Distribution function of electrons scattered with a large energy loss in an electric field

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Expressions are obtained for the distribution function of the electrons of a gas or semiconductor plasma in an electric field E, when the scattering is determined by one mechanism with a fixed energy loss E_0. At $kT \ll eE\lambda \ll E_0$, where T is the lattice-atom temperature and λ is the electron mean free path, the distribution is strongly asymmetrical. The distribution function can be represented as a sum of two terms. The first predominates at small angles θ between the momentum p and the field direction, and decreases rapidly with increasing angle in proportion to exp $(-\theta^2/\theta_0^2(E))$. The second term depends less on the angle (like $(1-\cos\theta)^{-1}$) and predominates at large angles. The energy dependence is proportional to exp $(-\int^{\mathscr{E}} d\mathscr{E}' eE\lambda(\mathscr{E}'))$. The calculation is performed for cases when the scattering probability w does not depend on the momentum transfer Q and when $w \sim Q^{-2}$.

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1. INTRODUCTION

Simple situations are possible, in which the scattering of the electrons of a gas or semiconductor plasma situated in a constant homogeneous electric field E is determined by a single mechanism, with a fixed energy loss $\mathscr{E}_0$. In semiconductors this corresponds to scattering by optical phonons ($\mathscr{E}_0 = \hbar\omega_0$), while in gases it corresponds to the case when the probability of excitation of levels having an energy $\mathscr{E}_0$ exceeds the probability of elastic scattering and of excitation of other levels.

The character of the distribution function $f(\mathbf{P})$ of the electrons turns out to be significantly different, depending on the relation between $\mathscr{E}_1$, which is the characteristic energy scale of the fall-off of the distribution function, and $\mathscr{E}_0$. The quantity $\mathscr{E}_1$ can be defined as

$$\mathscr{E}_1(\mathscr{E}) = -(d\ln f_0/d\mathscr{E})^{-1},$$

where $f_0(\mathscr{E})$ is the isotropic part of the distribution function. If $\mathscr{E}_1 \gg \mathscr{E}_0$, then the scattering can be regarded as quasielastic, and the Davydov approximation[1,2] is applicable. In this case the distribution function is almost isotropic. The gain or loss of energy in the elelctric field have the character of small random walks, and this corresponds to diffusion in energy, with a diffusion coefficient

$$D_\varepsilon = {}^1/_3(eE\lambda_T(\mathscr{E}))^2\nu_T(\mathscr{E}),$$

where λ_T and ν_T are the transport mean free path and the collision frequency. The kinetic equation reduces then to the diffusion equation. If the quantum absorption $\mathscr{E}_0$ is disregarded, the equation takes the form

$$\frac{d}{d\mathscr{E}}\mathscr{E}^{1/2}\left(D_\varepsilon\frac{df_0}{d\mathscr{E}} + \mathscr{E}\nu f_0\right) = 0, \tag{1}$$

where ν is the energy-loss frequency. Equating the energy flux to zero, we obtain

$$f_0 \sim \exp\left\{-\int^{\mathscr{E}}\frac{\mathscr{E}_0\nu(\mathscr{E}')}{D_\varepsilon(\mathscr{E}')}d\mathscr{E}'\right\}, \tag{2}$$

in which case

$$\mathscr{E}_1(\mathscr{E}) = D_\varepsilon(\mathscr{E})/\mathscr{E}_0\nu(\mathscr{E}). \tag{3}$$

If $\mathscr{E}_1 \ll \mathscr{E}_0$, however, the distribution is strongly elongated along the electric field. This case was considered qualitatively by Townsend for gases[3] and by Shockley for semiconductors.[4] They obtained $\mathscr{E}_1 \sim eE\lambda$. The form of the distribution function for high energies in the case of isotropic scattering was obtained by Keldysh.[5]

