

Sound absorption in metals above the Debye temperature

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The absorption of longwave sound in a metal single crystal is investigated above the Debye temperature. It is shown that absorption in the scattering of electrons by phonons is due mainly to the electron thermal conductivity and increases linearly with the temperature. For certain directions of propagation and polarization of the sound wave, the thermal conductivity absorption is absent. In such cases the dissipation of the sound is much lower, the absorption now being due to the phonon viscosity, and the corresponding coefficient is independent of the temperature and has the same form and order of magnitude as in dielectrics.

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1. INTRODUCTION

The first detailed quantum mechanical investigation of sound absorption in metals was carried out in the classical work of A. I. Akhiezer in 1938.¹ However, this work contains certain inaccuracies, of which the most significant is that the change in the order of magnitude of the right side of the "electron" kinetic equation was not taken into account in its averaging over the constant energy surface (for details see Sec. 2 below). These inaccuracies led to a qualitative distortion of the description of the sound dissipation in metals above the Debye temperature. In particular, it followed from Ref. 1 that at $T \gg \Theta$, where Θ is the Debye temperature, the contributions made to the sound attenuation by the viscosity and by the thermal conductivity turned out to be of the same order, while the electron viscosity was always large in comparison with the phonon viscosity. In actuality, this is not the case: it will be shown below that the thermal conductivity damping is as a rule large in comparison with the viscous damping and over a wide temperature range $\Theta \ll T \ll \varepsilon_F(\Theta/\varepsilon_F)^{1/3}$, where ε_F is the chemical potential of the electron gas,¹⁾ the phonon viscosity is large in comparison with the electron viscosity. Such a non-correspondence with the results of Ref. 1 is explained by the inaccuracies mentioned above; however, in spite of the more than forty-year history of the problem, no investigation of the sound absorption in metals at $T \gg \Theta$, free of the inadequacies mentioned, has been carried out, so far as we know, a fact that accounts for the present paper.

The statement of the problem is completely identical with that of Ref. 1: We consider longwave sound (such that we can regard the external field as quasistationary in relation to the electron-phonon system), that propagates in a metal single crystal. The elementary excitations in the crystal reduce only to electrons and phonons, which interact by scattering from one another; the geometric (macroscopic) dimensions of the sample are large in comparison with the free path lengths of the electrons and phonons. We can thus eliminate from consideration in the mechanism of scattering from impurities size effects, phenomena that take place on grain boundaries of a polycrystalline structure, and so on.

In the linear approximation, the different mechanisms of energy dissipation make additive contributions to the

sound absorption; therefore, we can introduce for each such mechanism its own absorption coefficient. $\Gamma_i(T)$ after which the actual absorption coefficient $\Gamma(T)$ is determined as the sum over all Γ_i . In this connection, we first investigate the role of viscous damping. In Sec. 2, we study the properties of the kinetic equations for the nonequilibrium increments to the electron and phonon distribution functions. The coefficient of viscous damping $\Gamma_\eta(T)$ is expressed in Sec. 3 in terms of the solution of these equations, and is compared with the coefficient of thermal conductivity damping $\Gamma_\kappa(T)$, which is connected by a phenomenological formula with the coefficient of thermal conductivity κ and can be determined for known $\kappa(T)$ without solution of the corresponding kinetic equations. Section 4 is devoted to a discussion of the results.

2. THE KINETIC EQUATIONS

In the setup of the problem under consideration, the field of the acoustic wave leads to modulation of the energy of the electrons and phonons, so that its values turn out to be dependent on the strain tensor of the crystal u_{ij} . In the approximation in linear u_{ij} ,

$$\varepsilon(\mathbf{p}) = \varepsilon_0(\mathbf{p}) + \mu_{ij}(\mathbf{p})u_{ij}, \quad (1)$$

$$\omega(\mathbf{k}, g) = \omega_0(\mathbf{k}, g) \{1 + \lambda_{ij}(\mathbf{k}, g)u_{ij}\}, \quad (2)$$

where ε_0 and ω_0 are the energy of the electron with quasimomentum \mathbf{p} and the frequency of the phonon with quasimomentum \mathbf{k} in the absence of sound, g denotes the polarization of the phonon, $\mu_{ij} \sim \varepsilon_F$ and $\lambda_{ij} \sim 1$ are the tensors of the electron and phonon deformation potentials.

We choose the nonequilibrium increments to the electron and phonon distribution functions n and N in the form

$$\delta n = -\frac{\partial n_0}{\partial \varepsilon} \varphi, \quad \delta N = -\frac{\partial N_0}{\partial \hbar \omega} \chi,$$

where

$$n_0 = \left[\exp\left(\frac{\varepsilon - \varepsilon_F}{T}\right) + 1 \right]^{-1}, \quad N_0 = \left[\exp\left(\frac{\hbar \omega}{T}\right) - 1 \right]^{-1},$$

while the ε and ω therein refer to the deformed state of the crystal and are determined by formulas (1) and (2).

