg}(\operatorname{tg}(Vd/v_F\alpha+\chi))}{\sin(Vd/v_F\alpha+\chi)},\tag{4.13}$$

$$\chi=\arcsin(V/d),\quad v_F/d\ll e|V|,\quad \Delta-e|V|\gg v_F/d;$$

$$j_1^a\left(x_0=\pm\frac{d}{2}\right)=\frac{2\pi v_F}{eRd}\int_0^1\alpha^4d\alpha\sum_{n=0}^{N}\frac{(\Delta^2-\varepsilon_n^2)^{1/2}}{\sin(2\varepsilon_nd/v_F\alpha)[\Delta^2-(\varepsilon_n-eV)^2]^{1/2}},$$
$$e|V|<\Delta;$$

$$j_1^a(x_0=0)=\frac{4\pi v_F}{eRd}\int_0^1\alpha^4d\alpha\sum_{m=0}^{M}\frac{(\Delta^2-\varepsilon_n^2)^{1/2}}{\sin(\varepsilon_md/v_Fd)}\left\{[\Delta^2-(\varepsilon_m-eV)^2]^{1/2}\right.$$
$$\left.\times\cos\frac{(\varepsilon_m-eV)d}{v_F\alpha}+(eV-\varepsilon_m)\sin\frac{(\varepsilon_m-eV)d}{v_F\alpha}\right\}^{-1},\quad e|V|<\Delta.$$

Here ε_n and ε_m are the positive roots of the equations

$$\operatorname{tg}\frac{2\varepsilon_nd}{v_F\alpha}=\frac{(\Delta^2-\varepsilon_n^2)^{1/2}}{\varepsilon_n},$$
$$\operatorname{tg}\frac{\varepsilon_md}{v_F\alpha}=\frac{(\Delta^2-\varepsilon_m^2)^{1/2}}{\varepsilon_m},\tag{4.14}$$

and N and M are maximal numbers, such that ε_N, $\varepsilon_M\leqslant e|V|$. Under the condition $v_F/d\ll e|V|\ll\Delta$, expressions (4.14) take the form

$$j_1^a\left(x_0=\pm\frac{d}{2}\right)=\frac{2v_F^2}{e^2RVd^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\sin\frac{2eVd}{v_F}(2n+1),$$
$$j_1^a(x_0=0)=\frac{2v_F}{eRd}\ln\left|\frac{1+\cos(eVd/v_F)}{1-\cos(eVd/v_F)}\right|.\tag{4.15}$$

It is seen that the current-voltage characteristics of *SNINS* systems is far from trivial already at $e|V|<\Delta$, and its form also differs substantially for different positions of the insulating barrier. The normal part j_1^r of the Josephson current in a *SINS*-system increases monotonically with increasing V and reaches a maximum at $eV=\Delta$. The increase of the current at $eV\sim\Delta$ is a consequence of the presence of a square-root singularity in the state density of the superconductor. The anamolous term j_1^a of such a system oscillate with a period $\pi v_F/d$, since the position of the singularities in the state density in the normal-metal layer changes with increasing potential difference across the barrier. The current j_1^r $(x_0=0)$ remains practically unchanged in magnitude $(e|V|<\Delta)$ and only oscillates weakly with a period $2\pi v_F/d$. The formal expression for the anomalous part of the Josephson current contains in this case, as seen from (4.15), logarithmic divergences at the points $V=\pi v_Fn/ed$. This is caused by the fact that the Green's functions of the system have pole singularities that reflect the presence of discrete levels. This behavior of the current $j_1(V)$ in a *SNINS* system recalls in a certain sense the behavior of the analogous current in *SIS* junctions, for which the expression diverges logarithmically if $eV=2\Delta$.[22,23] This singularity of the voltage dependence of the Josephson current (called the Riedel peak[24]) in *SIS* junctions is due to the presence of a square-root divergence in the state density of the superconductor at energies close to the gap in the spectrum. The divergence in the expression for the current j_1 in a *SNINS* system results from the presence of a discrete spectrum on both sides of the insulating barrier.

