Stability of the nonequilibrium states of a superconductor with a finite difference between the populations of the electron- and hole-like spectral branches

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The stability of the nonequilibrium states of a superconductor with a finite difference between the populations of the electron- and hole-like spectral branches is investigated. It is shown that an instability similar to the Cooper instability of a normal metal arises at a sufficiently large value of the imbalance. This eliminates the imbalance within quantum-mechanical (nonkinetic) time periods. The consistency of the allowance for the imbalance in the nonequilibrium Ginzburg-Landau equations is discussed.

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1. INTRODUCTION

The purpose of the present communication is to show that the nonequilibrium state of a superconductor is unstable against order-parameter fluctuations when the imbalance between the populations of the electron- and hole-like branches of the excitation spectrum is sufficiently large. The physical cause of the instability consists in the following.

Let an imbalance between the populations of the electron- and hole-like spectral branches be produced in a superconductor. For simplicity, we limit ourselves to the spatially homogeneous case with $T_c - T \ll T_c$. (Here T_c and T are respectively the superconducting-transition and sample temperatures.) As shown by Tinkham,[1] in such a situation the imbalance relaxation time τ_Q is much longer than the energy-relaxation time τ_ε of the quasiparticles:

$$\tau_\varepsilon \approx \Theta_D^2/T^3; \quad \tau_Q \approx \tau_\varepsilon T/\Delta \gg \tau_\varepsilon.$$

Here Θ_D is the Debye energy and Δ is the energy gap in the superconductor. As a result, there is established in the superconductor over a time period of the order τ_ε a quasiparticle distribution function[1-4] described when $\xi_p \gg \Delta$ by the formula

$$n_p = \left(\exp\frac{\varepsilon_p - \nu \operatorname{sign}\tilde{\xi}_p}{T} + 1\right)^{-1}; \tag{1}$$

$$\tilde{\xi}_p = \xi_p + \Phi = p^2/2m - \varepsilon_F + \Phi, \quad \varepsilon_p = (\tilde{\xi}_p^2 + \Delta^2)^{1/2},$$

where $\Phi = \frac{1}{2}\pi\dot{\chi} + e\varphi$ is a gauge-invariant scalar potential, χ being the phase of the order parameter and φ the electrostatic potential.

It follows from the quasineutrality condition that the chemical potential ν of the quasiparticles is equal to Φ when small corrections of the order of $\Delta/T_0 \ll 1$ are neglected. By substituting (1) in the self-consistency equation, we can verify[2] that the presence of the imbalance leads to a decrease of the gap:

$$\Delta^2(\Phi) = \Delta^2(0) - 2\Phi^2. \tag{2}$$

From this it follows that the gap should vanish when $\Phi = \Phi_1 = \Delta(0)\sqrt{2}$. On the other hand, when $\Delta = 0$ and $\Phi = \Phi_1 \neq 0$, we in fact have a normal metal: Φ_1 has the meaning of a shift of the excitation energy reference level, and should not enter into any observable quantities. But a normal metal at $T < T_c$ is unstable against Cooper pairing; it is important that the increment of this instability be finite at $\Delta = 0$. Therefore, the instability sets in earlier, at those values of Φ at which Δ is finite [the instability point is that value of Φ ($=\Phi_c$) at which the instability growth rate vanishes].

We shall see that the development of the instability leads to restructuring of the spectrum of the superconductor and, as a result, to elimination of the imbalance. The characteristic time of this restructuring turns out to be much shorter than τ_Q. The foregoing arguments imply that: (1) the imbalance of the populations of the branches of the energy spectrum cannot completely suppress the superconducting gap; (2) at some critical value, the imbalance relaxes rapidly on account of quantum-mechanical (nonkinetic) mechanisms. The purpose of the calculation below is to verify these ideas quantitatively.

2. DISPERSION EQUATION FOR THE ORDER-PARAMETER OSCILLATIONS

We wish to investigate the dynamics of the order parameter of a superconductor over time periods much shorter than τ_ε. For the case of distribution functions that are even in $\tilde{\xi}_p$ (i.e., in the absence of an imbalance), this problem has been considered by Aronov and Gurevich[5] through the investigation of the poles of the two-particle Green function. It will be convenient for us to use another method, which is similar to the method used by Volkov and Kogan[6] to analyze the equilibrium case.