The kinetic equations for the distribution functions of

the electrons and phonons in the quasistationary field of the sound wave do not contain the derivatives $\partial\varphi/(\partial t, \partial\mathbf{r}, \partial\mathbf{p}), \partial\chi/(\partial t, \partial\mathbf{r}, \partial\mathbf{k})$ explicitly and have the following form in the approximation that is linear in φ and χ :¹

$$I_{e,ph}(\varphi, \chi) = \dot{n}_e, \quad (3)$$

$$I_{ph,e}(\varphi, \chi) + I_{ph,ph}(\chi) = \dot{N}_e, \quad (4)$$

$$\dot{n}_s = \frac{\partial n_0}{\partial x} \left(\frac{\mu_{ij} - \langle \mu_{ij} \rangle_F}{T} - x \langle \lambda_{ij} \rangle \right) \dot{u}_{ij}, \quad (5)$$

$$\dot{N}_s = \hbar\omega \frac{\partial N_0}{\partial \hbar\omega} (\lambda_{ij} - \langle \lambda_{ij} \rangle) \dot{u}_{ij}, \quad (6)$$

where $x = (\varepsilon - \varepsilon_F)/T$, the dot denotes differentiation with respect to time, $\langle \lambda_{ij} \rangle$ is the tensor λ_{ij} averaged over \mathbf{k} and g , $\langle \mu_{ij} \rangle_F$ is the tensor $\mu_{ij}(\mathbf{p})$ averaged over the Fermi surface:

$$\langle \lambda_{ij} \rangle = \left(3 \int d^3\mathbf{k} \right)^{-1} \sum_g \int \lambda_{ij} d^3\mathbf{k},$$

$$\langle \mu_{ij} \rangle_F = \oint_{\varepsilon_F} \mu_{ij} \frac{d\sigma_p}{|\nabla_p \varepsilon|} / \oint_{\varepsilon_F} \frac{d\sigma_p}{|\nabla_p \varepsilon|}.$$

In writing (5) and (6), we have taken into account the conservation of the number of electrons and of the energy, so that the conditions

$$\sum_p \dot{n}_s = 0, \quad \sum_p \varepsilon_p \dot{n}_s + \sum_{\mathbf{k}} \hbar\omega \dot{N}_s = 0$$

are satisfied identically.

The collision operators (3) and (4) have the form²

$$I_{e,ph}(\varphi, \chi) = I_{e,ph}^{(1)}(\varphi) + I_{e,ph}^{(2)}(\chi) = - \sum_g \int \frac{\partial N_0}{\partial \hbar\omega} w(n_0' - n_0)$$

$$\times \{ (\varphi_{p'} - \varphi_p + \chi_k) \delta(\varepsilon_p - \varepsilon_{p'} - \hbar\omega_k) - (\varphi_{p'} - \varphi_p - \chi_{-k}) \delta(\varepsilon_p - \varepsilon_{p'} + \hbar\omega_k) \} \frac{d^3\mathbf{k}}{(2\pi\hbar)^3},$$

$$I_{ph,e}(\varphi, \chi) = I_{ph,e}^{(1)}(\varphi) + I_{ph,e}^{(2)}(\chi) = \frac{\partial N_0}{\partial \hbar\omega} \int w(n_0' - n_0) (\varphi_{p'} - \varphi_p + \chi_k) \quad (8)$$

$$\times \delta(\varepsilon_p - \varepsilon_{p'} - \hbar\omega_k) \frac{2d^3\mathbf{p}}{(2\pi\hbar)^3}, \quad (9)$$

$$I_{ph,ph}(\chi) = \frac{1}{T} \int \left\{ \frac{1}{2} \sum_{\omega_1, \omega_2} W N_0 (N_{\omega_1+1}) (N_{\omega_2+1}) (\chi_1 + \chi_2 - \chi) \delta(\omega_1 + \omega_2 - \omega) \right. \\ \left. + \sum_{\omega_1, \omega_2} W N_0 N_{\omega_1} (N_{\omega_2+1}) (\chi_2 - \chi_1 - \chi) \delta(\omega + \omega_1 - \omega_2) \right\} \frac{d^3\mathbf{k}_1}{(2\pi\hbar)^3}, \quad (10)$$

where the quasimomenta of the electrons and phonons in (8)–(10) are connected by the usual conservation laws.

We note that it follows from the properties of the first Born approximation, in which the collision operators (8)–(10) are written, and from the invariance of the equations of mechanics relative to the replacement $t \rightarrow -t$, that

$$w(\mathbf{p}', \mathbf{k} | \mathbf{p}) = w(\mathbf{p} | \mathbf{p}', \mathbf{k}) = w(-\mathbf{p}', -\mathbf{k} | -\mathbf{p}), \quad (11)$$

$$W(\mathbf{k}_1, \mathbf{k}_2 | \mathbf{k}) = W(\mathbf{k} | \mathbf{k}_1, \mathbf{k}_2) = W(-\mathbf{k}_1, -\mathbf{k}_2 | -\mathbf{k}),$$

where the quantum numbers of the final state are written on the left and the quantum numbers of the initial state, on the right. It then follows that the operators

$$I_{e,ph}^{(1)}(\varphi), \quad I_{ph,e}^{(2)}(\chi), \quad I_{ph,ph}(\chi)$$

do not change the parities of the functions φ relative to the variable \mathbf{p} and of the function χ relative to the variable \mathbf{k} , while the operators

$$I_{e,ph}^{(2)}(\chi), \quad I_{ph,e}^{(1)}(\varphi)$$

respectively transform a function of definite parity in \mathbf{k} into a function of the same parity in \mathbf{p} and a function

of definite parity in \mathbf{p} into a function of the same parity in \mathbf{k} .

If we now take it into account that the right sides of Eqs. (3) and (4) are invariant under the substitutions $\mathbf{p} \rightarrow -\mathbf{p}$ and $\mathbf{k} \rightarrow -\mathbf{k}$ (we emphasize here that the \dot{u}_{ij} does not depend on \mathbf{k} , since the quasimomentum \mathbf{k} of the phonon, is no way connected with the wave number of the sound wave described by the tensor u_{ij}), then it follows from the above that $\varphi_p = \varphi_{-p}$, $\chi_k = \chi_{-k}$.