The current divergences in *SNINS* systems (just as in *SIS* systems) are, however, only formal. Actually the discrete levels should be somewhat broadened by inelastic relaxation, which can be due to various causes (for example, inhomogeneity of the *NS* boundaries, the presence of a small number of impurities, etc.). Equation (4.15) is therefore not valid in the vicinities of the divergence points. It can be replaced near such points, with logarithmic accuracy, by

$$j_1^a(x_0=0)=\pm\frac{4v_F}{eRd}\ln\frac{d}{\tau_{\varepsilon N}v_F}.\tag{4.16}$$

Here $\tau\varepsilon_N$ is the characteristic time of the energy relaxation of the quasiparticles in the N layer. In addition, all the results are valid only if the characteristic tunneling frequency ν_T, which has the meaning of the reciprocal time of the electron "jumping" through the insulating barrier, in comparison with $1/\tau\varepsilon_N$ and the analogous quantity $1/\tau\varepsilon_S$ for a superconductor, thereby ensuring smallness of the nonequilibrium effects. In other words, it was assumed that the current could be calculated by perturbation theory. If (4.16) is large, so that the indicated conditions are not satisfied, then the employed calculation method is not valid in the vicinity of the points $V=\pi v_Fn/ed$. We shall not dwell here in greater detail on this question.

The result (4.13) is quite understandable. Its explanation is that at $e|V|>\Delta$ there are not quasiparticle excitations in the superconductor. Therefore an electron that has tunnelled through the insulating barrier from the Fermi surface of the normal-metal layer may turn out to be above the superconductor gap only under the condition[1)] $e|V|>\Delta$. We recall for comparison that a similar situation obtains in an *SIS* junction. The normal current j_0 and the current j_2 (frequently called the quasiparticle and pair interference current) are in this case (likewise $T=0$) different from zero at voltages $e|V|>2\Delta$, inasmuch as under this condition a pair can break in one superconductor and electrons can tunnel into the continuous spectrum above the gap of the other superconductor. We note also that the result (4.13) is valid at any position of the insulator layer inside the normal metal. We emphasize likewise that (4.13) is valid only in the absence of impurities $(l>d)$. The normal-metal layer acquires in a certain sense the properties of a superconductor even if the *BCS*-interaction constant is zero, since there are no reasons that could lead to a destruction of the superconducting correlation between the Cooper pairs that pass through the N layer. If, however, impurities are now introduced into the normal-metal layer, the situation changes markedly. Thus, at $l \ll d$ the presence of the superconducting edges will have little effect on the state density in the normal metal near the insulating interlayer. In this case the result for the normal current in a *SINS* system coincides in the principal approximation with the well-known expression for the current in an *SIN* junction:

$$j_0 = \frac{1}{2eR}\int_{-\infty}^{\infty}\left(\operatorname{th}\frac{\varepsilon+eV}{2T} - \operatorname{th}\frac{\varepsilon}{2T}\right)\frac{|\varepsilon|\Theta(|\varepsilon|-\Delta)}{(\varepsilon^2-\Delta^2)^{1/2}}d\varepsilon,$$

and in a *SNINS* system the current-voltage characteristic will differ little from Ohms law.

Let us return, however, to the case when there are no impurities. We present now expressions for the currents in the *SINS* system ($x_0 = \pm d/2$) at $v_F/d \ll e|V| - \Delta \ll \Delta$ $(T=0)$:

$$j_0 = \frac{2\operatorname{sign}V}{3eR}\frac{e|V|-\Delta}{e|V|+\Delta}(e|V|+2\Delta) + \frac{V}{R(e|V|-\Delta)}$$
$$\times\left(\frac{\pi v_F^3}{2d^3(e|V|+\Delta)}\right)^{1/2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{3/2}}\sin\frac{4nd(e|V|-\Delta)}{v_F};$$
$$j_1^r = \frac{v_F^2}{6eRd^2}\frac{\Delta}{(e|V|-\Delta)(e^2V^2-\Delta^2)^{1/2}}G, \qquad (4.17)$$