We shall, in the spatially homogeneous situation of interest to us, proceed from the system of equations for the Green functions

$$g(\mathbf{p}, t) = -i\langle [a_{p\uparrow}(t), a_{p\uparrow}^+(t)]\rangle, \tag{3}$$

$$f(\mathbf{p}, t) = -i\langle [a_{p\uparrow}(t), a_{-p\downarrow}(t)]\rangle, \tag{4}$$

in the form[6]

$$i\frac{\partial g}{\partial t} + \tilde{\Delta} f^* + \tilde{\Delta}^* f = 0, \quad \operatorname{Re} g = 0, \tag{5}$$

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$$\left(i\frac{\partial}{\partial t}-2\xi_p\right)f+2\tilde{\Delta}ge^{2e\varphi t}=0, \tag{6}$$

$$\tilde{\Delta}=i\frac{\lambda}{2}\int_{-\theta_D}^{\theta_D}d\xi_p f. \tag{7}$$

Here $\tilde{\Delta}$ is the complex order parameter and λ is the electron–electron interaction contant ($\hbar\equiv1$). Equation (7) is the self-consistency equation.

Equation (6) differs somewhat in form from the corresponding equation given in Volkov and Kogan's paper.[6] We assume that the potential φ, which does not depend on the space coordinates and the time, is included in the chemical potential $\varepsilon_F+e\varphi$ of the electrons. Therefore the term $2e\varphi$ does not occur in the round brackets, but the factor $e^{2e\varphi t}$ enters in the second term instead.

Let us, as usual, linearize Eqs. (5)–(7). We seek the solution in the form

$$g(\xi_p,t)=g_0(\xi_p)+g_1(\xi_p,t),\qquad f(\xi_p,t)=(f_0(\xi_p)+f_1(\xi_p,t))e^{2i\Phi t},$$
$$\tilde{\Delta}(t)=(\Delta+\Delta_1(t))e^{2i\Phi t},$$
$$g_0(\xi_p)=-i\frac{\tilde{\xi}_p}{\varepsilon_p}(1-2n_p),\quad f_0(\xi_p)=-i\frac{\Delta}{\varepsilon_p}(1-2n_p). \tag{8}$$

Here g_0, $f_0e^{2i\Phi t}$, and $\Delta e^{2i\Phi t}$ are the values of the Green functions and the complex order parameter in the steady state characterized by the quasiparticle distribution function n_p. The quantity Φ is connected with n_p by the neutrality condition[2,9]:

$$\Phi=\int\frac{\tilde{\xi}_p}{\varepsilon_p}n_p\,d\xi_p.$$

Assuming that all the contributions are proportional to $e^{i\omega t}$, we obtain from the system (5)–(7) the following dispersion equation determining the frequency of the spatially homogeneous oscillations in the system, given the quasiparticle distribution function:

$$\left(\gamma+\frac{\lambda}{2}\int_{-\theta_D}^{\theta_D}d\xi_p\frac{\omega^2-4\Delta^2}{\omega^2-4\varepsilon_p^2}\frac{1-2n_p}{\varepsilon_p}\right)\left(\gamma+\frac{\lambda}{2}\int_{-\theta_D}^{\theta_D}d\xi_p\frac{\omega^2}{\omega^2-4\varepsilon_p^2}\frac{1-2n_p}{\varepsilon_p}\right)$$
$$=\left(\frac{\lambda}{2}\int_{-\theta_D}^{\theta_D}d\xi_p\frac{2\tilde{\xi}_p\omega}{\omega^2-4\varepsilon_p^2}\frac{1-2n_p}{\varepsilon_p}\right)^2, \tag{9}$$
$$\gamma=1-\frac{\lambda}{2}\int_{-\theta_D}^{\theta_D}d\xi_p\frac{1-2n_p}{\varepsilon_p}.$$

