# Dispersion of the velocities of transverse and longitudinal sound in tin subjected to a magnetic field

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Moscow State University (Submitted 8 July 1981) Zh. Eksp. Teor. Fiz. 81, 2234–2246 (December 1981)

A contactless method was used to study the excitation and propagation of transverse and longitudinal sound along the principal crystallographic directions in tin. A study was made of the monotonic dependences of the velocity of sound on the applied magnetic field (Alpher-Rubin effect). Measurements were made of the amplitudes of quantum oscillations of the velocity of sound associated with certain extremal sections of the Fermi surface. The ratios of the amplitudes of the quantum oscillations of the velocities of transverse and longitudinal sound along the principal crystallographic directions were used to determine the deformation anisotropy of the Fermi surface in the third and sixth energy bands of tin.

PACS numbers: 72.55. + s, 71.25.Hc, 43.35.Rw

### INTRODUCTION

The dispersion of the velocity of sound in metals is, like the absorption of sound, due to the interaction of the lattice with the conduction electrons. A magnetic field H alters greatly the nature of this interaction. Depending on the ratio of the parameters of the sound wave ( $\omega$  is the frequency and  $\lambda$  is the wavelength of sound) and the main parameters of the electron subsystem ( $\nu = \tau^{-1}$  is the collision frequency,  $\omega_c$  is the cyclotron frequency, l is the mean free path,  $\delta$  is the skin layer thickness, R is the Larmor radius, etc.) in metals subjected to magnetic fields, one can have a great variety of magnetoacoustic phenomena. They include the Alpher-Rubin effect, Bömmel-Pippard magnetoacoustic oscillations, acoustic cyclotron resonance, giant quantum oscillations of the absorption of sound, etc. When the conditions  $\hbar \omega_c \gg kT$  and  $\omega_c \tau > 1$  are satisfied, the absorption of sound exhibits oscillations which are a direct analog of the de Haas-van Alphen effect (in contrast to the giant quantum oscillations, which should be regarded rather as effects of the resonance absorption of phonons by the conduction electrons at the Fermi level).

The magnetoacoustic effects which appear in the absorption of sound alter also the velocity of sound because of the Kramers-Kronig dispersion relations. An investigation of the dispersion of the velocities of transverse and longitudinal sound in a metal subjected to a magnetic field can give information on the Fermi surface geometry, cyclotron masses of carriers, and asymptotic behavior of the magnetoresistance along various crystallographic directions.

The propagation of a sound wave in a metal may be regarded as a mechanical motion of charged particles in a conducting medium. The application of a magnetic field distorts the paths of these particles and this gives rise to a transverse current analogous to the Hall current. Interaction of this current with a magnetic field gives rise to additional forces which combine with the usual elastic forces in a metal. According to the magnetohydrodynamic theory of Alpher and Rubin,<sup>1</sup> changes in the velocities of longitudinal  $S_t$  and transverse  $S_t$ sound in a magnetic field are described by the expressions

$$S_{l} = S_{l_{0}} \left[ 1 + \frac{B^{2} \sin^{2} \theta}{8\pi \rho S_{l_{0}}^{2} (1 + 4\pi^{4} \delta^{4} / \lambda^{4})} \right], \quad \mathbf{q}_{l} \perp \mathbf{H},$$

$$(1)$$

$$S_{t} = S_{t_{0}} \left[ 1 + \frac{B^{2} \cos^{2} \theta}{8\pi\rho S_{t_{0}}^{2} (1 + 4\pi^{4} \delta^{4} / \lambda^{4})} \right], \quad \mathbf{q}_{t} \| \mathbf{H},$$

$$\tag{2}$$

where  $\theta$  is the angle between the direction of propagation of sound and the magnetic field;  $\rho$  is the density of the metal. In the first approximation, the magnitude of this effect is governed by the ratio of the magnetic field energy  $B^2/8\pi$  to the elastic energy  $\rho S^2$ . Measurements of Galkin and Korolyuk carried out on polycrystalline tin<sup>2</sup> and of Alers and Fleury on single crystals of silver, gold, etc.<sup>3</sup> are in good agreement with calculations based on the Alpher-Rubin theory.