We obtain in this paper expressions for the distribution function in the nearby energy region for two particular cases: isotropic scattering with probability w independent of the energy (scattering by deformation optical phonons), and scattering by polar optical phonons with $w(Q) \sim Q^{-2}$, where $\mathbf{Q}$ is the momentum transfer. In these cases the integral for the transport cross section converges, so that the anisotropy of the distribution function is connected with the discrete character of the energy loss, and not with the scattering anisotropy as in the case of Coulomb collisions.[6,7] It is shown that at small angles θ between the momentum $\mathbf{P}$ and the field direction, the distribution function is governed mainly by the particles accelerated by the field from low energies $\mathscr{E} < \mathscr{E}_0$ (from the passive region) and experience in this case not a single collision. This highly anisotropic needle-shaped part of the distribution function makes the main contribution to $f_0(\mathscr{E})$ and decreases with increasing θ as $\exp\{-(\theta/\theta_0(\mathscr{E}))^2\}$. The particles from the "needle," however, land in the large-angle region ("halo") after experiencing the last collision at high energy $\approx \mathscr{E} + \mathscr{E}_0$. The distribution function falls off with increasing θ much slower in the halo than in the needle: $f \sim (1-\cos\theta)^{-1}$. The distribution at high energies is determined by the character of the scattering.

In the case of scattering by deformation phonons, starting with a certain energy, the number of particles in the needle becomes smaller than in the halo, and the results of Ref. 5 are obtained. This means that the contribution made to $f_0(\mathscr{E})$ by the electrons that arrive from the passive region ($\mathscr{E} < \mathscr{E}_0$) is small. In the angular dependence, however, the needle-shaped component $\sim\exp\{-(\theta/\theta_0(\mathscr{E}))^2\}$ can be traced also at high energies.

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In scattering by polar phonons, the mean free path increases with increasing $\mathscr{E}$, and starting with a certain energy $\tilde{\mathscr{E}}$, the quantity $eE\lambda(\mathscr{E})$ becomes larger than $\mathscr{E}_0$. At $\mathscr{E} \gg \tilde{\mathscr{E}}$ the distribution function becomes almost isotropic.

2. SCATTERING BY DEFORMATION PHONONS

Neglecting the thermal motion of the atoms of the gas or of the lattice ($\mathscr{E}_0$, $\mathscr{E}_1 \gg kT$), we write down the Boltzmann equation in the form

$$eE\frac{df}{dP_z}=\int w(\mathbf{Q})\{\delta(\mathscr{E}(\mathbf{P}+\mathbf{Q})-\mathscr{E}(\mathbf{P})-\mathscr{E}_0)f(\mathbf{P}+\mathbf{Q}) -\delta(\mathscr{E}(\mathbf{P})-\mathscr{E}(\mathbf{P}-\mathbf{Q})-\mathscr{E}_0)f(\mathbf{P})\}d^3\mathbf{Q}, \tag{4}$$

where the z axis is directed along the electric field and $\mathbf{Q}$ is the phonon momentum. In scattering by deformation optical phonons we have $w(\mathbf{Q}) = \text{const}$. We rewrite (4) in terms of the dimensionless variables $\varepsilon = \mathscr{E}/\mathscr{E}_0$, $\mathbf{p} = \mathbf{P}/(2m\mathscr{E}_0)^{1/2}$, and $\alpha = \mathscr{E}_0/eE\lambda$, where $\lambda = 1/4\pi m^2 w$ is the mean free path of the electron at $\varepsilon \gg 1$:

$$\partial f/\partial p_z=-2\alpha(\varepsilon-1)^{1/2}f(\mathbf{p})+2\alpha(\varepsilon+1)^{1/2}f_0(\varepsilon+1), \qquad f_0(\varepsilon)=\frac{1}{4\pi}\int f(\mathbf{p})\,d\Omega. \tag{5}$$

We are interested in the case $\alpha \gg 1$. It is more convenient then to reduce (5) to an integral equation.

