Critical dynamics of ferromagnets above T_c in a magnetic field

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The critical dynamics of cubic ferromagnets above T_c in a magnetic field is considered on the basis of the dynamic scaling hypothesis. The expressions for the Green functions in the hydrodynamic and critical regions are analyzed in the limiting cases of weak and strong fields. The homogeneous dynamic susceptibility in a magnetic field is considered in detail. It is shown, in particular, that, because of the existence of spin diffusion, the critical damping depends nontrivially on the temperature and the field in weak fields.

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1. INTRODUCTION

Thus far, the theoretical investigation of the critical dynamics of ferromagnets (see, for example, Refs. 1–5) has been limited largely to the simplest case of zero magnetic field. But quite a large number of experiments devoted both to the investigation of paramagnetic resonance above T_c (see, for example, Refs. 6–8) and to the measurement of longitudinal magnetization relaxation in a magnetic field[9-11] have been performed in recent years, and, because we do not have a proper picture of critical phenomena in a field, the results of these experiments have not really been discussed. Furthermore, in their recent experiments on critical neutron scattering in iron located in a magnetic field and at a temperature above the Curie point, Okorokov *et al.*[12] detected polarization effects that, as shown in Ref. 13, allow us to obtain nontrivial information about the dynamics of critical fluctuations.

In the present paper we consider the critical dynamics of ferromagnets in a magnetic field, taking both the exchange and the dipole interactions between the atomic spins into account, but completely neglecting the effects of the crystal anisotropy. The entire analysis is based on the dynamic-scaling hypothesis,[1] and has a phenomenological character.

As is well known (see, for example, Ref. 14), in the static theory, a field is considered to be weak if the following condition is fulfilled[1)]:

$$g\mu H \ll T_c(\varkappa a)^{(5-\eta)/2}, \qquad (1)$$

where H is the internal magnetic field, $\varkappa^{-1}=R_c(\tau)=a\tau^{-\nu}$ is the correlation length, $\tau=(T-T_c)/T_c$, a is a length of the order of the interatomic distance, $\nu\approx 2/3$, and η is the Fisher exponent, which is small, and will be neglected below. If the inequality (1) is replaced by the opposite inequality, then the field is strong, and the correlation length ceases to depend on the temperature, the field-dependent correlation length being given by the formula

$$R_c(H)=\varkappa_H^{-1}=a(T_c/g\mu H)^{2/5}. \qquad (2)$$

Thus, if $g\mu H \gg T_c(\varkappa a)^{5/2}$, then all the static quantities are primarily functions of H, and depend weakly on τ. At the same time, the effect of the field on the critical dynamics can be appreciable even in the case in which the condition (1) is fulfilled. Indeed, if we neglect the dipole forces, then the magnetization in a ferromagnet located in a magnetic field executes an undamped uniform precession. All the perturbations connected both with the inhomogeneity and with the dipole–dipole interaction lead to the damping (of constant Γ), and a change in the frequency, of the precession. Therefore, the magnetic field begins to govern the critical dynamics only when the Larmor frequency becomes of the same order of magnitude as the damping constant Γ. If at the same time the inequality (1) is fulfilled, then Γ is a function of τ, and weakly dependent on H. If, on the other hand, $g\mu H \gg T_c(\varkappa a)^{5/2}$, then the damping, like the static quantities, is primarily determined by the field strength, and varies slowly with the temperature.

In the second section of the paper we give expressions for the Green functions of the critical fluctuations perpendicular and parallel to the field, and analyze their behavior in the limiting cases of weak and strong fields in the exchange temperature region. In this region, defined[3] by the inequality $4\pi\chi_0 \ll 1$, where χ_0 is the magnetic susceptibility of the material, the dipole forces can be taken into account within the framework of perturbation theory. In the third section, a similar analysis is performed for the dipole temperature region, in which $4\pi\chi_0 \gg 1$. Finally, the fourth section of the paper is devoted to the question of the homogeneous dynamic susceptibility in a magnetic field. In this section we show, in particular, that, owing to the presence of spin diffusion, in a weak field the position and width of the paramagnetic resonance line and also the longitudinal absorption depend on the quantity H in an irregular fashion. The experimental investigation of such a dependence would, in principle, allow us to determine the coefficient of spin diffusion by radio-frequency methods.

2. SPIN-SPIN CORRELATIONS IN A FERROMAGNET ABOVE T_c IN A MAGNETIC FIELD. THE EXCHANGE APPROXIMATION

In this section we give for the pair spin Green functions $G_{\alpha\beta}(\mathbf{q},\omega,\mathbf{H})$ formulas generalizing the dynamic-scaling expressions[1] to the case of a nonzero magnetic field:

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$$G_{\alpha\beta}(\mathbf{q},\omega,\mathbf{H})=i\int_0^\infty dte^{i\omega t}\langle[S_\mathbf{q}^\alpha(t),S_{-\mathbf{q}}^\beta(0)]\rangle, \tag{3}$$

$$S_\mathbf{q}^\alpha=N^{-1/2}\sum_i e^{i\mathbf{q}\mathbf{r}_i}S_i^\alpha,$$

where the S_j^α are the atomic-spin components. The averaging is performed over the states of the magnet, which can be described by the Hamiltonian

$$\mathscr{H}=\mathscr{H}_e+\mathscr{H}_d+\mathscr{H}_m,$$

$$\mathscr{H}_e=-\frac{1}{2}\sum_{i\neq j}V_{ij}\mathbf{S}_i\mathbf{S}_j=-\frac{1}{2}\sum_\mathbf{q}V_\mathbf{q}S_\mathbf{q}S_{-\mathbf{q}}, \tag{4}$$

$$\mathscr{H}_d=\frac{1}{2}(g\mu)^2\sum_{i\neq j}\{\mathbf{S}_i\mathbf{S}_j r_{ij}^2-3(\mathbf{r}_{ij}\mathbf{S}_i)(\mathbf{r}_{ij}\mathbf{S}_j)\}r_{ij}^{-5} \tag{4a}$$

$$=\frac{\omega_0}{2}\sum_\mathbf{q}S_\mathbf{q}^\alpha S_{-\mathbf{q}}^\beta\left(n_\alpha n_\beta-\frac{1}{3}\delta_{\alpha\beta}\right), \tag{4b}$$

$$\mathscr{H}_m=-g\mu\sum_i\mathbf{S}_i\mathbf{H}_{ex}. \tag{4c}$$

Here $\mathscr{H}_e$ and $\mathscr{H}_d$ are respectively the exchange and dipole parts of the Hamiltonian; $\mathscr{H}_m$ describes the interaction of the magnet with the external magnetic field H_{ex}; V_{ij} is the exchange integral; $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$, the $\mathbf{r}_i$ being the coordinates of the atoms; $g\mu S$ is the effective magnetic moment of the atom; $\omega_0=4\pi(g\mu)^2v_0^{-1}$ is the characteristic energy of the dipole interaction, v_0 being the volume of the unit cell; and $\mathbf{n}=\mathbf{q}q^{-1}$.

In this section we discuss the exchange temperature region, defined, according to Maleev,[3] by the condition $4\pi\chi_0\ll 1$. Then, we can, on excluding the region of very small q values (see below), neglect the dipole forces in the first approximation. Then in the absence of a field, the tensor $G_{\alpha\beta}$ is isotropic, and is usually written in the following form:

$$G_{\alpha\beta}(q,\varkappa,\omega)=G_0(q,\varkappa)F\left(\frac{q}{\varkappa},\frac{\omega}{\Omega(q,\varkappa)}\right)\delta_{\alpha\beta}=G_0(q,\varkappa)\frac{i\Gamma(q,\varkappa,\omega)}{\omega+i\Gamma(q,\varkappa,\omega)}\delta_{\alpha\beta}. \tag{5}$$

The static Green function

$$G_0(q,\varkappa)=T_c^{-1}(\varkappa a)^{-2}g(q/\varkappa)$$

is given with a high degree of accuracy by the Ornstein–Zernike formula, according to which

$$g(q/\varkappa)=Z(1+q^2/\varkappa^2)^{-1},$$

where $Z\sim S(S+1)/3$. The behavior of the characteristic critical-fluctuation energy $\Omega(q,\varkappa)$ and the function $\Gamma(q,\varkappa,\omega)$ is known only in the various limiting cases. Thus, in the hydrodynamic region, i.e., for $q\ll\varkappa$ and $\omega\ll\Omega_e(\varkappa)=T_c(\varkappa a)^{5/2}$, the dynamic form factor $F(q,\varkappa,\omega)$ has the diffusional form, i.e., $\Gamma(q,\varkappa)=D(\varkappa)q^2$, where $D(\varkappa)\sim T_c(\varkappa a)^{1/2}a^2$ is the coefficient of spin diffusion.