When the magnetic field is increased, quantum oscillations appear against the background of the monotonic Alpher-Rubin dependence. These oscillations of the velocity of sound are related in a simple manner to oscillations of the elastic moduli of a metal:

$$C^{\rm osc} = \rho S^2 - \rho S_0^2 \approx 2\rho S_0 \Delta S^{\rm osc}, \quad \Delta S^{\rm osc} / S_0 \ll 1.$$
(3)

The elastic moduli are the second derivatives of the free energy F with respect to the strain  $\epsilon_{ij}$ :

$$C_{ij} = \partial^2 F / \partial \varepsilon_{ij}^2. \tag{4}$$

The components of the strain tensor in an acoustic wave are

$$\varepsilon_{ij} \approx (e_i q_j + e_j q_i)/2, \tag{5}$$

where the components of the unit vectors  $\hat{\mathbf{q}} = \mathbf{q}/q$  and  $\hat{\mathbf{e}} = \mathbf{e}/e$  describe the directions of propagation and of the polarization of sound.

Following the thermodynamic approach of Testardi and Condon,<sup>4</sup> we find that the free energy of a metal can be written in the form

$$F = F' + \frac{i}{2\varepsilon} C_0 \varepsilon + \Omega(B, \varepsilon) + \frac{B^2}{8\pi},$$
(6)

where F' is independent of the strain and  $C_0$  is the tensor of the elastic moduli which includes all the nonoscillatory effects that are not described by the term  $\Omega(B, \varepsilon)$ . Litshitz and Kosevich<sup>5</sup> showed that the free energy of an electron gas  $\Omega(B, \varepsilon)$  is an oscillatory function of the type

$$\Omega(B,\varepsilon) = \Phi(B,\varepsilon) \cos \frac{chA(\varepsilon)}{eB}, \qquad (7)$$

where  $\Phi(B,\varepsilon)$  is a slowly varying function and  $A(\varepsilon)$  is an extremal section of the Fermi surface dependent on the strain.

The magnetization M is found from  $\Omega$  using the relationship  $M = -\partial \Omega / \partial B | \epsilon$ . The derivatives of the function (7) with respect to the strain and magnetic induction are related by the following expression applicable to the static case:

$$\frac{\partial}{\partial \varepsilon_{ij}}\Big|_{B} = -BD_{ij}\frac{\partial}{\partial B}\Big|_{\varepsilon_{ij}}, \qquad (8)$$

where the parameter

 $D_{ij} = \partial \ln A(\varepsilon) / \partial \varepsilon_{ij}$ 

determines the corresponding component of the deformation potential tensor of the metal:

$$\Lambda_{ij} = \frac{dE}{d\varepsilon_{ij}} = \left(\frac{dE}{dA}\right) \left(\frac{dA}{d\varepsilon_{ij}}\right) = \frac{\hbar^2 D_{ij} A}{2\pi m^*}.$$
 (9)

We have allowed here for the fact that the effective mass of an electron is  $m^* = (\hbar^2/2\pi)(dA/dE)$ .

In general, when an acoustic wave travels in a metal, the variables B and  $\varepsilon_{ij}$  cannot be regarded as independent. The total derivative with respect to the strain can be written in the form

$$\frac{d}{d\epsilon_{ij}} = \frac{\partial}{\partial \epsilon_{ij}} \Big|_{s} + \frac{\partial B}{\partial \epsilon_{ij}} \frac{\partial}{\partial B} \Big|_{\epsilon_{ij}}.$$
 (10)

In the case of longitudinal sound traveling across the magnetic field and of transverse sound traveling along the magnetic field we find from Eqs. (4) and (6)-(10) that the amplitudes of the quantum oscillations of the elastic moduli are given by

$$C_{ij}^{\text{osc}} = -B^2 (D_{ij} + \delta_{ij})^2 \frac{\partial M}{\partial B}, \quad \mathbf{q}_i \perp \mathbf{B},$$
(11)

$$C_{ij}^{\text{osc}} = -B^2 (D_{ij} - \delta_{ij})^2 \frac{\partial M}{\partial B}, \quad \mathbf{q}_i || \mathbf{B}.$$
 (12)

We can thus see that the amplitudes of the quantum oscillations of the velocity of acoustic waves traveling along a given crystallographic direction are governed by the corresponding components of the deformation potential tensor of the metal. The difference between the deformation interactions of electrons with longitudinal and transverse acoustic waves is due to the fact that in the former case it is governed by the diagonal components, whereas in the latter case it is governed by the nondiagonal components of the tensor  $\Lambda_{ij}$ . The signs of the corresponding components of the deformation in the deformation in the deformation in the diagonal components of the tensor  $\Lambda_{ij}$ .