$$j_1^a = \frac{2\pi v_F}{eRd}\int_0^1 \alpha^4 d\alpha \sum_{n=N_1}^{N_2}\frac{1}{\sin(2\varepsilon_n d/v_F\alpha)}\left(\frac{\Delta^2-\varepsilon_n^2}{\Delta^2-(\varepsilon_n-eV)^2}\right)^{1/2} + I_c(V),$$
$$j_2 = \frac{2\Delta\operatorname{sign}V}{eR(e|V|-\Delta)}\left(\frac{\pi v_F^3}{d^3(e|V|+\Delta)}\right)^{1/2} \qquad (4.18)$$
$$\times\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^{3/2}}\cos\left\{(2n+1)\frac{2d(e|V|-\Delta)}{v_F}\right\}.$$

Here G is Catalan's constant, ε_n is determined from (4.20), N_1 is a minimal number such that $\varepsilon_{N_1} \geqslant eV - \Delta$ and N_2 is a maximal number such that $\varepsilon_{N_2} \leqslant \Delta$. The expression for the term $I_c(V)$ in (4.18), which determines the contribution made by the continuous spectrum to the current, will be written out below.

Similar expressions hold also in the case $x_0 = 0$. We shall not present these corresponding equations, so as not to encumber the exposition. Attention should be called, however, to the fact that at $e|V| > \Delta$ the expressions for the currents j_0 and j_2 contain terms that oscillate as function of V with respective periods $\pi v_F/2d$ and $\pi v_F/d$. Oscillations of this type on current-voltage characteristics (albeit in systems with a somewhat different geometry) were first investigated experimentally by Tomash[25] and theoretically by McMillan and Anderson[26]. The physical cause of the oscillations on the plot of the current against the voltage is an interference effect that leads to spatial quantization of the energy of the excitations. In a *SINS* system such an effect was noted in a number of experiments[27-29]. An oscillatory $j(V)$ dependence was indicated also in theoretical papers[30,31] The cited references were devoted to the nonstationary Josephson effect in a *SINS* system. The Green's functions of the system were calculated the method of eigenfunction expansion of the single-particle problem, followed by a solution of Gor'kov's equations.[2)]

With further increase of V, the expressions for the currents j_0 and j_2 becomes somewhat more complicated. In this case a contribution to the current is made not only by the lower but also by the higher energy levels of the excitations in the normal-metal layer, and also by the continuous spectrum at $eV > 2\Delta$ $(T=0)$. The resultant equations are all of the same type. We write down, for example, the result for j_0 in the *SINS* system at $eV > \Delta$ $(T \ll T_c)$:

$$j_0 = I_1 + I_2, \qquad (4.19)$$

$$I_1 = \frac{\pi v_F}{eRd}\int_0^1 \alpha^4 d\alpha\sum_{n=0}^{N}\left(\operatorname{th}\frac{\varepsilon_n}{2T} - \operatorname{th}\frac{\varepsilon_n - eV}{2T}\right)\frac{eV-\varepsilon_n}{[(eV-\varepsilon_n)^2-\Delta^2]^{1/2}}.$$

In this equation, ε_n is determined from (4.14) and N is in this case a maximal number such that $\varepsilon_N \leqslant eV - \Delta$. At $\varepsilon_n \ll \Delta$ and $T=0$ Eq. (4.19) goes over into (4.17). We have furthermore

$$I_2 = \frac{2}{eR}\Theta(eV-2\Delta)\int_0^1 \alpha^3 d\alpha\int_{\Delta}^{eV-\Delta} d\varepsilon\left(\operatorname{th}\frac{\varepsilon-eV}{2T} - \operatorname{th}\frac{\varepsilon}{2T}\right)$$
$$\times\frac{\varepsilon(\varepsilon-eV)}{[(\varepsilon-eV)^2-\Delta^2]^{1/2}}\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon^2-\Delta^2\cos^2(2\varepsilon d/v_F\alpha)}. \qquad (4.20)$$

