On the theory of the Fröhlich conductivity of a conductor with a commensurable charge density wave

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The electrical conductivity of a quasi-one-dimensional Peierls conductor with a charge density wave (CDW) whose wavelength is commensurable with the original lattice constant is studied theoretically. It is assumed in the model considered that the Fermi surface consists of two slightly curved planes, and that the order parameter $\Delta = |\Delta| e^{i\chi}$ is a continuous function of the coordinates. An expression is obtained for the current on the basis of the microscopic theory, and an equation is derived for the phase χ with allowance for the pinning of the commensurable CDW. It is shown that the state with the coordinate-independent phase χ is the energetially advantageous state. The conductivity of the conductor is investigated in detail in this case; it is shown that phenomena similar to the Josephson effect occur in the sample: in electric fields E weaker than the threshold field E_0 the contribution to the conductivity σ is made by the quasiparticles, while the CDW is slowed down; for $E > E_0$ the motion of the CDW leads to an increase in σ and to the generation of an alternating current with frequency connected with E. Also investigated are the inhomogeneous solutions for $\chi(x)$: chains of solitons and antisolitons. In the model considered the soliton is neutral, but it contributes to the current when it moves. The mobility of the solitons and their contribution to the conductivity are computed.

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1. INTRODUCTION

Experimental investigations carried out recently[1,2] have confirmed the theoretical predictions[2-4] that the charge density wave (CDW) produced during the Peierls transition in a quasi-one-dimensional crystal contributes to the conductivity of such a crystal. One of the proofs of this is the fact that the conductivity σ_1 of the crystal is a constant in fields of intensities E lower than some threshold value E_0, but increases with the field when $E > E_0$, attaining a value $\sigma_\infty > \sigma_1$. This behavior of the conductivity is interpreted as follows. In fields $E < E_c$ the conductivity is due to the quasiparticles, the CDW being prevented from moving by its pinning, which is due to the interaction with the impurities in the case of an incommensurable CDW or the commensurability effects in the case of a commensurable CDW. If the field E is higher than the threshold field E_0, then the CDW begins to move and contributes to the conductivity of the sample. The motion of the CDW is accompanied by nonstationary effects. Thus, the following effects have been observed in $NbSe_3$, which possesses a nonlinear current-voltage characteristic (CVC) due to an incommensurable CDW[1,2]: noise generation with several isolated frequencies in the nonlinearity region of the CVC[5,6] and the appearance of singularities on the CVC upon the application of an alternating field $E_1 \sin \omega t$ to the sample.[7] A nonlinear CVC has been observed also in TaS_3, in which the CDW period is four times longer than the parent-lattice constant[8] ($Q = \pi/4a$, where Q is the wave vector of the CDW and a is the parent-lattice constant in the direction of the filaments).

The theoretical analysis of the Fröhlich mechanism for the conductivity of quasi-one-dimensional conductors with allowance for the pinning of the CDW has been performed largely with the use of the phenomenological approach. This approach has been used to estimate the CDW-breakaway threshold field E_0 (Refs. 9 and 10) and to compute the contribution, due to the CDW fluctuations, to the conductivity.[11,12] Guyer and Miller[11] have investigated uniform fluctuational CDW displacements, while Rice *et al.*[12] have analyzed the nonuniform fluctuational distortions of the commensurable CDW—the phase solitons, i.e., the local nonlinear distortions of the CDW phase. In those papers in which the conductivity of a conductor with a CDW is computed on the basis of the microscopic theory,[13,14] the effects of the pinning of the CDW are neglected.

In the present paper we generalize the microscopic equations obtained in our previous paper[14] by taking into account the effects of the pinning of the commensurable CDW, i.e., we consider the case in which the wave vector Q of the CDW and the reciprocal lattice vector are commensurable. The equations used are valid provided we can neglect the one-dimensional fluctuations and the localization effects (due, for example, to the influence of the three-dimensionality). We shall derive an equation for the CDW phase χ on the basis of the equations of the microscopic theory.

This equation describes, in particular, the slipping of the CDW as a whole when $E > E_0$ [the phase χ depends only on the time: $\chi = \chi(t)$]. The motion of the CDW is not uniform in this case: in the motion with average velocity

$$u \sim \left\langle \frac{\partial \chi}{\partial t} \right\rangle$$

the CDW executes oscillations with frequency ω equal in order of magnitude to

$$\omega = el(E^2 - E_0^2)^{1/2},$$

where $l = v\tau$ is the mean free path. The current component due to the CDW undergoes the same oscillations. The differential equation (which is of first order in the time) for χ in this case has exactly the same form as

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the equation of the resistance model of the Josephson junction for the phase difference (see, for example, Ref. 15). Therefore, there occur in a conductor with a CDW effects similar to the nonstationary Josephson effects that occur in superconducting contacts.[1)]

The equation for the phase χ has, besides homogeneous solutions, inhomogeneous solutions: $\chi = \chi(x, t)$. Some of these solutions are solitons ($\partial\chi/\partial x > 0$) and antisolitons ($\partial\chi/\partial x < 0$). Phase solitons in conductors with commensurable CDW were proposed by Rice *et al.*[12] They concluded that solitons and antisolitons could be treated as oppositely charged quasiparticles that should be created in pairs on account of the electrical neutrality of the system. Such quasiparticles contribute to the current. Our conclusions concerning the solitons and antisolitons differ significantly from the results obtained in Ref. 12. The point is that a soliton (antisoliton) is a macroscopic formation, and its dimension is, as a rule, significantly greater than the Thomas–Fermi screening distance r_{TF} (we have in mind the screening distance r_{TF} due to both the quasiparticles and the CDW, since the CDW also screens off the electric field E). Therefore, the charge arising in a soliton as a result of the spatial variation of the phase is screened off, and, in consequence, the total charge of a static soliton (antisoliton) is equal to zero (the potential drop across it is also equal to zero). But there exists across a moving soliton (antisoliton) a potential drop whose sign is opposite to that of the total potential across the sample, and the solitons make a contribution to the conductivity, specifically, they increase it. This contribution is proportional to the number of solitons in the sample. In real samples of macroscopic dimensions, the solitons spread to many filaments, and the energy necessary for their formation is significantly higher than the thermal energy. Thus, the production of solitons (φ particles) as a result of the thermal vibrations is highly improbable,[2)] and the dominant contribution to the conductivity of the sample is made by the uniform slipping of the CDW. Nevertheless, we shall find the solutions for the solitons, and compute their mobility, since they can, in principle, be produced near defects,[19] or be excited with the aid of external influences. In the present paper we shall assume that the characteristic vibration frequencies ω in the system are smaller than the energy gap $|\Delta|$.

2. THE BASIC EQUATIONS

Let us derive the equations that describe a quasi-one-dimensional conductor with a commensurable CDW. To do this, let us generalize the equations obtained in our previous paper.[14] In that paper we considered a conductor with a commensurable CDW and an energy spectrum described by the formula

$$\varepsilon_p = \frac{p_\parallel^2}{2m} + \eta(\mathbf{p}_\perp) - \varepsilon_F, \quad (1)$$

where $\mathbf{p}_\parallel$ and $\mathbf{p}_\perp$ are the components of the electron momentum in the directions parallel and perpendicular to the filaments and ε_F is the Fermi energy. It is assumed that $|\eta(\mathbf{p}_\perp)| \ll \varepsilon_F$, where the function $\eta(\mathbf{p}_\perp)$ describes the deformation of the Fermi surfaces, which are nearly flat. At the same time, $|\eta| > \tau^{-1}$, which allows us to neglect the effects of the localization of the electrons in the disordered potentials of the impurities (here τ is the characteristic time of the scattering of the momentum by the impurities).