In the critical region, where $q\gg\varkappa$, we have

$$\Omega_e(q,\varkappa)=\Omega_e(q)=T_c(qa)^{5/2}\sim\Gamma(q,0).$$

The form of the function $\Gamma(q,\omega)$ for finite ω is unknown; we can only assert that the real and imaginary parts of Γ are of the same order of magnitude when $\omega\sim\Omega_e(q)$. The properties of $\Gamma(q,\omega)$ are studied in Ref. 15 in the limit $\omega\gg\Omega_e(q)$.

The fluctuations become anisotropic when the magnetic field is switched on. The two transverse Green functions G_{+-} and G_{-+} describe the fluctuations of the magnetization in the direction perpendicular to the field, and coincide in the static limit (i.e., $G_{0+-}=G_{0-+}=G_{0xx}=G_{0yy}=G_{0\perp}$), while the longitudinal Green function $G_{0zz}=G_{0\parallel}$ corresponds to the fluctuations along the field $\mathbf{H}$. Generalizing the dynamic-scaling hypothesis[1] to the case of nonzero magnetic fields, we can write for each of these functions by analogy with (5) the expression

$$G_\lambda(q,\varkappa,\omega,H)=G_{0\lambda}(q,\varkappa,H)F_\lambda(q/\varkappa,\omega/\Omega_e(\varkappa),g\mu H/\Omega_e(\varkappa)), \tag{6}$$

where λ denotes "$+-$", "$-+$", or zz.

The static Green functions $G_{0\lambda}$ can be represented in the form

$$G_{0\lambda}(q,\varkappa,H)=T_c^{-1}(\varkappa a)^{-2}g_\lambda(q/\varkappa,g\mu H/\Omega_e(\varkappa)). \tag{7}$$

For $H=0$, the functions $g_\lambda(q/\varkappa)=g(q/\varkappa)$. As is well known,[14] the g_λ can be expanded in series in powers of $[g\mu H/\Omega_e(\varkappa)]^2$ in weak fields, i.e., when $g\mu H\ll\Omega_e(\varkappa)$.

Let us elucidate the behavior of the quadratic (in the field) correction to $G_{0\lambda}$ as a function of q and $\varkappa$. For this purpose, let us note that the first term of the expansion of $G_{0\lambda}(H)$ in powers of H^2 is connected as follows with the fourth-order vertex part $\mathscr{T}_4(q,\varkappa)$, in which two momenta are equal to q and the two others are equal to zero:

$$G_{0\lambda}(q,\varkappa,H)-G_0(q,\varkappa,0)=(g\mu H)^2G_0^2(q,\varkappa)G_0^2(\varkappa)\mathscr{T}_{4\lambda}(q,\varkappa). \tag{8}$$

As is well known, for $q\ll\varkappa$, $\mathscr{T}_{4\lambda}\sim T_c(\varkappa a)$, and the difference

$$G_{0\lambda}(H)-G_0(0)\propto(g\mu H/\Omega_e(\varkappa))^2.$$

If, on the other hand, $q\gg\varkappa$, then, according to the Polyakov correlation-coalescence principle[16] (which is equivalent to the Polyakov–Kadanoff operator algebra[14]),

$$\mathscr{T}_{4\lambda}(q,\varkappa)\sim T_c(qa)^{1/2}(\varkappa a)^{1/2}. \tag{9}$$

It follows then from (8) and (9) that

$$G_{0\lambda}(q,\varkappa,H)-G_0(q,\varkappa,0)\sim G_0(q)(\varkappa/q)^{3/2}(g\mu H/\Omega_e(\varkappa))^2. \tag{10}$$

It is easy to see that, in the region $q\gg\varkappa$, each term of the expansion of $G_{0\lambda}(q,H)$ in powers of $[g\mu H/\Omega_e(\varkappa)]^2$ has a factor of the order of $(\varkappa/q)^{3/2}G_0(q)$ attached to it.

The static Green functions are regular in τ (see Ref. 14) in strong fields, i.e., when $g\mu H\gg\Omega_e(\varkappa)$, and also in q^2 when $g\mu H\gg\Omega_e(q)$. Therefore, for $q,\varkappa\ll\varkappa_H$ [which is equivalent to the inequalities $g\mu H\gg\Omega_e(q),\Omega_e(\varkappa)$], the functions $g_\lambda(q/\varkappa,\varkappa_H/\varkappa)$ from (7) can be written in the form of the expansion:

$$g_\lambda(q/\varkappa,\varkappa_H/\varkappa)=(\varkappa/\varkappa_H)^2\{g_{0\lambda}+g_{1\lambda}(\varkappa/\varkappa_H)^{1/2}+g_{2\lambda}(q/\varkappa_H)^2+\ldots\}, \tag{11}$$

where $g_{0\lambda}$, $g_{1\lambda}$, and $g_{2\lambda}$ are numbers of the order of unity. Let us note right away that, since by definition $\chi_{0\parallel}=\partial M/\partial H$, while $\chi_{0\perp}=M/H$, then $g_{0\perp}=5g_{0\parallel}$.

In the other limiting case of a strong field, when $\varkappa\ll\varkappa_H\ll q$, instead of the formulas (10) or (11), we have

$$g_\lambda(q/\varkappa,\varkappa_H/\varkappa)=(\varkappa_H/q)^2\{\tilde{g}_{0\lambda}+\tilde{g}_{1\lambda}(\varkappa_H/q)^{1/2}+\ldots\}. \tag{12}$$

Let us now consider the behavior of the Green functions in a field when the ω's are nonzero. First of all, it is necessary to take into account the fact that the component of the total spin along the direction of the field is conserved in the exchange approximation. This leads to the occurrence of an undamped homogeneous precession, i.e., at $q=0$

$$G_{+-}(\omega, H)=-\frac{\langle S_z\rangle}{\omega-g\mu H+i\delta}=-G_{0\perp}(H)\frac{g\mu H}{\omega-g\mu H+i\delta},$$
$$G_{-+}(\omega, H)=G_{+-}(\omega, -H). \tag{13}$$

At the same time, on account of this same conservation law, for $\omega\neq 0$ and $q=0$, the longitudinal function $G_{\parallel}(0, \omega, H)=0$.

Requiring that the expression (6) for the Green functions in a field coincide with the formula (5) when $H=0$ and with the formula (13) when $q=0$, we write it in the following form:

$$G_{+-}(q, \varkappa, \omega, H)=G_{0\perp}(q, \varkappa, H)\frac{-g\mu H+i\Gamma_{+-}(q, \varkappa, \omega, H)}{\omega-g\mu H+i\Gamma_{+-}(q, \varkappa, \omega, H)}, \tag{14}$$

$$G_{\parallel}(q, \varkappa, \omega, H)=G_{0\parallel}(q, \varkappa, H)\frac{i\Gamma_{\parallel}(q, \varkappa, \omega, H)}{\omega+i\Gamma_{\parallel}(q, \varkappa, \omega, H)}, \tag{15}$$

where $\Gamma_{+-}=\Gamma_{\parallel}$ in zero field.

