Point-contact spectroscopy of electron-phonon coupling in metals with a small electron mean free path

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It is shown that for an arbitrary electron elastic scattering length l_i and for $d < (l_i l_\varepsilon)^{1/2}$ (d is the point contact diameter and l_ε is the electron-phonon relaxation length) the second derivative of the current with respect to voltage at the point contact is proportional to the electron-phonon coupling (EPC) transport function $G^{(\omega)} = \hat{\alpha}^2(\omega)F(\omega)$, which differs from the isotropic EPC function $g(\omega) = \alpha^2(\omega)F(\omega)$ (Éliashberg function) by the presence of a K-factor. The mean value of the latter is $\langle K\rangle = \langle\alpha^2\rangle/\alpha^2 = 0.589 l_i/d$ in the dirty limit $l_i \ll d$, whereas for the model with an opening in a pure metal ($l_i \gg d$) we have $\langle K\rangle = 0.25$. The results signifies that point-contact spectroscopy should be feasible in a metal with an arbitrarily small elastic scattering length. The magntiude of the K-factor is expressed in terms of the probability for scattering of electrons with a momentum $\mathbf{p}$ at a point $\mathbf{r}$ [Eq. (2.29)] and for arbitrary values of the Knudsen parameter l_i/d.

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1. INTRODUCTION

The nonlinear dependence of the current and the voltage in point contacts of normal metals[1,2] makes it possible to reconstruct the electron-phonon coupling (EPC) function of the metal $G(\omega) = \hat{\alpha}^2(\omega)F(\omega)$, where $F(\omega)$ is the phonon state-density function and $\alpha^2(\omega)$ is the squared matrix element, averaged over the Fermi surface, of the electron-phonon coupling. As shown in Ref. 3, the d^2I/dV^2 is proportional to the function $G(\omega)$ for pure single-crystal point contacts, and the definition of $G(\omega)$ contains the K-factor that characterizes the probability that the electron will pass through the opening between the metals.

According to the theory,[3] for a round opening[1)]

$$\frac{1}{R}\frac{dR}{dV} = \frac{16}{3\pi}\frac{ed}{v_F}\int_0^\infty \frac{d\omega}{T} G(\omega) S\left(\frac{\hbar\omega - eV}{T}\right), \tag{1.1}$$

$$G(\omega) = \sum_\alpha \int \frac{dS_\mathbf{p}}{v_\perp} \int \frac{dS_{\mathbf{p}'}}{v_\perp'(2\pi\hbar)^3} W^\alpha_{\mathbf{p}-\mathbf{p}'} K(\mathbf{p},\mathbf{p}')\delta(\omega - \omega^\alpha_{\mathbf{p}-\mathbf{p}'}) \Big/ \int \frac{dS_\mathbf{p}}{v_\perp} = \hat{\alpha}^2(\omega)F(\omega), \tag{1.2}$$

where $W^\alpha_\mathbf{p}$ is the squared modulus of the matrix element of the interaction of the electrons with the branch α of the phonon spectrum,

$$S(x) = \frac{d^2}{dx^2}\frac{x}{e^x - 1}, \tag{1.3}$$

$$K(\mathbf{p},\mathbf{p}') = K_0(\mathbf{n},\mathbf{n}') = \frac{|n_z n_z'|\theta(-n_z n_z')}{|n_z\mathbf{n}' - n_z'\mathbf{n}|}, \quad \mathbf{n} = \frac{\mathbf{p}}{p_F}, \quad \mathbf{n}' = \frac{\mathbf{p}'}{p_F}, \tag{1.4}$$

$F(\omega)$ is the state density of the phonons, $\hat{\alpha}^2(\omega)$ differs from the square matrix element $\alpha^2(\omega)$ averaged over the Fermi surface in that Eq. (1.2) contains the function $K(\mathbf{p},\mathbf{p}')$. The known function $g(\omega) = \alpha^2(\omega)F(\omega)$ [expression (1.2) without the K-factor] is the isotropic Éliashberg EPC function. In contrast to the latter, $G(\omega)$ depends on the direction in the crystal and makes it possible to study the anisotropy of its phonon spectrum.[4] At absolute zero temperature, the function (1.1) takes the form

$$\frac{1}{R}\frac{dR}{dV} = \frac{16ed}{3\pi v_F\hbar} G\left(\frac{eV}{\hbar}\right). \tag{1.5}$$

Here d is the orifice diameter, $R = dV/dI$ is the dynamic resistance of the contact. The foregoing expressions are valid if the impurity scattering length is large compared with the opening diameter. It will be shown below that they remain in force [with the form of $K(\mathbf{p},\mathbf{p}')$ modified] also in a more general case.

Verkin *et al.*[5] have shown that, depending on the ratio of the contact diameter and the elastic (l_i) and inelastic (l_ε) electron scattering lengths, three spectroscopic regimes can be realized: ballistic ($d \ll l_i, l_\varepsilon$), diffusion [$l_i \ll d \ll (l_i l_\varepsilon)^{1/2}$], and thermal ($d \gg l_i, l_\varepsilon$). A theory of the thermal regime is also presented in that reference. The diffusion regime was considered in a preceding paper[6] where it was shown that the form of the derivative $R^{-1}dR/dV$ describes as before the spectrum of the electron-phonon coupling. The treatment in Ref. 6, however, was based on a qualitative model of a channel with a specific carrier scattering law. It was noted that the maximum of the phonon state density corresponds to maxima of the derivative of the differential resistance with respect to voltage. Thus, Ref. 6 led to the important possibility of studying, with the aid of point-contact spectroscopy,[4] not only perfect single crystals, but also of alloys of strongly deformed or amorphous metals. An experimental study of point-contact centers in alloys was carried out in Ref. 7 and confirmed this conclusion.

The purpose of the present article is to develop a consistent theory of point-contact spectroscopy in the diffusion regime, i.e., in metals with finite elastic-scattering length in the limit

$$d \ll \lambda, \tag{1.6}$$

where $\lambda = (l_i l_\varepsilon)^{1/2}$ is the electron "cooling" length, i.e., the length over which the energy acquired by the electron from the field is consumed by the excitation of the phonon spectrum. In contrast to the preceding paper,[6] where a model collision integral was used, we consider in the present paper a more general form of electron elastic scattering, both from point defects (§2.1) and from delocalized inhomogeneities due to random elas-

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tic-deformation fields (§2.2). Just as in the preceding investigations[3,6] we neglect the capture (reabsorption) of the nonequilibrium phonons in the point contact, which produce according to Ref. 8 a background for the function $R^{-1}dR(V)/dV$, but also multiphonon processes that lead to replicas of the spectrum at multiple frequencies.[9] An expression that generalizes the results of the theory of Ref. 6 is obtained for the K-factor. This expression is valid for point contact in the form of a long impurity-containing channel which connects pure metallic conductors. We consider in the present paper the electric conductivity of point contacts of varying cross sections, so that the transition from the model with a round opening to the model of a long channel can be tracked (§§3, 4). An interpolation procedure is proposed that makes it possible, if the condition $d \ll \lambda$ is satisfied, to calculate the elastic and inelastic components of the resistance of the point contact at arbitrary values of the Knudsen parameter l_i/d (§4). It is also shown that in the diffusion limit the average value of the K-factor can be represented in universal form that is valid for a multicontact system of arbitrary geometry (§3).

