Magnetic properties of an excitonic dielectric with a real spectrum

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(Submitted 18 December 1980; resubmitted 17 June 1981)
Zh. Eksp. Teor. Fiz. **81**, 1803–1810 (November 1981)

The role played by spin-orbit interaction in the formation of band magnetism following an excitonic transition in IV-VI substances is discussed. It is shown that a spontaneous moment perpendicular to the principal symmetry axis of the crystal arises in a noncentrosymmetric phase with nonzero imaginary order parameter. A new ferromagnetism mechanism is thus predicted, caused by the spin-orbit interaction and by the appearance of a complex order parameter in a IV-VI crystal with anisotropic spectrum.

PACS numbers: 75.10.Lp

1. INTRODUCTION

Depending on the phase of the order parameter, three types of order are possible in electron-hole pairing in a semimetal or in a narrow-gap semiconductor.[1] Real order parameters, depending on the spin structure, describes states with a charge density wave ($\Delta_{sR} \neq 0$) or spin density wave ($\Delta_{tR} \neq 0$), while imaginary ones describe states with current density wave ($\Delta_{sI} \neq 0$) or with spin current density wave ($\Delta_{tI} \neq 0$).

Usually in the analysis of dielectric pairing one uses a simple two-band model with an isotropic parabolic spectrum. The properties of this model for the excitonic state are well known. It is of interest at present to identify the substance in which phase transitions are most adequately described by a scheme of excitonic transitions. To answer this question it would be necessary to ascertain whether the phase transition has some qualitative singularity due either to the specifics of the excitonic transition (e.g., to the appearance of an imaginary order parameter), or to the specifics of the particular substance assumed to be an excitonic dielectric.

The present paper is devoted to the study of the specifics of a phase transition in real *IV-VI* substances suspected of exhibiting the properties of an excitonic dielectric. An important role in *IV-VI* crystals is played by the spin-orbit interaction, as well as by the non-parabolicity, due to **kp** hybridization in the Luttinger-Kohn representation, of the spectrum near the L point of the Brillouin zone.

The influence of the spin-orbit interaction of the character of the electron-hole pairing in the simple two-band model was considered in Ref. 2. The additional coupling constants due to the spin-orbit interaction lift the degeneracy of the current states, making the critical temperatures of the states with current charge density and spin-current charge density distinguishable.

Allowance for the spin-orbit interaction in the single-particle spectrum and in the four-fermion part of the Hamiltonian H_{int} is a computationally difficult problem. For this reason, the spin-orbit interaction was previously[2] not taken into account in the single-particle Hamiltonian. Evidence favoring this approach was the absence of spin splitting of the spectrum in the centrosymmetric phase.

In the present paper we solve the alternate problem: we consider a real *IV-VI* spectrum and disregard for simplicity the Coulomb vertices with spin flip, due to the spin-orbit interaction. It turns out that simultaneous allowance of the nonparabolicity and of the spin-orbit interaction in the single-particle spectrum leads to the following qualitative result: the onset of ferromagnetism in a non-centrosymmetric phase with imaginary gap, i.e., in a mixed state $\Delta_{sR} \neq 0$, $\Delta_{sI} \neq 0$. A nonzero contribution to the spontaneous moment of the electron system is made in this case by electrons with momenta that are not small, such that account must be taken of at least the third order of **kp** perturbation theory in the calculation of the spectrum. Such a spectrum was obtained by Martinez.[3] The qualitative consequences of the allowance for the spin-orbit interaction in H_{int} do not differ in any way from those considered in Ref. 2 and have no bearing on the result mentioned above.

It is known that the coexistence of charge- and spin-density waves leads to a spin splitting of the single-particle spectrum and to excitonic ferromagnetism when the electron and hole Fermi surface are not completely congruent.[4,5] In the present paper we note the possibility of a similar ferromagnetism when imaginary gaps coexist.