We shall also neglect the one-dimensional fluctuations, which are suppressed by the effects connected with the three-dimensional character of the electron or phonon spectrum,[20] and the effects connected with the pinning of the CDW to the impurities. It follows from a number of experimental[1,2,8] and theoretical[9] investigations that, at least in fairly pure samples, the CDW pinning caused by the commensurability is stronger than the pinning to the impurities. In particular, the threshold-field strength E_0 is, as a rule, much higher in the case of commensurable CDW than in the incommensurable CDW case, in which the pinning is due only to the impurities. Efetov and Larkin[21] have shown that the phase fluctuations caused by the impurities do not destroy the long-range order, and, consequently, do not cause significant pinning of the phase when

$$1/\tau \ll \lambda\varepsilon_F\gamma\left(\frac{d^2\omega^2(\mathbf{k})}{dk_\perp^2}\right)_{\mathbf{k}=\mathbf{Q}} / [\omega(\mathbf{Q})d]^2,$$

where λ is the dimensionless electron-phonon interaction constant; $\gamma \sim (\Delta/\varepsilon_F)^{n-2}$, n being the commensurability order; $\omega(\mathbf{k})$ is the phonon dispersion law; and d is the filament spacing.

Let us also note that, although we take only the electron-impurity collision integral into consideration, the results of the calculations can be applied also to the case of momentum scattering by phonons if the temperature is much higher than the Debye temperature. In this case the scattering is almost elastic, and the integrals for the collisions with the phonons and the impurities have the same form.[13] Since the interaction with the phonons does not lead to the pinning of the CDW, we can neglect the above limitation of τ in the case in which τ is determined by the scattering on the phonons.

We shall, for simplicity, assume that the wave vector $\mathbf{Q}$ of the CDW is parallel to the filaments: $\mathbf{Q} \parallel \mathbf{p}_\parallel$.

To generalize the equations of Ref. 14 to the case of commensurable CDW, it is sufficient to derive the equations for the Green functions in the absence of external fields. This is due to the fact that the commensurability effects give rise to corrections of the order of $(\Delta/\varepsilon_F)^{n-2}$, which are small when $n > 2$ (the case of period doubling, i.e., the $n = 2$ case, is a special one, and we shall not consider it here). Consequently, we can, in computing the response to an external field, neglect the commensurability effects, and use the well-known results, χ, of the linear-response calculation for a conductor with an incommensurable CDW.[14] This does not mean that we shall analyze only the linear problem, since the equations obtained are nonlinear with respect to the CDW phase χ [we should, in computing the response of the system to an external field, separate out the phase factors $e^{i\chi}$, and limit ourselves to the linear terms containing the coordinate (x) and time (t) derivatives of the order parameter $\Delta = |\Delta|e^{i\chi}$].

Let us consider, for example, the retarded Green function $G^R(\mathbf{p},\mathbf{p}')$. By proceeding from the Fröhlich Hamiltonian, we can derive for G^R the equation

$$(\varepsilon-\varepsilon_p)G^R(\mathbf{p},\mathbf{p}')=\Sigma g G^R(\mathbf{p}-\mathbf{q},\mathbf{p}')\langle b_q+b_{-q}^+\rangle+\delta_{pp'}. \qquad (2)$$

In the presence of a CDW the anomalous averages

$$\langle b_{nQ}+b_{-nQ}^+\rangle$$

are nonzero. In the case of commensurable CDW we should take into account not only the functions $G^R(\mathbf{p}\pm\mathbf{Q},\mathbf{p})$, but also the functions $G^R(\mathbf{p}\pm n\mathbf{Q},\mathbf{p})$. As a result, we obtain an equation for the $(n\times n)$-matrix Green function $\hat{G}^R(\mathbf{p},\mathbf{p}')$. This equation is given in Ref. 4 under the assumption that

$$\langle b_{nQ}+b_{-nQ}^+\rangle=0$$

for all n except $n=1$. But the appearance of a CDW commensurable with the wave vector $\mathbf{Q}$ automatically leads to the appearance of the CDW harmonics with wave vectors $2\mathbf{Q}$, $3\mathbf{Q},\ldots,(n-2)\mathbf{Q}$.[22] The equations for the $\hat{G}^R(\mathbf{p},\mathbf{p}')$ then become complicated. We shall, for simplicity, limit ourselves to the consideration of the cases with $n=3$ and $n=4$ (the results for all the cases of different multiplicities $n\geqslant 3$ are qualitatively identical; they differ mainly in the numerical coefficients in front of the threshold-field strength E_0).

From (2) we obtain for the case with $n=4$ the equation[21]

$$[\hat{G}^R]^{-1}\hat{G}^R(\mathbf{p},\mathbf{p}')=\delta_{pp'},$$

$$[\hat{G}^R]^{-1}=\begin{bmatrix}\varepsilon-\varepsilon_1 & -\Delta & -\Delta_2 & -\Delta^*\\ -\Delta^* & \varepsilon-\varepsilon_2 & -\Delta & -\Delta_2\\ -\Delta_2 & -\Delta^* & \varepsilon-\varepsilon_0 & -\Delta\\ -\Delta & -\Delta_2 & -\Delta^* & \varepsilon-\varepsilon_0\end{bmatrix}; \qquad (3)$$

$$\Delta=\langle g(b_Q+b_{-Q}^+)\rangle,\quad \Delta_2=\langle g(b_{2Q}+b_{-2Q}^+)\rangle,$$

$$\varepsilon_1=\varepsilon(Q/2+k),\quad \varepsilon_2=\varepsilon(-Q/2+k),$$

$$\varepsilon_0=\varepsilon(3Q/2+k)=\varepsilon(5Q/2+k)\gg\Delta\gg\Delta_2,\quad |\mathbf{k}|\ll p_F\sim Q.$$

Only one order parameter Δ arises in the case of threefold commensurability (i.e., in the $n=3$ case).

Besides the functions $\hat{G}^{R(A)}$, let us also consider the Green function $\hat{G}$ (see, for example, Ref. 14). The function $\hat{G}$ consists of a regular and an anomalous part:

$$\hat{G}=\hat{G}^{(r)}+\hat{G}^{(a)},$$

$$\hat{G}^{(r)}_{\varepsilon\varepsilon'}=\hat{G}^R\,\mathrm{th}\frac{\varepsilon'}{2T}-\mathrm{th}\frac{\varepsilon}{2T}\hat{G}^A.$$

In the equilibrium case

$$\hat{G}^{(a)}=0,\quad \hat{G}^{(r)}=(\hat{G}^R-\hat{G}^A)\,\mathrm{th}\frac{\varepsilon}{2T}.$$

As in Ref. 14, it is convenient for us to introduce the Green functions integrated over the variable $\xi=(p_\parallel-p_F)Q/2m$:

$$\tilde{g}_{ik}=i\int\frac{d\xi}{\pi}G_{ik}(p,p'). \qquad (4)$$

The tilde indicates that we should take account of the commensurability effects in computing the $\tilde{g}_{ik}$, i.e., take account of the small terms of the type $\Delta\varepsilon_0$ in inverting the matrix $(\hat{G}^R)^{-1}$ in (3).