At $eV \gg \Delta$ and $T=0$ we get

$$I_1 \approx \Delta/eR. \qquad (4.21)$$

To investigate the integral I_2 we use the following circumstance. Assume that we have the integral

$$I = \int_1^A f(x)\frac{dx}{x^2-\cos^2 Bx}.$$

Here $A > 1$ and $B \gg 1$. It is easy to show that

$$I = \int_1^A \frac{f(x)}{x}\frac{dx}{(x^2-1)^{1/2}} + O\left(\frac{1}{B}\right).$$

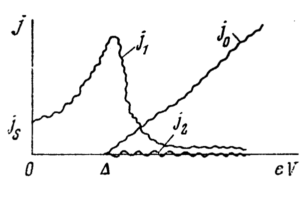
Using this equation, we easily obtain for I_2 $(T=0)$

$$I_2 = (e^2V^2 - 2eV\Delta)^{1/2}/eR. \qquad (4.22)$$

At $eV \gg \Delta$ we have from (4.21) and (4.22)

$$j_0 = V/R - \Delta^2/2e^2VR. \qquad (4.23)$$

Thus, at large V the current-voltage characteristics of *SINS* systems yield Ohms law. A similar result holds also for *SNINS* systems. We note also that in Eqs. (4.21)–(4.23) we used the strong inequality $v_F/d \ll \Delta$ and left out the corresponding small terms.



We now write down the expression for the term $I_c(V)$ in (4.18), which determines the contribution made to the current j_1^a by the continuous spectrum:

$$I_c(V)=\frac{\Theta(eV-\Delta)}{eR}\int_0^1\alpha^3d\alpha\int_M^{eV}d\varepsilon\left(\text{th}\frac{\varepsilon}{2T}-\text{th}\frac{\varepsilon-eV}{2T}\right)\times\frac{(\varepsilon^2-\Delta^2)^{1/2}\Delta^2\cos(2\varepsilon d/v_F\alpha)}{[\Delta^2-(\varepsilon-eV)^2]^{1/2}[\varepsilon^2-\Delta^2\cos^2(2\varepsilon d/v_F\alpha)]}. \quad (4.24)$$

Here $M=\max\{\Delta, eV-\Delta\}$. At $T=0$ and $eV\gg\Delta$, Eq. (4.24) yields

$$I_c(V)=\frac{(\pi/2)^{1/2}}{e^3V^2R}\left(\frac{\Delta v_F}{d}\right)^{1/2}\sin\frac{2(eV-\Delta)d}{v_F}. \quad (4.25)$$

The general form of the voltage dependences of the currents j_0, j_1, and j_2 in a *SINS* system at $T=0$ is shown in Fig. 2. The currents j_0 and j_2 in a *SNINS* system behave in similar fashion. It is difficult to plot $j_1(V)$ for *SNINS* systems in view of the considerable irregularity of this function. We indicate only that this current component has logarithmic singularities of the type (4.15) at $eV<2\Delta$. At large values of V the behavior of $j_1(V)$ in *SNINS* and *SINS* systems is of the same type.

We present now certain equations that determine the nonstationary Josephson effect in the considered systems in the temperature region $v_F/d\ll T\ll T_c$. At $e|V|<\Delta$ the currents j_0 and j_2 are exponentially small. For example, for j_0 in a *SINS* system we have

$$j_0\left(x_0=\pm\frac{d}{2}\right)=\frac{\exp\{-(\Delta-e|V|)/T\}}{eR}\left\{\frac{1}{2}[\pi T(2\Delta+T)]^{1/2}+(\Delta-e|V|)^{-1}\left(\frac{\pi\Delta v_F^3}{2d^3}\right)^{1/2}\sum_{n=1}^{\infty}\frac{\sin\{4nd(\Delta-eV)/v_F+\pi/4\}}{n^{3/2}}\right. \quad (4.26)$$

At $e|V|>\Delta$ we have

$$j_{0(2)}(T\gg v_F/d)={}^1/_2 j_{0(2)}(T=0),\quad 0<e|V|-\Delta\ll\Delta. \quad (4.27)$$