FIG. 1. Fermi surface of tin in the third hole (a) and sixth electron (b) bands.

tion potential tensor cannot be found by measuring the amplitudes of the quantum oscillations of the velocity of sound. The absolute values of  $\Lambda_{ij}$  can be found by simultaneous measurements of the differential magnetic susceptibility. If the values of  $\partial M/\partial B$  are not available, the ratio of the amplitudes of the quantum oscillations of the various elastic moduli of a metal can be used to determine the relationships between the various values of  $D_{ij}$ , i.e., to find the deformation anisotropy of the Fermi surface.

We investigated the influence of a magnetic field on the velocities of transverse and longitudinal sound in tin single crystals. The multiply connected Fermi surface of tin has small regions very sensitive to the lattice deformation and this gives rise to large amplitudes of oscillations of the elastic moduli. We investigated mainly the quantum oscillations due to extremal cross sections in the third hole and sixth electron bands of tin. These cross sections (based on the Craven model of the Fermi surface of tin<sup>6</sup>) are shown in Fig. 1. The deformation characteristics of the Fermi surface in these bands have been investigated earlier using the magnetostriction oscillations<sup>7</sup> and hydrostatic compression.<sup>8,9</sup> Thé change in the extremal sections of the Fermi surface due to hydrostatic pressure is

$$d\ln A = D_{ij}S_{ij}d\sigma, \tag{13}$$

where  $S_{ij} = C_{ij}^{-1}$  are the elastic (compliance) constants. Under hydrostatic pressure the diagonal components of the stress tensor are equal to the pressure  $\sigma_{ii} = -P$  and the nondiagonal components vanish. Table I gives the logarithmic derivatives with respect to pressure of the extremal sections of the Fermi surface of tin in the third and sixth bands.

It should be noted that the values of  $\partial \ln A/\partial P$  obtained in the hydrostatic compression experiments are calculated on the assumption that the compressibility of a metal is isotropic. However, the compressibility of tin is strongly anisotropic and, therefore, the deformation parameters given below should be regarded as approximate.

White tin has the tetragonal bcc crystal structure. Propagation of pure longitudinal and transverse acoustic waves in a lattice of this type is possible only along the principal crystallographic directions: [100], [110], and [001]. Figure 2 shows the propagation and polarization vectors of acoustic waves along these directions and the corresponding components of the tensor of the elastic moduli. The values of the elastic moduli at T = 4.2 % are given, for example, in Ref. 10.

TABLE I. Logarithmic derivatives with respect to pressure  $\partial(\ln A)/\partial P$  for some extremal sections of Fermi surface of tin.

	Extremal	$\partial \ln A / \partial P$			
Ħ	section	[7]	[8]	[9]	
[001]	$\left\{\begin{array}{c} \delta_{i'}\\ \delta_{i^2}\end{array}\right.$	-4.5 -2.0	$^{+4,8}_{+0,6}$	-2.94 -0.25	
[100]	τ2'		-	-2,55	



FIG. 2. Principal crystallographic directions of the propagation of sound and the corresponding components of the tensor of the elastic moduli.

#### **MEASUREMENT METHOD**

Single crystals of tin used in our investigation were grown from the melt in a demountable quartz polished mold by the method of Khaikin *et al.*<sup>11</sup> The quality of the material used in the preparation of the samples was characterized by the resistivity ratio  $\rho(300 \text{ K})/\rho(4.2 \text{ K})$ =  $8 \cdot 10^4$ . The samples were disks 1.8 cm in diameter and either 0.100 or 0.135 cm thick. Measurements were carried out on samples for which the normal to the surface coincided with one of the principal crystallographic directions: [100], [110], and [001]. The samples were placed in protective Plexiglas boxes with annular paper spacers. Two measuring coils parallel to one another were wound on a box. Measurements were made at T = 4.2 K in magnetic fields up to 85 kOe.