In the passive region I ($\varepsilon < 1$, see Fig. 1), there are no collisions and

$$f_{\mathrm{I}}(\mathbf{p})=2\alpha\int\limits_{-(1-\varepsilon_\perp)^{1/2}}^{p_z}(\varepsilon'+1)^{1/2}f_0(\varepsilon'+1)\,dp_z' +\alpha\int\limits_1^\infty\left(\frac{\varepsilon'+1}{\varepsilon'-\varepsilon_\perp}\right)^{1/2}\exp\left\{-\alpha\int\limits_1^{\varepsilon'}\left(\frac{\varepsilon''-1}{\varepsilon''-\varepsilon_\perp}\right)^{1/2}d\varepsilon''\right\}f_0(\varepsilon'+1)\,d\varepsilon'. \tag{6}$$

Here $\varepsilon_\perp = \varepsilon - p_z^2$ is the transverse energy. At $\varepsilon > 1$, $p_z > 0$, $\varepsilon_\perp < 1$ (region II in the figure) we have

$$\begin{aligned}f_{\mathrm{II}}(\mathbf{p})=&2\alpha\int\limits_{\varepsilon_\perp}^{1}\left(\frac{\varepsilon'+1}{\varepsilon'-\varepsilon_\perp}\right)^{1/2}\exp\left\{-\alpha\int\limits_1^{\varepsilon}\left(\frac{\varepsilon''-1}{\varepsilon''-\varepsilon_\perp}\right)^{1/2}d\varepsilon''\right\}f_0(\varepsilon'+1)\,d\varepsilon'\\ &+\alpha\int\limits_1^{\varepsilon}\left(\frac{\varepsilon'+1}{\varepsilon'-\varepsilon_\perp}\right)^{1/2}\exp\left\{-\alpha\int\limits_{\varepsilon'}^{\varepsilon}\left(\frac{\varepsilon''-1}{\varepsilon''-\varepsilon_\perp}\right)^{1/2}d\varepsilon''\right\}f_0(\varepsilon'+1)\,d\varepsilon'\\ &+\alpha\int\limits_1^{\infty}\left(\frac{\varepsilon'+1}{\varepsilon'-\varepsilon_\perp}\right)^{1/2}\exp\left\{-\alpha\int\limits_1^{\varepsilon'}\left(\frac{\varepsilon''-1}{\varepsilon''-\varepsilon_\perp}\right)^{1/2}d\varepsilon''\right.\\ &\left.-\alpha\int\limits_1^{\varepsilon}\left(\frac{\varepsilon''-1}{\varepsilon''-\varepsilon_\perp}\right)^{1/2}d\varepsilon''\right\}f_0(\varepsilon'+1)\,d\varepsilon'.\end{aligned} \tag{7}$$

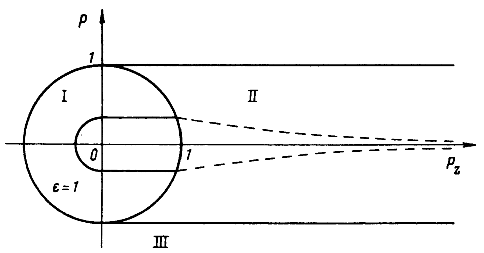

FIG. 1. Regions in phase space; the dashed line corresponds to $f^{(n)} = f^{(h)}$.

In the remaining region III,

$$f_{\mathrm{III}}(\mathbf{p})=2\alpha\int\limits_{-\infty}^{p_z}(\varepsilon'+1)^{1/2}\exp\left\{-2\alpha\int\limits_{p_z'}^{p_z}(\varepsilon''-1)^{1/2}dp_z''\right\}f_0(\varepsilon'+1)\,dp_z'. \tag{8}$$