The pair current $j_1^{(r)}$ decreases exponentially at $T\gg v_F/d$. For example, in a *SNINS* system the expression for this current is

$$j_1^r(x_0=0)=\frac{16\pi T}{eR\Delta d}\frac{\exp\{-2d/\xi_T\}}{4\pi^2T^2+e^2V^2}\left\{[2\pi T(\Delta^2-e^2V^2)^{1/2}-e^2V^2]\cos\frac{eVd}{v_F}-eV[2\pi T+(\Delta^2-e^2V^2)^{1/2}]\sin\frac{eVd}{v_F}\right\},\quad \frac{v_F}{d}\ll e|V|\leqslant\Delta. \quad (4.28)$$

The anomalous j_1^a has no factor that decreases exponentially with temperature. Thus, at $v_F/d\ll e|V|\ll\Delta$ we have

$$j_1^a\left(T\gg\frac{v_F}{d}\right)=\begin{cases}\dfrac{e|V|}{2T}j_1^a(T=0), & e|V|\ll T\\ {}^1/_2 j_1^a(T=0), & e|V|\gg T\end{cases}. \quad (4.29)$$

The value of $j_1^a(T=0)$ is determined by Eqs. (4.15). When necessary, it is easy to calculate the expressions for the currents in other particular cases, too.

From the general equations (4.12) obtained by us it is easy to derive an expression for the current in a *NINS* system, i.e., in the case when the order parameter Δ vanishes on one of the edges. It is clear that in this case only the normal current differs from zero:

$$j_0=\frac{1}{eR}\int_0^1\alpha^3d\alpha\int_{-\infty}^{\infty}d\varepsilon\left(\text{th}\frac{\varepsilon}{2T}-\text{th}\frac{\varepsilon-eV}{2T}\right)\times\left\{\frac{g^R(\varepsilon)c(\varepsilon)+is(\varepsilon)}{c(\varepsilon)+ig^R(\varepsilon)s(\varepsilon)}-\frac{g^A(\varepsilon)c(\varepsilon)+is(\varepsilon)}{c(\varepsilon)+ig^A(\varepsilon)s(\varepsilon)}\right\}$$
$$c(\varepsilon)=\cos(2\varepsilon d/v_F\alpha)\quad s(\varepsilon)=\sin(2\varepsilon d/v_F\alpha). \quad (4.30)$$

The current-voltage characteristic of such system at $eV\gg v_F/D$ and $T=0$ leads to Ohms law. In the case $T\gg v_F/d$ we have

$$j_0=\begin{cases}\dfrac{V^2}{2TR}\,\text{sign}\,V, & e|V|\ll T\\ V/2R, & T\ll e|V|\ll\Delta\end{cases}. \quad (4.31)$$

At such temperatures, the $j_0(V)$ dependence in *NINS* systems becomes Ohmic only at large voltages $eV\gg\Delta$.

Let us return, however, to the study of *SNINS* systems. Let now an alternating voltage $V(t)=V+V_1\cos\omega_1 t$ be applied to the barrier in addition to the dc voltage V. In this case

$$W(\omega)=\sum_{n=-\infty}^{\infty}J_n\left(\frac{eV_1}{\omega_1}\right)\delta(\omega-eV-n\omega_1),$$

J_n is a Bessel function of order n. The current is given by

$$j(t)=\text{Im}\sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}J_n\left(\frac{eV_1}{\omega_1}\right)\left\{J_{k+n}\left(\frac{eV_1}{\omega_1}\right)j_q(eV+n\omega_1)e^{ik\omega_1 t}+\exp[i\varphi+2iVt]J_{k-n}\left(\frac{eV_1}{\omega_1}\right)j_p(eV+n\omega_1)e^{ik\omega_1 t}\right\}. \quad (4.32)$$

Here $j_q(\omega)$ and $j_p(\omega)$ are determined by Eqs. (4.12). It is clear that in this case, too, we get oscillations of the current as a function of voltage, as well as logarithmic peaks of the Josephson component of the current (in *SNINS* systems). In addition, effects of this type appear here also in the dependence of the alternating component of the voltage on the frequency ω_1.