2. NONLINEAR ELECTRIC CONDUCTIVITY OF MICROCONTACT IN THE PRESENCE OF SCATTERERS (GENERAL RESULTS)

An analysis of the electric conductivity of a dirty point contact should include an examination of the interaction of the electrons with other scatterers. Depending on the nature of these scatterers one can consider the interaction of electrons with point defects of the structure and with impurities, or else their motion in stochastic elastic-deformation fields. In this section we consider both elastic-scattering mechanisms.

1. Electric conductivity of point contacts with pointlike elastic-scattering centers

The kinetic equation for the electron distribution function in the presence of elastic scattering by pointlike defects as well as inelastic scattering by phonons is of the form

$$\mathbf{v}\frac{\partial f_\mathbf{p}}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f_\mathbf{p}}{\partial \mathbf{p}} - I_i\{f_\mathbf{p}(\mathbf{r})\} = I_{ph}\{f_\mathbf{p}(\mathbf{r})\}, \tag{2.1}$$

where $I_i\{f_\mathbf{p}\}$ is the elastic-collision integral,

$$I_i\{f_\mathbf{p}\} = \int\limits_{\varepsilon_\mathbf{p}=\varepsilon_{\mathbf{p}'}} \frac{dS_{\mathbf{p}'}}{v_\perp'} W_{\mathbf{pp}'}^{(i)}\{f_{\mathbf{p}'} - f_\mathbf{p}\}, \tag{2.2}$$

and $I_{ph}\{f_\mathbf{p}\}$ is the integral of the collisions with the phonons:

$$I_{ph}\{f_\mathbf{p}\} = \sum_\alpha \int d\tau_\mathbf{q} W_\mathbf{q}^\alpha \{[f_{\mathbf{p}+\mathbf{q}}(1-f_\mathbf{p})(N_\mathbf{q}+1) - f_\mathbf{p}(1-f_{\mathbf{p}+\mathbf{q}})N_\mathbf{q}]\delta(\varepsilon_{\mathbf{p}+\mathbf{q}} - \varepsilon_\mathbf{p} - \omega_\mathbf{q}^\alpha) + [f_{\mathbf{p}+\mathbf{q}}(1-f_\mathbf{p})N_\mathbf{q} - f_\mathbf{p}(1-f_{\mathbf{p}+\mathbf{q}})(N_\mathbf{q}+1)]\delta(\varepsilon_{\mathbf{p}+\mathbf{q}} - \varepsilon_\mathbf{p} + \omega_\mathbf{q}^\alpha)\}, \tag{2.3}$$

$W_{\mathbf{pp}'}^{(i)}$ is the squared matrix element of the scattering from the impurity, and $W_\mathbf{q}^\alpha$ is the squared matrix element of the electron-phonon coupling. The electric field $e\mathbf{E} = -\nabla\Phi(\mathbf{r})$, determined from the electroneutrality condition, is expressed in terms of the voltage V applied to the point contact:

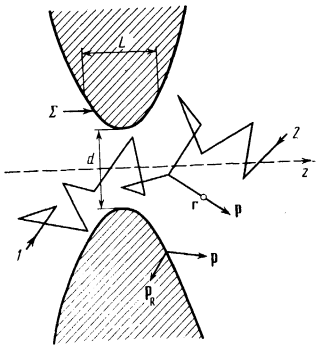

FIG. 1. Trajectory of electron in a point contact in the diffusion limit. 1—Trajectory arriving at the point r from $+\infty$; 2—trajectory of flight from $-\infty$.

$$\int d\tau_\mathbf{p}\{f_\mathbf{p} - n_F(\varepsilon_\mathbf{p})\} = 0; \quad \Phi(\mathbf{r}\to\infty) = \frac{eV}{2}\operatorname{sign} z, \tag{2.4}$$
$$n_F(\varepsilon) = [1+\exp(\varepsilon-\mu)]^{-1}.$$

Since the current spreads out with increasing distance from the point contact, the disequilibrium of the electron system decreases, so that the distribution function should satisfy the boundary condition

$$f_\mathbf{p}(\mathbf{r}\to\infty) = n_F(\varepsilon_\mathbf{p}). \tag{2.5}$$

The boundary condition in the surface Σ of the point contact (Fig. 1) is determined by the character of the electron reflection. Hereafter, however, we shall confine ourselves mainly to extremely dirty contacts, $l_i \ll d$. In this case the physical results should not depend on the character of the scattering of the electrons by the surface Σ. We therefore assume for simplicity the scattering by the surface Σ to be specular (the subscript R means specular reflection of the vector p in the plane tangent to the contact surface)[2]:

$$f_\mathbf{p}(\mathbf{r}\in\Sigma) = f_{\mathbf{p}_R}(\mathbf{r}\in\Sigma). \tag{2.6}$$

Point-contact spectroscopy is possible because of the relatively weak intensity of the electron-phonon relaxation, at which the inequalities $d,\ l_i \ll l_\varepsilon$ are satisfied. In this limit it is possible to formulate a perturbation theory in terms of the electron phonon collision integral $I_{ph}\{f_\mathbf{p}\}$. We represent the distribution function $f_\mathbf{p}$ in the form of a series in the parameter d/l_ε:

$$f_\mathbf{p}(\mathbf{r}) = f_\mathbf{p}^{(0)}(\mathbf{r}) + f_\mathbf{p}^{(1)}(\mathbf{r}) + \ldots. \tag{2.7}$$

For the functions $f_\mathbf{p}^{(0)}$ $f_\mathbf{p}^{(1)}$ we obtain from the kinetic equation (2.1) the chain of equations

$$\mathbf{v}\frac{\partial f_\mathbf{p}^{(0)}}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f_\mathbf{p}^{(0)}}{\partial \mathbf{p}} - I_i\{f_\mathbf{p}^{(0)}\} = 0, \tag{2.8}$$

$$\mathbf{v}\frac{\partial f_\mathbf{p}^{(1)}}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f_\mathbf{p}^{(1)}}{\partial \mathbf{p}} - I_i\{f_\mathbf{p}^{(1)}\} = I_{ph}\{f_\mathbf{p}^{(0)}\} - e\mathbf{E}_1\frac{\partial f_\mathbf{p}^{(0)}}{\partial \mathbf{p}}. \tag{2.9}$$

We have introduced here an increment $e\mathbf{E}_1 = -\Phi_1$ to the self-consistent field $e\mathbf{E}$; of the first order in the parameter d/l_ε.

The elastic and inelastic components of the point-contact current, I_0 and I_1 are given by the relations

$$\mathbf{I}_{0,1} = e\int dS \int d\tau_\mathbf{p} f_\mathbf{p}^{(0,1)}(\mathbf{r})\mathbf{v}. \tag{2.10}$$

It is convenient to represent the zeroth-approximation distribution function $f_p^{(0)}$ in the form[6]

$$f_p^{(0)}=\alpha_p(\mathbf{r})f_0^+ +[1-\alpha_p(\mathbf{r})]f_0^-, \tag{2.11}$$

$$f_0^\pm = n_F[\varepsilon_p+\Phi(\mathbf{r})\mp eV/2], \tag{2.12}$$

where $1-\alpha_p(\mathbf{r})$ is the probability that the electron, which has at the point r a momentum p, arrives there from $-\infty$ (the left half-space), whereas $\alpha_p(\mathbf{r})$ is the probability of arrival at this same point from $+\infty$ (the right half-space) (see Fig. 1).