Let us apply this equation to the case $\Delta=0$, $\Phi\neq0$ ($T<T_c$) discussed in the Introduction. From (9) we have

$$1-\frac{\lambda}{2}\int_{-\theta_D}^{\theta_D}d\xi_p\frac{1-2n_p}{\tilde{\xi}_p\pm\omega/2}\operatorname{sign}\tilde{\xi}_p=0\ . \tag{10}$$

Using the distribution function (1) with $\Delta=0$, we obtain from (10) the relations

$$\operatorname{Re}\omega_N=\mp2\Phi, \tag{11a}$$
$$|\operatorname{Im}\omega_N|=\Delta^2(0)/10T_c. \tag{11b}$$

This solution describes an instability with a finite growth rate $\operatorname{Im}\omega_N>0$, the growing order-parameter fluctuations $\Delta_1e^{2\Phi t}$ being, on account of (11a), independent of Φ.

To find the point where the instability appears (i.e., the point where $\operatorname{Im}\omega=0$), we must investigate Eq. (9) in the superconducting state. In this case the relation between Δ and n_p is determined by the self-consistency equation, which in our notation has the form $\gamma=0$. We then have from (9)

$$(\omega^2-4\Delta^2)\left[\int_{-\theta_D}^{\theta_D}d\xi_p\frac{1-2n_p}{\varepsilon_p(\omega^2-4\varepsilon_p^2)}\right]^2=\left[\int_{-\theta_D}^{\theta_D}d\xi_p\frac{2\tilde{\xi}_p(1-2n_p)}{\varepsilon_p(\omega^2-4\varepsilon_p^2)}\right]^2. \tag{12}$$

If the distribution function n_p is even in $\tilde{\xi}_p$ the right-hand side of Eq. (12) vanishes, and we arrive at the dispersion equation obtained by Aronov and Gurevich.[5]

Using again the self-consistency equation $\gamma=0$, we can reduce Eq. (12) to a form most convenient for us and applicable to both the superconductor and the normal metal:

$$1-\frac{\lambda}{2}\int_{-\theta_D}^{\theta_D}d\xi_p\frac{1-2n_p}{\tilde{\xi}_p\pm[(\omega/2)^2-\Delta^2]^{1/2}}\frac{\tilde{\xi}_p}{\varepsilon_p}=0. \tag{13}$$

It can be seen from the dispersion equation (13) that the characteristic values ξ_p are greater than or of the order of $\max(\omega,\Delta)$. Then the characteristic values of $\operatorname{Re}\omega\sim\Delta(0)$, i.e., much greater than $\Delta(\Phi)$. Therefore the region of small values of $\xi_p\sim\Delta$ is unimportant, and we can use for n_p the function (1). Substituting (1) in (13) and expanding in powers of the small parameters $\Delta/T\ll1$ and $\Delta/T_c\ll1$, we obtain from (13) the relations[1)]

$$\operatorname{Re}\omega=\mp2\Phi,\quad|\operatorname{Im}\omega|=\frac{2}{\pi}((T_c-T)-\Delta(\Phi)). \tag{14}$$

It should be borne in mind that Δ and Φ are connected by the relation (2).

From (14) we obtain the instability point at which $\operatorname{Im}\omega=0$:

$$\Phi_c=\Phi_1-\frac{\Delta_c^2}{2^{1/2}\Delta(0)},\quad\Delta_c=\frac{\Delta^2(0)}{9T_c},\quad\Delta(0)=3.09[T_c(T_c-T)]^{1/2}. \tag{15}$$

Figure 1 shows the $\Delta(\Phi)$ dependence and the characteristic values of Φ and Δ.

Thus, we have found that instability sets in at $\Delta=\Delta_c>0$. Notice that the instability considered is of the same type as the normal Cooper instability that converts a normal metal into a superconductor at $T<T_c$.