If the condition $2eV=n\omega_1$ is satisfied the systems in question are subject to exactly the same effect as *SIS* junctions (Shapiro steps[32]), in which direct current flows through the junction. For *SINS* junctions at $T=0$ this current in the principal approximation (we assume for simplicity that $eV\ll\Delta$ and $\omega_1\ll\Delta$) is equal to

$$j_n=\frac{\pi v_F}{5eRd}(-1)^nJ_n\left(\frac{2eV}{\omega_1}\right)\sin\varphi. \quad (4.33)$$

If there is no dc voltage on the barrier ($V=0$), and the frequency and amplitude of the alternating voltage satisfy the condition $eV_1\ll\omega_1\ll\Delta$, the expression for the current in the system takes the simple form ($T<v_F/d$):

$$j(t)=j_s\sin\varphi+2j_1(\omega_1)\frac{eV_1}{\omega_1}\sin\omega_1 t\cos\varphi. \quad (4.34)$$

Here j_s is the density of the stationary current in the

SNINS junction[2]. In *SINS* systems, the expression (4.34) yields

$$j(t)=\frac{\pi v_F}{5eRd}\left\{\sin\varphi+\frac{2eV_1}{\omega_1}\left[1+\frac{\omega_1^2}{4\Delta^2}+\frac{10v_F}{\pi\omega_1 d}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\sin\left(\frac{2\omega_1 d}{v_F}(2n+1)\right)\right]\sin\omega_1 t\cos\varphi\right\}. \qquad (4.35)$$

A similar expression (containing unfortunately errors) was obtained by another method in Ref. 33.

In a *SNINS* system, the expression for the current $j(t)$ become logarithmically large if the frequency satisfies the condition $\omega_n=\pi v_F n/ed$ (in the presence of a dc voltage component: $eV+\omega_n=\pi v_F n/ed$). A photon of frequency ω_n is then resonantly absorbed by the tunneling electron and the conductivity of the junction becomes large.

Thus, the nonstationary properties of *SINS* and *SNINS* systems differ greatly, owing to the different structures of the energy spectra of these systems. We note also that the results obtained by us from the general equations (4.12) are valid under the conditions

$$\nu_T\ll 1/\tau_{\varepsilon N}\ll v_F/d,\quad \nu_T\ll 1/\mu_{\varepsilon S}\ll\Delta. \qquad (4.36)$$

These inequalities ensure smallness of the nonequilibrium effects and at the same time make it possible to speak of the presence of a discrete excitation spectrum in the normal-metal layer. They also make it possible to use expressions obtained for the Green's functions in the BCS model. Furthermore, the first of these inequalities imposes a restriction on the value of d. The current-voltage characteristics of the investigated systems likewise differ substantially from the analogous characteristics of *SIS* junctions[22,23].

We note that the general equations (4.12) do not lead in the case $d\to\infty$ to the simple Ohms law $j=V/R$, as might be expected for an *NIN* junction: they retain a contribution from the discrete spectrum. The reason is that the formal transition $d\to\infty$ is not allowed in our equations. Actually, our analysis presupposes smallness of the Andreev-level broadening ($\sim 1/\tau_{\varepsilon N}$) in comparison with the distance between such levels ($\sim v_F/d$). At sufficiently large d, for real systems, this condition no longer holds and the spectrum in the junction actually becomes continuous. At the same time, by making $\tau\varepsilon_N$ arbitrarily large we restrict by the same token [as is seen from (4.36)] the validity of our calculations to small current ($\nu_T\to 0$), since no account is taken in them of the nonequilibrium effects that are due to the change of the distribution function on account of the current flow, and manifest themselves under the condition $\nu_T\tau\varepsilon_N$ 1. To be able to take the limit as $d\to\infty$ it is necessary, as indicated above, to take into account the finite nature of the mean free path l. It is easy to verify that in this case we obtain naturally, the usual Ohms law.