2. ELECTRON SPECTRUM AND BASIC EQUATIONS

We investigate here the Coulomb interaction in a two-band model with a real *IV-VI* spectrum. The electron spectrum of such a crystal constitutes two closely lying bands with extrema at the point L, and is described by the Martinez model.[3] The Green's function corresponding to the zeroth Hamiltonian is nondiagonal:

$$G_0^{-1}(\mathbf{p},\ \omega_n) = i\omega_n + \mu - \beta\varepsilon_p - i\beta\gamma^5 Z + \alpha_x Y - \alpha_y X, \tag{1}$$

where $\beta, \gamma^5, \alpha_x, \alpha_y$ are Dirac matrices in the standard representation, μ is the chemical potential,

$$\varepsilon_p = \frac{\varepsilon_g}{2} + \frac{p_z^2}{2m} + \frac{p_\perp^2}{2m_\perp},\quad Z = v_z p_z + \frac{w}{v_z}(p_x - 3p_y)p_z^2,$$

$$X, Y = \left(v_\perp + \frac{w_1}{v_\perp} p_z^2\right) p_{x,y}.$$

 0038-5646/81/110955-04$02.40

The wave functions of the c- and v-band were chosen to be real. The parameters of the matrix (1) are defined in Ref. 3. The presence in (1) of the matrix element Z is due to kp hybridization, and the elements X and Y are due to the spin-orbit interaction. The hybridization is due to the choice of the Kohn-Luttinger basis[6] and describes the deviation of the spectrum from parabolic. This deviation is due to the presence of two closely lying terms at the point L of the Brillouin zone in the IV-VI crystals. The OZ axis is directed along the principal symmetry axis of the crystal.

The group D_{3d} of the wave vector at the point L contains inversion, and the spectrum in the non-centrosymmetric phase comprises two bands that are doubly degenerate in spin and can be found from (1). If we confine ourselves in the calculation of the determinant to the fourth powers of the momentum, we obtain the Martinez spectrum[3]

$$\omega_{1,2}=-\mu\pm[\varepsilon_p^2+v_z^2p_z^2+v_\perp^2(p_x^2+p_y^2)+2w(p_x-3p_y)p_xp_z+2w_1p_z^2(p_x^2+p_y^2)]^{1/2}. \tag{2}$$

As noted in the Introduction, scattering with spin flip will not be considered in the Hamiltonian H_{int} corresponding to the Coulomb-interaction energy. Thus

$$H_{int}=\frac{1}{2}\sum_{pp'q}V_{ijlm}a_{\alpha i}^{+}(\mathbf{p}+\mathbf{q})a_{\beta j}^{+}(\mathbf{p}'-\mathbf{q})a_{\beta m}(\mathbf{p}')a_{\alpha l}(\mathbf{p}), \tag{3}$$

where α and β are the spin indices and i, j, l, and m are the band indices; summation over the repeated indices in (3) and elsewhere is implied. The matrix elements of the Coulomb interaction are calculated in the Luttinger-Kohn basis for the point L of the Brillouin zone:

$$V_{ijlm}=\int d\mathbf{r}\,d\mathbf{r}'\varphi_{ik}^{*}(\mathbf{r})\varphi_{jk}^{*}(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\varphi_{lk}(\mathbf{r}')\varphi_{mk}(\mathbf{r}). \tag{4}$$

The system of Gor'kov equations for the matrix Green's function over the bands and spins is of the form

$$(G_0^{-1})_{ik}^{\alpha\gamma}G_{kj}^{\gamma\beta}-\Sigma_{ik}^{\alpha\gamma}G_{kj}^{\gamma\beta}=\delta_{ij}\delta_{\alpha\beta}. \tag{5}$$

Here G_0^{-1} is known from (1), while Σ is determined by the self-consistency condition

$$\Sigma_{ij}^{\alpha\beta}=(\Sigma_{ji}^{\beta\alpha})^{*}=-T\Omega\sum_n\int\frac{d^3p}{(2\pi)^3}\{V_{ijlm}G_{lm}^{\alpha\beta}(p,\omega_n)-\delta_{\alpha\beta}V_{imjl}G_{ml}^{\alpha\alpha}(p,\omega_n)\}, \tag{6}$$

where Ω is the volume of the basic region of the crystal.

We note here the assumptions usually employed in the theory of excitonic dielectrics:

$$\begin{gathered}V_{1111}=V_{1212}=V_{2121}=V_{2222}\equiv V,\\ V_{1112}=V_{1121}=V_{1211}=V_{2111}=V_{2212}=V_{2122}=V_{1222}\equiv W,\\ V_{1122}=V_{2211}=V_{1221}=V_{2112}\equiv U.\end{gathered} \tag{7}$$

In the Bloch representation allowance for interaction with umklapp of one particle W leads to a refinement of the phase diagram of an excitonic ferromagnet.[7,8] For our present purposes allowance for W is of no fundamental importance. In addition, in the Luttinger-Kohn representation W merely renormalizes the kp hybridization,[9] and we shall disregard it.