The exponential factors of the form

$$\exp\left\{-\alpha\int\limits_{\varepsilon_1}^{\varepsilon_2}\left(\frac{\varepsilon-1}{\varepsilon-\varepsilon_\perp}\right)^{1/2}d\varepsilon\right\}$$

have the meaning of the probability that a particle with energy ε_1 will acquire in the electric field an energy ε_2 without experiencing a collision. At $\alpha \gg 1$ the bulk of the particles is contained in the passive region. The electric field transfers them into the region II, and most of them experience collisions at $0 < \varepsilon - 1 \lesssim \alpha^{-2/3}$ and return to the passive region. Therefore the principal role in (7) is played at $\varepsilon - 1 \ll 1$ by the first term (the remaining terms are here exponentially small):

$$\begin{aligned}f_{\mathrm{II}}(\mathbf{p})&=2\alpha\int\limits_{\varepsilon_\perp}^{1}\left(\frac{\varepsilon'+1}{\varepsilon'-\varepsilon_\perp}\right)^{1/2}f_0(\varepsilon'+1)\,d\varepsilon'\exp\left\{-\alpha\int\limits_1^{\varepsilon}\left(\frac{\varepsilon''-1}{\varepsilon''-\varepsilon_\perp}\right)^{1/2}d\varepsilon''\right\}\\ &=f(\varepsilon=1,\varepsilon_\perp)\exp\left\{-\alpha\int\limits_1^{\varepsilon}\left(\frac{\varepsilon''-1}{\varepsilon''-\varepsilon_\perp}\right)^{1/2}d\varepsilon''\right\}.\end{aligned} \tag{9}$$

In region I it suffices likewise to take into account only the first term. Averaging (9) over the angles, substituting in (6) the expression obtained for f_0, and putting $\varepsilon = 1$, we obtain an equation for $\varphi(\varepsilon_\perp) = f(\varepsilon = 1, \varepsilon_\perp)$:

$$\varphi(\varepsilon_\perp)=\alpha\int\limits_0^{(1-\varepsilon_\perp)^{1/2}}dx\int\limits_0^1 dy\,\frac{\varphi(y)}{(1+x^2+\varepsilon_\perp-y)^{1/2}}\times\exp\left\{-\alpha\int\limits_1^{1+x^2+\varepsilon_\perp}\left(\frac{\varepsilon'-1}{\varepsilon'-y}\right)^{1/2}d\varepsilon'\right\}. \tag{10}$$

Accurate to quantities $\sim\alpha^{-1/3}$ we can write in place of (10)

$$\varphi(\varepsilon_\perp)=\alpha\int\limits_0^\infty dx\int\limits_0^\infty dy\,\varphi(y)\exp\left\{-\frac{2}{3}\alpha(\varepsilon_\perp+x^2)^{3/2}\right\}. \tag{11}$$

Thus, the integration with respect to y drops out and

$$\varphi(\varepsilon_\perp)=A\int\limits_0^\infty\exp\left\{-\frac{2}{3}\alpha(\varepsilon_\perp+x^2)^{3/2}\right\}dx. \tag{12}$$

The constant A is determined by the normalization condition. Subsitutting (12) and (9) in (6) and neglecting the contribution made to the normalization integral by the regions II and III ($\sim\alpha^{-1/3} \ll 1$), we obtain

$$A=\alpha n/\pi(2m\mathscr{E}_0)^{3/2}, \tag{13}$$

where n is the particle concentration. Substituting (9), (12), and (13) in (6) we obtain the distribution function in region I:

$$f_{\mathrm{I}}(\mathbf{p})=\frac{\alpha n}{2\pi(2m\mathscr{E}_0)^{3/2}}\int\limits_{-\infty}^{p_z}\exp\left\{-\frac{2}{3}\alpha(\varepsilon_\perp+x^2)^{3/2}\right\}dx. \tag{14}$$

At $p_z > 0$ it decreases relative to $\varepsilon_\perp$ like $\exp(-2/3\,\alpha\varepsilon_\perp^{3/2})$, and at $p_z > \alpha^{-1/3}$ it is practically independent of p_z. At negative $p_z < -\alpha^{-1/3}$, there are practically no particles. The distribution function in the passive region was investigated in Ref. 8.

