Quantum oscillations of the rf impedance of cadmium plates

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An experimental investigation was made of quantum oscillations of the rf surface impedence of a cadmium plate subjected to a strong magnetic field perpendicular to the surface. When the external excitation wave was symmetric with respect to the electric field, the amplitude of these oscillations was several tens of times greater than the amplitude in the case of antisymmetric excitation. A theoretical analysis of the skin effect was made for compensated metals and a comparison with the experimental results established that the main contribution to these quantum oscillations of the impedance was due to the Shubnikov–de Haas effect. The experimental dependence of the oscillation amplitude confirmed this conclusion.

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It is usual to assume[1] that quantum oscillations of the surface impedance of a metal may be due to the Shubnikov–de Haas effect [oscillations of the conductivity $\sigma(H_0)$] and the de Haas–van Alphen effect [oscillations of the magnetic moment $M(H_0)$]. At very low and also at sufficiently high frequencies the Shubnikov–de Haas oscillations predominate. At intermediate frequencies the oscillations of the surface impedance are dominated by the magnetic moment.

Some experimental results demonstrate that this general approach is not always valid. For example, Vol'skiĭ and Petrashov[2] investigated experimentally the helicon spectrum and concluded that the main contribution to the impedance oscillations was made by the de-Haas–van Alphen effect right down to the lowest frequencies of the order of hundreds of hertz. Therefore, the question which of these effects dominates the impedance must be settled separately in each specific case.

We shall report an experimental investigation of the surface impedance oscillations exhibited by thin [$d \sim 1, d/2 \leq \delta(H_0)$, where d is the plate thickness, l is the mean free path of electrons, and $\delta(H_0)$ is the depth of the skin layer] single-crystal plates of a compensated metal (cadmium) in which helicons did not appear and which was subjected to a magnetic field perpendicular to the surface. We observed a strong dependence of the amplitude of the quantum oscillations on the method of excitation of an rf wave. For example, the oscillation amplitude observed in a strong magnetic field in the case of symmetric (in respect of the electric field) excitation by the external wave was several tens of times greater than the amplitude of the corresponding oscillations in the antisymmetric excitation case.

In the experiments carried out at helium temperatures using an autodyne detector[3] and the modulation method we recorded the derivatives of the real part of the surface impedance $Z = R + iX$. The sample was a disk of cadmium $d = 0.2$ mm thick with a diameter of 11 mm. The resistivity ratio was $\rho_{300}/\rho_{4.2} \sim 5 \cdot 10^4$. A static magnetic field H_0 was created by a superconducting solenoid and it was modulated at a frequency of ~18 Hz. The various excitation methods were realized as described in our earlier investigation.[4] A system of two identical coils between which the sample was placed was located in a device in which the sample could be rotated in two mutually perpendicular planes.

The sensitivity of the apparatus was checked as a function of the parameters of the experiment. The autodyne detector sensitivity was compared in the symmetric and antisymmetric excitation configurations by checking the equality of the amplitudes of the doppleron oscillations in the range of magnetic fields in which the depth of the skin layer was less than the half-thickness of the plate and the difference between the two excitation methods was unimportant.[5]

The different effects of the destabilizing factor (magnetic field) on the autodyne output in the case of symmetric and antisymmetric excitation configurations was manifested only in strong magnetic fields and resulted in a difference between sensitivities not exceeding 10%.

Figure 1 shows the experimental records of d^2R/dH_0^2 obtained for the antisymmetric (1) and symmetric (2) excitation methods. Both records begin with doppleron oscillations which transform gradually (on increase in the magnetic field) into the Sondheimer oscillations. In the case of diffuse scattering of electrons by the surface the amplitude of these oscillations contains as a factor the square of the modulus of the smooth part of the impedance $|Z_s^{sk}|^2$ governed by the skin effect.[6] The dependence of this quantity on the magnetic field in the case of symmetric $|Z_s^{sk}|$ (2) and antisymmetric $|Z_s^{sk}|$ (1) excitation methods, plotted using the experimental results of our earlier study,[4] is also included in Fig. 1. In the range of fields $H_0 < 20$ kOe the magnetic field dependences of $|Z_s^{sk}|$ and $|Z_a^{sk}|$ are similar, and in this respect the properties of the plate do not differ from those of a semi-infinite metal. In the case of stronger fields the skin layers on the opposite sides of the plate interact with one another and determine the difference in the behavior of $|Z_s^{sk}|$ and $|Z_a^{sk}|$ in these two cases. In a magnetic field of $H_0 \approx 35$ kOe the ratio of these two quantities is $|Z_s^{sk}|^2/|Z_a^{sk}|^2 \approx 3$. A similar ratio of the amplitudes of the Sondheimer oscillations is obtained in the case of symmetric and antisymmetric excitation