It will be shown below that, because of the interaction of the hydrodynamic modes, the dependence of the functions Γ_λ in weak fields on $g\mu H/\Omega_e(\varkappa)$ is, in contrast to the corresponding dependence of the static functions, generally not regular. In strong fields the quantities Γ_λ can be expanded in a regular fashion in powers of the small parameter $\tau(T_c/g\mu H)^{3/5}$, and depend only on H in the first-order approximation.

Let us consider in detail the behavior of the functions Γ_λ in the hydrodynamic region $q\ll\varkappa$, $\omega\ll\Omega_e(\varkappa)$. In this region, guided by scaling considerations, and knowing the limiting values of Γ_λ for $H=0$, we can write

$$\Gamma_\lambda(q, \varkappa, \omega, H)=D_\lambda(\varkappa, \omega, H)q^2=D(\varkappa)d_\lambda\left(\frac{\omega}{\Omega_e(\varkappa)}, \frac{g\mu H}{\Omega_e(\varkappa)}\right)q^2, \tag{16}$$

where $D(\varkappa)$ is the coefficient of spin diffusion. First of all, it is not difficult to verify that in weak fields the dependence on q of the dynamical factor of the Green functions is determined by the spin diffusion, i.e., in the first approximation the dependence of d_λ on H and ω is insignificant, and $d_\lambda=d_\lambda(0, 0)=1$. This is due to the fact the spin-diffusion coefficient is determined by the region of characteristic frequencies $\omega\sim\Omega_e(\varkappa)$ and momenta $q\sim\varkappa$, which, in the first approximation, is negligibly affected by weak fields. In fact, this is a basic result concerning the dynamics in weak fields.

Let us now give the first few terms of the expansion of the d_λ in weak fields and explain their physical meaning:

$$d_{+-}=1+id_{0\perp}\frac{g\mu H}{\Omega_e(\varkappa)}+d_{1\perp}\left(\frac{g\mu H}{\Omega_e(\varkappa)}\right)^2+id_{2\perp}\frac{\omega}{\Omega_e(\varkappa)}+d_{3\perp}\left(i\frac{g\mu H-\omega}{\Omega_e(\varkappa)}\right)^{3/2}+\ldots, \tag{17}$$

$$d_{\parallel}=1+d_{1\parallel}\left(\frac{g\mu H}{\Omega_e(\varkappa)}\right)^2+id_{2\parallel}\left(\frac{\omega}{\Omega_e(\varkappa)}\right)+d_{3\parallel}\left(-\frac{i\omega}{\Omega_e(\varkappa)}\right)^{3/2}+\ldots. \tag{18}$$

Here the $d_{i\perp,\parallel}$ are real numbers of the order of unity, and it can be concluded from the requirement that the excitation energy be positive that $d_{0\perp}>0$. The term linear in H in the expansion of d_{+-} clearly corresponds to the consideration of the spatial nonuniformity of the precession. Its appearance can most easily be understood on the basis of the molecular-field approximation if we note that the term corresponding to it in the expansion of D_{+-} is

$$D(\varkappa)g\mu H/\Omega_e(\varkappa)\sim T_c a^2\langle S_z\rangle.$$

Let us emphasize that the nonuniformity of the precession is much weaker than the diffusional damping.

The terms in d_λ that are proportional to H^2 correspond to a slight renormalization of the spin-diffusion coefficient in a field, while the ω-dependent terms determine the frequency dispersion of D_λ. The irregular character of the dispersion is due to the contribution to D_λ of the interaction of the hydrodynamic modes (14), (15) [with allowance for (16)–(18)]. The corresponding corrections can be obtained by the same procedure used in Ref. 17 (see also Ref. 18), where the frequency dispersion of $D(\omega)$ in zero field is analyzed. It can be seen from the expression (17) that the irregularity of the dispersion leads to irregular dependence of D_{+-} on the magnetic field.

It should be noted that Vaks *et al.*[19] have found the spatial dispersion and the damping of the precession above T_c to occur in the molecular field approximation. In this case for $q\neq 0$ the damping in weak fields in the hydrodynamic region turns out to be proportional to H^{-3}, which does not allow passage to the $H=0$ limit. The point is that Vaks *et al.*[19] chose as the zeroth approximation the undamped precession with a variance $\langle S_z\rangle q^2$, which exists only at $H\neq 0$. The damping, which was computed within the framework of perturbation theory with the use of a long interaction range as a parameter, arose as a result of the scattering of these "precessional" modes by each other. But we should, in considering the critical dynamics in weak fields, take account of the spin diffusion, which occurs in zero field. Therefore, it is natural to proceed from the diffusionally-damped-uniform-precession approximation. Then we arrive at the described picture of the hydrodynamics in a weak field.

In a strong field, the hydrodynamic region is defined by the condition $q\ll\varkappa_H$. Naturally, we should first of all consider the field dependence of $d_{\parallel}$ for $\omega=0$ in the case of the longitudinal Green function $G_{\parallel}$ and the H dependence of d_{+-} for $\omega=g\mu H$ in the case of the transverse function G_{+-}. The asymptotic forms of the functions $d_\lambda[g\mu H/\Omega_e(\varkappa)]$ from (16) are such that in strong fields D_λ does not to first order depend on τ, i.e.,

$$d_\lambda\left(\frac{g\mu H}{\Omega_e(\varkappa)}\right)=\left(\frac{g\mu H}{T_c}\right)^{1/5}\tau^{-1/3}\tilde{d}_\lambda\left[\tau\left(\frac{T_c}{g\mu H}\right)^{3/5}\right]$$
$$=\left(\frac{g\mu H}{T_c}\right)^{1/5}\tau^{-1/3}\left\{\tilde{d}_{0\lambda}+\tilde{d}_{1\lambda}\tau\left(\frac{T_c}{g\mu H}\right)^{3/5}+\ldots\right\}, \tag{19}$$

where $\tilde{d}_\lambda(\tau)$ is a regular function of τ; the $\tilde{d}_{i\parallel}$ are real numbers, while the $\tilde{d}_{i+-}$ are, generally speaking, complex numbers.

Thus, the dynamical factor of $G_{\parallel}$ is determined by the spin diffusion both in weak and in strong fields. But now the spin-diffusion coefficient depends only on the field in the first approximation. We shall not discuss the weak frequency dispersion of $\Gamma_{\parallel}(\omega)$ here. Concerning Γ_{+-}, we should make two comments. First, as a result of the fact that the $\tilde{d}_{0+-}$ are complex quantities, the orders of magnitude of the spatial dispersion and the damping of the precession are found in Ref. 19 to be the same, being given in the first approximation by

$$\operatorname{Re}\Gamma_{+-}\sim\operatorname{Im}\Gamma_{+-}\sim T_c\langle S_z\rangle(qa)^2.$$

Secondly, since the characteristic critical-fluctuation energy $T_c(\varkappa_H a)^{5/2}\sim g\mu H$ in a strong field, the frequency dependence of $\Gamma_{+-}(\omega)$ is important in the region $\omega\sim g\mu H$ (cf. the analysis in Ref. 17). But near the $\omega\approx g\mu H$ resonance for $q\ll\varkappa_H$, both the damping and the spatial dispersion are small in comparison with $g\mu H$, and therefore we shall not discuss the ω dependence of Γ_{+-} here. Let us note that similar results are obtained in Ref. 19 for the high-field Γ_{+-} for ferromagnets with a large interaction radius.