The smallness of Δ_c and $\Phi_1-\Phi_c$ is due only to the specific nature of the $T_c-T\ll T_c$ case. Equations (9) and (10) are valid for an arbitrary distribution function. For $T\lesssim T_c$ the distribution function n_p no longer has the simple form (1), since no hierarchy of relaxation times exists in this case. Nevertheless, the gap width $\Delta(0)$

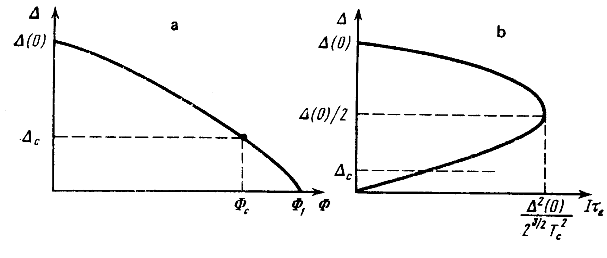

FIG. 1.

is the only characteristic energy scale of the problem. It can therefore be asserted that at $T \lesssim T_c$ all the phenomena will be qualitatively the same, and $\Phi_c \sim \Phi_1 \sim \Delta(0)$.

It is important to note that $\mathrm{Re}\,\omega_c = -2\Phi_c$, i.e., that the $\tilde{\Delta}$ fluctuations that build up are precisely those which correspond to the steady equilibrium state in the absence of an imbalance.

In conclusion let us, in the light of the results obtained, discuss the time variation of the order parameter of a superconductor in the presence of a finite imbalance. Here we focus our attention on the simplest and at the same time often encountered case of tunneling injection. [Let us note, however, that from our point of view Eqs. (9) and (12) and their corollaries are valid for spatially inhomogeneous systems as well if all the characteristic scales of the spatial variations are much greater than the coherence length ξ. Therefore, the results obtained above are applicable to a much broader class of systems.]

We begin with the elucidation of the conditions under which the distribution function (1) is formed in the steady state.[2)] We assume that the collisions of the quasiparticles with the phonons are responsible for the relaxation of the imbalance, and that $T_c - T \ll T_c$. As is well known, the quasiparticle–phonon collision integral includes two types of processes: 1) absorption (emission) of a phonon by a quasiparticle, the probability for which is proportional to

$$1+\frac{\tilde{\xi}_p\tilde{\xi}_{p'}-\Delta^2}{\varepsilon_p\varepsilon_{p'}},$$

and 2) quasiparticle-pair production (recombination) with the emission (absorption) of a phonon, the probability for which is proportional to

$$1-\frac{\tilde{\xi}_p\tilde{\xi}_{p'}-\Delta^2}{\varepsilon_p\varepsilon_{p'}},$$

where $\mathbf{p}' = \mathbf{p} + \hbar\mathbf{q}$, $\mathbf{q}$ being the phonon wave vector. It can be seen that, for $\Delta \ll \tilde{\xi}_p$, $\tilde{\xi}_{p'}$, the most probable of the processes of the type 1) are transitions within the same branch, i.e., the processes for which $\tilde{\xi}_p, \tilde{\xi}_{p'} > 0$, while the most probable of the processes of the type 2) are those for which $\tilde{\xi}_p\tilde{\xi}_{p'} < 0$. Therefore, the processes of the type 1) establish over a time period $\sim\tau_\varepsilon$ a distribution with fixed branch chemical potentials ν_+ and ν_-. The condition for the vanishing of that part of the collision integral which corresponds to the processes of the type 2) for $\Delta \ll \tilde{\xi}_p$, $\tilde{\xi}_{p'}$ then requires that $\nu_+ = -\nu_- = \nu$. From the quasineutrality condition it follows that $\nu = \Phi$. These arguments are inapplicable to the small region $\tilde{\xi}_p \lesssim \Delta$. It is precisely this region that is responsible for the relaxation of the imbalance, and it is precisely because of this that $\tau_Q \approx \tau_\varepsilon T/\Delta \gg \tau_\varepsilon$.