The commensurability effects should be taken into consideration only when the self-consistency condition is fulfilled. This condition can easily be obtained on the basis of the equations of motion of the operators b_q and b_{-q}^+. We obtain

$$\left(1+\omega_Q^{-2}\frac{\partial^2}{\partial t^2}\right)\Delta=\lambda\langle\tilde{g}_{12}+\tilde{g}_{23}+\tilde{g}_{34}+\tilde{g}_{41}\rangle, \qquad (5)$$

$$\left(1+\omega_{2Q}^{-2}\frac{\partial^2}{\partial t^2}\right)\Delta_2=2\lambda_2\,\mathrm{Re}\langle\tilde{g}_{13}+\tilde{g}_{24}\rangle, \qquad (6)$$

$$\lambda=g^2S(2\pi\omega_Q v)^{-1}=\lambda_2\omega_{2Q}/\omega_Q.$$

Here ω_Q is the frequency of the phonons with wave vector $\mathbf{Q}$ and S is the area of the Brillouin zone's cross section in the plane $p_\parallel-p_F=0$. The angle brackets denote averaging over $\mathbf{k}_\perp$ and integration over ε:

$$\langle(\ldots)\rangle=\int\frac{d\mathbf{k}_\perp}{S}\int d\varepsilon(\ldots).$$

We are interested in the frequencies $\omega\ll\omega_{2Q}$; therefore, we can discard the time derivative in Eq. (6). Further, we should find the regular function $\hat{G}^{(r)}$ with the aid of (3) and (4) and substitute it into Eqs. (5) and (6). The commensurability effects will then lead to the appearance of terms of the type

$$\lambda\left\langle(\tilde{g}_{21}^R-\tilde{g}_{21}^A)\,\mathrm{th}\frac{\varepsilon}{2T}\right\rangle\left(\frac{\Delta^*}{\varepsilon_0}\right)^2\approx\Delta^*\left(\frac{\Delta^*}{\varepsilon_0}\right)^2.$$

Here we used the self-consistency condition in the zeroth order in the parameter Δ^*/ε_0. Then the Eqs. (5) and (6) can be rewritten in the form

$$\left(1+\omega_Q^{-2}\frac{\partial^2}{\partial t^2}\right)\Delta=4\frac{\Delta^{*3}}{\varepsilon_0^2}+16\frac{\lambda_2}{\lambda}\frac{\Delta^*|\Delta|^2}{\varepsilon_0^2}\cos 2\chi+\lambda\langle g_{12}^{(0)}\rangle, \qquad (7)$$

$$\Delta_2=4\frac{\lambda_2}{\lambda}\frac{|\Delta|^2\cos 2\chi}{\varepsilon_0}. \qquad (8)$$

Here $g_{12}^{(0)}$ is the Green function without allowance for the commensurability effects and χ is the CDW phase. We neglect the corrections to the spectrum that arise as a result of the commensurability.

Let us separate the imaginary and real parts of (7). After simple transformations, we obtain for the phase χ and the modulus of the order parameter in the semiclassical approximation in the parameter $\omega/|\Delta|$ the equations:

$$-\omega_Q^{-2}\left[2\frac{\partial\chi}{\partial t}\frac{\partial|\Delta|}{\partial t}+|\Delta|\frac{\partial^2\chi}{\partial t^2}\right]=\gamma|\Delta|\sin n\chi+i\,\mathrm{Sp}(\sigma_x\hat{g}), \qquad (9)$$

$$\omega_Q^{-2}\left[\frac{\partial^2}{\partial t}|\Delta|-|\Delta|\left(\frac{\partial\chi}{\partial t}\right)^2\right]=\gamma|\Delta|\cos n\chi-i\,\mathrm{Sp}(\sigma_y\hat{g}), \qquad (10)$$

where ω is the oscillation frequency; for $n=4$

$$\gamma=\frac{4|\Delta|^2}{\varepsilon_0^2}\left(1+2\frac{\lambda_2}{\lambda}\right),$$

while for $n=3$, $\gamma=3|\Delta|/\varepsilon_0$, where $\varepsilon_0=\varepsilon(\frac{3}{2}Q)$; for $n>4$ we have in order of magnitude $\gamma\approx|\Delta|^{n-2}/\varepsilon_0^{n-2}$; $\hat{g}$ is the Green function whose phase factors have been separated out. It is connected with the function $\hat{g}$ by the transformation

$$\hat{g}=\hat{s}\hat{\sigma}_z\hat{g}\hat{s}^+,\quad \hat{s}=\cos(\chi/2)+i\hat{\sigma}_z\sin(\chi/2). \qquad (11)$$

The function $\hat{g}$ satisfies an equation that can easily be derived from Eq. (18) of Ref. 14. We shall also write down the equations for the matrices $\hat{g}^{R(A)}$, which are defined in much the same way as (11). We can write the necessary equations in the form of a single equation by introducing the supermatrix

$$\check{g}=\begin{pmatrix}\hat{g}^R & \hat{g}\\ 0 & \hat{g}^A\end{pmatrix}. \qquad (12)$$

In the semiclassical approximation with respect to $\omega/|\Delta|$, the equation for the matrix $\check{g}$ has the form

$$-iv\frac{\partial}{\partial x}\check{g}+\frac{i}{2}\frac{\partial}{\partial t}[\check{\sigma}_z,\check{g}]_+-\tilde{\varepsilon}[\check{\sigma}_z,\check{g}]_--i|\Delta|[\check{\sigma}_y,\check{g}]_-$$
$$+[\check{h},\check{g}]_-+[\check{\Sigma},\check{g}]_--i\frac{\nu_2}{8T}\frac{\partial\chi}{\partial t}\mathrm{ch}^{-2}\left(\frac{\varepsilon}{2T}\right)\left[\check{g},\left(\check{\sigma}_z\frac{\partial\check{g}}{\partial\varepsilon}+\frac{\partial\check{g}}{\partial\varepsilon}\check{\sigma}_z\right)\right]_+=0, \quad (13)$$

$$\check{\Sigma}=\begin{pmatrix}\hat{\Sigma}^R & \hat{\Sigma}\\ 0 & \hat{\Sigma}^A\end{pmatrix},\quad \check{\sigma}_z=\begin{pmatrix}\hat{\sigma}_z & 0\\ 0 & \hat{\sigma}_z\end{pmatrix},\quad [\check{a},\check{b}]_\pm=\check{a}\check{b}\pm\check{b}\check{a},$$
$$\hat{\Sigma}=-\frac{i}{2}\int\frac{d\mathbf{k}'_\perp}{S}\Big\{\nu_1(\mathbf{k}_\perp-\mathbf{k}'_\perp)\hat{\sigma}_z\hat{g}(\mathbf{k}'_\perp)\hat{\sigma}_z-\frac{1}{2}\nu_2(\mathbf{k}_\perp-\mathbf{k}'_\perp)$$
$$\times[\hat{\sigma}_x\hat{g}(\mathbf{k}'_\perp)\hat{\sigma}_x+\hat{\sigma}_y\hat{g}(\mathbf{k}'_\perp)\hat{\sigma}_y]\Big\},\quad \check{h}=\frac{v}{2}\frac{\partial\chi}{\partial x}\check{\sigma}_z+\varphi\check{\sigma}_z+\frac{1}{2}\frac{\partial\chi}{\partial t}\check{1},$$

where $\hat{\Sigma}$ is the mass operator describing the scattering by the impurities, $\nu_1=1/\tau_1$ and $\nu_2=1/\tau_2$ are the rates of the collisions not involving and involving a change in the direction of the longitudinal momentum, φ is the electrostatic potential,

$$\tilde{\varepsilon}=\varepsilon-\eta(\mathbf{p}_\perp),\quad \eta(\mathbf{p}_\perp)=[\varepsilon(\mathbf{p})+\varepsilon(\mathbf{p}+\mathbf{Q})]/2.$$

Moreover, the matrix $\check{g}$ satisfies the normalization relation

$$\check{g}\check{g}=\check{1}. \quad (14)$$

The current density j is defined, as in the incommensurable case, according to the formula

$$j=Nv^2\left[\mathrm{Sp}\langle g\rangle+\frac{\partial\chi}{\partial t}\left\langle\frac{\partial g_0^R}{\partial\varepsilon}\right\rangle\right]\Big/8. \quad (15)$$

The charge density ρ is found from the formula

$$\rho=\frac{k_0^2}{4\pi}[\mathrm{Sp}(\hat{\sigma}_z\langle g\rangle)-\varphi]. \quad (16)$$

Here $k_0^{-1}=r_{\mathrm{TF}}$ is the screening distance for a static electric field E in the normal state, i.e., for $T>T_p$ (the field E is directed along the vector $\mathbf{Q}$). The second term in (16) arose because of the integration over high energies ($\varepsilon\gg\Delta, T$). The expression for ρ in the case of superconductors contains a similar term.[23, 24]

3. THE INHOMOGENEOUS STATIC SOLUTIONS FOR THE CDW PHASE (PHASE SOLITONS)

Let us consider the simplest case of a stationary CDW in the absence of an external field, and let us analyze the possible solutions for the CDW phase χ. Let us assume that the impurity concentration is low ($\nu_{1,2}\ll\Delta$) and also that the deformation of the Fermi surfaces is slight ($\eta\ll\Delta$). Then we can neglect η in the region of appreciable energies $\varepsilon\approx\Delta$, and thus eliminate from the equations the dependence on the transverse momentum $\mathbf{k}_\perp$.