Since our aim is to determine the conditions for the appearance of spontaneous magnetization M in the system, there is likewise no need to take into account pairing with spin flip $\Sigma^{\uparrow\downarrow}$, since such quasi-mean values only increase M, which as will be shown, is produced by other factors. With allowance for the assumptions made above from (5) and (6), the system of equations for the order parameters

$$\Delta_s=\tfrac{1}{2}(\Sigma_{12}^{\uparrow\uparrow}+\Sigma_{12}^{\downarrow\downarrow}),\quad \Delta_t=\tfrac{1}{2}(\Sigma_{12}^{\uparrow\uparrow}-\Sigma_{12}^{\downarrow\downarrow})$$

takes the form

$$\Delta_{sR}=\frac{T\Omega(V-3U)}{(2\pi)^3}\sum_n\int\frac{d^3p}{\mathrm{Det}}\{\Delta_{sR}[(i\omega_n+\mu)^2-\varepsilon_p^2+X^2+Y^2]$$
$$-\tfrac{1}{2}|iZ-\Delta_+|^2\Delta_{-R}-\tfrac{1}{2}|iZ-\Delta_-|^2\Delta_{+R}\}, \tag{8}$$
$$\Delta_{sI}=\frac{T\Omega(V-U)}{(2\pi)^3}\sum_n\int\frac{d^3p}{\mathrm{Det}}\{[(i\omega_n+\mu)^2-\varepsilon_p^2-X^2-Y^2](Z-\Delta_{sI})$$
$$-\tfrac{1}{2}|iZ-\Delta_+|^2(Z-\Delta_{-I})-\tfrac{1}{2}|iZ-\Delta_-|^2(Z-\Delta_{+I})\}, \tag{9}$$
$$\Delta_{tR}=\frac{T\Omega(V+U)}{(2\pi)^3}\sum_n\int\frac{d^3p}{\mathrm{Det}}\{\Delta_{tR}[(i\omega_n+\mu)^2-\varepsilon_p^2-X^2-Y^2]$$
$$-\tfrac{1}{2}|iZ-\Delta_+|^2\Delta_{-R}-\tfrac{1}{2}|iZ-\Delta_-|^2\Delta_{+R}\}, \tag{10}$$
$$\Delta_{tI}=\frac{T\Omega(V-U)}{(2\pi)^3}\sum_n\int\frac{d^3p}{\mathrm{Det}}\{\Delta_{tI}[(i\omega_n+\mu)^2-\varepsilon_p^2+X^2+Y^2]$$
$$+\tfrac{1}{2}|iZ-\Delta_-|^2(Z-\Delta_{+I})-\tfrac{1}{2}|iZ-\Delta_+|^2(Z-\Delta_{-I})\}. \tag{11}$$

In Eqs. (8)–(11) we used the notation

$$\begin{gathered}\Delta_\pm=\Delta_s\pm\Delta_t,\quad \Delta_R=\mathrm{Re}\,\Delta,\quad \Delta_I=\mathrm{Im}\,\Delta,\\ \mathrm{Det}=(i\omega_n-\omega_1)(i\omega_n-\omega_2)(i\omega_n-\omega_3)(i\omega_n-\omega_4);\\ \omega_{1,2}=-\mu+(\varepsilon_p^2+E_\pm^2)^{1/2},\quad \omega_{3,4}=-\mu-(\varepsilon_p^2+E_\pm^2)^{1/2},\end{gathered} \tag{12}$$
$$E_\pm^2=\Delta_{sR}^2+\Delta_{tR}^2+(Z-\Delta_{sI})^2+\Delta_{tI}^2+X^2+Y^2\pm A(X,Y,Z), \tag{13}$$
$$A(X,Y,Z)=2\{[\Delta_{sR}\Delta_{tR}+(Z-\Delta_{sI})\Delta_{tI}]^2+(X^2+Y^2)[\Delta_{sR}^2+\Delta_{tI}^2]\}^{1/2}.$$

It is easy to change over in the spectrum $\omega(p)$ (13) with the spin splitting $\pm A$ to the case of an excitonic ferromagnet. For this purpose we neglect in the Bloch basis ($Z=0$) the spin-orbit interaction ($X=Y=0$) and retain only the ordering due to the charge- and spin-density waves. Then

$$E_\pm^2=(\Delta_{sR}\pm\Delta_{tR})^2,$$

which agrees with the results of Ref. 4.

Disregarding the imaginary order parameters ($\Delta_I=0$), we can obtain for the spectrum another limiting case,[10] wherein spin polarization of the electrons is possible if electric current is made to flow through the sample.