We note that the described properties of the collision operators follow from the general properties of electron-phonon interactions and are universal in this sense. The conditions $\varphi_p = \varphi_{-p}$, $\chi_k = \chi_{-k}$, aside from the symmetry of the collision operators, are essentially based on the symmetry of the right-hand sides of the kinetic equations (3) and (4) and therefore are generally characteristic only for the considered problem.

In the case of interest to us,

$$T \gg \Theta, \quad \partial N_0 / \partial \hbar\omega \approx -T / (\hbar\omega)^2,$$

and the remaining functions that enter into the integrands in (8) and (9) can be expanded in powers of $\hbar\omega/T$. It is not difficult to see that only odd powers of such an expansion enter into the expression that determines $\hat{I}_{e,ph}^{(1)}(\varphi)$. Therefore, the term corresponding to the elastic electron-phonon collisions will have the form

$$I_{e,ph}^{(1)}(\varphi) = 2 \int w \frac{\partial N_0}{\partial \hbar\omega_k} \frac{\partial n_0}{\partial \varepsilon_{p'}} (\varphi_{p'} - \varphi_p) \delta(\varepsilon_{p'} - \varepsilon_p) \hbar\omega_k \frac{d^3\mathbf{k}}{(2\pi\hbar)^3} \\ = 2 \oint_{\varepsilon_F} w \frac{\partial N_0}{\partial \hbar\omega_{p-p'}} \frac{\partial n_0}{\partial \varepsilon_{p'}} (\varphi_{p'} - \varphi_p) \hbar\omega_{p-p'} \frac{d\sigma_{p'}}{|\nabla_p \varepsilon|}. \quad (12)$$

In the expression for $\hat{I}_{e,ph}^{(2)}(\chi)$, however, the term that is linear in $\hbar\omega/T$ enters with a factor $(\chi_k - \chi_{-k})$ that is equal to zero by virtue of the parity of the function χ noted above, so that the first nonvanishing term will be quadratic in $\hbar\omega/T$.

In addition to the parity of the operators $\hat{I}_{e,ph}(\varphi, \chi)$ relative to the substitution $\mathbf{p} \rightarrow -\mathbf{p}$, their parity in the variable x is also important. Here x is the deviation, introduced above of the energy of the electrons from ε_F , divided by the temperature. Since the probability $w(\mathbf{p}', \mathbf{k} | \mathbf{p})$ is a slowly changing function of the energy of the electrons (with scale of the order of ε_F), it is customary to assume, taking the degeneracy of the electron gas into account that $w(\mathbf{p}', \mathbf{k} | \mathbf{p}) \approx w(\mathbf{p}_F', \mathbf{k} | \mathbf{p}_F)$, and accordingly the operator $\hat{I}_{e,ph}(\varphi, \chi)$ turns out to be even in x , i.e., it does not change the parity of the function φ in this variable.² However, we note that in the region $\Theta \ll T \ll \varepsilon_F$ there exists, besides the small parameter associated with the Debye temperature and equal to Θ/T , also the parameter T/ε_F . Although both quantities are small in comparison with unity, the relation between them can be arbitrary. Therefore, besides the expansion in powers of $\hbar\omega/T$ (which actually leads to a series in powers of Θ/T), we must also take into account the expansion of the slowly changing functions in powers of $(\varepsilon - \varepsilon_F)/\varepsilon_F$, which leads to the appearance of the factor T/ε_F . After expansion of the quantity w in powers of $(\varepsilon - \varepsilon_F)/\varepsilon_F$, the operator $\hat{I}_{e,ph}$ breaks up into the sum

$$I_{e,ph}(\varphi, \chi) = I_{e,ph+}(\varphi, \chi) + (T/\varepsilon_F) I_{e,ph-}(\varphi, \chi),$$

where $\hat{I}_{e,ph+}$ corresponds to the usually employed approximation and $\hat{I}_{e,ph-}$ reverses the parity of $\varphi(x)$. The factor T/ε_F is separated in explicit form in order that the kernels of the operators \hat{I}_+ and \hat{I}_- have the same order of magnitude.

We emphasize that the presence of two comparable small parameters connected with the temperature makes the considered range of values of T substantially different from the case $T \ll \Theta$, where the corresponding small parameters T/Θ and T/ε_F are connected by the relation $T/\Theta \gg T/\varepsilon_F$ and no expansion in powers of $(\varepsilon - \varepsilon_F)/\varepsilon_F$ can be carried out.

We now average Eq. (3) over the constant-energy surface $\varepsilon_p = \varepsilon$ [the averaging is carried out in a fashion similar to (7) with the replacement of the condition $\varepsilon_p = \varepsilon_F$ by the condition $\varepsilon_p = \varepsilon$]. It is easy to see that after such averaging the operator $\hat{I}_{e,ph}^{(2)}(\chi)$ turns out to be of the same order as the non-averaged operator. So far as $\hat{I}_{e,ph}^{(1)}(\varphi)$ is concerned, in correspondence with the expression (12) we have $\langle \hat{I}_{e,ph}^{(1,1)}(\varphi) \rangle = 0$ so that the first nonzero term will correspond to the third order in $\hbar\omega/T$, i.e., the kernel of the operator $\langle \hat{I}_{e,ph}^{(3)}(\varphi) \rangle$ will be smaller by a factor of $(\Theta/T)^2$ than the kernel of the unaveraged operator.²⁾ It is important that this averaging, changes also the order of right-hand side of Eq. (3). The even \hat{n}_{s+} and odd \hat{n}_{s-} parts of the right-hand side of Eq. (3) are of the order of