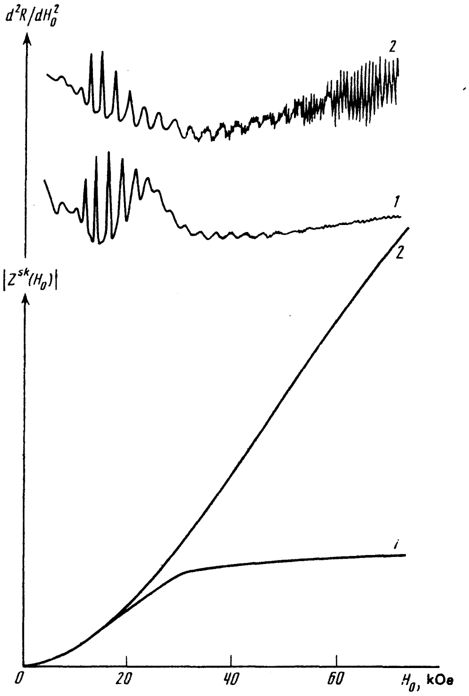

FIG. 1. Experimental records of the derivative d^2R/dH_0^2 and of the smooth dependence of $|Z^{sk}(H_0)|$ for a cadmium plate in the case of antisymmetric (1) and symmetric (2) wave excitation methods; $f = 3.6$ MHz, $d = 0.2$ mm, $H_0 \| C_6$, $T = 4.2$°K.

methods.

In a magnetic field $H_0 > 30$ kOe we find that quantum oscillations of the impedance with different periods due to several sections of the Fermi surface appear against the background of the size effect. In the symmetric excitation method the amplitude of these ocillations is considerably greater than in the antisymmetric case. For example, in a magnetic field of $H_0 \approx 65$ kOe the ratio of these two amplitudes is 20–40.

The nature of the observed effects can be interpreted and the cause of such a strong dependence of the oscillation amplitude on the excitation method can be found by solving the problem of the surface impedance of a metal plate allowing for the quantization of the orbital electron motion in a static magnetic field H_0. Then, the Maxwell equations for nonferromagnetic metals should allow for the difference between the electromagnetic wave induction B and the magnetic field H.

Let us assume that an external wave travels along the z axis and is characterized by the following electric and magnetic field vectors: $\mathbf{E}\{E_x, 0, 0\}$, $\mathbf{H}\{0, H_y, 0\}$; a static magnetic field H_0 is applied in the zy plane and makes an angle θ with the z axis. If the magnetic moment associated with the induction vector $\mathbf{B}$ of the external wave is expressed in the form $\mathbf{m} = \mathbf{M}(\mathbf{B}_0 + \mathbf{B}) - \mathbf{M}(\mathbf{B}_0)$ and if allowance is made for the fact that $B \ll B_0$ (B_0 is the induction corresponding to the static magnetic field H_0), we can write down the Maxwell equations in the form[2]

$$\operatorname{rot}\mathbf{B}\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)=\frac{4\pi}{c}\mathbf{j}(z),\quad \operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},\qquad (1)$$

where $\partial M/\partial B_0$ is the differential magnetic susceptibility which is an oscillatory function of the magnetic field H_0.

Solution of the system (1) for a compensated metal with a weak spatial dispersion in the case of specular reflection (allowance for the diffused nature of the reflection has little effect on the results) gives the following dispersion equation:

$$k^2E=\frac{4\pi i\omega}{c^2}\sigma E\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)^{-1},\qquad (2)$$

where $\sigma = Nec\gamma/H_0$; N is the carrier density; e is the electron charge; c is the velocity of light; $\gamma = \nu/\Omega$; ν is the frequency of collisions with scatterers; Ω is the cyclotron frequency.

The impedance of a metal follows from Eq. (1):

$$Z=E_x(0)\Big/\int_0^d j(z)\,dz=\frac{4\pi i\omega\sigma}{k^2c^2}\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)^{-1}E_x(0)\Big/\int_0^d E_x(z)\,dz.\qquad (3)$$

The wave number k is the skin number k_{sk}, i.e., $k = k_{sk}$. Other roots of the dispersion equation which make a small relative contribution to the impedance will be ignored. In accordance with Ref. 3, the skin wave number is given by

$$k_{sk}\approx(1+i)\gamma^{1/2}k_H,\quad k_H^2=4\pi\omega Ne/H_0c.$$

The distribution of the field in a metal plate, which is different for the symmetric and antisymmetric excitation methods, is

$$E_s=-E_0\cos k_{sk}(d/2-z),\quad E_a=E_0\sin k_{sk}(d/2-z),\qquad (4)$$

and the expression for the impedance (3) transforms in these two cases as follows:

$$\begin{aligned}Z_s&=\frac{4\pi i\omega}{c^2}\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)^{-1}\frac{1}{k_{sk}}\operatorname{ctg}\frac{k_{sk}d}{2},\\ Z_a&=-\frac{4\pi i\omega}{c^2}\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)^{-1}\frac{1}{k_{sk}}\operatorname{tg}\frac{k_{sk}d}{2}.\end{aligned}\qquad (5)$$