We consider now Eq. (7) for the solution in region II. The third term corresponds to particles that have after the last collision $\varepsilon' > 1$ and $p'_z < 0$, and are then drawn by the field into region II without collisions. The number of such particles is negligibly small, so that this term can be left out. The first term

$$f_{\rm II}^{(n)}(\varepsilon, \varepsilon_\perp) = \varphi(\varepsilon_\perp) \exp\left\{-\alpha \int_1^{\varepsilon} \left(\frac{\varepsilon'-1}{\varepsilon'-\varepsilon_\perp} \right)^{1/2} d\varepsilon' \right\} \tag{15}$$

describes the distribution of the particles accelerated without collisions from the passive region. The function $\varphi(\varepsilon_\perp)$ decreases in accordance with (12) like $\sim\exp\{-2/3\alpha\varepsilon_\perp^{3/2}\}$. At $\varepsilon \gg 1$, the second factor in (12) decreases with increasing $\varepsilon_\perp$ much more rapidly than the first, so that in this case, setting the argument of the function $\varphi(\varepsilon_\perp)$ equal to zero and then using (12) and (13), we obtain

$$f_{\rm II}^{(n)} = \frac{n\Gamma(1/3)}{3\pi} \left(\frac{3\alpha^2}{2} \right)^{1/3} (2m\mathscr{E}_0)^{-3/2} \exp\{-\alpha(\varepsilon(\varepsilon-1))^{1/2} + \alpha \ln(\varepsilon^{1/2} + (\varepsilon-1)^{1/2})\} \exp\{-\alpha\varepsilon_\perp (\ln 2\varepsilon^{1/2} - 1)\}. \tag{16}$$

This expression is in fact the distribution function corresponding to the Townsend-Shockely mechanism. Its principal dependence on the energy takes the form $\sim\exp(-\alpha\varepsilon) = \exp(-\mathscr{E}/eE\lambda)$. The last factor describes the abrupt Gaussian decrease of this needle-shaped function with increasing angle: $\sim\exp(-\theta^2/\theta_0^2(\varepsilon))$, where $\theta_0^{-2}(\varepsilon) = \alpha\varepsilon(\ln 2\varepsilon^{1/2} - 1)$. The probability that a particle emitted from the passive region will acquire without collisions an energy ε is determined by the length of its path, which increases logarithmically with increasing $\varepsilon_\perp$. This is the reason for the narrowing of the needle width relative to $\varepsilon_\perp$ with increasing ε.

The second term in (7) corresponds to particles that have experienced the last collision at an energy $\varepsilon > 1$. The distribution function in the region III also describes such particles. At not too high energies these two parts of the distribution function (halo) make a small contribution to f_0. It is therefore possible to claculate $f_0(\varepsilon)$ using expressions (12), (13), and (15) for the needle. At $\varepsilon \gg 1$ we obtain

$$f_0^{(0)}(\varepsilon) = \frac{n\Gamma(1/3)}{6\pi\varepsilon \ln(4\varepsilon)} (2m\mathscr{E}_0)^{-3/2} \left(\frac{2\alpha}{3} \right)^{-1/3} \times \exp\{-\alpha(\varepsilon(\varepsilon-1))^{1/2} + \alpha \ln(\varepsilon^{1/2} + (\varepsilon-1)^{1/2})\}. \tag{17}$$

Substituting (17) in (7) and (8) we can verify that in the halo region the main contribution to the integral is made by the vicinity of the upper limit, and the distribution function is of the form

$$f^{(h)}(\varepsilon, \theta) = f_0^{(0)}(\varepsilon+1)/(1-\cos\theta). \tag{18}$$

Thus, the angular dependence of the distribution function in the regions of small and large angles differ substantially. At small angles there is an abrupt Gaussian decrease with increasing θ, while at large angles there is a much smoother variation in accordance with (18). The boundary between the needle and the halo corresponds to the angle[1)]

$$\theta^2(\varepsilon) \approx 1/\varepsilon \ \ln 2\varepsilon^{1/2}, \tag{19}$$

which is obtained from the equation $f^{(h)}(\varepsilon, \theta) = f^{(n)}(\varepsilon, \theta)$. The total distribution function is the sum of $f^{(n)}(\varepsilon, \theta)$ and $f^{(h)}(\varepsilon, \theta)$.