$$\hat{n}_{s+} \sim T^{-1} \varepsilon_F \hat{\xi}_s, \quad \hat{n}_{s-} \sim \hat{\xi}_s, \quad (13)$$

where $\hat{\xi} \sim \Lambda_{ij} \hat{\mu}_{ij}$, Λ_{ij} is a certain symmetric tensor with components of the order of unity. In the averaging over the constant-energy surface, the difference $\langle \mu_{ij} \rangle - \langle \mu_{ij} \rangle_F$ appears in the right side of (3). But $\langle \mu_{ij} \rangle = \langle \mu_{ij}(\varepsilon) \rangle$ with a characteristic scale of change of ε of the order of ε_F . Therefore, the difference $\langle \mu_{ij} \rangle - \langle \mu_{ij} \rangle_F$ can be expanded in powers of $(\varepsilon - \varepsilon_F)/\varepsilon_F$, so that we obtain the following estimate for the right side of the averaged equation:

$$\langle \hat{n}_{s+} \rangle \sim T \hat{\xi}_s / \varepsilon_F, \quad \langle \hat{n}_{s-} \rangle \sim \hat{\xi}_s. \quad (14)$$

We now proceed to discuss Eq. (4). We can again carry out the operator $\hat{I}_{ph,e}$ an expansion in powers of $\hbar\omega/T$. Here we take it into account that the equation

$$\begin{aligned} & \frac{2}{(2\pi\hbar)^3} \int w(\mathbf{p}', \mathbf{k}|\mathbf{p}) \varphi_{\mathbf{p}'} \frac{\partial n_0}{\partial \varepsilon} \delta(\varepsilon - \varepsilon') d^3\mathbf{p} \\ &= \frac{2}{(2\pi\hbar)^3} \int w(\mathbf{p}', \mathbf{k}|\mathbf{p}) \varphi_{\mathbf{p}} \frac{\partial n_0}{\partial \varepsilon} \delta(\varepsilon - \varepsilon') d^3\mathbf{p} \end{aligned} \quad (15)$$

is valid. In the derivation of (15), we used the transformation of the quasimomentum $\mathbf{p} \rightarrow -\mathbf{p}'$, the condition (11), the parity of the function $\varphi_{\mathbf{p}}$ in the variable \mathbf{p} , and the equality

$$w(\mathbf{p}', \mathbf{k}|\mathbf{p}) \delta(\varepsilon - \varepsilon') = w(\mathbf{p}'|\mathbf{p}, -\mathbf{k}) \delta(\varepsilon - \varepsilon')$$

(see Ref. 2), which means that the emission, by an electron, of phonon with quasimomentum \mathbf{k} and absorption of a phonon with quasimomentum $-\mathbf{k}$, with neglect of the energy of the phonon, are equally probable processes.

It follows from Eq. (15) that the linear term in the expansion of $\hat{I}_{ph,e}$ in powers of $\hbar\omega/T$ vanishes and the expansion begins with terms $\sim (\hbar\omega/T)^2$.

3. DISSIPATION OF ENERGY

At $T \gg \Theta$ the contribution of the phonon-electron collisions to the formation of the function $\chi_{\mathbf{k}}$ is negligibly small, so that the phonon kinetic equation can be written approximately in the form

$$\hat{I}_{ph,ph}(\chi) = \dot{N}_s. \quad (16)$$

Thus dragging of the phonons by the nonequilibrium electrons does not occur: the nonequilibrium increment to the phonon distribution function in metals does not depend on the electron distribution function and has the same form as in dielectrics (at $T \gg \Theta$).

However, in spite of the simplifications that have been achieved, it is hardly possible to obtain an exact solution of the kinetic equations in the case of an arbitrary dispersion law for the electrons and phonons. We therefore limit ourselves to order-of-magnitude estimates of the characteristic values of the function χ and φ . Here we must take explicitly into account the fact that establishment of equilibrium in the electron subsystem only through the most probable elastic electron-phonon collisions is not possible. For this purpose we consider the equation

$$\langle I_{e,ph}^{(1)}(\varphi) \rangle + \langle I_{e,ph}^{(2)}(\chi) \rangle = \langle \hat{n}_s \rangle \quad (17)$$

in addition to Eq. (3). Here χ is a solution of Eq. (17). A solution of Eq. (3) is simultaneously a solution of Eq. (17). The converse is generally not true, since Eq. (17) has an infinite set of solutions having different dependences on the directions of the vector $\mathbf{e} = \mathbf{p}/|\mathbf{p}|$. However, in the class of the functions which depend only on x , Eq. (17) has a unique solution. Let $\varphi_1 = \varphi_1(x)$ be such a solution. We shall seek a solution of Eq. (3) in the form $\varphi = \varphi_1(x) + \varphi_2(x, \mathbf{e})$. Then φ_2 should satisfy the equation

$$I_{e,ph}^{(1)}(\varphi_2) = \hat{n}_s - I_{e,ph}^{(2)}(\chi) - I_{e,ph}^{(1)}(\varphi_1). \quad (18)$$

We note that φ_1 , as well as any arbitrary function that depends only on the electron energy and not on the direction of its quasimomentum, reduces the elastic-collision operator to zero: $I_{e,ph}^{(1,1)}(\varphi_1(x)) = 0$ [see (12)]. Therefore, $\varphi_1(x)$, defined as the solution of Eq. (17), describes the deviation of the electron subsystem from equilibrium, due to the inelasticity of the electron-phonon interaction, and the separation of φ into a sum of two terms: $\varphi_1 + \varphi_2$ corresponds to the separation in explicit form of the contributions of the elastic and inelastic processes to the formation of the electron distribution function.