Let us now elucidate the behavior of the Green functions in the critical region $q\gg\varkappa$. First of all, let us note that in this region the relations $g\mu H\ll\Gamma(q)\sim T_c(qa)^{5/2}$ are always valid in weak fields. Furthermore, there exists a strong-field region, $\varkappa\ll\varkappa_H\ll q$, in which $g\mu H$ is small compared to $\Gamma(q)$, and the Green function (14) can be expanded in powers of H. Limiting ourselves to the linear approximation, we have for $G_{+-}(q,\omega,H)$ in a weak field the expression

$$\begin{aligned}G_{+-}(q,\omega,H)-G(q,\omega)&=G_0(q)\frac{\omega g\mu H}{(\omega+i\Gamma(q,\omega))^2}\left[i\frac{\partial\Gamma_{+-}(q,\omega,H)}{g\mu\partial H}-1\right]\\&=G^2(q,\omega)\mathcal{T}_3^{+z-}(q,\omega;0,0)G_0(0)g\mu H,\end{aligned}\tag{20}$$

where $\mathcal{T}_3^{+z-}$ is the third-order dynamical vertex with two coinciding frequencies and momenta, introduced in Ref. 20 (see also Ref. 13). Using the correlation-coalescence principle as generalized to the case of the dynamical vertex and the dynamic-scaling hypothesis, we find for $\mathcal{T}_3$ in the case in which $q\gg\varkappa$ the expression[20,21]

$$\mathcal{T}_3^{+z-}(q,\varkappa,\omega)=T_c\frac{\omega}{\Omega_e(q)}\gamma_3\left(\frac{\omega}{\Omega_e(q)}\right)(qa)^{1/2}\varkappa a.\tag{21}$$

Here $\gamma_3(0)$ is a real number; the factor $\omega/\Omega_e(q)$ has been separated out as a result of the dynamical character of the vertex ($\mathcal{T}_3=0$ at $\omega=0$), while the factor $q^{1/2}\varkappa$ is due to the correlation-coalescence principle.

By comparing the expressions (20) and (21), we find the correction to Γ_{+-} in H:

$$\Gamma_{+-}(q,\varkappa,\omega,H)=\Gamma(q,\varkappa,\omega)+ig\mu H\frac{q}{\varkappa}\gamma_{+-}\left(\frac{\omega}{\Omega_e(q)}\right),\tag{22}$$

where $\gamma_{+-}(0)$ is a real positive number. The sign of $\gamma_{+-}(0)$ can be determined from the requirement that the excitation energy be positive.

A natural generalization of the formula (22) to the entire field region $g\mu H\ll\Omega_e(q)$ is the following expression:

$$\Gamma_{+-}(q,\varkappa,\omega,H)=\Gamma(q,\varkappa,\omega)\left\{1+\frac{ig\mu H}{\Omega_e(\varkappa)}\left(\frac{\varkappa}{q}\right)^{3/2}\gamma_{+-}\left(\frac{g\mu H}{\Omega_e(\varkappa)},\frac{\omega}{\Omega_e(q)}\right)\right\},\tag{23}$$

which assumes in strong fields for which $\varkappa\ll\varkappa_H\ll q$ the form

$$\Gamma_{+-}(q,\omega,H)=\Gamma(q,\omega)\left\{1+\left(\frac{\varkappa_H}{q}\right)^{3/2}\varphi_{+-}\left(\frac{\omega}{\Omega_e(q)}\right)\right\};\quad|\varphi_{+-}(0)|\sim1.\tag{24}$$

We can verify in exactly the same fashion the fact that in weak fields the H-induced correction to $\Gamma_\parallel$ is proportional to

$$q/\varkappa(g\mu H/\Omega_e(\varkappa))^2$$

[cf. (22)]. Here, as before, the factor $q/\varkappa$ arises as a result of the correlation-coalescence principle. In strong fields for which $\varkappa_H\ll q$, the formula (24) is valid for $\Gamma_\parallel$ provided the function φ_{+-} is replaced by $\varphi_\parallel$. If, on the other hand, the field is so strong that $g\mu H\gg\Omega_e(q)$, i.e., if $\varkappa_H\gg q$, then we return to the above-considered case of hydrodynamics in a strong field.

3. CRITICAL DYNAMICS IN A MAGNETIC FIELD WITH ALLOWANCE FOR THE DIPOLE FORCES

Thus far, in discussing the picture of the critical phenomena in a magnetic field, we have assumed that the spin-spin interaction in a ferromagnet is a purely exchange interaction, i.e., we have neglected the term $\mathscr{H}_d$, (4b), in the Hamiltonian (4). The dipole-dipole magnetic interaction energy in magnets is, as a rule, small compared to the exchange energy. But if we consider the dynamics of sufficiently long-wave excitations, we cannot completely neglect the dipole forces. Indeed, in contrast to the exchange forces, the dipole forces do not conserve the total spin of the magnet, and, thus, ensure the relaxation of the magnetization fluctuations and the eventual dying down of the precession in the $\mathbf{q}\to0$ homogeneous limit. Furthermore, since the dipole interaction is anisotropic, its consideration leads to the anisotropy of the fluctuations even in zero field. Finally, because of the fact that the dipole forces are long-range forces, there arise demagnetization effects, as a result of which the susceptibility of a body, $\tilde{\chi}_{\alpha\beta}$, differs from the susceptibility of the material, $\chi_{\alpha\beta}$, and the internal magnetic field $\mathbf{H}$ does not coincide with the external field $\mathbf{H}_{ex}$.

As is well known,[3,15] when allowance is made for the dipole interaction, the Green function $G_{\alpha\beta}$, (3), satisfies the relation

$$G_{\alpha\beta}=G_{\alpha\beta}^{(1)}-\omega_0G_{\alpha\mu}^{(1)}n_\mu n_\nu G_{\nu\beta},\tag{25}$$

where $\mathbf{n}=\mathbf{q}q^{-1}$ and $G_{\alpha\beta}^{(1)}$ possesses the symmetry of the original Hamiltonian without the dipole forces, and is proportional to the magnetic susceptibility of the material $\chi_{\alpha\beta}$: $\omega_0G_{\alpha\beta}^{(1)}=4\pi\chi_{\alpha\beta}$. In the $\mathbf{q}\to0$ homogeneous limit, the tensor $n_\mu n_\nu$ (for a body of ellipsoidal shape) goes over into the tensor whose elements are the demagnetizing factors $N_{\mu\nu}$.

Solving Eq. (25), we obtain

$$G_{\alpha\beta}=G_{\alpha\beta}^{(1)}-\omega_0G_{\alpha\nu}^{(1)}n_\nu n_\mu G_{\mu\beta}^{(1)}/(1+\omega_0G_{\nu\mu}^{(1)}n_\nu n_\mu).\tag{26}$$

The internal magnetic field $\mathbf{H}$ is connected with the external field $\mathbf{H}_{ex}$ also by the well-known relation

$$\mathbf{H}=\mathbf{H}_{ex}-4\pi\hat{N}\mathbf{M},\quad\mathbf{M}=\chi_{0\perp}\mathbf{H}.\tag{27}$$

It is shown in Ref. 3 that, if $4\pi\chi_0\ll1$, then we can take the dipole forces into account with the aid of perturbation theory when computing the dipole-induced damping constant Γ_0. Then, according to Ref. 2,

$$\Gamma_0 \sim \frac{\omega_2^0}{T_c}\tau^{-1} \sim \Omega_e(\varkappa)\left(\frac{q_0}{\varkappa}\right)^4,$$

where $q_0 \sim (\omega_0/T_c)^{1/2}a^{-1}$ is the characteristic dipole momentum, which is determined from the condition $4\pi\chi_0(q_0)=1$. Since the diffusion-induced damping and the dipole-induced relaxation are relatively independent, when the dipole forces in a weak field are taken into account, we have for the Γ_λ in place of (16) the expressions

$$\Gamma_\lambda(q,\varkappa,\omega,H)=D_\lambda(\varkappa,\omega,H)q^2+\Gamma_{d\lambda}(\varkappa,\omega,H), \tag{28}$$

where, to first order in $\omega/\Omega_e(\varkappa)$ and $g\mu H/\Omega_e(\varkappa)$, the damping constant

$$\Gamma_0=\gamma_0\Omega_e(\varkappa)(q_0/\varkappa)^4.$$

The allowance for the dependence of Γ_λ on ω and H will be discussed in detail in the following section of the paper; just now let us proceed to consider the dipole region, where $4\pi\chi_0 \gg 1$ ($\varkappa \ll q_0$). To begin with, let us recall the situation in the case of zero field.[3] According to the formula (26), when $\varkappa \ll q_0$, the anisotropy of the Green function is substantial, only the components of the function that are perpendicular to the momentum $\mathbf{q}$ (for $\mathbf{q}\to 0$) now becoming infinite at $\tau=0$. Further, from the analysis performed in Ref. 3 we can obtain the following expression for the characteristic energy with allowance for the dipole scaling:

$$\Omega_d=\frac{1}{2}[G_0^{(1)}]^{-1}\{\alpha(q_0a)^{1/2}+[\alpha^2(q_0a)+4\beta T_cG_0^{(1)}(q_0a)^3]^{1/2}\}, \tag{29}$$

where α and β are numbers of the order of unity, and, clearly, in the limit $T\to T_c$ (for $q=0$)

$$\Omega_d=T_c\beta(q_0a)^{3/2}\varkappa a. \tag{30}$$

Nevertheless, if $\beta \ll \alpha^2$, then the so-called normal dipole dynamics with a dipole energy $\Omega_d \sim \chi_0^{-1}$ will be observed in real experiments in the temperature region in which $(\alpha^2/\beta)q_0^2 \ll \varkappa^2 \ll q_0^2$, i.e., in the region not too close to T_c. And only when $\varkappa^2 \ll (\alpha^2/\beta)q_0^2$ will the dipole energy be given by the formula (30), i.e., will the dynamics become rigid. It is possible that Shiino and Hashimoto[22] observed in their investigation of the longitudinal relaxation in EuS the beginning of the transition from the regime of normal dipole dynamics to the rigid dynamics regime.

The bases for the existence of the inequality $\beta \ll \alpha^2$ are the following. The term with β in the formula (29) occurs because of the allowance for the contribution to Ω_d of the processes of rescattering of the critical fluctuations by each other, and therefore should, as noted in Ref. 3, contain a numerical smallness. Furthermore, it is shown in Ref. 21 that, because of the presence of three-point correlations, the critical dynamics should depend on the magnitude of the atomic spin S. And since the rescattering due to the three-point correlations is suppressed at large S values, the region of normal dynamics in ferromagnets with small S should be narrower than in the $S \gg 1$ case.

Let us now give the expressions for the $\Gamma_\lambda^{(1)}(H)$ in a weak field. Since in the dipole region $\Omega_e(\varkappa) \ll \Omega_d(\varkappa)$ and $g\mu H$ should be comparable to $\Omega_e(\varkappa)$, $g\mu H \ll \Omega_d(\varkappa)$ all the more in a weak field, and for the $\Gamma_\lambda^{(1)}$ we have

$$\Gamma_{+-}^{(1)}(\varkappa,\omega,H)=\Gamma(\varkappa,\omega)\left\{1+i\frac{g\mu H}{\Omega_e(\varkappa)}\gamma_{+-}^{(1)}\left(\frac{\omega}{\Omega_d(\varkappa)}\right)\right\}, \tag{31}$$

$$\Gamma_{\parallel}^{(1)}(\varkappa,\omega,H)=\Gamma(\varkappa,\omega)\left\{1+\left(\frac{g\mu H}{\Omega_e(\varkappa)}\right)^2\gamma_{\parallel}^{(1)}\left(\frac{\omega}{\Omega_d(\varkappa)}\right)\right\}. \tag{32}$$

Here $\Gamma(\varkappa,0,0)\sim\Omega_d(\varkappa)$ and $|\gamma_{+-}^{(1)}(0)|\sim\gamma_{\parallel}^{(1)}(0)\sim 1$.

If $\varkappa \ll q \ll q_0$, then, using the correlation-coalescence principle for the third-order vertex, we obtain in place of (23) the formulas

$$\Gamma_{+-}^{(1)}(q,\omega,H)=\Gamma(q,\omega)\left\{1+\frac{ig\mu H}{\Omega_e(\varkappa)}\left(\frac{\varkappa}{q}\right)^{3/2}\tilde{\gamma}_{+-}^{(1)}\left(\frac{\omega}{\Omega_d(q)}\right)\right\}, \tag{33a}$$

$$\Gamma_{\parallel}^{(1)}(q,\omega,H)=\Gamma(q,\omega)\left\{1+\left(\frac{g\mu H}{\Omega_e(\varkappa)}\right)^2\left(\frac{\varkappa}{q}\right)^{3/2}\tilde{\gamma}_{\parallel}^{(1)}\left(\frac{\omega}{\Omega_d(q)}\right)\right\}. \tag{33b}$$

Notice that this behavior of $\Gamma_{+-}^{(1)}(H)$ guarantees the possession of a small peak by $G_{+-}^{(1)}$ as a function of H at

$$g\mu H\sim\omega\Omega_e(\varkappa)/\Omega_d(\varkappa),$$

if $q \ll \varkappa$ and $\omega \ll \Gamma(\varkappa,0)$, or at

$$g\mu H\sim\omega\frac{\Omega_e(\varkappa)}{\Omega_d(\varkappa)}\left(\frac{q}{\varkappa}\right)^{3/2},$$

if $\varkappa \ll q \ll q_0$ and $\omega \ll \Gamma(q,0)$. The nature of the temperature dependence of this peak is connected with the form of the characteristic critical-fluctuation energy in the dipole region, and is determined also by the correlation-coalescence principle for the third-order dynamical vertex when $\varkappa \ll q$.

In a strong field the situation is entirely similar to the situation considered above in the exchange case. If

$$\Omega_e(q_0)>\Omega_e(q)>g\mu H>\Omega_e(\varkappa),$$

then $\varkappa$ in the formulas (33a) and (33b) should be replaced by $\varkappa_H$, while for $q_0>\varkappa_H>q$ the functions $\Gamma_\lambda^{(1)}$ are given by the formulas (31), (32), in which $\varkappa$ should also be replaced by $\varkappa_H$. Further increase in the field should take us from the dipole regime to the high-field exchange regime.

4. PARAMAGNETIC RESONANCE AND LONGITUDINAL RELAXATION IN THE CRITICAL REGION ABOVE T_c

Paramagnetic resonance in ferromagnets and longitudinal relaxation in a magnetic field in the critical region have been quite intensively studied in experiments (see, for example, Refs. 6–10). But thus far no detailed theoretical discussion of the homogeneous dynamic susceptibility in an external field has been published. The aim of this section is to consider this question.

As a rule, in experiment we measure the susceptibility $\tilde{\chi}_{\alpha\beta}$ of a body and control the external field $\mathbf{H}_{ex}$ applied to the sample. But of physical importance are the internal field $\mathbf{H}$ and the susceptibility of the material $\chi_{\alpha\beta}$, which, because of the demagnetization effects, differ from $\mathbf{H}_{ex}$ and $\tilde{\chi}_{\alpha\beta}$. We shall discuss the effect of the demagnetization below; for now let us consider the behavior of $\chi_{\alpha\beta}$ as a function of H and τ.

As in the preceding section, we represent the longitudinal and transverse susceptibilities in the following form:

$$\chi_{\parallel}(\tau,\omega,H)=\chi_{0\parallel}(\tau,H)\,\frac{i\Gamma_{\parallel}(\tau,\omega,H)}{\omega+i\Gamma_{\parallel}(\tau,\omega,H)}, \tag{34}$$

$$\chi_{+-}(\tau,\omega,H)=-\chi_{0\perp}(\tau,H)\,\frac{g\mu H-i\Gamma_{+-}(\tau,\omega,H)}{\omega-g\mu H+i\Gamma_{+-}(\tau,\omega,H)}, \tag{35}$$
$$\chi_{-+}(H)=\chi_{+-}(-H),$$

where the $\chi_{0\lambda}$ are the static susceptibilities and $\Gamma_\lambda(\omega,H)$ is the critical-damping constant; at $\omega=0$ and $H=0$ the real positive quantity $\Gamma_\lambda(0,0)=\Gamma_0$.