The equation for $\alpha_p(\mathbf{r})$ can be obtained from the kinetic equation (2.9). If account is taken of the fact that the function $f_0^\pm$ depends only on the electron energy ε_p, as well as of the definition $e\mathbf{E}=-\nabla\Phi$, this equation can be written in the form

$$\mathbf{v}\frac{\partial\alpha_p}{\partial\mathbf{r}}+e\mathbf{E}\frac{\partial\alpha_p}{\partial\mathbf{p}}-I_i\{\alpha_p\}=0. \tag{2.13}$$

As will be shown below, $\alpha_p(\mathbf{r})$ varies in momentum space in a region of size $p\sim p_F$, while in coordinate space it varies over lengths $r\sim d$. The presence of the field term $e\mathbf{E}\partial\alpha_p/\partial\mathbf{p}$ in Eq. (2.13) leads therefore to small corrections for the function α_p, inasmuch as in the real case the inequality $eV/\varepsilon_F\ll 1$ is always satisfied. As a result, Eq. (2.13) can be rewritten in the form

$$\mathbf{v}\frac{\partial\alpha_p}{\partial\mathbf{r}}-I_i\{\alpha_p\}=0. \tag{2.14}$$

This does not include the electric field. The boundary conditions for α_p follow from the analogous relations (2.5) and (2.6) for the distribution function

$$\alpha_p(\mathbf{r}\to\infty)=\theta(z);\quad \alpha_p(\mathbf{r}\in\Sigma)=\alpha_{p_R}(\mathbf{r}\in\Sigma). \tag{2.15}$$

The inelastic increment $f_p^{(1)}$ to the electron distribution function is obtained from the solution of Eq. (2.9) with zero conditions at infinity

$$f_p^{(1)}(\mathbf{r}\to\infty)=0,\quad \Phi_1(\mathbf{r}\to\infty)=0. \tag{2.16}$$

At $eV/\varepsilon_F\ll 1$ the kinetic equation (2.9) can be represented in the form

$$\mathbf{v}\frac{\partial\chi_p}{\partial\mathbf{r}}-I_i\{\chi_p\}=I_{ph}\{f_p^{(0)}(\mathbf{r})\}, \tag{2.17}$$

$$f_p^{(1)}(\mathbf{r})=\Phi_1(\mathbf{r})\frac{\partial n_F}{\partial\varepsilon}+\chi_p(\mathbf{r}). \tag{2.18}$$

The boundary conditions for $\chi_p(\mathbf{r})$ are of the form

$$\chi_p(\mathbf{r}\to\infty)=0;\quad \chi_p(\mathbf{r}\in\Sigma)=\chi_{p\bar{R}}(\mathbf{r}\in\Sigma). \tag{2.19}$$

The solution of the boundary-value problem for the function χ_p can be represented with the aid of the Green's function

$$g_{pp'}(\mathbf{r},\mathbf{r}')=g_{-p'-p}(\mathbf{r}',\mathbf{r}),$$

which satisfies the relations

$$\mathbf{v}'\frac{\partial}{\partial\mathbf{r}'}g_{pp'}(\mathbf{r},\mathbf{r}')+I_i\{g_{pp'}(\mathbf{r},\mathbf{r}')\}=-\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{r}-\mathbf{r}'), \tag{2.20}$$

$$g_{pp'}(\mathbf{r},\mathbf{r}'\to\infty)=0,\quad g_{pp'}(\mathbf{r},\mathbf{r}'\in\Sigma)=g_{ppp'_R}(\mathbf{r},\mathbf{r}'\in\Sigma), \tag{2.21}$$

where

$$I_i\{g_{pp'}\}=\int\limits_{\varepsilon_{p'}=\varepsilon_{p''}}\frac{dS_{p''}}{v''_\perp}W^{(i)}_{p'-p''}\{g_{pp''}-g_{pp'}\}. \tag{2.22}$$

Using Eqs. (4.5) and (4.6) it is easy to obtain expressions for the inelastic increments to the distribution function $f_p^{(1)}$ and to the point-contact current I_1:

$$\chi_p(\mathbf{r})=\int d\mathbf{r}'\,d\mathbf{p}'\,g_{pp'}(\mathbf{r},\mathbf{r}')I_{ph}\{f_{p'}^{(0)}(\mathbf{r}')\}, \tag{2.23}$$

$$I_1=\frac{2e}{(2\pi\hbar)^3}\int d\mathbf{r}\,d\mathbf{p}\,G_p(\mathbf{r})I_{ph}\{f_p^{(0)}(\mathbf{r})\}, \tag{2.24}$$

where the introduced function $G_p(\mathbf{r})$ is of the form

$$G_p(\mathbf{r})=\int dS'\int d\mathbf{p}'v_z'g_{p'p}(\mathbf{r}',\mathbf{r}). \tag{2.25}$$

Starting from Eqs. (2.20)–(2.22) and relation (2.25), we find that the function $G_p(\mathbf{r})$ is a solution of the following boundary-value problem:

$$\mathbf{v}\frac{\partial G_p}{\partial\mathbf{r}}+I_i\{G_p(\mathbf{r})\}=-v_z\delta(z), \tag{2.26}$$

$$G_p(\mathbf{r}\to\infty)=0,\quad G_p(\mathbf{r}\in\Sigma)=G_{pR}(\mathbf{r}\in\Sigma). \tag{2.27}$$

Comparing these equations with the boundary-value problem (2.14) and (2.15) for the elastic electric conductivity, we easily find a connection between the Green's function $G_p(\mathbf{r})$ and the previously introduced function $\alpha_p(\mathbf{r})$:

$$G_p(\mathbf{r})=\alpha_{-p}(\mathbf{r})-\theta(z). \tag{2.28}$$

Equation (2.24) allows us to represent the second derivative of the current-voltage characteristic of the contact in the form (1.1)–(1.3). By using in this case Eq. (2.28) for the factor $K(\mathrm{p},\mathrm{p}')$, an expression can be obtained that is valid at an arbitrary value of the parameter l_i/d and for an arbitrary geometry of the contact. This expression is of the form

$$K(\mathrm{p},\mathrm{p}')=\frac{3\pi}{32}\frac{v_F}{d}\int d^3\mathbf{r}[\alpha_p(\mathbf{r})-\alpha_{p'}(\mathbf{r})][\alpha_{-p}(\mathbf{r})-\alpha_{-p'}(\mathbf{r})]\Big/\int d\mathrm{S}\langle v\alpha_p\rangle, \tag{2.29}$$

where S is the cross section of the contact. In the sections that follow we shall obtain specific equations for the K-factor under special assumptions concerning the geometry of the contact and its purity.