The solution of the problem (16)–(18) is carried out in the following sequence: the function χ is determined from (16) and substituted in (17), after which (17) can be solved relative to φ_1 ; finally, χ and φ_1 , which are now known functions, are substituted in (18), after which φ_2 is obtained from it.

For an estimate of the characteristic values of $\varphi_{1,2}$ and χ , we write down Eqs. (16)–(18) in the τ approximation. Here we need not worry about the signs of the corresponding expressions, choosing the plus sign everywhere, since the terms having the form of a difference of two identical or nearly identical quantities

have been revealed from the preceding analysis. It is simpler to choose the correct sign at the end of the calculations by starting out from the fact that the quantity $\dot{T}S$, where S is the entropy of the system, should be positive (the positiveness of $\dot{T}S$ follows rigorously from the properties of collision operators; for more details see below).

We note further that $\langle \hat{I}_{e,ph}^{(1)}(\varphi_1) \rangle \sim \hat{I}_{e,ph}^{(1)}(\varphi_1)$, since $\hat{I}_{e,ph}^{(1,1)}(\varphi_1) = 0$. Therefore, with account of what was said previously, the right-hand side of (18) can be rewritten in the form

$$\begin{aligned} \dot{n}_e + I_{e,ph}^{(2)}(\chi) + I_{e,ph}^{(1)}(\varphi_1) &\sim \dot{n}_e + I_{e,ph}^{(2)}(\chi) \\ + \langle I_{e,ph}^{(1)}(\varphi_1) \rangle &\sim \dot{n}_e + I_{e,ph}^{(2)}(\chi) + \langle \dot{n}_e \rangle + \langle I_{e,ph}^{(2)}(\chi) \rangle \sim \dot{n}_e + I_{e,ph}^{(2)}(\chi), \end{aligned}$$

i.e., φ_2 turns out to be independent of φ_1 .

The order of magnitude of the collision operators is now easily determined by standard methods (see, for example, Ref. 2), as a result of which we arrive at the expressions

$$\delta N / \tau^{III} \sim T \dot{\zeta} / \Theta, \quad (19)$$

$$\left(1 + \frac{T}{\varepsilon_F} x\right) \left(\frac{\Theta}{T}\right)^2 \frac{\delta n_1}{\tau_p} \sim x \left(1 + \frac{T}{\varepsilon_F} x\right) \left[\frac{\delta N}{\tau_L} + \dot{\zeta}\right], \quad (20)$$

$$\left(1 + \frac{T}{\varepsilon_F} x\right) \frac{\delta n_2}{\tau_p} \sim x \left(1 + \frac{T}{\varepsilon_F} x\right) \frac{\delta N}{\tau_L} + \left(\frac{\varepsilon_F}{T} + x\right) \dot{\zeta}; \quad (21)$$

$$\tau^{III} \sim \frac{\hbar M s^2}{\Theta T}, \quad \tau_p \sim \frac{\hbar}{T}, \quad \tau_L \sim \frac{\hbar T^2}{\Theta^2},$$

where M is the mass of the elementary cell and s is the speed of sound. In writing down (19)–(21), we have transformed from φ and χ to δn and δN in order that the corresponding τ have the meaning of the actual relaxation time:

$$\delta n = -\varphi \delta n_0 / \partial \varepsilon \sim \varphi / T, \quad \delta N = -\chi \delta N_0 / \partial \hbar \omega \sim T \chi / \Theta^2,$$

since at $T \gg \Theta$ the dominant role is played by phonons with maximal energy: $\hbar \omega \sim \Theta$.

The forms (19)–(21) are conditional in the sense that only the order of magnitude of the terms contained in the parentheses is correct; that is, each of them still contains a factor of the order of unity which is different in the different cases. Therefore, for example, it is not possible to cancel $1 + T x / \varepsilon_F$ on the right and left sides of Eq. (20). It is not difficult to determine from (19)–(21) the orders of magnitude of the quantities δN and $\delta n_{1,2}$, and, consequently of χ and $\varphi_{1,2}$. We have

$$\varphi_{1+} \sim \hbar \frac{T}{\varepsilon_F} \left(\frac{T}{\Theta}\right)^2 \left(1 + \frac{\Theta \varepsilon_F}{T^2}\right) \dot{\zeta}, \quad \varphi_{1-} \sim \hbar \left(\frac{T}{\Theta}\right)^2 \left(1 + \frac{\Theta \varepsilon_F}{T^2}\right) \dot{\zeta}, \quad (22)$$

$$\varphi_{2+} \sim \hbar \frac{\varepsilon_F}{T} \dot{\zeta}, \quad \varphi_{2-} \sim \hbar \left(1 + \frac{\Theta \varepsilon_F}{T^2}\right) \dot{\zeta}, \quad \chi \sim \frac{\hbar M s^2}{T} \dot{\zeta},$$

where φ_+ is an even and φ_- an odd function of x .

In the factor $1 + \Theta \varepsilon_F / T^2$ entering in $\varphi_{1\pm}$ and φ_{2-} , the term $\Theta \varepsilon_F / T$ results from δN , which is taken into account on the right sides of Eqs. (20) and (21), i.e., it describes the effect of the dragging of the electrons by the nonequilibrium phonons. In the considered temperature range, $\Theta \varepsilon_F / T^2$ can be either large and small in comparison with unity. Therefore, both terms are retained in the expressions for $\varphi_{1\pm}$ and φ_{2-} . The contribution to the scattering by the nonequilibrium phonons is always negligibly small, so that the order of magnitude of φ_{2+} does not depend on the drag effect.