The equilibrium and spatially homogeneous CDW state is described by the functions

$$\hat{g}_{eq}^{R(A)}=g^{R(A)}\hat{\sigma}_z+f^{R(A)}i\hat{\sigma}_y,$$

which satisfy the Eq. (13) and the normalization relation (14); here

$$g^{R(A)}=\varepsilon/\xi^{R(A)},\quad \xi^{R(A)}=\pm[(\varepsilon\pm i0)^2-|\Delta|^2].$$

From Eq. (13) we find the corrections to the equilibrium matrix $\hat{g}_{eq}$ that are due to the presence in the crystal of the self-consistent electric potential $\varphi(x)\sim\varphi e^{ikx}$ and the phase $\chi(x)\sim\chi e^{ikx}$. The correction entering into the equation (9) for the phase has the form

$$\delta g_x=\mathrm{Sp}(\sigma_x\delta\hat{g})=(\delta g_x^R-\delta g_x^A)\,\mathrm{th}\frac{\varepsilon}{2T},$$
$$\delta g_x^{R(A)}=\frac{|\Delta|kv}{2}\frac{ikv\chi/2-\varphi}{\xi^{R(A)}[(\xi^{R(A)})^2-(kv/2)^2]}. \quad (17)$$

The correction determining the charge density (13) is equal to

$$\delta g_z=\mathrm{Sp}(\hat{\sigma}_z\delta\hat{g})=(\delta g_z^R-\delta g_z^A)\,\mathrm{th}\frac{\varepsilon}{2T},$$
$$\delta g_z^{R(A)}=\frac{|\Delta|^2(-\varphi+ikv\chi/2)}{\xi^{R(A)}[(\xi^{R(A)})^2-(kv)^2/4]}. \quad (18)$$

Let us substitute δg_x from (17) into (9). We obtain an equation for the phase χ:

$$-L_0^2\left(\frac{\partial^2\chi}{\partial x^2}+\frac{2E}{v}\right)+\sin(n\chi)=0, \quad (19)$$

where $L_0^2=\lambda v^2N_S/4|\Delta|^2\gamma$ and N_S is the fraction of condensed electrons. It is given by the formula

$$N_S=-\frac{|\Delta|^2}{4}\int d\varepsilon[(\xi^R)^{-3}-(\xi^A)^{-3}]\,\mathrm{th}\frac{\varepsilon}{2T}$$
$$\approx\begin{cases}1-(8\pi|\Delta|/T)^{1/2}e^{-|\Delta|/T}, & |\Delta|\gg T\\ 7\zeta(3)|\Delta|^2/4\pi^2T^2, & |\Delta|\ll T\ .\end{cases} \quad (20)$$

An equation of the type (19) is derived from phenomenological arguments in Ref. 12, where it is used to analyze solitons. The latter are those solutions to (19) (with $E=0$) in which the phase χ varies over the distance L_0 from one constant value to another:

$$\chi=\pm\frac{4}{n}\,\mathrm{arctg}\left[\frac{n^{1/2}}{2}e^x\right].$$

The plus sign corresponds to a $2\pi/n$ phase change (soliton), while the minus sign corresponds to a $-2\pi/n$ phase change (antisoliton), when x is increased. Rice *et al.*[12] assigned a definite charge $\rho\sim\partial\chi/\partial x$ to the soliton and $\rho\sim-\partial x/\partial x$ to the antisoliton. They assumed on the basis of the neutrality of the crystal as a whole that the solitons and antisolitons are produced in pairs. But to find the soliton (or antisoliton) charge, we must compute the charge density ρ with the aid of the formulas (18) and (16), and then solve simultaneously the Poisson equation and the equation (19) for the phase. In the case of highly conducting quasi-one-dimensional conductors, the length of a soliton is significantly greater than the Thomas–Fermi screening distance[3] r_{TF}. Therefore, the Poisson equation reduces to the quasineutrality condition $\rho=0$.

The charge-density calculation yields

$$\rho=-\frac{k_0^2}{4\pi}\left[\varphi(1-N_S)+\frac{1}{2}N_Sv\frac{\partial\chi}{\partial x}-\frac{v^2|\Delta|}{3}\frac{\partial(N_S/|\Delta|^2)}{\partial|\Delta|}\frac{\partial^2\varphi}{\partial x^2}\right]. \quad (21)$$

Let us determine the derivative $\partial\varphi/\partial x$ from the condition for local neutrality ($\rho=0$) and substitute it into (19) [the last term in (21) is negligible to first order in the commensurability parameter γ]. We obtain the equation for the phase

$$L^2\frac{\partial^2\chi}{\partial x^2}+\sin n\chi=0,\quad L^2=L_0^2N_S(1-N_S)^{-1}. \quad (22)$$

A similar equation can be derived in the case of a gapless conductor with a high impurity concentration ($\eta\gg\nu\gg\Delta$). Then $L^2=8|\Delta|^2\gamma/a^2v^2\lambda$, where

$$a=\left\langle\frac{i|\Delta|(f^R+f^A)}{8T\nu_2\,\text{ch}^2(\varepsilon/2T)}\right\rangle\approx\begin{cases}|\Delta|^2/\nu_2T, & T\gg\eta,\\ |\Delta|^2/\nu_2\eta, & T\ll\eta,\end{cases}$$
$$b=\frac{1}{2}\left\langle\text{th}\frac{\varepsilon}{2T}\frac{\partial}{\partial\varepsilon}(g^R-g^A)\right\rangle\approx\begin{cases}|\Delta|^2/T^2, & T\gg\eta\\ |\Delta|^2/\eta^2, & T\ll\eta.\end{cases}\quad(23)$$

Equation (22) describes both isolated solitons (antisolitons) and soliton (or antisoliton) chains of different densities. In these solutions the coordinate dependence of the phase $\chi(x)$ has the form of a staircase, each step of which gives a $2\pi/n$ phase increment (for the solitons). Furthermore, there exist solutions in the form of a sequence of alternating solitons and antisolitons; these solutions are described by a periodic $\chi(x)$ function. It is precisely these solutions that are regarded in Refs. 11 and 12 as satisfying the total-neutrality condition for the crystal. In fact, for $L>r_{TF}$ all the solutions to Eq. (22) should satisfy the local-neutrality (or, more precisely, quasineutrality) condition, which was required by us in the derivation of Eq. (22). The total charge of each soliton (antisoliton) is equal to zero. The soliton dimension ($\sim L$) is greater than the correlation length v/Δ because of the factor λ/γ, and at low temperatures the soliton dimension is even greater on account of the fact that $N_S\to 1$ at $T=0$.