2. Electric conductivity of point contact in the presence of random elastic-deformation forces

Contamination of a point contact can be due in a real case also to the presence of scatterers that are not localized in space. Such "defects" are the fields of the elastic stresses and of dislocations in metals, or the charged impurities in semi-metals and semiconductors. In the present section we show that in this case the calculation of the point-contact spectrum can be reduced to a determination of the function α_p, which characterizes the elastic electric conductivity of the contact.

In the presence of extended defects, their influence cannot be accounted for by introducing an elastic-collision integral that is local in space. If the potential of the scatterers varies slowly in space in the scale of the de Broglie wavelength of the electron, their presence can be taken into account by introducing into the dynamic part of the Boltzmann equation the force $\mathbf{F}_i$ that acts on the conduction electrons. The kinetic equation then takes the form [cf. Eq. (2.1)]

$$\mathbf{v}\frac{\partial f_p}{\partial\mathbf{r}}+e\mathbf{E}\frac{\partial f_p}{\partial\mathbf{p}}+\mathbf{F}_i\frac{\partial f_p}{\partial\mathbf{p}}=I_{ph}\{f_p\}. \tag{2.30}$$

The influence of force $\mathbf{F}_i = -\nabla U_i$ reduces to a distortion of the trajectories of the electron motion, which can take a rather tangled form in the complicated potential of the scatterers.

In exactly the same manner as in the preceding section, we can formulate a perturbation theory in terms of the electron-phonon relaxation, and introduce distribution functions $f_{\mathbf{p}}^{(0)}$ and $f_{\mathbf{p}}^{(1)}$ of the zeroth and first order in the parameter d/l_ε, respectively. These functions are satisfied by Eqs. (2.8) and (2.9) if the elastic-collision integral $I_i\{f_{\mathbf{p}}\}$ in these equations is replaced by the field term $-\mathbf{F}_i \partial f_{\mathbf{p}}/\partial \mathbf{p}$. We shall find it convenient to introduce the transition probability density $P(\mathbf{r}, \mathbf{p}|\mathbf{r}', \mathbf{p}'; \tau)$ of an electron moving from the vicinity of a phase-space point $\mathbf{r}', \mathbf{p}'$ into the vicinity of the point $\mathbf{r}, \mathbf{p}$ within a time τ. Using this quantity, we easily rewrite Eqs. (2.8) and (2.9) in integral form

$$f_{\mathbf{p}}^{(0)}(\mathbf{r}) = \int d\mathbf{p}' d\mathbf{r}' P(\mathbf{p}, \mathbf{r}|\mathbf{p}', \mathbf{r}'; \tau) f_{\mathbf{p}'}^{(0)}(\mathbf{r}'), \tag{2.31}$$

$$f_{\mathbf{p}}^{(1)}(\mathbf{r}) = \Phi_1(\mathbf{r}) \frac{dn_F}{d\varepsilon} + \int d\mathbf{p}' d\mathbf{r}' \tilde{g}(-\mathbf{p}', \mathbf{r}'|-\mathbf{p}, \mathbf{r}) I_{ph}\{f_{\mathbf{p}'}^{(0)}(\mathbf{r}')\}, \tag{2.32}$$

$$\tilde{g}(\mathbf{p}', \mathbf{r}'|\mathbf{p}, \mathbf{r}) = \int_0^\infty d\tau P(\mathbf{p}', \mathbf{r}'|\mathbf{p}, \mathbf{r}; \tau). \tag{2.33}$$

We have assumed that the motion of the electrons in the field U_i can be regarded as an aggregate of uncorrelated elastic scatterings through small angles. At sufficiently high scatterer density, the main contribution to the integral (2.33) is made by times $\tau \gtrsim d/v_{\mathbf{dr}}$, which greatly exceed the time τ_0 of the correlated motion. The changes of the momentum over different time intervals that are separated from one another by a time longer than τ_0 are statistically independent. The state density of the transition $P(\mathbf{p}', \mathbf{r}'|\mathbf{p}, \mathbf{r}; \tau)$ at $\tau \gg \tau_0$ satisfies in that case the closed integral equation (the Smoluchowski equation[10,11])[3)]

$$P(\mathbf{p}, \mathbf{r}|\mathbf{p}', \mathbf{r}'; \tau) = \int d\mathbf{p}'' d\mathbf{r}'' P(\mathbf{p}, \mathbf{r}|\mathbf{p}'', \mathbf{r}''; \tau - \tau_i) P(\mathbf{p}'', \mathbf{r}''|\mathbf{p}', \mathbf{r}'; \tau_i). \tag{2.34}$$

The assumption that the scattering is relatively weak on the correlated-motion segments (the assumption of the "softness" of the scatterers) means that the time τ_i in which an appreciable change takes place in the electron velocity greatly exceeds the time τ_0; $\tau_i \gg \tau_0$. This condition means the existence of a time scale such that the changes in the electron velocity becomes uncorrelated, whereas the characteristic changes of the coordinate $\mathbf{p}, \mathbf{r}$ are still small. The last circumstance makes it possible to obtain, with the aid of the Smoluchowski equation (2.34), a differential Fokker-Planck equation. For an isotropic electron dispersion law this equation takes the form[4)]

$$\mathbf{v} \frac{\partial f_{\mathbf{p}}}{\partial \mathbf{r}} + e\mathbf{E} \frac{\partial f_{\mathbf{p}}^{(0)}}{\partial \mathbf{p}} - \frac{1}{2\tau_i} \Delta_\omega f_{\mathbf{p}}^{(0)} = 0, \tag{2.35}$$

$$\left\{ \mathbf{v} \frac{\partial}{\partial \mathbf{r}} - \frac{1}{2\tau_i} \Delta_\omega \right\} \tilde{g}(\mathbf{p}, \mathbf{r}|\mathbf{p}', \mathbf{r}') = \delta(\mathbf{p}-\mathbf{p}') \delta(\mathbf{r}-\mathbf{r}'). \tag{2.36}$$

We have left out of the last equation the field term $e\mathbf{E}\partial\tilde{g}/\partial\mathbf{r}$, which is small in terms of the parameter $eV/\varepsilon_F \ll 1$ [see (2.13) and (2.14)].

Equations (2.35) and (2.36) do not describe the change of the kinetic energy of an electron moving in the scatterer field U_i. This conservation law is satisfied accurate to fluctuations of the random potential U_i, which we assume to be small compared with the characteristic energies of the problem. The quantity τ_i in (2.35) and (2.36) is a phenomenological parameter that determines the rate of fall-off of the electron velocity correlator on the trajectory of the random motion. The operator Δ_ω constitutes the dimensionless angular part of the Laplace operator in the electron-momentum space. The boundary conditions for Eqs. (2.35) and (2.36) follow from the conditions for the spreading (2.5) and reflection (2.6) of the electrons on the point-contact surface.