If we neglect in the spectrum (13) the spin-orbit interaction ($X=Y=0$) and all the order parameters except Δ_{sI} (current-density wave), we obtain the spectrum given in Ref. 11. Thus, the system (8)–(11) and the spectrum (13) are generalizations of different cases realized in phase transitions in an excitonic dielectric. It is clear, however, that the total spectrum (13) is hardly realistic, since it presupposes the coexistence of four gaps. We shall be interested hereafter in such nontrivial particular cases that can lead to ferromagnetic ordering and at the same time do not reduce to the known excitonic ferromagnetism.

3. GENERAL EXPRESSION FOR THE MAGNETIC MOMENT IN THE EXCITONIC PHASE

The point of the phase transition into the ferromagnetic state can be determined from the divergence of the response of the system to an external magnetic

field $\chi = (M/H)_{H=0}$. The presence of a spontaneous moment can be established also by direct calculation in the ordered phase. We obtain **M** from the expression

$$\mathbf{M} = \frac{\mu_B T\Omega}{(2\pi\hbar)^3}\sum_{ni}\int d^3p\,\mathrm{Sp}[\sigma G_{ii}(\mathbf{p},\omega_n)], \tag{14}$$

where μ_B is the Bohr magneton, $i = v$ or c, and G_{cc} and G_{vv} are diagonal Green's-function blocks.

Calculating the Green's function from (5) and substituting in (14), we obtain for the components of the magnetic moment

$$M_z = \frac{\mu_B\Omega}{(2\pi\hbar)^3}\int d^3pA^{-1}(X,Y,Z)\,[\Delta_{sR}\Delta_{tR}+\Delta_{tI}(\Delta_{sI}-Z)]\,(f_1+f_3-f_2-f_4), \tag{15}$$

$$M_x = \frac{\mu_B\Omega}{(2\pi\hbar)^3}\int d^3pA^{-1}(X,Y,Z)\left\{Y\Delta_{sR}(f_2+f_4-f_1-f_3) + X\Delta_{tI}\varepsilon_p\left(\frac{f_1-f_3}{(\varepsilon_p^2+E_+^2)^{1/2}}+\frac{f_4-f_2}{(\varepsilon_p^2+E_-^2)^{1/2}}\right)\right\}, \tag{16}$$

$$M_y = \frac{\mu_B\Omega}{(2\pi\hbar)^3}\int d^3pA^{-1}(X,Y,Z)\left\{-X\Delta_{sR}(f_2+f_4-f_1-f_3) + Y\Delta_{tI}\varepsilon_p\left(\frac{f_1-f_3}{(\varepsilon_p^2+E_+^2)^{1/2}}+\frac{f_4-f_2}{(\varepsilon_p^2+E_-^2)^{1/2}}\right)\right\}, \tag{17}$$

$$f_i = \left[1+\exp\frac{\omega_i}{T}\right]^{-1}.$$

We proceed now to the possibilities when $\mathbf{M}\neq 0$.

a) It is easy to change over in (15)–(17) to the case of an excitonic ferromagnet, by putting $X = Y = 0$, Δ_{sR}, $\Delta_{tR}\neq 0$. Then $M_x = M_y = 0$ and

$$M_z = \frac{\mu_B\Omega}{(2\pi\hbar)^3}\int d^3p\,(f_1+f_3-f_2-f_4); \tag{18}$$

$$\omega_{1,2} = -\mu+[\varepsilon_p^2+(\Delta_{sR}\pm\Delta_{tR})^2]^{1/2}, \quad \omega_{3,4} = -\mu-[\varepsilon_p^2+(\Delta_{sR}\pm\Delta_{tR})^2]^{1/2}. \tag{19}$$

As seen from (18), equality of the effective masses of the electrons and holes in the absence of doping ($\mu = 0$) leads to $\mathbf{M} = 0$, i.e., ferromagnetism takes place only if the electron and hole Fermi surfaces are not fully congruent.[4,5]

b) By way of a less trivial example, we note the analogy with the excitonic ferromagnet in the current state (Δ_{sI}, $\Delta_{tI}\neq 0$). In this case $\mathbf{M}\neq 0$, but the spin-orbit interaction is of no principal significance, and from (13) and (15) with $X = Y = 0$ we obtain

$$M_z = \frac{\mu_B\Omega}{(2\pi\hbar)^3}\int d^3p\,\{\mathrm{sign}(Z-\Delta_{sI})\,(f_1+f_3-f_2-f_4)\}, \tag{20}$$

$$\omega_{1,2} = -\mu+[\varepsilon_p^2+(\Delta_{sI}-Z\pm\Delta_{tI})^2]^{1/2}, \qquad \omega_{3,4} = -\mu-[\varepsilon_p^2+(\Delta_{sI}-Z\pm\Delta_{tI})^2]^{1/2}. \tag{21}$$