Let us first consider the behavior of $\chi_\lambda(\omega,H)$ in the exchange region $4\pi\chi_{0\lambda}(\tau,H)\ll 1$. Then, as has already been noted above, in the case of small frequencies and weak fields, i.e., for $\omega, g\mu H\ll\Omega_e(\varkappa)$, we have in the first approximation $\chi_0\propto\varkappa^{-2}$ and $\Gamma_0\sim\omega_0^2T_c^{-1}\tau^{-1}$. We can then expand the $\chi_{0\lambda}(\tau,H)$ in a regular fashion in powers of $[g\mu H/\Omega_e(\varkappa)]^2$. It is clear that under these conditions the power series expansion of $\Gamma_\lambda(\omega,H)$ in $\omega/\Omega_e(\varkappa)$ and $g\mu H/\Omega_e(\varkappa)$ contains regular terms. But, as we shall now verify, the dependence $\Gamma_\lambda(\omega,H)$ in the region of low frequencies and weak fields is determined by the presence of spin diffusion in the system, the corrections due to the spin diffusion exceeding the regular terms of the expansion and being irregular when $\Gamma_0\lesssim\omega$ and $g\mu H\ll\Omega_e(\varkappa)$.

To compute the $\Gamma_\lambda(\tau,\omega,H)$, let us use the generalizations to the nonzero-H case of the well-known formulas for Γ in the exchange region[3]:

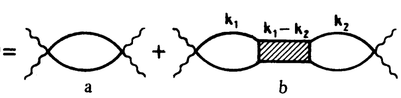
$$\Gamma_\lambda(\omega,H)=G_{0\lambda}^{-1}[\Phi_\lambda(\omega,H)-\Phi_\lambda(0,H)](i\omega)^{-1}, \tag{36}$$

where $\omega_0G_{0\lambda}=4\pi\chi_{0\lambda}$ and for Φ_λ we have

$$\Phi_{\alpha\beta}(\omega,H)=i\int_0^\infty dt\,e^{i\omega t}\langle[\dot S_0^\alpha(t),\dot S_0^\beta(0)]\rangle, \tag{37}$$

$$\dot S_0^\alpha=\omega_0N^{-1/2}\sum_{\mathbf{k}}\frac{k_\mu k_\nu}{k^2}S_{\mathbf{k}}^\mu S_{-\mathbf{k}}^\rho\varepsilon_{\alpha\nu\rho}. \tag{38}$$

It is necessary to represent the expression for the $\Gamma_\lambda(\omega,H)$ in the form of the following two diagrams:

$\Gamma_\lambda(\omega,H)=$ 

(39)

where the lines correspond to the Green function; the vertices, to the operator $\dot S_0^\alpha$; and the hatched block, to the total four-particle vertex $\mathcal{T}_4$.

In Ref. 3, it is shown in the course of the determination of Γ_0 that the diagram (39a) and the diagram (39b), which takes the rescattering into account, have one and the same order of magnitude, and that they add up to Γ_0. Below we shall verify that the dependence of Γ_λ on H and ω is determined by the diagram (39a) only, and that the contribution from the rescattering is small. Using the results of Ref. 3, we obtain after simple computations the following expression for the $\Gamma_\lambda(\omega,H)$ from the diagram (39a):

$$\Gamma_{+-}(\omega,H)=\frac{1}{15}\,\omega_0^2G_0^{-1}\frac{T_cv_0}{(2\pi)^3}\{2[I(\omega-2g\mu H)+I(\omega+g\mu H)]$$
$$+3[I(\omega-g\mu H)+I(\omega)]\}, \tag{40}$$

$$\Gamma_{\parallel}(\omega,H)=\frac{1}{15}\,\omega_0^2G_0^{-1}\frac{T_cv_0}{(2\pi)^3}\{I(\omega-g\mu H)+I(\omega+g\mu H)$$
$$+4[I(\omega-2g\mu H)+I(\omega+2g\mu H)]\}, \tag{41}$$

$$I(\omega)=\int dq\,\frac{1}{i\pi^2}\int\frac{dx_1dx_2}{x_1x_2}\,\frac{\operatorname{Im}G_q(x_1)\operatorname{Im}G_q(x_2)}{x_1+x_2-\omega-i\delta}, \tag{42}$$

$$G_q(\omega)=G_q(0)\,\frac{i\Gamma_q}{\omega+i\Gamma_q}. \tag{43}$$

The damping constant Γ_q is, generally speaking, a function of H and ω. But, since here we are interested in the small H- and ω-related corrections to Γ_0, it is not necessary to take into account the dependence of Γ_q on ω and H in the Green formulas (42) and (43). It is easy to see that the dominant contribution to the x_1, x_2, and q integrals for $I(\omega)-I(0)$ is made by the hydrodynamic region $q\ll\varkappa$, $x_1, x_2\ll\Omega_e(\varkappa)$, in which $\Gamma_q=Dq^2+\Gamma_0$, where $D\sim T_c(\varkappa a)^{1/2}a^2$ is the spin-diffusion coefficient. Substituting (43) into (42), we have

$$I(\omega)-I(0)=-2\pi G_0^2\int_0^\infty\frac{q^2\,dq}{Dq^2+\Gamma_0}\,\frac{\omega}{\omega+2i(Dq^2+\Gamma_0)}$$
$$=\frac{\pi^2G_0^2\Gamma_0^{1/2}}{D^{3/2}}\left[1-\left(1-\frac{i\omega}{\Gamma_0}\right)^{1/2}\right]=\frac{\pi^2G_0^2\Gamma_0^{1/2}}{D^{3/2}}\left[\Psi_1\left(\frac{\omega}{2\Gamma_0}\right)+i\Psi_2\left(\frac{\omega}{2\Gamma_0}\right)\right],$$

$$\Psi_1(x)=1-2^{-1/2}[(1+x^2)^{1/2}+1]^{1/2},\quad \Psi_2(x)=2^{-1/2}x/[(1+x^2)^{1/2}+1]^{1/2}. \tag{44}$$

According to the formulas (44), (40), and (41), the corrections to the Γ_λ are complex. For the transverse susceptibility this implies the appearance of both a resonance-frequency shift $\Delta\omega_r$ and a correction $\Delta\Gamma_{+-}$ to the damping constant; for the longitudinal susceptibility, the appearance of dispersion in $\Gamma_\parallel(\omega,H)$. From the expressions (44) and (40), (41) we find that

$$\Delta\omega_r=\omega_r-g\mu H=E_0\left[\Psi_2\left(\frac{g\mu H}{2\Gamma_0}\right)+2\Psi_2\left(\frac{g\mu H}{\Gamma_0}\right)\right], \tag{45}$$

$$\Delta\Gamma_{+-}(\omega_r)=E_0\left[5\Psi_1\left(\frac{g\mu H}{2\Gamma_0}\right)+2\Psi_1\left(\frac{g\mu H}{\Gamma_0}\right)\right], \tag{46}$$

$$\Delta\Gamma_\parallel(\omega,H)=E_0\left\{\Psi\left(\frac{\omega-g\mu H}{2\Gamma_0}\right)+\Psi\left(\frac{\omega+g\mu H}{2\Gamma_0}\right)\right.$$
$$\left.+4\left[\Psi\left(\frac{\omega-2g\mu H}{2\Gamma_0}\right)+\Psi\left(\frac{\omega+2g\mu H}{2\Gamma_0}\right)\right]\right\}. \tag{47}$$

$$E_0=\frac{\pi^2}{15}\,\frac{v_0}{(2\pi)^3}\,\frac{\omega_0^2G_0T_c\Gamma_0^{1/2}}{D^{3/2}},\quad \Psi(x)=\Psi_1(x)+i\Psi_2(x). \tag{48}$$

It can be seen from the formulas (45)–(48) that the corrections $\Delta\Gamma_\lambda$, $\Delta\omega_r\propto D^{-3/2}$; therefore, their experimental study allows us, in principle, to determine the spin-diffusion coefficient. It also follows from the expressions (45)–(47) that the resonance frequency increases with increasing field intensity, whereas the line width and the longitudinal relaxation, which is determined by $\operatorname{Re}\Gamma_\parallel(0,H)$, decrease.