Just as in the preceding section, the function $f_{\mathbf{p}}^{(0)}$ can be expressed in terms of the probability $\alpha_{\mathbf{p}}(\mathbf{r})$ of the arrival of an electron at a point $\mathbf{r}, \mathbf{p}$ from the region $r = \infty$, $z > 0$. This connection is determined by Eqs. (2.11) and (2.12). The function $\alpha_{\mathbf{p}}(\mathbf{r})$ is then the solution of the following boundary-value problem:

$$\mathbf{v} \frac{\partial \alpha_{\mathbf{p}}}{\partial \mathbf{r}} - \frac{1}{2\tau_i} \Delta_\omega \alpha_{\mathbf{p}} = 0,$$
$$\alpha_{\mathbf{p}}(\mathbf{r} \to \infty) = \theta(z); \quad \alpha_{\mathbf{p}}(\mathbf{r} \in \Sigma) = \alpha_{\mathbf{p}R}(\mathbf{r} \in \Sigma). \tag{2.37}$$

The inelastic component of the point-contact current can be represented in the form

$$I_i = \frac{2e}{(2\pi\hbar)^3} \int d\mathbf{p} d\mathbf{r} \bar{G}_{\mathbf{p}}(\mathbf{r}) I_{ph}\{f_{\mathbf{p}}^{(0)}(\mathbf{r})\}, \tag{2.38}$$

$$\bar{G}_{\mathbf{p}}(\mathbf{r}) = \int dS' \int d\mathbf{p}' v_z' \tilde{g}(-\mathbf{p}, \mathbf{r}|-\mathbf{p}', \mathbf{r}'). \tag{2.39}$$

Comparing the equations for $\bar{G}_{\mathbf{p}}$ and $\alpha_{\mathbf{p}}$ with the analogous equations of the preceding section, it is easy to note that they are similar in structure. The only difference is that in the case of distributed scatterers the corresponding "scattering operator" is not integral but differential. The functions $\bar{G}_{\mathbf{p}}$ and $\alpha_{\mathbf{p}}$ are also connected by a relation similar to (2.28):

$$\bar{G}_{\mathbf{p}}(\mathbf{r}) = \alpha_{-\mathbf{p}}(\mathbf{r}) - \theta(z). \tag{2.40}$$

The general expression (2.29) for the K-factor, obtained in the preceding section, remains the same. In the present case, however, the function $\alpha_{\mathbf{p}}$ must be determined from the boundary-value problem (2.37) and can differ from the function $\alpha_{\mathbf{p}}$ for the problems with point scatterers. In the next section it will be shown that in the diffusion limit ($v_F \tau_i \ll d$) the form of the function $\alpha_{\mathbf{p}}$ is independent of the character of the elastic scattering.

3. PHONON SPECTROSCOPY IN THE DIFFUSION LIMIT

We consider the case of extremely dirty point contacts, for which the condition $l_i/d \ll 1$ is satisfied. In this case the kinetic equations (2.8) and (2.9) can be solved by expanding in the small parameter l_i/d. If it is recognized, however, that these equations themselves are the results of an expansion of the initial equation (2.1) in the small parameter of the electron-phonon relaxation d/l_ε, then for the perturbation theory indicated above to be correct it is necessary to satisfy the additional inequality $l_i/d \gg d/l_\varepsilon$ which is equivalent to the criterion $d \ll (l_i l_\varepsilon)^{1/2}$ given in §1.

We represent the quantity $\alpha_p(\mathbf{r})$ by a series in the small parameter l_i/d. With the aid of (2.14) we then obtain

$$\alpha_p(\mathbf{r})=\alpha_0(\mathbf{r})-\tau_i v\frac{\partial\alpha_0}{\partial \mathbf{r}}. \tag{3.1}$$

We have confined ourselves in (3.1) to consideration of elastic scattering in the isotropic model ($W^{(i)}_{pp'}=W^{(i)}_{|p-p'|}$) and of a spherically symmetrical carrier dispersion law. We have introduced in (3.1) the transport mean free path time τ_i:

$$\frac{1}{\tau_i}=\int\limits_{\varepsilon_p=\varepsilon_{p'}=\varepsilon_F}\frac{dS_{p'}}{v'_\perp}W^{(i)}_{|p-p'|}[1-\cos(\widehat{pp'})]. \tag{3.2}$$

The zeroth approximation $\alpha_0(\mathbf{r})$ in terms of the parameter l_i/d should be obtained from the solution of the following boundary-value problem (n is the normal to the contact surface Σ):

$$\Delta\alpha_0(\mathbf{r})=0,\quad \alpha_0(\mathbf{r}\to\infty)=\theta(z);\quad \frac{\partial}{\partial \mathbf{n}}\alpha_0(\mathbf{r}\in\Sigma)=0. \tag{3.3}$$

According to (2.11), $\alpha_p(\mathbf{r})$ determines the electron distribution function $f_p^{(0)}$ and, with the aid of the electroneutrality condition (2.4), the distribution of the electric field

$$e\mathbf{E}=-\nabla\Phi(\mathbf{r})=-eV\nabla\alpha_0(\mathbf{r}). \tag{3.4}$$

We note that the solution of Eq. (2.37) for the function $\alpha_p(\mathbf{r})$ that corresponds to elastic scattering by delocalized center in the diffusion limit $v_F\tau_i\ll d$, can also be represented in the form (3.1) and (3.2). Thus, all the results obtained below for dirty contacts are independent of the character of the elastic scattering of the electrons.

As seen from relations (3.1) and (3.4), as well as from Eq. (2.11), in the dirty limit the filling of the electronic states in momentum space is characterized by two concentric Fermi spheres

$$\varepsilon_p=\varepsilon_F-\Phi+eV/2,\quad \varepsilon_p=\varepsilon_F-\Phi-eV/2.$$

Inside the smaller sphere, the function $f_p^{(0)}$ is equal to unity, and outside the larger sphere it is equal to zero. In the remaining part of p-space the filling is not in equilibrium, is anisotropic, and $0<f_p^{(0)}<1$. The distribution function is symmetrical with respect to rotations of the momentum p about the electric-field intensity vector E drawn through the point $p=0$ (Fig. 2).

The boundary-value problem (3.3) can be solved in the sufficiently general case of a contact in the form of a single-cavity hyperboloid of revolution[13] (see Fig. 1). The function $\alpha_0(\mathbf{r})$ for such a contact is of the form (r is the radius vector drawn from the center of the contact)

$$\alpha_0(\mathbf{r})=\theta(z)-\varphi_0(\mathbf{r})\operatorname{sign} z, \tag{3.5}$$
$$\varphi_0(\mathbf{r})=\frac{1}{\pi}\operatorname{arctg}\left\{\left[\frac{r^2}{2b^2}-\frac{1}{2}+\left(\left(\frac{r^2}{2b^2}-\frac{1}{2}\right)^2+\frac{z^2}{b^2}\right)^{1/2}\right]^{-1/2}\right\}.$$

The result makes it possible to calculate directly both the elastic and inelastic components of the current in the point contact.