In magnetic fields corresponding to intense quantum oscillations the value of $k_{sk}d/2$ is small compared with unity. For example, estimates made in accordance with Ref. 3 show that in a magnetic field of $H_0 = 65$ kOe we have $k_{sk}d/2 \approx 0.2$. Expanding as series the functions $\tan(k_{sk}d/2)$ and $\cot(k_{sk}d/2)$, we obtain the following expressions for the surface impedance:

$$Z_s=\frac{2\pi i\omega d}{c^2}\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)^{-1}\frac{4}{k_{sk}^2d^2}\left(1-\frac{k_{sk}^2d^2}{12}\right),\qquad (6)$$

$$Z_a=-\frac{2\pi i\omega d}{c^2}\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)^{-1}\left(1+\frac{k_{sk}^2d^2}{12}\right).\qquad (7)$$

We shall now analyze these expressions. Bearing in mind that the dispersion equation in our case is

$$k_{sk}^2=k_{0sk}^2(\sigma)\left(1-4\pi\sin^2\theta\frac{\partial M}{\partial B_0}\right)^{-1},\qquad (8)$$

we find that Eq. (6) for the impedance in the symmetric case does not contain the de Haas–van Alphen oscillations in the first approximation with respect to $k_{sk}d/2$ but it does include implicitly quantum oscillations of the conductivity $\sigma(H_0)$ (Shubnikov–de Haas effect). In the antisymmetric excitation case the quantum oscillations of the impedance considered in the zeroth approximation are due to oscillations of the magnetic moment, which may be small because of the smallness of the angle θ. The ratio of the contribution of the de

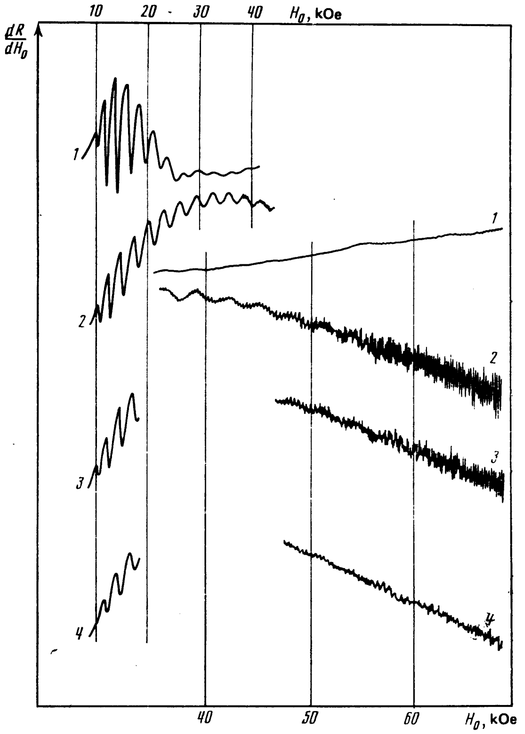

FIG. 2. Records of oscillations of dR/dH_0 plotted as a function of the magnetic field for a cadmium plate in the case of antisymmetric (1) and symmetric (2–4) excitation methods: 2) $\theta \approx 0°$; 3) $\theta \approx 3°$; 4) $\theta \approx 6°$. The lower part of the figure shows on the right (on a scale enlarged in respect of H_0) the angular dependence of the amplitude of the same quantum oscillations: $f = 3.0$ MHz, $d = 0.2$ mm, $H_0 \| C_6$, $T = 2°K$.

Haas–van Alphen oscillations to the impedance deduced from Eqs. (6) and (7) for the symmetric and antisymmetric excitation methods is $Z_s/Z_a \approx \frac{1}{3}$, whereas in the case of the Shubnikov–de Haas effect the corresponding value is $Z_s/Z_a \approx 50$ ($k_{sk}d \approx 1$).

Moreover, in contrast to the de Haas–van Alphen effect, the amplitude of the Shubnikov–de Haas oscillations should be independent of the angle. A weak dependence can occur only as a result of a change in the area S of a section of the Fermi surface characterized by a momentum $p_z - \partial^2 S/\partial p_z^2$. In the case of oscillations of the magnetic moment the amplitude dependence should be strong (since $\sin^2\theta$). For example, the ratio of the amplitudes of the quantum oscillations corresponding to the angles $\theta = 6°$ and $\theta \approx 1°$ should be ≈ 36.

Figure 2 shows the records of the quantum oscillations of the impedance of a cadmium plate obtained for several angles θ in the symmetric excitation case. For each fixed angle we observe in weaker fields the first three doppleron oscillations whose amplitude is strongly affected by the slope of the magnetic field. There is a slight reduction in the quantum oscillation amplitude when the magnetic field is inclined. We can thus see that a comparison with the experimental results demonstrates that the quantum oscillations of the surface impedance are dominated by the Shubnikov–de Haas effect.

We can summarize our results by stating that the application of different methods of excitation of a wave to thin $[d < \delta(H_0, \omega)]$ metal plates makes it possible to separate the various quantum oscillation effects contributing to the hf impedance.

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