Integrating (18) over the angles from $\theta(\varepsilon)$ to π, we obtain the correction to $f_0(\varepsilon)$ needed to account for the halo:

$$f_0^{(1)}(\varepsilon) = f_0^{(0)}(\varepsilon+1) \ln(4/\theta^2(\varepsilon)). \tag{20}$$

The solution method develops here is valid under the condition $f_0^{(1)}(\varepsilon)/f_0^{(0)}(\varepsilon) \ll 1$. Comparing (20) and (17), we obtain a criterion in the form

$$\varepsilon \ll \varepsilon_{\rm cr} = 1/4 \exp(e^{\alpha}). \tag{21}$$

At energies exceeding $\varepsilon_{\rm cr}$, the main contribution to $f_0(\varepsilon)$ is determined by the halo. The kinetic equation for this case was solved by Keldysh[5]:

$$f^{(h)}(\varepsilon, \theta) = B \frac{\varepsilon^{(\alpha-1)/2} e^{-\alpha s(\varepsilon+1)}}{1 - s\cos\theta}, \quad B = \frac{n(m\mathscr{E}_0)^{-3/2}(4e)^{\alpha/2}}{(2\pi)^3}, \quad s = 1 - 2\exp(-2e^{\alpha}). \tag{22}$$

The total distribution function at $\varepsilon > \varepsilon_{\rm cr}$ is the sum of (22) and (16).

3. SCATTERING BY POLAR PHONONS

In this case $w = w_0/Q^2$. Substituting this expression in (4) and changing over to dimensionless variables, we obtain

$$\frac{\partial f(\mathbf{p})}{\partial p_z} = -\frac{2\tilde{\alpha}}{\tau(\varepsilon)} f(\mathbf{p}) + 2\tilde{\alpha}(\varepsilon+1)^{1/2} F(\varepsilon+1, \theta). \tag{23}$$

Here

$$\tilde{\alpha} = \frac{2\pi m w_0}{eE}, \quad \frac{1}{\tau(\varepsilon)} = \frac{\Theta(\varepsilon-1)}{4\varepsilon^{1/2}} \ln \frac{2\varepsilon-1+2(\varepsilon(\varepsilon-1))^{1/2}}{2\varepsilon-1-2(\varepsilon(\varepsilon-1))^{1/2}},$$

and $\Theta(x)$ is the step function.

In view of the anisotropy of the scattering, the arrival term in the kinetic equation is determined not by the isotropic part of the distribution function $f_0(\varepsilon)$, but by

$$F(\varepsilon+1, \theta) = \frac{1}{4\pi} \int \frac{f(\varepsilon+1, \theta')\, d\Omega'}{2\varepsilon+1-2(\varepsilon(\varepsilon+1))^{1/2} \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos(\varphi-\varphi')}. \tag{24}$$

We solve (23) by the method developed in Sec. 2 above. For the distribution function in region I we obtain

$$f_{\rm I}(\mathbf{p}) = \frac{A}{2} \int_{-\infty}^{p_z} (1+2x) \exp\left\{-\frac{2}{3}\tilde{\alpha}(x^2+\varepsilon_\perp)^{3/2}\right\} dx, \quad A = \frac{n\tilde{\alpha}}{\pi(2m\mathscr{E}_0)^{3/2}(1-(2/3)^{1/3}\Gamma(5/3)\tilde{\alpha}^{-2/3})}. \tag{25}$$