The corrections $\Delta\Gamma_\lambda$ and $\Delta\omega_r$ increase with increasing ω and H, and depend on ω and H in the region

$$\Gamma_0\lesssim\omega,\quad g\mu H\ll\Omega_e(\varkappa)$$

in an irregular fashion. Noting that

$$E_0\sim(\Gamma_0/\Omega_e(\varkappa))^{1/2}\Gamma_0,$$

we can determine their orders of magnitude:

$$\frac{\Delta\omega_r}{g\mu H}\sim\left(\frac{\Gamma_0}{\Omega_e(\varkappa)}\right)^{1/2}\min\left\{1,\left(\frac{\Gamma_0}{g\mu H}\right)^{1/2}\right\},$$
$$\frac{\Delta\Gamma_{+-}}{\Gamma_0}\sim-\frac{(g\mu H)^2}{(\Omega_e(\varkappa)\Gamma_0^3)^{1/2}}\min\left\{1,\left(\frac{\Gamma_0}{g\mu H}\right)^{3/2}\right\}. \tag{49}$$

The terms obtained in the expansion of Γ_λ are greater than the regular corrections:

$$\Delta\omega_r/g\mu H\sim\Gamma_0/\Omega_e(\varkappa),\quad \Delta\Gamma_\lambda/\Gamma_0\sim(g\mu H/\Omega_e(\varkappa))^2.$$

Let us also note that, as follows from the formula (47), $\Gamma(\omega)$ undergoes dispersion in zero field, and that this dispersion has an irregular character in the region $\Gamma_0 \lesssim \omega \ll \Omega_e(\varkappa)$.

It must be said that the structural corrections of (45)–(48) occur not only in ferromagnets in the vicinity of T_c, but also in all magnetic systems in which spin diffusion occurs together with the homogeneous dipole relaxation. Similar phenomena occurring in quasi-two-dimensional systems are being intensively studied (see, for example, Ref. 23).

Let us now consider the diagram, (39b), taking account of the rescattering. To begin with, let us note that, if we neglect the dependence of $\mathscr{T}_4$ on the momentum $\mathbf{k}_1 - \mathbf{k}_2$, then the diagram vanishes. Indeed, in that case the angle integrations at the two dipole vertices can be performed independently, and the operator $\dot{S}_0^\alpha$ integrated over the angle is equal to zero. To determine the nonzero contribution, it is sufficient to expand $\mathscr{T}_4$ in powers of $(\mathbf{k}_1 - \mathbf{k}_2)^2 \varkappa^{-2} < 1$, and estimate the contribution from the first nonvanishing term of the expansion, this term being proportional to $(\mathbf{k}_1 \cdot \mathbf{k}_2)^2 \varkappa^{-4}$. Here it is first of all necessary to determine the contribution of the diagram with ω- and H-dependent two-particle intermediate states, e.g., with the ω- and H-dependent functions G_{k_2}. As shown above, in this case the low-momentum ($k_2 \ll \varkappa$) and low-frequency $[\omega_2 \ll \Omega_e(\varkappa)]$ dependence of G_{k_2} is important, and therefore the integral of $G_{k_1}^2 \mathscr{T}_4$ over k_1 can be estimated, using the static G_{k_1} and $\mathscr{T}_4$. This integral is determined by the region $k_1 \sim \varkappa$, where $\mathscr{T}_4 \sim \varkappa$, and is in order of magnitude equal to unity. The remaining integral over k_2, in comparison with the contribution (44) of the diagram (39a), contains the factor $k_2^2 \varkappa^{-2}$, which derives from the factor $(\mathbf{k}_1 \cdot \mathbf{k}_2)^2 \varkappa^{-4} (k_1 \sim \varkappa)$. Therefore, taking account of the fact that the characteristic $k_2 \sim \max(\omega, \Gamma_0) D^{-1}$, we find that, as compared to the diagram (39a), the contribution from the rescattering will at least be of the order of $\max(\omega, \Gamma_0)/\Omega_e(\varkappa)$ in smallness. It is not difficult to verify that the diagrams with ω- and H-dependent many-particle intermediate states will make even smaller contributions. Thus, the expressions (45)–(48) completely determine $\Delta\Gamma_\lambda$ in the region ω, $g\mu H \ll \Omega_e(\varkappa)$. But when ω or $g\mu H$ has the same order of magnitude as $\Omega_e(\varkappa)$, all the corrections to Γ_0 are important, and we do not have a simple expression for $\Gamma_\lambda(\omega, H)$.

As has already been discussed above, in a strong field, all the physical quantities depend only on the magnetic field. In particular,

$$\chi_{0\lambda} \sim \omega_0 T_c^{-1} (\varkappa_H a)^{-2} \propto H^{-4/5}, \quad \chi_{0\perp}(H) = 5\chi_{0\parallel}(H).$$

In this case the $\Gamma_\lambda(H)$ are given by the expression for Γ_0 with $\varkappa$ replaced by $\varkappa_H$. Then $\Gamma_\parallel(0, H)$ is real, $\Gamma_{+-}(H)$ has a real and an imaginary part of the same order of magnitude, and

$$\Gamma_\parallel(0, H) \sim \mathrm{Re}\, \Gamma_{+-}(H) \sim \mathrm{Im}\, \Gamma_{+-}(H) \sim \frac{\omega_0^2}{T_c} \left(\frac{T_c}{g\mu H} \right)^{3/5}. \tag{50}$$

It can be seen from this expression that $g\mu H \gg \Gamma_\parallel(0, H)$, $|\Gamma_{+-}(H)|$, and that, as the field intensity increases, the resonance line narrows down and the longitudinal absorption decreases. It is not difficult to show that in this case $\Gamma_\lambda(\tau, H)$ can be expanded in a regular fashion in a series in powers of the small parameter $\tau(T_c/g\mu H)^{3/5}$.

Let us discuss in greater detail the behavior of $\Gamma_\lambda(\tau, H)$ in the entire transition region from weak to strong fields, excluding the above-considered case of very weak fields, i.e., assuming that $g\mu H \gg |\Gamma_\lambda(\tau, H)|$. Here we shall, in considering $\chi_{+-}(\omega)$, be interested in the neighborhood of the resonance frequency $\omega \approx \omega_r \approx g\mu H$; in considering $\chi_\parallel(\omega)$, in the frequencies $\omega \ll \Omega_e(\tau, H)$, neglecting the slight frequency dispersion of $\Gamma_\lambda(\omega)$. Then, using perturbation theory in terms of ω_0 and the scaling hypothesis for the case of a nonzero field, we obtain for Γ_λ the expression

$$\Gamma_\lambda(\tau, H) = \frac{\omega_0^2}{T_c} \frac{1}{\tau} \gamma_\lambda \left(\frac{\tau}{h^{3/5}} \right), \quad h = \frac{g\mu H}{T_c}. \tag{51}$$

The imaginary and real parts of $\Gamma_{+-}(\tau, H)$ determine respectively the resonance-frequency shift and the line width. The function $\Gamma_\parallel(\tau, H)$ is real and determines the longitudinal absorption. In fact, above we found the asymptotic forms of $\gamma_\lambda(x)$ for $x \ll 1$ and $x \gg 1$.

Let us discuss the behavior of the damping constant $\mathrm{Re}\Gamma_\lambda$ as a function of τ in a fixed field, bearing in mind the experimental investigations reported in Refs. 8 and 11. It is clear that the damping first increases with decreasing τ, and then this growth is restrained by the field. And since there is one characteristic variation scale $\tau \sim h^{3/5}$, it is natural to suppose that the damping either remains a monotonically increasing function right down to $\tau = 0$, or has a maximum at $\tau \sim h^{3/5}$. We cannot say which of these possibilities is realized, since it is not possible to compute the sign of the derivative

$$\frac{\partial}{\partial \tau} \mathrm{Re}\, \Gamma_\lambda(\tau)$$

at $\tau = 0$. The most interesting situation is the one in which the behavior of the damping constant is nonmonotonic, especially as we can assume on the basis of the experimental investigations reported in Refs. 7, 8, and 11 that it is precisely this situation that is realized. The necessary condition for a maximum can be written in the form

$$\mathrm{Re}\, \gamma_\lambda(x) - x\, \mathrm{Re}\, \gamma_\lambda'(x) = 0, \quad x = \tau h^{-3/5}. \tag{52}$$

On account of (46) and (49), for $x \gg 1$

$$\mathrm{Re}\, \gamma_\lambda(x) = c_{0\lambda} - c_{1\lambda} x^{-5/4},$$

where $c_{1\lambda} > 0$. If, on the other hand, $x \ll 1$, then, as follows from (50), $\mathrm{Re}\gamma_\lambda(x) \sim x$. It is clear that, since in these limiting cases, $\mathrm{Re}\gamma_\lambda' > 0$, while $\mathrm{Re}\gamma_\lambda(x)$ is positive, this equation can have a solution at $\tau > 0$.