Under the condition $eV/\varepsilon_F\ll 1$, the elastic component of the current satisfies Ohm's law. The resistance R of the contact is of the form ($\sigma_0=ne^2\tau_i/m$ is the conductivity of the metal and $a=d/2$)

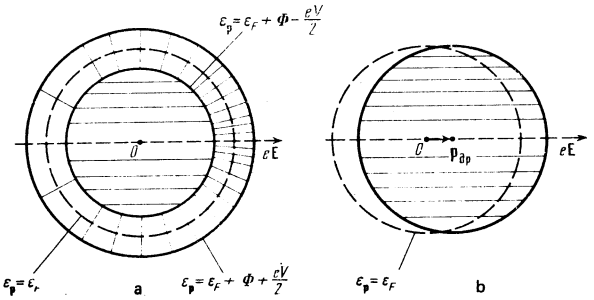

FIG. 2. Filling of electronic states (shown hatched) in p-space: a) in a point contact, b) in a uniform current state.

$$R^{-1}=2\sigma_0[b-(b^2-a^2)^{1/2}], \tag{3.6}$$
$$b=\frac{1}{\pi}L\left(1+\frac{\pi S}{4L^2}\right). \tag{3.7}$$

We have introduced in (3.7) the area $S=\pi d^2/4$ of the smallest contact cross section and the effective length of the contact L (see Fig. 1):

$$L=\sigma_0 RS. \tag{3.8}$$

We note that according to (3.6) we obtain, in the limit of a flat round contact for which $b=d/2$ (see Fig. 1), the well-known result $R^{-1}=\sigma_0 d$ (Ref. 14).

The form factor $K(\mathbf{n},\mathbf{n}')$ is calculated by substituting in the general expression (2.29) the function α_p obtained for the considered contact (Fig. 1) [see (3.1), (3.5)]. After simple calculations we obtain ($\mathbf{n}=\mathbf{p}/p_F$, $\mathbf{n}'=\mathbf{p}'/p_F$)

$$K(\mathbf{n},\mathbf{n}')\equiv K(\mathbf{n}-\mathbf{n}')$$
$$=\frac{9\pi}{32}\frac{l_i}{d}\frac{(n_z-n_z')^2(1-{}^1\!/_2x^2)+{}^1\!/_2(\mathbf{n}-\mathbf{n}')^2x^2}{1+x^2}. \tag{3.9}$$

The parameter x in (3.9) is determined by the geometry of the contact and is expressed in terms of its diameter d and effective length L [see (3.8)]:

$$x=\pi d/4L,\quad 0\leqslant x\leqslant 1. \tag{3.10}$$

The variation of x is restricted by the limiting values for a long channel ($x=0$) and a round opening ($x=1$).

As seen from (3.9), in the case of a long channel the anisotropy of the form factor $K(\mathbf{n}-\mathbf{n}')$ is pronounced most strongly. In this limit, we obtain from (3.9) the expression

$$K_{ch}(\mathbf{n}-\mathbf{n}')=\frac{9\pi}{32}\frac{l_i}{d}(n_z-n_z')^2. \tag{3.11}$$

For a round opening, the K-factor depends also on the component, perpendicular to the contact axis, of the vector $\mathbf{n}-\mathbf{n}'$:

$$K_{op}(\mathbf{n}-\mathbf{n}')=\frac{9\pi}{128}\frac{l_i}{d}[(n_z-n_z')^2+(\mathbf{n}-\mathbf{n}')^2]. \tag{3.12}$$

In conclusion, we present the values of the K factors averaged over the Fermi surface:

$$\langle K\rangle=\int\frac{dS_p}{v_\perp}\int\frac{dS_{p'}}{v_{\perp'}}K(\mathbf{p},\mathbf{p}')\Big/\left(\int\frac{dS_p}{v_\perp}\right)^2, \tag{3.13}$$

where we consider, to complete the picture, both a dirty ($l_i\ll d,L$) and a pure ($l_i\gg L,d$) channel (of length

L) and opening (of diameter d). In the case of a clean opening we have[3]

$$\alpha_p(\mathbf{r})=\theta[\mathbf{p}\in\Omega(\mathbf{r})], \quad z<0, \tag{3.14}$$

i.e., α_p is equal to unity if the vector p lies within the solid angle $\Omega(\mathbf{r})$ at which the opening is seen from the point r, and to zero when $\mathbf{p}\bar{\in}\Omega(\mathbf{r})$. Calculation of the integral of (2.29) yields in this case the known result[3]

$$K(\mathbf{n},\mathbf{n}')=\frac{|n_z n_z'|}{|n_z\mathbf{n}'-n_z'\mathbf{n}|}\theta(-n_z n_z'). \tag{3.15}$$

Taking the azimuthal symmetry into account, we obtain, after substituting (3.15) in (3.13),

$$\langle K\rangle=\frac{1}{2\pi}\int_0^1 dx\int_0^t dy\int_0^{2\pi} d\varphi\frac{xy}{[x^2+y^2-2x^2y^2-2xy(1-x^2)^{1/2}(1-y^2)^{1/2}\cos\varphi]^{1/2}}. \tag{3.16}$$

Calculation of this integral yields[16]

$$\langle K\rangle_{\text{pure op.}}=1/4. \tag{3.17}$$

For a pure channel [see (4.2) below] the average K-factor is[5)]

$$\langle K\rangle_{\text{pure op.}}=\frac{3\pi}{16}\frac{L}{d}=0.589\frac{L}{d}\quad L\gg d. \tag{3.18}$$

It is proportional to the length of the channel, a reflection of the fact that the length of the inelastic interaction of the electrons with the phonons increases with increasing channel length L. One must not attach too much importance to this circumstance, however, since a long channel with specularly reflecting walls is patently an unrealistic model of a point contact. Much more realistic is the model with the opening. This is evidenced, in particular, by the good agreement between known data and the value of the electron-phonon interaction in point contacts, as determined from the intensity of the point-contact (PC) spectrum[16] in accordance with (1.5).

In the dirty limit, the values of the K-factor, averaged over the orientations of the vectors n and n', coincide for an opening and a channel, and are equal to [see (3.11) and (3.12)]

$$\langle K\rangle_{\text{dirt}}=\frac{3\pi}{16}\frac{l_i}{d}=0.589\frac{l_i}{d}. \tag{3.19}$$

In the diffusion case, this result is more general and does not depend on the concrete geometry of the contact. Indeed, changing over to the quantity

$$q(\mathbf{n}-\mathbf{n}')=\frac{d}{l_i}K(\mathbf{n}-\mathbf{n}')$$

we obtain in accordance with (3.1) and (2.29)

$$q(\mathbf{n}-\mathbf{n}')=\frac{9\pi}{32}\int d\mathbf{r}|(\mathbf{n}-\mathbf{n}')\nabla\alpha_0|^2/\int d\mathbf{S}\nabla\alpha_0, \tag{3.20}$$

where $d\mathbf{S}$ is an area element of an arbitrary section of the contact. After averaging over the orientations of the vectors n and n', Eq. (3.20) yields

$$\langle q(\mathbf{n}-\mathbf{n}')\rangle=\frac{3\pi}{16}\int d\mathbf{r}|\nabla\alpha_0|^2/\int d\mathbf{S}\nabla\alpha_0=\frac{3\pi}{16}. \tag{3.21}$$

The validity of the last equation follows from the fact that the function $\alpha_0(\mathbf{r})$ is harmonic and from the boundary conditions, which are valid in the diffusion limit [see (3.3)]. It follows in particular from (3.21) that the isotropic PC spectra, which are proportional to the averaged K-factor, depend in the diffusion limit only on the mean free path l_i and are insensitive to the geometry of the contact region, including the possible presence of many contacts in this region.