Not all the solutions are stable against weak perturbations. To analyze the stability, we should generalize Eq. (22) to the nonstationary case. Then we obtain in place of (22) an equation of the type

$$\Omega^2\frac{\partial^2\chi}{\partial t^2}+\Gamma\frac{\partial\chi}{\partial t}-L^2\frac{\partial^2\chi}{\partial x^2}+\sin n\chi=0,\quad(24)$$

where the frequencies Ω and Γ will be computed in Sec. 4. Let us linearize Eq. (24) with respect to the deviations, $\delta\chi(x,t)=\chi(x,t)-\chi_0(x)$, from the steady-state solutions $\chi_0(x)$ to Eq. (22). We obtain for $\delta\chi(x,t)\sim\chi_1e^{-\lambda t}$ the equation

$$-L^2\frac{\partial^2\chi_1}{\partial x^2}+n\cos n\chi_0=\mathscr{E}\chi_1,\quad \mathscr{E}=\lambda(1-\lambda).\quad(25)$$

Equation (25) has the solution $\chi_1\sim\partial\chi_0/\partial x$, for which $\mathscr{E}=0$. If the function $\partial\chi_0/\partial x$ does not have nodes (as obtains for chains consisting of solitons or antisolitons), then the solution corresponds to the ground state. Then the remaining solutions correspond to $\mathscr{E}>0$ and $\lambda>0$. Consequently, the soliton or antisoliton chains are stable against weak perturbations. If the function $\partial\chi_0/\partial x$ has nodes, then the state with $\mathscr{E}=0$ is not the ground state, and there exist solutions χ_1 for which $\mathscr{E}<0$ and $\lambda<0$. Consequently, a chain made up of an alternating sequence of solitons and antisolitons is unstable.

Finally, let us compute the soliton energy. We shall proceed for the Lagrangian

$$\mathscr{L}=\int dV\left\{\frac{E^2}{8\pi}-\rho\varphi+\frac{k_0^2N_S}{2v}\left[\frac{v}{2}\left(\frac{\partial\chi}{\partial x}\right)^2+\frac{v^2}{2L_0^2}\frac{1-\cos n\chi}{n}\right]\right\}.\quad(26)$$

By varying $\mathscr{L}$ with respect to χ and φ, we can derive the equation (19) for the phase and the Poisson equation. Let us substitute into (26) the solution, $\chi(x)$ and $E(x)$, corresponding to an isolated soliton. Performing the integration, we obtain for the soliton energy ε_s the following order-of-magnitude estimate:

$$\varepsilon_s\approx k_0^2W\left(\frac{\gamma}{\lambda}\right)^{1/2}\frac{N_S}{(1-N_S)^{1/2}}|\Delta|,\quad(27)$$

where W is the area of the sample cross section in the direction perpendicular to the vector **Q**. For samples of macroscopic dimensions the magnitude of the coefficient in the brackets is very large, since the screening distance r_{TF} in good conductors is of the order of the interatomic distances. Thus, in the model of a continuous medium (and not of individual weakly interacting chains) under consideration, the energy necessary for the excitation of a soliton is much higher than the thermal energy. Such solitons, which extend over the entire sample (in the transverse direction), can, apparently, be produced only with the aid of external influences.

The potential drop across each static soliton is equal to zero. This follows from the expression (21) for ρ. Thus, the electric field E in a soliton is sign-variable, and varies over a large distance L. Besides this characteristic variation length for the field E, there exists in a Peierls dielectric the small screening distance r_{TF}. It is precisely this latter distance that determines the screening of the static field E parallel to Q in an open sample. Indeed, if the scale of variation of E is small compared to L, then it follows from Eq. (19) that

$$\varphi=\frac{v}{2}\frac{\partial\chi}{\partial x}.$$

Then we find from (16) in the leading approximation that $\rho=-(k_0^2/4\pi)\varphi$.

The Poisson equation yields

$$\frac{\partial^2\varphi}{\partial x^2}=k_0^2\varphi.\quad(28)$$

Consequently, the electric field is screened off over a distance equal to the Thomas–Fermi screening distance r_{TF}, irrespective of the number of uncondensed electrons.

4. THE SLIPPING OF A CDW UNDER THE ACTION OF A CONSTANT ELECTRIC FIELD

The analysis performed shows that, in the adopted model, the homogeneous state of the conductor cannot be destroyed by the thermal fluctuations. In this section we study the spatially homogeneous state of the crystal in the presence in it of a constant electric field E, and compute the conductivity of the crystal in this state. To do this, we should find from Eq. (13) the linear response $\delta\check{g}$ to a perturbation of the form $\varphi(x,t)\sim\exp(-i\omega t+ikx)$. It is convenient to represent the matrix $\delta\check{g}$ in the form of a sum of a regular

$$\delta g^{(r)}=\delta\hat{g}^R\,\text{th}\frac{\varepsilon'}{2T}-\delta\hat{g}^A\,\text{th}\frac{\varepsilon}{2T}$$

and an anomalous $\hat{g}^{(a)}$ part. Let us again consider small $\nu\ll\Delta$ and a slight deformation of the Fermi surfaces. For $g^{(a)}$ we obtain from (13) an algebraic equation whose solution has the form

$$g_x^{(a)}\equiv\text{Sp}(\hat{\sigma}_x\hat{g}^a)=\frac{\alpha}{2}\frac{\omega\omega_1|\Delta|(\varepsilon/\xi)[i\chi(\omega\omega_2-k^2v^2)-2kv\varphi]}{\xi^2(\omega\omega_2-k^2v^2)+|\Delta|^2\omega\omega_1},$$
$$g_y^{(a)}\equiv\text{Sp}(\hat{\sigma}_y\hat{g}^{(a)})/i=-2\alpha(|\Delta|^2/\varepsilon\xi)\delta\Delta,\quad(29)$$
$$g_1^{(a)}\equiv\text{Sp}\,\hat{g}^{(a)}=2\xi^2g_x/(|\Delta|\omega_1),\quad |\varepsilon|>|\Delta|,$$
$$\alpha^{-1}=2T\,\text{ch}^2(\varepsilon/2T),\quad \omega_1=\omega+2iv\frac{\varepsilon}{\xi},\quad \nu=\nu_1+\frac{\nu_2}{2},\quad \omega_2=\omega+2i\nu_2\frac{\varepsilon}{\xi},$$

where $\delta\Delta$ is the deviation of the gap from the equilibrium value. The deviations of the functions $\hat{g}^{R(A)}$ from the equilibrium functions have the form

$$\delta g_x^{R(A)}=\frac{|\Delta|kv}{2(\xi^{R(A)})^3}\left(\varphi+\frac{ikv\chi}{2}\right), \tag{30}$$

$$\delta g_1^{R(A)}=\frac{\omega kv|\Delta|^2\varphi}{4(\xi^{R(A)})^5},\quad \delta g_v^{R(A)}=\frac{\varepsilon^3}{(\xi^{R(A)})^3}\delta\Delta.$$

With the aid of the expressions (29) and (30) found in the homogeneous case (i.e., for $k\to 0$, but $ik\varphi=E\neq 0$) we find from (9) and (10) the equations

$$\frac{|\Delta|}{\omega_Q^2}\frac{\partial^2\chi}{\partial t^2}+\frac{2}{\omega_Q^2}\frac{\partial\chi}{\partial t}\frac{\partial\delta\Delta}{\partial t}+|\Delta|\gamma\sin n\chi=|\Delta|\gamma\left(\frac{E}{E_0}-\frac{1}{\omega_0}\frac{\partial\chi}{\partial t}\right) \tag{31}$$

$$\frac{1}{\omega_Q^2}\frac{\partial^2\delta\Delta}{\partial t^2}-\frac{\delta\Delta}{\omega_Q^2}\left(\frac{\partial\chi}{\partial t}\right)^2-|\Delta|\gamma\cos n\chi=-\lambda c\delta\Delta. \tag{32}$$

From the formula (15) we find the current

$$j=\sigma_1 E-\frac{\sigma_N|\Delta|\nu_2}{6}\frac{\partial}{\partial|\Delta|}\left(\frac{N_S}{|\Delta|^2}\right)\frac{\partial E}{\partial t}+\frac{\sigma_2}{l}\frac{\partial\chi}{\partial t}. \tag{33}$$