A contactless method was used for the excitation and detection of transverse and longitudinal sound. This method is based on the fact that an electromagnetic wave incident on a metal-vacuum interface generally excites acoustic vibrations of the same frequency in a metal.<sup>12</sup> In the presence of a static magnetic field the main excitation mechanism is the Lorentz interaction of a current induced in the skin layer with the magnetic induction in the sample. Longitudinal acoustic waves are excited in a field H parallel to the surface of the metal and in this case we have  $q_i \perp H$ . When a magnetic field is perpendicular to the surface of a metal, transverse sound with  $q_t \parallel H$  is excited. The polarization of the transverse sound is governed by the orientation of the rf magnetic field relative to the crystallographic axes.

The amplitude of the sound excited by a contactless method is proportional to the intensities of the static and rf magnetic fields, and it is inversely proportional to the density of the metal, velocity of sound, and its frequency. The efficiency of conversion of an electromagnetic wave into sound is very low: for example, in the case of longitudinal sound in tin of f = 1 MHz frequency it amounts to  $\eta = 1.4 \cdot 10^{-12}$  H<sup>2</sup> at T = 4.2 °K.

The excitation of sound in a plane-parallel plate of thickness d is more efficient than on the surface of a semi-infinite metal. This is due to the fact that at frequencies  $f_n$  satisfying the condition

$$f_n = \frac{\omega_n}{2\pi} = \frac{S}{d} \left( n + \frac{1}{2} \right), \quad n = 0, 1, \dots$$
 (14)

standing acoustic waves are established across the plate thickness. The Q factor of the plate acting as an

acoustic resonator should be sufficiently high for the detection of standing acoustic waves. This imposes an additional condition on the plate thickness:

γ*d*≪1,

(15)

where  $\gamma$  is the attenuation coefficient of sound.

These conditions are easily satisfied in the rf range in metals at low temperatures. Standing acoustic waves across the plate thickness are accompanied by resonance singularities in the frequency dependences of the surface impedance of the plate.

The dispersion of the velocity of sound was determined by us from changes in the frequencies of acoustic resonances in a plate caused by the application of a magnetic field. Monotonic dependences of the velocity of sound on the magnetic field were recorded using apparatus which was a variant of the rf Bloch bridge.<sup>13</sup> The physical principle of the bridge method of measuring the resonance singularities of the surface impedance is the great change in the skin layer thickness due to the excitation of standing acoustic waves in a plate. The appearance of a resonance correction to the self-induction signal between the coils is due to the fact that the magnetic flux passing through a metal sample at a frequency  $f_n$  includes a term proportional to the magnetic induction in the sample multiplied by the acoustic wave amplitude.

When measurements were made by the bridge method, one of the coils surrounding a sample was subjected to a signal from a commercial rf oscillator; a record was obtained while the frequency of this resonator was varied continuously. In general, an acoustic resonance signal appearing in the detector coil on passage through  $f_n$  included the absorption and dispersion components. These signals were separated by the method of hf phase detection and the apparatus made it possible to record simultaneously signals due to the real and imaginary parts of the surface impedance of the plate.<sup>14</sup> The acoustic wave velocity of



FIG. 3. Record of Im Z for an acoustic resonance of transverse sound in a sample with  $n \parallel 100$ . H = 70 kOe, T = 4.2 K,  $q \parallel 100$ ,  $e \parallel 100$ .

suitable polarization was deduced from the maximum of the signal Re  $\Delta Z$  or from the central frequency of the dispersion signal Im  $\Delta Z$ . Measurements were made at the frequencies of the main acoustic resonances (n = 0) and for all the investigated samples these resonance frequencies were in the range 0.5-2MHz. An example of a record of an acoustic resonance of the transverse sound in a sample whose normal coincided with the [100] crystallographic axis is shown in Fig. 3.