5. CONCLUSION

We have investigated the influence of external fields and of impurities on the Josephson behavior of superconducting systems containing insulator and a normal-metal interlayers. The expressions for the superconducting current at various impurity densities ($l\gg\xi_0$ and $l\ll\xi_0$) turn out to be qualitatively different. In the case of a small number of impurities, the quantity that plays the role of the reciprocal coherence length for the normal-metal layer takes the form $2/\xi_T+1/l$, while at $l\ll\xi_0$ the analogous quantity is $(D/2\pi T)^{1/2}$. In the former case under the condition $l\ll d$ (but $l<\xi_0$) the amplitude of the current is exponentially small even at zero temperature, whereas when the number of impurities is large the current does not have an exponentially small factor at zero temperatures. We note that similar results are obtained also in the case of *SNS* junctions which do not contain an insulating interlayer (cf. the results of Refs. 10 and 11). The mechanism whereby the impurities influence the current is the smearing of the wave functions of the Cooper pairs that penetrate into the normal-metal layer. This is taken into account automatically by shifting the Matsubara frequency along the real axis. It has also been established that the superconducting current depends significantly on the location of the insulator layer inside the normal metal.

We investigated the influence of the magnetic field on the stationary Josephson effect. It was shown in Ref. 2 that the presence of a magnetic field in a *SNINS* system leads to a shift of the Andreev levels. Using this result, it is easy to calculate the superconducting current. In this case there likewise takes place a shift of the Matsubara frequency, but now along the imaginary axis. The critical current density in a *SNINS* junction in the presence of a magnetic field in the case of $T=0$ increases somewhat and oscillates weakly as a function of the magnetic field, while in a *SINS* junction it remains unchanged. At high temperatures $T\gg v_F/d$ the shift of the Andreev levels in a magnetic field has little influence on the current. We recall that in *SNS* junctions the spatial quantization is destroyed by a magnetic field,[11,12] and as a result the Josephson effects have a different character under these conditions.[13]

A theory was developed for the nonstationary Josephson current in *SNINS* systems. It turned out that the behavior of such systems in the presence of a voltage on the insulating layer is far from trivial. The current-density characteristics exhibit oscillations due to the presence of discrete levels of the electronic excitations in the normal layer. The Josephson current $j_1(V)$ in a *SNINS* system at $eV<2\Delta$ has logarithmic peaks spaced $\pi v_F/ed$ apart. This effect is attributed to the possibility of resonant tunneling of the electrons at voltage values corresponding to coincidence of the energy levels on the right and on the left of the insulating barrier. Thus, the nonstationary behavior of *SNINS* systems differs noticeably from the analogous behavior of *SIS* junctions. At the same time, there are also many common properites of these two types of weakly coupled superconducting systems.

Thus, the properties of *SNINS* systems turn out to be quite complicated and varied, owing to the large number of factors that influence their behavior (complexity of the geometry, presence of a discrete spectrum, and others).

The experimental investigation of the dependence of the normal current on the voltage in *SINS* systems is the subject of Refs. 27–29. In these studies, experimental proof was obtained of the presence of a discrete excitation spectrum in *SINS* systems and of the associate oscillations of the state density in the N layer. These oscillations manifest themselves, in particular, in the oscillatory dependence of the current on the junction voltage, which was observed in Refs. 27–29 at low as well as at sufficiently high ($T > v_F/d$) temperatures. The distances between the maxima and minima of the corresponding experimental curves are of the order of $\Delta V \sim v_F/ed$, providing direct evidence of the presence of discrete levels in the system.

No experiments for the general case of *SNINS* systems have been reported so far. From our point of view, it is of interest to carry out further experimental investigation of the stationary and nonstationary Josephson effects in *SNINS* systems. In particular, it would be of interest to study in detail the dependence of the stationary Josephson current on the temperature, on the location of the insulator layer, on the impurity density, and on the magnetic field, as well as to investigate a number of nonstationary effects (oscillations of the current component j_0, j_1, and j_2, logarithmic peaks on the plots of j_1 against the voltage and the frequency, etc.). Such investigations would be useful for further study of the proximity effect.

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1) We note that this result is valid only in first-order expansion of the current in $1/R$. In the succeeding orders of such an expansion, the normal current can be the results of the existence of a nonequilibrium mechanism of penetration of the electric field into the superconductor. The contribution of such a mechanism can be neglected because of the large value of R.

2) There is no quantitative agreement between our results for the currents in a *SINS* system and the analogous results of Refs. 30 and 31. It can be verified that the final expressions in these references are inaccurate.

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