This means ferromagnetism in the mixed phase, when current density and spin current density waves coexist.

c) A nontrivial case is that of complex Δ_s: $\Delta_{sR}\neq 0$ and $\Delta_{sI}\neq 0$. The electron spectrum in the non-centrosymmetric phase ($\Delta_{sR}\neq 0$) is split in spin and at $\Delta_{sI}\neq 0$ it is asymmetrical ($\omega(\mathbf{p})\neq\omega(-\mathbf{p})$). This leads to a nonzero spontaneous moment, with $M_z = 0$ and $M_{x,y}\neq 0$. We calculate M_x in the next section.

4. MAGNETIC MOMENTS IN A PHASE WITH COMPLEX Δ_s AT $T = 0$

We carry out a direct calculation of one of the components of the magnetization vector, e.g., M, which is defined in (16). This expression can be simplified by assuming $\Delta_{tI} = 0$. Then the form of Eq. (16) allows us to make certain general statements with respect to M_x. The magnetization can take place only if the electron and hole Fermi surfaces are not fully congruent, e.g., in the case of nonzero doping. We assume hereafter that the Fermi level μ lies in the conduction bands $\omega_{1,2}$.

For $M_x\neq 0$ it is necessary and sufficient that the spectrum be asymmetrical with respect to any of the reflections $p_x\to -p_x$, $p_y\to -p_y$, and also with respect to the inversion $\mathbf{p}\to -\mathbf{p}$. For this reason we can disregard the term $\sim w_1$ in the bare spectrum (2), whereas the combination $\sim w$ is essential and the entire effect exists, (i.e., $M_x\neq 0$), since $w\neq 0$. Taking the foregoing into account, we write down that part of (16) which makes a nonzero contribution to M_x at $T = 0$:

$$M_x = \frac{\mu_B\Omega}{2(2\pi\hbar)^3}\int d^3p\,\frac{p_y}{(p_x^2+p_y^2)^{1/2}}[f(E_2-\mu)-f(E_1-\mu)], \tag{22}$$

where the spectrum $E_{1,2}$ is obtained from (13) accurate to the fourth powers of the momentum,

$$E_{1,2} = [\varepsilon_p^2+v_\perp^2(p_x^2+p_y^2)+(\Delta_{sI}-v_zp_z)^2+\Delta_{sR}^2 + 2w(v_zp_z-\Delta_{sI})(p_z-3p_y)p_x^2\pm 2\Delta_{sR}v_\perp(p_x^2+p_y^2)^{1/2}]^{1/2}. \tag{23}$$

At $w = 0$ we have $M_x = 0$ and obtain in first order in w from (22)

$$M_x = -\mu_B\Omega\frac{3w}{64\pi^2\hbar^3}\int_0^\infty \rho^4\,d\rho\int_{-\infty}^{\infty}dp_z\left[\frac{\delta(E_1-\mu)}{E_1}-\frac{\delta(E_2-\mu)}{E_2}\right](v_zp_z-\Delta_{sI}), \qquad \rho = (p_x^2+p_y^2)^{1/2}. \tag{24}$$

In (24), $E_{1,2}$ denotes the spectrum (23) at $w = 0$.

To calculate (24) we make some simplifications that are not of fundamental significance: $v_z = v_\perp = v$, $m_\perp\to\infty$. Then $E_{1,2}$ in (24) take the form

$$E_{1,2} = [(\varepsilon_g/2+p_z^2/2m)^2+(\Delta_{sR}\pm v\rho)^2+(\Delta_{sI}-vp_z)^2]^{1/2} \tag{25}$$

and the integral (24) can be calculated exactly. Integrating with respect to ρ with δ-functions, we obtain

$$M_x = \frac{3\mu_B\Omega W}{64\pi^2\hbar^3}\int_{p_{z2}}^{p_{z1}}\frac{(vp_z-\Delta_{sI})\,[(\rho^+)^4-(\rho^-)^4]\,dp_z}{[\mu^2-\varepsilon_g^2/4-(vp_z-\Delta_{sI})^2-p_z^2\varepsilon_g/2m]^{1/2}}; \tag{26}$$