Here N_S is defined in (20),

$$c=1-\frac{1}{2}\int\limits_{|\Delta|}^{\infty}\frac{(\varepsilon^2-|\Delta|^2)^{1/2}}{\varepsilon}\alpha d\varepsilon,$$

$$\omega_0^{-1}=\frac{\lambda}{\gamma}\int\limits_{|\Delta|}^{\infty}\alpha\frac{\varepsilon^2 d\varepsilon}{\xi^2(\nu_2\xi^2+|\Delta|^2\nu)} \tag{34}$$

$$E_0^{-1}=\frac{\lambda v}{\gamma}\left[\frac{N_S}{2|\Delta|^2}+\int\limits_{|\Delta|}^{\infty}\frac{\alpha\nu d\varepsilon}{\xi(\nu_2\xi^2+|\Delta|^2\nu)}\right],$$

σ_N is the conductivity of the conductor in the normal state,

$$\sigma_1=\sigma_N\int\limits_{|\Delta|}^{\infty}\alpha\frac{\xi^2 d\varepsilon}{\xi^2+|\Delta|^2\nu\tau_2},$$

$$\sigma_2=\sigma_N\left(N_S+\int\limits_{|\Delta|}^{\infty}\alpha\frac{|\Delta|^2\nu d\varepsilon}{\xi(\xi^2\nu_2+|\Delta|^2\nu)}\right). \tag{35}$$

The Eqs. (31)–(33) describe the variation of the phase and the amplitude of the commensurable CDW in the presence of an electric field E. It can be seen from the formula (33) that the current j consists of a normal-electron current, a CDW-polarization current, and a current generated by the slipping of the CDW. The last term can be written as $j_{sl}=en_s u$, where

$$u=Q^{-1}\frac{\partial\chi}{\partial t},\quad n_s=2\sigma_2 m\nu_2.$$

This way of writing it emphasizes the physical meaning of this term, and it is the way often adopted in the literature (see, for example, Refs. 7 and 8).

Let us give the expressions for the parameters entering into Eqs. (31)–(33) in the limits of high and low temperatures[4)]:

$$\text{a)}\quad T\gg|\Delta|;\quad c=\pi\frac{|\Delta|}{T},\quad \omega_0=\frac{4|\Delta|T\tau_2\gamma}{\lambda\ln(|\Delta|/\eta)},$$

$$E_0=\frac{8|\Delta|T}{\pi v}\left(\frac{\tau}{\tau_2}\right)^{1/2}\frac{\gamma}{\lambda},\quad \sigma_1=\sigma_N,\quad \sigma_2=\sigma_N\frac{\pi}{4}\frac{|\Delta|}{T}\left(\frac{\tau_2}{\tau}\right)^{1/2}; \tag{36}$$

$$\text{b)}\quad T\ll|\Delta|;\quad c=1,\quad \omega_0=\left(2|\Delta|T\tau_2\gamma/\lambda\ln\frac{T}{\eta}\right)\exp\left(\frac{|\Delta|}{T}\right),$$

$$E_0=\frac{2|\Delta|^2\gamma}{v\lambda},\quad \sigma_1=\sigma_N\frac{2\pi^{1/2}T\tau}{|\Delta|\tau_2}\exp\left(-\frac{|\Delta|}{T}\right),\quad \sigma_2=\sigma_N. \tag{37}$$

The logarithms in (36)–(37) arose because of the truncation of the diverging integrals at small values of ξ, where the deformation of the Fermi surfaces should be taken into account.

From Eqs. (31)–(33) we can easily obtain the frequencies of the small natural oscillations of the phase χ and the amplitude $|\Delta|$. At $T=0$ we have

$$\omega_\chi=(^3/_2\lambda+\gamma)^{1/2}\omega_Q,\quad \omega_\Delta=\lambda^{1/2}\omega_Q. \tag{38}$$

Let us now consider the behavior of a conductor with a CDW in a constant field E. If the field E is weaker than the threshold field E_0, then it follows from (31)–(33) that χ and Δ do not depend on the time, and that

$$\chi=\frac{1}{n}\arcsin\left(\frac{E}{E_0}\right).$$

The current in this case is due to the uncondensed electrons: $j=\sigma_1 E$ (its difference from zero at $T=0$ is due to the deformation of the Fermi surfaces). If $E>E_0$, then the equations do not have steady-state solutions: there occurs in the system a slipping of the CDW accompanied by phase oscillations and small oscillations of the amplitude.

Let the frequencies of the oscillations be low compared to the natural frequencies (38). Then we can neglect the first two terms in (31)–(33) and the polarization current in (33). As a result, we obtain an equation similar to the Josephson equation for a superconducting bridge:

$$\frac{1}{\omega_0}\frac{\partial\chi}{\partial t}+\sin n\chi=\frac{E}{E_0},\quad j=\sigma_1 E+\frac{\sigma_2}{l}\frac{\partial\chi}{\partial t}. \tag{39}$$

The Eqs. (39) have been investigated in detail in the theory of the Josephson junction (see, for example, Refs. 25 and 26). Here we shall give only some results.

In fields $E<E_0$, the CVC of the system is linear: $j=\sigma_1 E$. For $E>E_0$, slipping of the CDW occurs and the CVC is nonlinear, its shape being dependent upon the resistance Z_ω of the external circuit at the oscillation frequency ω in the system. Thus, for $Z_\omega\ll R$ (R is the resistance of the sample), i.e., for a given field E, the shape of the CVC and the oscillation frequency ω are given by the expressions

$$j=\sigma_1 E+\sigma_2\frac{\omega_0}{l}\left[\left(\frac{E}{E_0}\right)^2-1\right]^{1/2}\theta(E-E_0),\quad \omega=n\omega_0\left[\left(\frac{E}{E_0}\right)^2-1\right]^{1/2}. \tag{40}$$

In the case in which $Z_\omega\gg R$, i.e., in the regime of a prescribed current, we have

$$\sigma_1 E=j-\frac{\sigma_2}{\sigma_1}\frac{\omega_0}{lE_0}\frac{(j^2-\sigma_1^2E_0^2)^{1/2}\theta(j-\sigma_1E_0)}{1+\sigma_2\omega_0/\sigma_1 lE_0},$$

$$\omega=n\omega_0\frac{(j^2-\sigma_1^2E_0^2)^{1/2}}{\sigma_1E_0+\sigma_2\omega_0/l}. \tag{41}$$

In this case the CVC is S-shaped.

In fields $E\gg E_0$, the CVC goes over into a straight line corresponding to Ohm's law:

$$j=\sigma_\infty E,\quad \sigma_\infty=\sigma_1+\sigma_2\omega_0/E_0 l. \tag{42}$$

The frequency and amplitude of the variable component of the current for $E\gg E_0$ are equal to

$$\omega=n\omega_0\frac{E}{E_0},\quad j_\omega=\frac{E_0 y(\omega)}{1+E_0 l[\sigma_1+y(\omega)]/\omega_0\sigma_2},\quad y(\omega)=\frac{L_{sam}}{Z_\omega W},$$

where L_{sam} and W are respectively the length and cross-

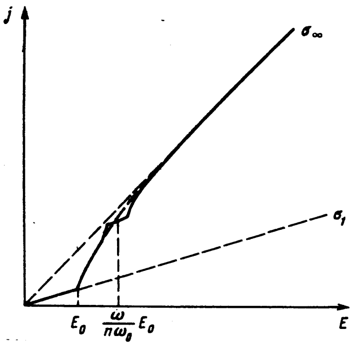

FIG. 1. Shape of the CVC in the absence of radiation (dashed curve) and in the presence of radiation of frequency ω (continuous curve).

sectional area of the sample.