4. POINT-CONTACT SPECTRUM AT ARBITRARY VALUE OF THE PARAMETER l_i/d

An essential parameter used in the preceding section was the homogeneity of contamination of the system [$\tau_i(\mathbf{r})=\text{const}$]. As a result of this homogeneity, the function $\alpha_p(\mathbf{r})$ could be represented in the form (3.1), with α_0 independent of the electron velocity v. The last circumstance made it possible to average the form factor $K(\mathbf{n},\mathbf{n}')$ over the orientations of the vectors n and n' in explicit form [see (3.20) and (3.21)]. In the case of a strongly inhomogeneous distribution of the impurities (including the contact between the pure and dirty regions of the metal), the function $\alpha_p(\mathbf{r})$ can no longer be represented in the form (3.1), and relation (3.21) may not hold for the averaged form factor. By way of example of such a system we consider a dirty channel of diameter d and length L, joining pure bulk metals. Such a channel was considered earlier[6] under the condition $L\gg d$ with a model used to take into account the elastic scattering of the electrons. Generalizing the expressions obtained in Ref. 6 for the functions α_p to include the case of arbitrary values of L/d, we obtain in accordance with the general relation (2.29) for the form factor of the channel

$$K_{\text{ch}}=\frac{4\lambda^2|n_z n_z'|\{1-\lambda^{-1}+(2\lambda^2)^{-1}\ln(1+2\lambda)\}^{-1}}{(1+2\lambda|n_z|)(1+2\lambda|n_z'|)}\Big\{K_0(\mathbf{n},\mathbf{n}') + \frac{3\pi}{32}\frac{L}{d}\frac{[n_z'-n_z+4\lambda n_z'|n_z|\theta(-n_z n_z')]^2-{}^1/_3(|n_z|-|n_z'|)^2}{|n_z n_z'|(1+2\lambda|n_z|)(1+2\lambda|n_z'|)}\Big\},\quad \lambda=\frac{l_i}{L}, \tag{4.1}$$

where $K_0(\mathbf{n},\mathbf{n}')$ is the form factor of a pure opening [see (1.4)].

Equation (4.1) describes the dependence of the anisotropy and of the form factor on the parameters L/d and l_i/d, which characterizes the geometry and degree of contamination of the contact. It takes a simpler form for the cases of dirty and pure channels[6)]:

$$K_{\text{pure ch}}=\theta(-n_z n_z')\left\{\frac{3\pi}{8}\frac{L}{d}+\frac{|n_z n_z'|}{|n_z\mathbf{n}'-n_z'\mathbf{n}|}\right\},\quad l_i\gg L; \tag{4.2}$$

$$K_{\text{dirt. ch}}=\frac{3l_i}{L}\left\{\frac{3\pi}{32}\frac{L}{d}\left[(n_z-n_z')^2-\frac{1}{3}(|n_z|-|n_z'|)^2\right]+\frac{|n_z n_z'|^2}{|n_z\mathbf{n}'-n_z'\mathbf{n}|}\theta(-n_z n_z')\right\},\quad l_i\ll L. \tag{4.3}$$

An analysis of the electric conductivity of homogeneously contaminated systems at an arbitrary ratio l_i/d is more complicated, since a model description of elastic scattering, similar to that considered earlier,[6] is not adequate in the present case. A more rigorous approach to this problem calls for an exact solution of Eq. (2.14) with the boundary conditions (2.15). In this case, when $W_{\mathbf{pp}'}^{(i)}=\text{const}$ [see (2.2)], the integro-differential equation (2.14) can be represented in the form

$$\alpha_p(\mathbf{r})=\frac{1}{\tau_i}\int_0^\infty dt\exp\left(-\frac{t}{\tau_i}\right)\bar{\alpha}(\mathbf{r}_{t,p});\quad \bar{\alpha}(\mathbf{r})=\frac{1}{4\pi}\int d\Omega_p\alpha_p(\mathbf{r}). \tag{4.4}$$

We have introduced in (4.4) the time t and the coordinate $\mathbf{r}_{t,\mathbf{p}}$ on the electron-trajectory that starts out from the point r, $-\mathbf{p}$, at the time $t=0$, with account taken of the specular reflection from the surface of the contact (Fig. 3). If (4.4) is integrated over the directions of the vector p ($\int d\Omega_{\mathbf{p}}/4\pi\ldots$), then we obtain for the function $\overline{\alpha}(\mathbf{r})$ an integral equation, to the solution of which the calculation of $\alpha_{\mathbf{p}}(\mathbf{r})$ reduces. The problem of finding $\overline{\alpha}(\mathbf{r})$ is still complicated and does not admit of an analytic solution. However, an interpolation procedure can be proposed, which yields the solution at an arbitrary value of the parameter l_i/d. If $\alpha_{\mathbf{p}}(\mathbf{r})$ is chosen in the form

$$\alpha_{\mathbf{p}}(\mathbf{r})=\frac{1}{\tau_i}\int_0^\infty dt\exp\left(-\frac{t}{\tau_i}\right)\alpha_0(\mathbf{r}_{t,\mathbf{p}}), \tag{4.5}$$

in which use is made of the function $\alpha_0(\mathbf{r})$, which is a solution of the boundary-value problem (3.2), then, as can be easily understood, we obtain the correct value for the distribution function $f_{\mathbf{p}}^{(0)}(\mathbf{r})$ in the dirty limit $l_i \ll d$.

It can be verified by a direct check that relation (4.5) determines the correct $f_{\mathbf{p}}^{(0)}(\mathbf{r})$ dependence also in the pure limit $l_i \gg d$. Thus, the choice of the distribution function $f_{\mathbf{p}}^{(0)}$ in the form (2.11) and (4.5) ensures correct limiting values of the resistance of the point contact in the pure and dirty limits. At an arbitrary ratio of the length l_i and d, Eq. (4.5) yields an interpolation relation for $R(l_i/d)$. This relation, calculated for a round opening in a plane partition with the aid of relations (2.10) and (2.4) is shown in Fig. 4. As seen from the figure, the form of the function $R(l_i/d)$ differs little from the results obtained by Wexler,[15] who determined the resistance of a round orifice by a variational method.