If we now substitute the values of χ_p and φ_p in the operator $\hat{I}_{ph,e}$, it turns out that

$$\frac{I_{ph,e}(\varphi, \chi)}{I_{ph,ph}(\chi)} \sim \left(\frac{\hbar \omega}{\Theta}\right) \frac{T}{\varepsilon_F} \ll 1.$$

This justifies the neglect of the phonon-electron collisions in the determination of the function χ .

The energy dissipated in a unit volume of the metal per unit time is connected with the nonequilibrium increments to the distribution functions by the expression

$$\begin{aligned} \delta Q = T \dot{S} = T \dot{S}_e + T \dot{S}_{ph} \\ = \frac{2}{(2\pi\hbar)^3} T \int \hbar \ln \frac{1-n}{n} d^3p + \sum_s \frac{T}{(2\pi\hbar)^3} \int N \ln \frac{1+N}{N} d^3k \\ = -\frac{2}{(2\pi\hbar)^3} \int \hbar \varphi d^3p - \sum_s \frac{1}{(2\pi\hbar)^3} \int N \chi d^3k. \end{aligned} \quad (23)$$

If we substitute the left sides of the kinetic equations in place of \dot{n} and \dot{N} , then the expression for $\delta \dot{Q}$ can be reduced to some quadratic form, thereby proving the positiveness of the expression (23). However, for specific estimates it is convenient to substitute not the right sides of Eqs. (3) and (4), but the left sides that are identically equal to them in solution space. Such an approach is especially convenient in those cases in which there are several scattering mechanisms, each of which makes its own contribution to the formation of the distribution function. When the expression for $\delta \dot{Q}$ is reduced to a quadratic form the result contains, in addition to the terms proportional to the squares of the different increments to the distribution function, also their bilinear combinations of the form $\varphi_i \varphi_j$ ($i \neq j$). The symmetry of the functions $\varphi_{i,j}$ and of the collision operators can then be such that the terms containing the bilinear combinations become equal to zero. Therefore, the question of the contribution of such terms to the expression for $T \dot{S}$ requires special consideration. Upon substitution of the right hand sides of the kinetic equations in place of \dot{n} and \dot{N} , the result turns out to be a linear function of φ and χ , so that the question of the contribution of the bilinear terms does not arise generally.

Using the expressions (22) for φ and χ , representing d^3p in the form

$$d\varepsilon \frac{d\sigma_p}{|\nabla_p \varepsilon|} = T dx \frac{d\sigma_p}{|\nabla_p \varepsilon|}$$

and taking it into account that the integral of the product of functions having different parity in x is equal to zero,³⁾ we get for the energy dissipation rates due to electron and phonon viscosities, respectively,

$$T \dot{S}_{n,e} \sim \frac{\hbar \dot{\zeta}^2}{a^3} \frac{\varepsilon_F}{T} \left(1 + \frac{T^2}{\Theta \varepsilon_F} + \frac{T^4}{\Theta^2 \varepsilon_F^2}\right), \quad (24)$$

$$T \dot{S}_{n,ph} \sim \frac{\hbar \dot{\zeta}^2}{a^3} \frac{M s^2}{\Theta} \sim \frac{\hbar \dot{\zeta}^2}{a^3} \frac{\varepsilon_F T}{\Theta}, \quad (25)$$

where a is of the order of the interatomic distance. The three terms in the parentheses in (24) have the following physical meaning: the first corresponds to the contribution φ_{2+} of elastic scattering of electrons by equilibrium phonons, $T^2 / \Theta \varepsilon_F$ corresponds to the effect of dragging of the electrons by nonequilibrium phonons in inelastic processes, and $T^4 / \Theta^2 \varepsilon_F^2$ corres-

ponds to the inelastic scattering of electrons by equilibrium phonons⁴⁾ (the functions $\varphi_{1\pm}$ make contributions of the same order to the last two terms). It is seen that the role of the dragging effect is almost always small. An exception is the region $T \sim (\Theta \varepsilon_F)^{1/2}$, when all three processes (dragging of the electrons, elastic and inelastic scattering) make comparable contributions to the sound dissipation due to electron viscosity.

Thus, at $\Theta \ll T \ll \varepsilon_F (\Theta / \varepsilon_F)^{1/2}$ the electron viscosity is determined by the electron-phonon collisions. The contribution from inelastic scattering by equilibrium phonons turns out to be comparatively small.

Sound dissipation due to phonon viscosity, as has already been seen from Eq. (16), has the same form as in dielectrics,^{2,3} and does not depend on the temperature. It is seen from a comparison of (24) and (25) that in the region $\Theta \ll T \ll \varepsilon_F (\Theta / \varepsilon_F)^{1/3}$ the phonon viscosity significantly exceeds the electron viscosity. The increase in the electron viscosity with increase in temperature is explained by the partial lifting of the degeneracy of the electron gas; however, the electron viscosity begins to play an important role only at $T \gg \varepsilon_F (\Theta / \varepsilon_F)^{1/3}$, which corresponds to $T \gg 10^4 \text{K}$ for good metals. For the problem considered, such high temperatures can scarcely be of practical interest. Therefore, at $T \gg \Theta$ the electron viscosity can always be neglected.