If there is applied to the sample, besides the constant field E, a variable field $E_{\sim}=E_1\sin\omega t$, then a step appears on the CVC, i.e., a straight section parallel to the straight line $j=\sigma_1 E$ (see Fig. 1). The position of the center of the step on the E axis for small E_1 is connected with the frequency ω by the relations (41) and (42). The height of the step for $\omega\gg\omega_0$ is equal to $E_1 n\omega_0/\omega$.

The gap is also perturbed in fields $E>E_0$. For example, for $E\gg E_0$ we find from (31)–(32) that

$$\delta\Delta=\omega_0^2E^2/\lambda\omega_Q^2E_0^2+\frac{|\Delta|\gamma}{\lambda c}\cos\left(n\omega_0\frac{E}{E_0}t\right).$$

If the oscillation frequency ω is comparable to the natural frequencies (38), then the first terms in Eqs. (31) and (32) are important. This circumstance leads to certain effects. For example, it is known from the theory of the Josephson effect that the presence of the second derivative $\partial^2\chi/\partial t^2$ gives rise to hysteresis on the CVC (in our case the hysteresis appears even in a prescribed E field). Furthermore, the oscillation amplitudes of the gap and, for example, the field E_1 will exhibit resonances. Using perturbation theory, we find from (31)–(33) that for $E\gg E_0$ and at low temperatures

$$\Delta_1=\frac{|\Delta|\gamma}{\lambda}\left[1-\frac{\omega^2}{\lambda\omega_Q^2}\left(1+\frac{1}{n^2}\right)\right]^{-1},\quad \omega=n\omega_0\frac{E}{E_0},$$

$$E_1=iE_0\left[\frac{2\omega^2}{n\lambda\omega_Q^2}\left(1-\frac{\omega^2}{n\lambda\omega_Q^2}\right)^{-1}-1\right]\left\{1-\frac{2\omega^2}{3\lambda\omega_Q^2}-i\omega\left(\frac{E_0v}{3\omega_0|\Delta|^2}+\frac{2|\Delta|^2\tau_2}{\lambda\omega_Q^2}\frac{\sigma_1+y(\omega)}{\sigma_2}\right)\right\}^{-1}. \qquad (43)$$

It can be seen from this that resonances occur at the natural oscillation frequencies. We have not computed the damping of the amplitude mode, since for this purpose we need to know the form of the Green functions for $\xi\sim\eta,\nu$. But at low temperatures this damping contains the factor $\exp(-|\Delta|/T)$, and is small.

Using Eqs. (31)–(33), we can easily compute the system's admittance $Y(\omega)=j_\omega/E_\omega$ as a function of the frequency in fields E weaker than the threshold field E_0: $E<E_0$. From the linearized equations (31) and (33) we

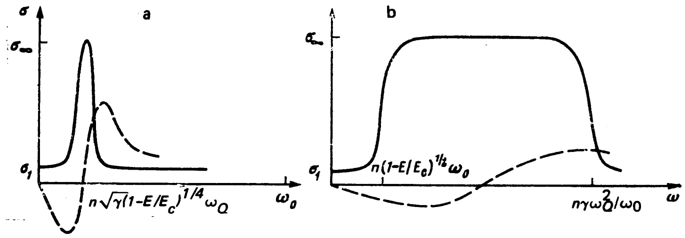

FIG. 2. Frequency dependence of the real (continuous curve) and imaginary (dashed curve) parts of the linear admittance in the cases: a) $\omega_0\gg\gamma^{1/2}\omega_0$; b) $\omega_0\ll\gamma^{1/2}\omega_0$.

find

$$Y(\omega)=\sigma_1+i\omega\frac{\sigma_N|\Delta|}{6\tau_2}\frac{\partial}{\partial|\Delta|}\left(\frac{N_s}{|\Delta|^2}\right)-i\sigma_2\omega\times\left[lE_0\left(n\frac{\omega^2}{\gamma\omega_Q^2}\left(1-\left(\frac{E}{E_0}\right)^2\right)^{1/2}-\frac{i\omega}{\omega_0}\right)\right]^{-1}. \qquad (44)$$

The last two terms in (44) give the contribution of the polarization and the slipping of the CDW to the conductivity of the system. The frequency dependence of $Y(\omega)$ in the cases of large and small magnitudes of the parameter $\omega_0/\gamma\omega_Q$ is shown in Fig. 2.

Let us also note that the gap is perturbed in the $E\neq 0$ case under consideration even in the approximation linear in the field E_ω:

$$\delta\Delta_\omega=-\mathrm{Re}\,n^2|\Delta|\gamma EE_\omega E_0^{-2}(\lambda c)^{-1}[1-\omega^2/\lambda c\omega_Q^2]^{-1}[n(1-E^2/E_0^2)^{1/2}-\omega^2/n\gamma\omega_Q^2-i\omega/\omega_0]^{-1}.$$

5. MOTION OF THE SOLITONS UNDER THE ACTION OF A CONSTANT ELECTRIC FIELD

The energy, (27), necessary for the production of a soliton of macroscopic dimensions is high, and such a soliton cannot be produced by the thermal vibrations. Solitons of this type can be evidently be excited with the aid of external influences. Furthermore, they can be produced in a crystal with an incommensurable CDW whose wave vector **Q** is close to the commensurable wave vector.[27]

Let us consider a soliton-containing crystal with a commensurable CDW. Upon the application to the sample of a constant field E, the solitons begin to move and thus contribute to the conductivity of the sample. To compute this contribution, we should generalize the equations to the case in which the phase χ and the potential φ depend on both the time and the coordinates. To do this, we should find from Eq. (13) the linear response $g(k,\omega)$ to the perturbation caused by the potential φ and the phase's derivatives $\partial\chi/\partial t$, etc. Then, using the Green functions, we should determine the current j, (15), and establish the form of the equation (9) for the phase. It is quite difficult to perform this task in the general case, since the form of the resulting expressions depends in a complex fashion on the relation between the parameters ω and Dk^2, whose $\omega\approx sk$, s is the soliton velocity, $k\sim L^{-1}$ or $k\sim d^{-1}$, d is the period of the soliton chain, and $D=v^2/2\nu_2$ is the diffusion coefficient. In all the cases the equations for the phase χ has the form of the sine-Gordon equation with ω- and k-dependent dissipative terms and an impressed force proportional to the current j flowing through the sample.

The computation of the coefficients of this equation in the pure case is especially complicated, since we must then know the behavior of the Green functions in the narrow energy range $|\varepsilon| - |\Delta| \sim \eta, \nu$. If we do not find the form of the functions $\hat{g}$ in this energy region, then we can only make order-of-magnitude estimates of the coefficients in the equation for χ. Such difficulties do not arise in the case of the gapless Peierls conductors when the conditions $\Delta \ll \nu \ll \eta$ are satisfied. At the same time, the case of the gapless conductor allows us to understand all the main properties of the system with a moving soliton chain. Therefore, we shall restrict ourselves to the consideration of precisely this case.[5)]

The Green functions $\hat{g}$ for a gapless Peierls conductor are computed in Ref. 14; the homogeneous (i.e., $k=0$) case is considered in that paper, but the generalization to the inhomogeneous ($k \neq 0$) case offers no difficulty, although it requires tedious calculations. We give only the final result: the expression for the current and the equation for the phase

$$j_{\omega,k}=\sigma_N\left\{E_{\omega,k}\frac{i\omega}{i\omega-Dk^2}\left[1-a\frac{i\omega}{i\omega-Dk^2}+\frac{Dk^2}{i\omega-Dk^2}\frac{b}{2}\right]-\frac{i\omega\chi_{\omega,k}}{l}(a+b/2)\right\}, \tag{45}$$

$$\frac{\omega_Q^{-2}}{\gamma}\frac{\partial^2\chi}{\partial t^2}+\sin n\chi=-\frac{\lambda\nu_2 a}{4|\Delta|^2\gamma}\sum_{\omega,k}\Big[-i\omega\chi_{\omega,k} -E_{\omega,k}l\frac{i\omega}{i\omega-Dk^2}-\frac{lb}{2a}\left(E_{\omega,k}-\frac{vk^2}{2}\chi_{\omega,k}\right)\Big]\exp(-i\omega t+ikx), \tag{46}$$

where a and b are given by the formulas (23).