The interpolation expression (4.5) can be used to calculate the K-factor at arbitrary values of l_i/d. It is convenient to do this for a long channel ($L \gg d$). In this case $\alpha_0(z)$ is of the form

$$\alpha_0(z)=\frac{1}{2}\left(1+\frac{2z}{L}\right),\quad |z|\leqslant\frac{L}{2}. \tag{4.6}$$

Calculating the function $\alpha_{\mathbf{p}}(z)$ from Eq. (4.5), we obtain the form factor in accordance with (2.29) in the form ($\lambda=l_i/L$)

$$K(\mathbf{n},\mathbf{n}')=\frac{9\pi}{32}\frac{L}{d}\frac{\lambda}{\gamma(\lambda)}\left\{(n_z-n_z')^2+n_z^2\exp\left(-\frac{1}{\lambda|n_z|}\right)\right.$$
$$\left.+n_z'^2\exp\left(-\frac{1}{\lambda|n_z'|}\right)+4\lambda(n_z'-n_z)\left[n_z^2\exp\left(-\frac{1}{2\lambda|n_z|}\right)\mathrm{sh}\frac{1}{2\lambda n_z}\right.\right.$$
$$\left.\left.-n_z'^2\exp\left(-\frac{1}{2\lambda|n_z'|}\right)\mathrm{sh}\frac{1}{2\lambda n_z'}\right]+4\lambda\frac{(n_zn_z')^2}{(n_z-n_z')}\exp\left(-\frac{1}{\lambda}\frac{|n_z|+|n_z'|}{2|n_zn_z'|}\right)\mathrm{sh}\frac{n_z'-n_z}{2\lambda n_zn_z'}\right]\right\};$$
$$\gamma(\lambda)=3\int_0^1 dt\,t^2\left[1-\exp\left(-\frac{1}{2\lambda t}\right)\right]. \tag{4.7}$$

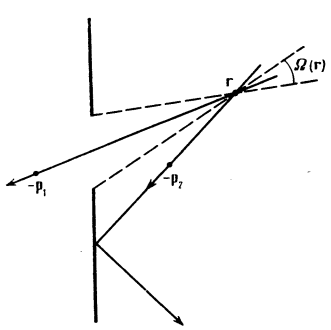

FIG. 3. Illustrating the calculation of the function $\alpha_{\mathbf{p}}(\mathbf{r})$.

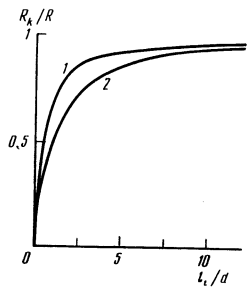

FIG. 4. Dependence of the reduced electric conductivity of a flat round contact on the Knudsen parameter l_i/d (R_c is the resistance of a pure point contact, d is its diameter). Curve 1—calculation by a variational method.[15] Curve 2—results obtained by the interpolation formula (4.5).

For an arbitrary geometry ($L/d \lesssim 1$) the interpolation procedure makes possible only a numerical calculation of the K-factor as a function of the parametry l_i/d. In accordance with the "theorem" (3.21) at small values, $\langle K\rangle$ is a linear function of l_i/d with slope that does not depend on the contact geometry. In the pure limit $l_i \gg d$ the function $\langle K\rangle$ tends to a limiting value $\langle K\rangle_\infty$ that depends on the parameter L/d [see (4.2)]:

$$\langle K\rangle_\infty=0.25+0.589L/d. \tag{4.8}$$

A qualitative plot of $\langle K\rangle$ against the parameter l_i/d is shown in Fig. 5.

5. DISCUSSION OF RESULTS

The results of the present paper pertain to the case when the electron energy scattering length $\lambda=(l_il_\varepsilon)^{1/2}$ is large compared with the characteristic dimension of the point contact. In this limit, an electron, passing through the localization region of the accelerating field, does not perturb the phonon system greatly. This makes possible phonon spectroscopy similar to that in the ballistic regime $l_i \gg d$ (Ref. 3). Calculation of the point-contact spectrum reduces to the calculation of the form factor $K(\mathbf{p},\mathbf{p}')$, which depends on the geometry of

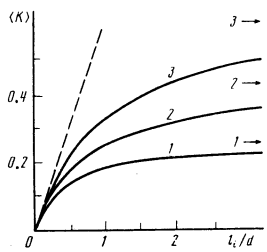

FIG. 5. Dependence of the average K factor on the parameter l_i/d for contacts with different values of the ratio L/d. At small l_i/d the function $\langle K\rangle$ takes the universal form $\langle K\rangle = 0.589\, l_i/d$. The arrows show the limiting values of the K-factor at $l_i/d=\infty$, corresponding to different values of L/d: 1) $L/d=0$, 2) $L/d=0.3$, 3) $L/d=0.6$.

the contact and on the degree of its contamination. In the most general case, the determination of the form factor K can be reduced to an analysis of the point-contact elastic electric conductivity characterized by the probability function $\alpha_p(r)$ [Eq. (2.29)]. Using the obtained relation (2.29), we can calculate the K-factor for different particular cases. In the diffusion limit $l_i \ll d$ the intensity of the point-contact structure decreases by a factor l_i/d compared with the pure limit. The value of the K-factor averaged over the directions of the momenta p and p′, according to theorem (3.21), depends linearly in the diffusion limit on the Knudsen parameter l_i/d, with a slope that is not sensitive to the geometry of the contact region, including the possibility of many contacts in this region. An analysis of the scattering by stochastic continuous inhomogeneities (strain fields) shows that in the diffusion regime their influence can be described by introducing the mean free path l_i and retaining the form of the K factor obtained in the model with point-like elastic scatterers. By way of example of an inhomogeneously contaminated contact, we considered the model of a dirty channel connecting pure metals.

The universality of the form of the function $\langle K\rangle(l_i/d)$ in the region of small mean free paths, and the dependence of the limiting value $\langle K\rangle_\infty$ at $l_i/d=\infty$ on the geometry of the contact, provide a favorable possibility of experimentally investigating the shape of the point contact. For this purpose it is necessary to normalize the measured spectrum in such a way that its dependence on the parameter l_i/d at small l_i/d has the necessary slope [independent of the shape of the point contact, see (3.21)]. From the intensity of the point-contact spectrum in the pure limit it is possible to determine uniquely the shape of the point contact [the parameter L/d, Eq. (4.8), Fig. 5].

This study of metals with small mean free paths by the method of point-contact spectroscopy is present only in the starting phase.[6,7] Our results explain the following experimental data given by Yanson and coworkers[7]: the existence in dirty contacts of a point-contract spectrum whose shape coincides with that of the spectrum in pure contacts; the proportionality of the intensity of the point-contact spectrum to the Knudsen parameter l_i/d, and others. Thus, point-contact spectroscopy method can yield information on the phonon spectra not only of pure single crystals but also of metals with arbitrarily small mean free path (alloys, films, metals subjected to strong plastic deformation).

[1)] Equation (1.3) is valid at $eV \gg T$. For a more general expression see Ref. 3.

[2)] We note that the analysis can be carried out also for an arbitrary scattering characterized by a scattering indicatrix $B(p, p')$, of electrons by a surface. The principal relations of the present section [see (2.28) and (2.29)] are valid also if the surface scattering has the symmetry property $B(p,p')=B(p',p)$. This property is possessed, in particular, by both specular and diffuse scattering from a surface.

[3)] Thus, trajectories of electrons in the field of scatterers are described as trajectories of a Markov random process.

[4)] The procedure for deriving these equations recalls the derivation of an equation for the distribution function of the orientation of a flexible polymer chain.[12]

[5)] This value differs by 25% from that obtained earlier[6] with a model collision integral.

[6)] We note that according to (4.2), in the case of a long pure channel the asymptotic function $G(\omega)$ [see (1.2)] at $\omega \ll \omega_D$ (ω_D is the Debye frequency) is of the form $G(\omega) \approx (\omega/\omega_D)^3$. This dependence differs from the relation $G(\omega) \approx (\omega/\omega_D)^4$, which is valid for a plane point contact with $L/d \ll \omega/\omega_D$.

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