In a number of dielectrics, the dispersion law of the phonons is such that collisions of longwave and shortwave phonons are forbidden. In this case the function χ_k contains a singularity as $\omega \rightarrow 0$, so that the corresponding path length diverges.^{4,5} The coefficients of thermal conductivity and sound absorption become infinitely large. The resultant divergences are removed by the fact that the integrals over ω are cut off at the lower limit at the value $\omega = \omega_0$, so that the path length governed by triple phonon-phonon collisions becomes comparable with the path length governed by other mechanisms. In the case of dielectrics, such mechanisms are four-fold collisions, scattering from impurities, and similar processes, which have significantly lower probability than the triple collisions of short wavelength phonons, in correspondence with which ω_0 turns out to be small in comparison with $\omega_D = \Theta / \hbar$, while the quantity Γ is anomalously large.⁴

In a similar situation in metals, the processes competing with triple phonon-phonon collisions are phonon-electron collisions, i.e., a process which has a significantly larger probability than the fourfold collisions and scattering of phonons by impurities. The quantity ω_0 in this case turns out to be comparable with the Debye frequency, which eliminates from physical meaning the division of phonons into short- and longwave, and obviates the need for special consideration of sound absorption in metals in the presence of hindrance rules.⁵⁾

Completing the consideration of the viscous mechanism of sound dissipation, we derive an expression for the absorption coefficient (the logarithmic damping decrement) corresponding to the expression (25). We have

$$\Gamma_{n, pA} \sim a^{-1} (\Omega / \omega_D)^2, \quad (26)$$

where Ω is the frequency of the sound wave and ω_D is the Debye frequency.

We now determine the contribution of the thermal-conductivity damping. The rate of energy dissipation per unit volume of the metal, which is governed by the thermal conduction process, is determined for monochromatic sound by the expression⁶

$$TS_\kappa = T \frac{\kappa_{ab} q_a q_b}{C^2 \Omega^2} (\lambda_{inij} \alpha_{in} \dot{u}_{ij})^2. \quad (27)$$

Here κ_{ab} is the thermal conductivity tensor, \mathbf{q} is the wave vector of a sound wave of frequency Ω , λ_{inij} is the elastic modulus tensor (not to be confused with λ_{ij} — the deformation potential tensor), α_{in} is the thermal expansion tensor, and C is the heat capacity per unit volume.

We estimate now Eq. (27). The quantity κ_{ab} is determined by the electron thermal conductivity and at high temperatures is of the order of

$$\kappa_{ab} \sim \hbar / m^* a^2,$$

where m^* is the effective mass of the electron. The heat capacity of the metal at $T \gg \Theta$ is determined by the phonon subsystem, so that the atomic heat capacity is equal to 3 and $C = 3/a^3$. The quantity $\lambda_{inij} \sim \rho s^2$, where $\rho = M/a^3$ is the density of the metal. Finally, we estimate α_{in} from the fact that ΔV becomes of the order of V for condensed matter at ΔT of the order of the binding energy of the atoms. Then $\alpha_{in} \sim (Ms^2)^{-1} \sim \varepsilon_F^{-1}$. Making use of the expression (27), we get

$$TS_\kappa \sim \frac{\hbar^2 \varepsilon_F T}{a^2 \Theta}. \quad (28)$$

The corresponding value of Γ is

$$\Gamma_\kappa \sim \frac{1}{a} \frac{T}{\Theta} \left(\frac{\Omega}{\omega_D} \right)^4. \quad (29)$$

Comparing (28) with (24) and (25), we come to the conclusion that the thermal conductivity absorption of sound in metals at $T \gg \Theta$ is always large in comparison with the viscous absorption. However, the thermal-conduction absorption does not always exist. There exist such directions of propagation and polarization of the sound wave for which the expression (27) goes strictly to zero, for example, for transverse sound propagating along a symmetry axis of the crystal. For these isolated directions, the damping of the sound is governed by the viscosity. Here the absorption coefficient is smaller by a factor of T/Θ , under typical conditions, than in the case of thermal conductivity damping [cf. (26) and (29)].

Thus, at $T \gg \Theta$, the sound absorption in metals is strongly anisotropic. In the general case, it is determined by the electron thermal conductivity and is T/Θ times larger than in dielectrics. For certain isolated directions of propagation and sound polarization, the thermal conduction mechanism is "turned off." Under these conditions, the absorption coefficient is greatly reduced and becomes of the same order as in dielectrics, and the sound absorption process itself is determined by the phonon viscosity.

We emphasize that the indicated anisotropy is not connected in any way with the topological features of the Fermi surface of the metal and is governed exclusively by the difference between the characteristic contributions made to the sound damping by the viscosity and the thermal conductivity. The latter distinguishes the considered case qualitatively from the sound absorption in dielectrics, where, at $T \gg \Theta$, the viscous and the thermal conductivity damping turn out to be of the same order,³ so that the "turning off" of the thermal conductivity mechanism of dissipation does not change the order of magnitude of the absorption coefficient.

4. DISCUSSION OF THE RESULTS

The results above can be clearly explained from the following qualitative considerations. It is well known (see, for example, Ref. 2) that at $T \ll \Theta$ two electron relaxation times exist in metals: τ_e —which describes the energy relaxation and τ_p —which describes the relaxation in the direction of the quasimomentum. Here the relaxation in the directions of \mathbf{p} was the character of diffusion over the constant-energy surface, so that τ_e and τ_p are connected by the relation $\tau_e \sim (T/\Theta)^2 \tau_p \ll \tau_p$. The quantity τ_e enters into the thermal conductivity coefficient, while the equation for the nonequilibrium increment to the electron distribution functions, which is determined by the conductivity of the metal, is obtained after averaging of the kinetic equation over \mathbf{x} ; therefore τ_p enters into the expression for the electrical conductivity. Both the energy relaxation and the relaxation over the directions of \mathbf{p} are important in the sound absorption and both relaxation times enter into the expression for the energy dissipated through the electron viscosity. In this case, the principal contribution is made by the process which has the larger τ , i.e., collisions changing the direction of \mathbf{p} .