In solving (45) and (46) we use the quasineutrality condition $\operatorname{div} j=0$, which allows us to eliminate the field E. Neglecting, where possible, the small a and b terms, we obtain an equation for the phase in fairly weak fields and for low soliton velocities s (i.e., for $\omega \approx sk \ll Dk^2$):

$$\frac{1}{\omega_Q^2\gamma}\frac{\partial^2\chi}{\partial t^2}-L^2\frac{\partial^2\chi}{\partial x^2}+\frac{1}{\omega_0}\frac{\partial\chi}{\partial t}+\sin n\chi=\frac{\langle E\rangle_x}{E_0}, \tag{47}$$

$$j=\sigma_1\langle E\rangle_x+\frac{\sigma_2}{l}\left\langle\frac{\partial\chi}{\partial t}\right\rangle_x, \tag{48}$$

where the angle brackets denote averaging over the coordinate and $L^2 = bD/2a\omega_0$.

Let us find the soliton velocity in weak fields $\langle E\rangle_x \ll E_0$. Let us find the solution to (47) in the form of a stationary wave $\chi = \chi(x - st)$. Then we obtain for the case of low s the equation

$$-L^2\frac{\partial^2\chi}{\partial x^2}-\frac{s}{\omega_0^2}\frac{\partial\chi}{\partial x}+\sin n\chi=\frac{\langle E\rangle_x}{E_0}. \tag{49}$$

We can, in the zeroth approximation, neglect the first derivative and the term on the right-hand side. Then we obtain an equation that coincides with (22), and describes, for example, soliton chains with different periods d. Let us denote the solution to this equation by χ_0. Let us now multiply Eq. (49) by $\partial\chi/\partial x$ and integrate over the period. For the soliton velocity s we obtain

$$\frac{s}{\omega_0}=-\frac{\langle E\rangle_x}{E}\int_0^d dx\left(\frac{\partial\chi_0}{\partial x}\right)\Big/\int_0^d dx\left(\frac{\partial\chi_0}{\partial x}\right)^2 =\mp\frac{\langle E\rangle_x}{E_0}\frac{\pi n^{1/2}}{4}\frac{kL}{\mathbf{E}(k)}, \tag{50}$$

where $\mathbf{E}(k)$ is the complete elliptic integral of the second kind. The coefficient k is connected with the period d of the chain by the relation

$$d=2k\mathbf{K}(k)L/n^{1/2}, \tag{51}$$

where $\mathbf{K}(k)$ is the complete elliptic integral of the first kind.

It follows from the expression (50) that, in a given mean field $\langle E\rangle_x$, the soliton and antisoliton move in opposite directions, since $\partial\chi_0/\partial x > 0$ in a soliton and $\partial\chi_0/\partial x < 0$ in an antisoliton. Substituting

$$\left\langle\frac{\partial\chi}{\partial t}\right\rangle_x=-s\left\langle\frac{\partial\chi}{\partial x}\right\rangle_x=-\frac{s}{d}2\pi n$$

into (48), we obtain the following expression for the current

$$j=\left[\sigma_1+\sigma_2\frac{\pi^2\omega_0 Lk}{2n^{1/2}ldE_0\mathbf{E}(k)}\right]\langle E\rangle_x=\left[\sigma_1+\sigma_2\frac{\pi^2 Lk}{2n^{1/2}d\mathbf{E}(k)}\right]\langle E\rangle_x. \tag{52}$$

From this it follows that the contribution made to the conductivity by the solitons in the gapless state is equal to $\sigma_2(L/d)(\pi^2/2n^{1/2})$ at low soliton densities (i.e., for $d \gg L$) and equal to σ_2 at high densities ($d \ll L$).

A similar analysis for the pure material shows that the correction made to the conductivity by a high-density soliton chain at low temperatures is, in order of magnitude, equal to $\sigma_1(\eta/|\Delta|(\nu_2/\nu)^{1/2})$.

6. CONCLUSION

The foregoing analysis shows that the most probable state of a Peierls conductor with a commensurable CDW is the state with a spatially homogeneous CDW phase χ. If the field E is not higher than the threshold field E_0, then the CDW is stationary and only the free quasiparticles contribute to the conductivity. In fields $E > E_0$ the CDW slips. This slipping is not uniform: as the CDW moves with a mean velocity $\bar{u}$ proportional to $(E^2 - E_0^2)^{1/2}$, it executes oscillations with frequency proportional to $\bar{u}$. The motion of the CDW makes to the conductivity a contribution that depends nonlinearly on the field E. Thus, there exists an analogy with the Josephson effect in point contacts: the CDW phase χ corresponds to the order-parameter phase difference φ in superconducting contacts, while the field E corresponds to the prescribed current j in the contacts. A Peierls conductor, like a Josephson junction, should emit radiation in fields stronger than the critical field E_0, and when it is acted upon by external radiation its CVC should exhibit steps. The difference from the Josephson contacts consists in the fact that the radiated power in the present case will be proportional to the volume of the sample.

The parameters of the Peierls conductor that determine the characteristic frequency ω_0 and the threshold-field strength E_0 depend on the Peierls-transition temperature T_P, the sample temperature T, the magnitude η of the deformation of the Fermi surfaces, etc. These parameters can vary greatly from material to material (e.g., T_P varies from a few degrees to hundreds of degrees), and, consequently, the quantities E_0 and ω_0 can be observed in, for example, TaS_3, in which a commensurable CDW ($n=4$) is produced at $T_P = 218$ K, and which has been found[8] to possess a nonlinear CVC. The characteristic fields were of the order of 100 V/cm. Such a high threshold-field value should yield a high value for the characteristic frequency ω_0 (up to frequencies in the infrared region). It is preferable to observe the effects considered in pure materials at low temperatures, where the conductivities σ_1 and σ_2 differ greatly from each other, and, consequently, the nonlinearity should be strongly pronounced.

Let us note finally that in the adopted model, in which the crystal spectrum (1) is prescribed, the phases on all the filaments are identical, and the CDW motion occurs synchronously in the entire volume of the sample. In principle, this may not be so if the coupling between the filaments is sufficiently weak. The question of the effect of the intensity of the interaction between the filaments on the effects considered requires a separate analysis.

The authors are grateful to S.A. Brazovskiĭ and A.V. Zaĭtsev for a discussion and useful comments.

[1)] Some of the results of these investigations are presented in Ref. 16. Let us note that the analogy with the Josephson effect is noted in Ref. 17, and is investigated in somewhat greater detail in Ref. 18. The phenomenological approach is used in both of these papers.

[2)] In Ref. 12 Rice *et al.* consider only one filament, and show that the energy required for the excitation of a soliton on such a filament is of the order of Δ (Δ is the energy gap in the spectrum of the Peierls dielectric).

[3)] Let us emphasize that the distance r_{TF} characterizes the screening in the temperature region above the Peierls transition point T_P, where the conductor is a metal. However, in the case of the temperatures $T < T_P$ being considered, the screening of the static field E is, as we shall see, also characterized by the distance r_{TF}, since the CDW also contributes to the screening.

[4)] Let us note that the expression for ω_0 in Ref. 16 contains a topographical error: $l\Delta/T$ was printed instead of $e^{\Delta/T}$.

[5)] In order to make the neglect of the pinning to the impurities also admissible when the condition $\nu > \Delta$ is fulfilled, we assume here that the large magnitude of ν is due to the scattering by the phonons, and not by the impurities.

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