It is clear from the foregoing arguments that the steady state value of Φ is established by tunneling injection within a time τ_Q, and is equal to $\Phi = IT\tau_Q$ (I is the imbalance-injection rate). Substituting this expression for Φ and $\tau_Q \approx \tau_\varepsilon T/\Delta$ in Eq. (2), we obtain for Δ^2 a quadratic equation whose solution is (cf. Ref. 12)

$$\Delta^2 = {}^1/_2\{\Delta^2(0) \pm [\Delta^4(0) - 8T_c^4(I\tau_\varepsilon)^2]^{1/2}\}.$$

This solution is depicted in Fig. 1(b). For $I\tau < \Delta_\varepsilon^2(0)/2^{3/2}T_c^2$ the equation has two solutions, one of which is always stable, while the second is stable when $\Delta > \Delta_c = \Delta^2(0)/9T_c$. Under injection conditions corresponding to $I\tau_\varepsilon > \Delta^2(0)/2^{3/2}T_c^2$ the system possesses no steady states. For fixed $I > \Delta^2(0)/2^{3/2}T_c^2\tau_\varepsilon$, the imbalance increases monotonically until it attains a level equal to Φ_c, after which the instability begins to develop. As can be seen from the formulas given, the characteristic value of the instability increment for $\Phi > \Phi_c$ is of the order of $\mathrm{Im}\,\omega \sim \tau_\Delta^{-1} = \Delta^2(0)/9T_c$. This means that for time periods short compared to the kinetic times, the system strives to go over into the state in which $\Phi = 0$. As already indicated, this assertion follows from the fact that $\mathrm{Re}\,\omega = -2\Phi$.

To identify decisively the state that the superconductor will go into during the time period $\tau_\Delta \ll t \ll \tau_\varepsilon$, we need to find the nonlinear solutions to the system (5)–(7), which we do not as yet know how to do. Let us note that all the schemes known to us[7–10] for describing nonequilibrium phenomena in superconductors are based from the very beginning on the assumption that the state of the condensate is established over a time period $t_1 \sim \tau_\Delta$. Meanwhile, the nonlinear equations (5)–(7) could, in principle, have oscillating solutions[3)] (which would die down over time periods equal to τ_ε). This circumstance, in our opinion, limits the applicability of the existing approaches to the solution of problems of the type considered.

Nevertheless, at low supercriticality levels we can qualitatively follow the dynamics of Δ and Φ. Indeed, the time t_1 does not in any case exceed τ_ε. But if the supercriticality is low, the characteristic time of the variation of the potential Φ turns out to be much longer than τ_ε. As a result, there arise Δ and Φ oscillations, which are schematically depicted in Fig. 2. These oscillations can be experimentally observed, e.g., in $\Delta(t)$ and $\Phi(t)$ measurements. Let us note that the characteristic frequencies of these oscillations are of the order of τ_ε^{-1}, i.e., fairly high.

We now discuss the experimental situation with tunneling injection as applied to our discussion. We have assumed equal quasiparticle and thermostat temperatures (i.e., the absence of quasiparticle warm-up). In actual experiments warm-up always occurs to one or another degree. But it is clear that if the warm-up does not suppress the superconductivity completely, all the qualitative phenomena described above are preserved. Furthermore, we have assumed that the im-

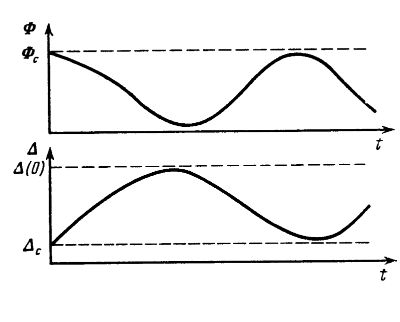

FIG. 2.

balance is relaxed by the phonons. But in the case of tunneling injection there can arise competing imbalance relaxation mechanisms, e.g., scattering by the impurities under conditions in which the condensate velocity v_s is finite. It seems to us that allowance for these mechanisms does not change the above-discussed qualitative picture.