The needle-shaped part of the distribution function takes the form

$$f^{(n)}(\varepsilon, \theta) = A \int_0^{\infty} \exp\left\{-\frac{2}{3}\tilde{\alpha}(\varepsilon_\perp + x^2)^{3/2}\right\} dx \times \exp\left\{-\frac{\tilde{\alpha}}{2} \int_1^{\varepsilon} \frac{d\varepsilon'}{(\varepsilon'(\varepsilon'-\varepsilon_\perp))^{1/2}} \ln \frac{\varepsilon'^{1/2} + (\varepsilon'-1)^{1/2}}{\varepsilon'^{1/2} - (\varepsilon'-1)^{1/2}}\right\}. \tag{26}$$

At $\varepsilon \gg 1$ the angular dependence of $f^{(n)}$ is, as before, Guassian $\sim\exp\{-\tilde{\alpha}\varepsilon\theta^2/2\}$, and the energy dependence is given by $\sim\exp\{-\tilde{\alpha}(\ln 4\varepsilon)^2/4\}$. In the halo region

$$f^{(h)}(\varepsilon, \theta) = \frac{f_0^{(n)}(\varepsilon+1)}{2\ln(4\varepsilon)[1-(1-\varepsilon^{-1})\cos\theta][1-(1-\varepsilon^{-2}/8)\cos\theta]}. \tag{27}$$

The substantial difference between this case and the preceding one is that the time between collisions increases with increasing energy, $\tau(\varepsilon)\sim\varepsilon^{1/2}/\ln\varepsilon$. The transport mean free path is $\lambda_T(\varepsilon)\sim\lambda_0\varepsilon$. Therefore at a certain $\varepsilon=\varepsilon_p\sim\tilde{\alpha}$ the energy acquired on $\lambda_T(\varepsilon_p)$ becomes comparable with $\mathscr{E}_0$. At $\varepsilon\gg\varepsilon_p$ the distribution function becomes close to isotropic and the Davydov approximation is applicable. This energy region was investigated in Ref. 9. It must be noted, however, that the Davydov distribution function (2) is not renormalizable at $\lambda_T(\varepsilon)\sim\varepsilon$, corresponding to the so-called energy runaway.[10] To obtain a physically meaningful result in this region it is necessary to take into account additional scattering mechanisms at high energies.

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[1)]Since the contribution of the halo to the total distribution function is small at $\theta<\theta(\varepsilon)$, expression (18) was written for $\theta\gg\theta(\varepsilon)/\alpha$.

[1]B. I. Davydov, Zh. Eksp. Teor. Fiz. **7**, 1069 (1937).
[2]B. I. Davydov and I. M. Shumushkevich, *ibid.* **10**, 1043 (1940).
[3]J. S. Townsend, Theory of Ionization of Gases by Collision, Oxford, 1910.
[4]W. Shockley, Czech. J. Phys. **B11**, 81 (1961).
[5]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **48**, 1692 (1965) [Sov. Phys. JETP **21**, 1135 (1965)].
[6]H. Dreicer, Phys. Rev. **115**, 238 (1959); **117** (329) (1960).
[7]A. V. Gurevich, Zh. Eksp. Teor. Fiz. **39**, 1296 (1960) [Sov. Phys. JETP **12**, 904 (1961)].
[8]I. B. Levinson, Dissertation, Semicond. Inst. USSR Acad. Sci., 1967.
[9]V. A. Chuenkov, Fiz. Tverd. Tela (Leningrad) **9**, 48 (1967) [Sov. Phys. Solid State **9**, 35 (1967)].
[10]F. G. Bass and Yu. G. Gurevich, Goryachie elektrony i sil'nye elektromagnitnye volny v plazme poluprovodnikov i gasovogo razryada (Hot Electrons and Strong Electromagnetic Waves in the Plasma of Semiconductors and of Gas Discharge), Nauka, M., 1965, p. 47.

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