$$\rho^\pm = \frac{1}{v}\left\{\Delta_{sR}\pm\left[\mu^2-\frac{\varepsilon_g^2}{4}-(vp_z-\Delta_{sI})^2-\frac{p_z^2\varepsilon_g}{2m}\right]^{1/2}\right\},$$

$$p_{z1,2} = \frac{1}{v+\varepsilon_g/2mv}\left\{\Delta_{sI}\pm\left[\Delta_{sI}^2+\left(1+\frac{\varepsilon_g}{2mv^2}\right)\left(\mu^2-\frac{\varepsilon_g^2}{4}-\Delta_{sI}^2\right)\right]^{1/2}\right\}. \tag{27}$$

The integration with respect to p_z breaks up into several regions, which are determined by the conditions $\mathrm{Im}\,\rho^\pm = 0$ and $\rho^\pm > 0$. The sum of the resultant expressions is written out in (26). Calculating (26) with allowance for (27), we obtain

$$M_x = \frac{\mu_B\Omega w\Delta_{sR}\Delta_{sI}\varepsilon_g b}{8\pi^2v^6(1+\varepsilon_g/2mv^2)^3mv^2\hbar^3}\left\{\left[2\left(\frac{\varepsilon_g^2}{4}-\mu^2\right)-3\Delta_{sR}^2\right]\left(1+\frac{\varepsilon_g}{2mv^2}\right)+\frac{\varepsilon_g\Delta_{sI}^2}{mv^2}\right\}, \tag{28}$$

$$b = \left[\Delta_{sI}^2+\left(1+\frac{\varepsilon_g}{2mv^2}\right)\left(\mu^2-\frac{\varepsilon_g^2}{4}-\Delta_{sI}^2\right)\right]^{1/2}.$$

M_y can be calculated from (17) in similar fashion.

5. DISCUSSION OF RESULTS

1. The main result of the present paper is a new ferromagnetism mechanism due to the nonparabolicity of

the spectrum and to spin-orbit interaction in a phase with complex Δ_s. We note that failure to take into account combinations of third order in p_i ($w = 0$) in the initial Green's function (1) would lead to symmetry of the spectrum with respect to the reflections $p_x \to -p_x$, $p_y \to -p_y$, meaning also to $M_{x,y} = 0$.

A spontaneous magnetic moment appears in a plane perpendicular to the interband dipole moment, which in *IV-VI* is parallel to the principal crystal symmetry axis *OZ*. Analagously, a directed magnetic moment arises in the model of orbital ferromagnetism.[12] It must be emphasized, however, that the ferromagnetism is produced in our case and in Ref. 12 by substantially different mechanisms. For orbital ferromagnetism, the spin-orbit interaction is not significant, whereas **M** in Eqs. (16) and (17) differs from zero only to the extent that the momentum matrix element $v_\perp$ connected with the spin-orbit interaction, differs from zero. In addition, in our case ferromagnetism exists in the case of doping, while orbital ferromagnetism takes place also in the undoped case.

2. We used in the present paper the approximation of pointlike electron-hole interaction, in which Δ does not depend on **p**. In semiconductors with low carrier density, the Coulomb interaction is nonlocal, and **kp** hybridization in the spectrum is the source of the antisymmetrical order parameter $\Delta_s^a(\mathbf{p}) = -\Delta_s^a(-\mathbf{p})$. In the ordered phase with $\Delta_s^a(\mathbf{p}) \neq 0$, the symmetry of the system with respect to the inversion $\mathbf{p} \to -\mathbf{p}$ is not violated, and ferromagnetism does not exist for this reason ($\mathbf{M} = 0$). At sufficiently low hybridization, however, a symmetrical component of a singlet order parameter can be produced[13] and its imaginary part makes the spectrum (13) asymmetric with respect to the substitution $\mathbf{p} \to -\mathbf{p}$ as well as the nonzero magnetic moment (28).

We note in conclusion that low-temperature ferromagnetism in $Pb_{1-x}Sn_xTe$ solid solutions without magnetic impurities was observed in Ref. 14. Recognizing that the narrow-gap semiconductors $Pb_{1-x}Sn_xTe$ are known to be prone to excitonic instability, the ferromagnetic-ordering mechanism proposed in the present paper must be borne in mind in the interpretation of experiments similar to those of Ref. 14.

The authors thank Yu. V. Kopaev for a discussion of the work and for helpful advice.

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Translated by J. G. Adashko