The problem of tunneling injection in the vicinity of T_c has been considered with allowance for the imbalance by Ivlev,[11] who predicts the instability of superconductors against spatially inhomogeneous fluctuations with a characteristic scale of the order of the diffusion length L_D. Under certain conditions (sufficiently high barrier transparency), this instability can arise sooner, i.e., at $I < \Delta_0^2/2^{3/2}T_c^2\tau_\varepsilon$. In this case the results can be applied to a small region of the superconductor with dimensions much smaller than L_D; they indicate that the gap cannot be made to vanish in the inhomogeneous situation also when the characteristic inhomogeneity dimension is greater than the coherence length ξ. Furthermore, the phenomena described by Ivlev[11] do not occur when $eV \gg \Delta$, where V is the potential difference across the tunnel junction. Let us note that an inhomogeneous state can arise in isolated, sufficiently long tunnel junctions as a result of the flow of a current in the plane of the junction.[12,13] We think that this inhomogeneity will not change the described qualitative picture.

In conclusion, we note that the instability considered can apparently manifest itself not only in the course of tunneling injections, but also in a much broader class of nonequilibrium states for a superconductor with a large imbalance between the populations of the electron- and hole-like spectral branches. The spatial homogeneity condition (i.e., the requirement that the characteristic scale of the inhomogeneity be much greater than the coherence length ξ) is the only serious limitation. In particular, we doubt the applicability of the nonstationary Ginzburg–Landau equations to a situation with a large imbalance. The point is that these equations, which describe the Cooper instability of a normal metal at $T < T_c$, do not reveal the instability of a normal metal at $\Phi = \Phi_1$, $T < T_c$. We believe that the imbalance is inconsistently taken into account in equations of this type.

We are grateful to A.G. Aronov, B.I. Ivlev, and V.S. Shumeĭko for useful discussions.

1) The condition Re $\omega = \mp 2\Phi$ is a consequence of the specific form of the distribution function (1), and may not hold in the general case. Separating the imaginary part of Eq. (13) at the instability point (Im$\omega = 0$), we obtain an equation for Re ω:

$$n(\tilde{\xi}_p = \mp[{}^1/_4(\mathrm{Re}\,\omega)^2 - \Delta^2]^{1/2}) = {}^1/_2,$$

which yields the indicated condition for the distribution function (1). A similar equation has been obtained by Aronov and Gurevich[5] in the case of even $n_\mathbf{p}$.

2) The distribution function (1) is derived in a number of papers.[1-4] We expound here the known qualitative picture to facilitate the understanding of the subsequent discussions.

3) We are grateful to V. S. Shumeĭko for demonstrating to us the existence of a solution of this type.

1. M. Tinkham, Phys. Rev. B **6**, 1747 (1972).
2. V. P. Galaĭko, Zh. Eksp. Teor. Fiz. **68**, 223 (1975) [Sov. Phys. JETP **41**, 108 (1975)].
3. A. Schmid and G. Schön, J. Low Temp. Phys. **20**, 207 (1975).
4. S. N. Artemenko and A. F. Volkov, Zh. Eksp. Teor. Fiz. **70**, 1051 (1976); **72**, 1018 (1977) [Sov. Phys. JETP **43**, 548 (1976); **45**, 533 (1977)].
5. A. G. Aronov and V. L. Gurevich, Zh. Eksp. Teor. Fiz. **65**, 1111 (1973) [Sov. Phys. JETP **38**, 550 (1974)].
6. A. F. Volkov and A. M. Kogan. Zh. Eksp. Teor. Fiz. **65**, 2038 (1973) [Sov. Phys. JETP **38**, 1018 (1974)].
7. L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
8. G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].
9. A. G. Aronov and V. L. Gurevich, Fiz. Tverd. Tela (Leningrad) **16**, 2656 (1974) [Sov. Phys. Solid State **16**, 1722 (1974)].
10. A. M. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)].
11. B. I. Ivlev, Zh. Eksp. Teor. Fiz. **75**, 1771 (1978) [Sov. Phys. JETP **48**, 893 (1978)].
12. B. I. Ivlev, Fiz. Tverd. Tela (Leningrad) **22**, 436 (1980) [Sov. Phys. Solid State **22**, 255 (1980)].
13. I. Iguchi, Phys. Rev. B **16**, 1954 (1977); J. Low Temp. Phys. **33**, 439 (1978).

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