The position of the maximum of the damping constant and the behavior of this constant at the maximum as τ and h are varied are determined in the following manner:

$$\tau h^{-3/5} = \mathrm{const}, \quad \mathrm{Re}\, \Gamma_{\lambda\, max} \propto h^{-3/5}. \tag{53}$$

The experimental verification of these two consequences would be of great interest.

Let us now briefly discuss the dipole region, where $4\pi\chi_{0\lambda} \gg 1$. The behavior of Γ_λ in weak and strong fields in this region has already been considered in the preceding section [see (31), (32)]. Therefore, here we shall limit ourselves to making only some additional remarks. First of all let us note that in the dipole region, in contrast to the exchange region, for $q \ll \varkappa$ the dependence of the dynamic susceptibility on q is unimportant. As a result, the regular expansions (31), (3 2) for the Γ_λ are valid in a weak field.

In the dipole region, even in a strong field, the characteristic energy $\Omega_d(H) > g\mu H$. At the same time

$$\mathrm{Re}\,\Gamma_{+-} \sim \mathrm{Im}\,\Gamma_{+-} \sim \Omega_d(H).$$

Therefore, here the resonance frequency is lower than, or of the order of, the damping constant. Let us emphasize again that we are talking about the susceptibility of the material, and not about the directly measurable susceptibility of a body. Finally, the dynamic part of the susceptibility in the dipole region cannot have the Lorentz form both for $H=0$ (Ref. 3) and for $H \neq 0$. In other words, the dispersion of $\Gamma_\lambda(\omega)$ should be substantial. Such dispersion in zero field has been experimentally observed.[22]

In conclusion of this section, let us, with magnetic-resonance experiments in mind, express the measurable susceptibility $\tilde{\chi}_{\alpha\beta}$ of a body in terms of the susceptibility $\chi_{\alpha\beta}$ of the material. Let us consider a body having the shape of an ellipsoid with the principal axes along the coordinate axes, and let us orient the external constant field along the z axis. Then, since $\chi_{zx} = \chi_{xz} = \chi_{yz} = \chi_{zy} = 0$, we have for the longitudinal part $\tilde{\chi}_{zz}$ and the internal field from (25) and (27) the expressions

$$\tilde{\chi}_{zz}(\omega) = \chi_{zz}(\omega)/[1+4\pi\chi_{zz}(\omega)N_z], \tag{54}$$

$$H = H_{ex}/(1+4\pi\chi_{0\perp}N_z). \tag{55}$$

Further, taking account of the fact that $\chi_{xx}(\omega) = \chi_{yy}(\omega)$ and $\chi_{xy}(\omega) = -\chi_{yx}(\omega)$, we easily obtain the transverse elements of the tensor:

$$\begin{gathered}
\tilde{\chi}_{xx}(\omega) = D^{-1}[\chi_{xx}(\omega)(1+4\pi N_y\chi_{xx}(\omega)) + 4\pi N_y\chi_{xy}^2(\omega)],\\
\tilde{\chi}_{yy}(\omega) = D^{-1}[\chi_{yy}(\omega)(1+4\pi N_x\chi_{yy}(\omega)) + 4\pi N_x\chi_{xy}^2(\omega)],\\
\tilde{\chi}_{xy}(\omega) = -\tilde{\chi}_{yx}(\omega) = \chi_{xy}(\omega)/D,\\
D = 1+4\pi(N_x+N_y)\chi_{xx}(\omega) + (4\pi)^2 N_x N_y(\chi_{xy}^2(\omega)+\chi_{xx}^2(\omega)).
\end{gathered} \tag{56}$$

Taking account of the fact $2\chi_{xx} = \chi_{+-} + \chi_{-+}$, $2i\chi_{xy} = \chi_{-+} - \chi_{+-}$, and using the formula (35), we obtain

$$\begin{aligned}
D = {} & [(\omega+i\Gamma_\perp)^2 - (\tilde{g}\mu H)^2]^{-1}\{(\omega+i\Gamma_\perp)^2 - (\tilde{g}\mu H)^2 - 4\pi\chi_{0\perp}(N_x+N_y)[(\tilde{g}\mu H)^2\\
& + \Gamma_\perp^2 - i\Gamma_\perp\omega] - (4\pi\chi_{0\perp})^2 N_x N_y[(\tilde{g}\mu H)^2 + \Gamma_\perp^2]\}.
\end{aligned} \tag{57}$$

Here $\Gamma_\perp = \mathrm{Re}\,\Gamma_{+-}$, and we have for convenience introduced the renormalized g-factor value:

$$\tilde{g} = \omega_r/\mu H = g - \Delta\omega_r/\mu H. \tag{58}$$

From (57) it is easy to find the resonance frequency $\tilde{\omega}_r$ and the damping constant $\tilde{\Gamma}_\perp$:

$$\begin{gathered}
\tilde{\omega}_r = \left\{(\tilde{g}\mu H)^2 \frac{\chi_{0\perp}^2}{\chi_{0xx}\hat{\chi}_{0yy}} - (\Gamma_\perp\chi_{0\perp})^2 \frac{N_x - N_y}{2}\right\}^{1/2},\\
\tilde{\Gamma}_\perp = \Gamma_\perp(1+2\pi\chi_{0\perp}(N_x+N_y)).
\end{gathered} \tag{59}$$

These expressions for the ellipsoid of revolution get greatly simplified when $N_x = N_y = N_\perp$:

$$\tilde{\omega}_r = \tilde{g}\mu H_{ex}\frac{1+4\pi N_\perp\chi_{0\perp}}{1+4\pi N_z\chi_{0\perp}}, \quad \tilde{\Gamma}_\perp = \Gamma_\perp(1+4\pi\chi_{0\perp}N_\perp). \tag{60}$$

The expressions (59) and (60) differ from those obtained in Ref. 24 in that they contain the renormalized g factor, $\tilde{g}$, which, as we have seen, depends on the temperature and the magnetic field.

As has already been repeatedly noted, the quantity $\Gamma_\perp$, as well as $\tilde{g}$, generally speaking depends on the frequency. In the exchange region this dispersion is slight, and can be neglected in the first approximation. At the same time, in the dipole region the dispersion of Γ_{+-} is important, and therefore these expressions can be used only to make estimates.

Finally, it is useful for practical purposes to discuss briefly the connection between the external and internal fields in a number of limiting cases. In the exchange region $H_{ex} \approx H$. In the dipole region, if the field H is weak and $4\pi N_z\chi_{0\perp} \gg 1$, then

$$H \sim H_{ex}(4\pi N_z\chi_{0\perp}(\varkappa))^{-1} \ll H_{ex}.$$

The condition for the external field to be weak then has the form

$$g\mu H_{ex} \ll 4\pi N_z\chi_{0\perp}(\varkappa)\Omega_e(\varkappa) \sim N_z(T_c\omega_0)^{1/2}(\varkappa a)^{1/2}. \tag{61}$$

On the other hand, in a strong field in the dipole region, if $4\pi N_z\chi_{0\perp}(H) \gg 1$, we have

$$H \sim H_{ex}\left(\frac{g\mu H_{ex}}{\omega_0 N_z}\right)^4 \frac{T_e}{\omega_0 N_z}. \tag{62}$$

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