In the region $T \gg \Theta$ the energy relaxation is a slow process for the electron gas, while the quasimomentum of the electron changes in each collision with a phonon by a quantity of the order of itself. Therefore τ_e and τ_p are connected as before by the relation $\tau_e \sim (T/\Theta)^2 \tau_p$ [see (20), (21)], but now $T/\Theta \gg 1$. It is important, however, that τ_e appears only in such processes for the description of which it is necessary to carry out averaging of the "electron" kinetic equation over the constant energy surface. Since the right hand side of the kinetic equation, which describes the thermal and electric conductivities of the metal, is linear in $\nabla_p \varepsilon$, it vanishes after such averaging. Therefore, only a single relaxation time, τ_e , enters into the expression for the coefficients of thermal and electrical conductivities. (But not in the expression for the dissipation of the sound energy! This contains again both relaxation times.) However, in contrast to the case $T \ll \Theta$, now the longer relaxation time (τ_e) enters with a small statistical weight, since the right side of the kinetic equation, even if it doesn't vanish upon averaging over the surface $\varepsilon(\mathbf{p}) = \text{const}$, does change in order of magnitude. Therefore, at not too high temperatures, the principal contribution to the sound dissipation is made by processes not with long but with short relaxation times, i.e., elastic electron-phonon collisions.

As for the relation between the viscous and thermal-conduction absorptions, for dielectrics both mechanisms are governed by phonons, i.e., by one and the same type of quasiparticle. For metals at $T \gg \Theta$ the viscosity and the heat capacity are determined by the phonon subsystem and the thermal conductivity by the electron subsystem. Hence the difference in the ratio of these two dissipation mechanisms.

The criteria of applicability of the considerations just given reduce to the conditions $\Omega \ll \nu$ and $l \ll \lambda_s$, which must be satisfied simultaneously. Here ν and l are the effective of collision frequency and the free path length of the quasiparticles responsible for the sound absorption relative to the process by which the dissipation is determined; λ_s is the sound wavelength, $\lambda_s = s/\Omega$.

In the case of thermal-conduction absorption, the sound dissipation is governed by the electron subsystem, so that ν is the frequency of the elastic electron-phonon collisions: $\nu_{e,ph} \sim \tau_p^{-1} \sim T/\hbar$. In this case the strictest condition is the condition $l \ll \lambda_s$, which leads to the limitation

$$\Omega \ll (\Theta/\varepsilon_e) (T/\hbar). \quad (30)$$

For the same directions of propagation and polarization of the sound for which the thermal conductivity absorption is "turned off," the dissipation is determined by the phonon viscosity. In these cases, ν is the frequency of triple phonon-phonon collisions: $\nu^{III} \sim \Theta T / Ms^2 \hbar$, and the conditions $\Omega \ll \nu$ and $l \ll \lambda_s$ are equivalent. As a result, we again arrive at the limitation (30).

Finally, the formula (27) itself was obtained under the assumption of adiabaticity of the deformation of the material by the field of the sound wave, which is valid under the condition $C\Omega \gg \kappa_{ab} q_a q_b$. This leads to the most severe limitation

$$\Omega \ll \Theta^2 / \hbar \varepsilon_e. \quad (31)$$

We note that the considered longwave approximation actually works all the way up to hypersonic frequencies. Thus, for typical metals at $T \sim 10^3$ K we have $\Omega \ll 10^{10}$ Hz from (31), which is explained by the large value of the frequency of collisions at high temperatures.

In conclusion, the authors consider it their pleasant duty to express their deep gratitude to I. M. Lifshitz for constant attention to the work, and also to Yu. M. Gal'perin, V. L. Gurevich, M. I. Kaganov and L. P. Pitaevskii for detailed discussions of the results and useful remarks.

¹In the considered problem, account of Fermi-liquid effects does not change the results, since the function which described the Fermi-liquid interaction of the conduction electrons $\Phi(\mathbf{p}, \mathbf{p}')$ does not enter in explicit fashion in the equations used below.²

²The vanishing of $\langle I_{e,ph}^{1,1}(\varphi) \rangle$ leads to the necessity of taking into account the inelasticity of the electron-phonon collisions, i.e., the next terms in the expansion of $I_{e,ph}^{(1)}$ in powers of $\hbar \omega / T$.¹

³We recall that the range of values of T that we have considered is limited by the relation $T \ll \varepsilon_F$, so that the finite limits can be replaced by infinite ones in the integration with re-

- spect to x , accurate to exponentially small corrections.
- ⁴ Comparing this term with the corresponding expression of Ref. 1 we see that the role of inelastic processes is overestimated in Ref. 1 by a factor of $(\epsilon_F/T)^2$.
- ⁵ It was pointed out in Ref. 4 that an additional numerical smallness can appear in the problem, and lead to a significant decrease of ω_0 . In such cases, the presence of hindrances leads to a strong increase in the viscous damping, with account of what was shown in Refs. 3 and 4, the quantity TS_7 can be estimated in such cases by a method completely analogous to that shown above.

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