Measurements of the amplitudes of quantum oscillations of the velocities of transverse and longitudinal sound were made with the aid of an "acoustic" oscillator, which was a device performing contactless excitation of sound in metals in a magnetic field. In this device the excitation and detection of acoustic vibrations in a plate were again achieved with the aid of two parallel coils surrounding the sample. The presence of resonance singularities in the transfer characteristic of such a system in a magnetic field made the system a self-excited oscillator operating at the resonance frequencies. It consisted of an amplifying stage coupled by a positive acoustic feedback to the sample and a variable attenuator controlled by a detector of a system for automatic regulation of the gain. One of the coils was used to excite acoustic waves in a plate and it was connected to the amplifier output. The other coil was connected to the input and it acted as a detector of an acoustic resonance. In the absence of a magnetic field the coefficient of the coupling between the coils was selected to be below the self-excitation threshold of the system. When the magnetic field was increased the coupling between the coils due to the mutual conversion of electromagnetic and acoustic energies in a metal increased and this resulted in oscillations at the frequency of standing acoustic waves across the thickness of the plate. The oscillation frequency was governed by the velocity of sound in the metal and the amplitude carried information on the attenuation of the acoustic waves. The system for



FIG. 4. Examples of records of the quantum oscillations of the velocity and attenuation of transverse sound in tin obtained by f' acoustic oscillator method.

automatic regulation of the gain in the self-excited oscillator was used to select correctly the signal proportional to the attenuation of sound and to prevent distortions of the useful signal on increase in the magnetic field. The operating principle of the device was described in detail in Ref. 15. Figure 4 shows simultaneous records of the quantum oscillations of the velocity and attenuation of the transverse sound in tin obtained by the acoustic oscillator method.

#### EXPERIMENTAL RESULTS AND DISCUSSION

#### 1. Monotonic dependences

We obtained information on the influence of magnetic fields on the velocities of propagation of transverse and longitudinal sound along all the principal crystallographic directions in tin. The dispersion of the velocity of longitudinal sound was studied in a magnetic field perpendicular to the direction of propagation  $q_1$  $\perp$  H; the dispersion of transverse sound was investigated in a field parallel to the direction of propagation  $\mathbf{q}_t \parallel \mathbf{H}$ . The main measurements were carried out in the field range 20-80 kOe. As shown in Fig. 3, when a standing acoustic wave was established in a plate, many additional resonance peaks were observed and these were excited by an rf magnetic field. The main reason for the fine structure of the acoustic resonances was the finite size of the plate in its plane. The problem of excitation of acoustic resonances in circular isotropic plates was solved in Ref. 16, but the problem of excitation of vibration resonances across the thickness of anisotropic disks was not solved. Consequently, a quantitative analysis of the experimentally recorded spectra of acoustic resonances was not carried out. When the magnetic field was increased, the resonance groups of each polarization of the excited sound were shifted toward higher frequencies. Measurements were made for one (strongest) resonance. The resonance frequency was determined to within  $\Delta f \approx 5 \cdot 10^{-6} f_n$ .

The experimentally determined dependences of the velocities of transverse and longitudinal sound on the square of the magnetic field are plotted in Fig. 5 for the crystallographic directions [001] and [110] directions. The same figure includes (dashed lines) the dependences calculated using the theory of Alpher and Rubin in the limit  $\delta/\lambda \ll 1$ . The values of the dispersion coefficient of the velocities of sound in a magnetic field

$$k = \frac{S(H) - S_0}{S_0 H^2} = \frac{\Delta S}{S_0 H^2}$$

were determined in each case from an analysis of the experimental dependences made by the least-squares method.

Table II gives the experimental values of the dispersion coefficients of the velocities of transverse and longitudinal sound along the principal crystallographic directions in tin, as well as the estimates obtained from the magnetohydrodynamic Alpher-Rubin theory.

Along all the principal crystallographic directions the dispersion coefficients of longitudinal sound were 15-20% greater than the theoretical values. The experi-



FIG. 5. a) Dispersion of the velocity of transverse  $q \parallel [001]$ ,  $e \parallel [100]$  (1) and longitudinal  $q \parallel [001]$ ,  $e \parallel [001]$  (2) sound in a magnetic field. b) Dispersion of the velocity of transverse  $q \parallel [110]$ ,  $e \parallel [11\overline{0}]$  (1) and longitudinal  $q \parallel [110]$ ,  $e \parallel [110]$  (2) sound in a magnetic field. Here, 1' and 2' are the theoretical dependences.

mental dispersion coefficients of transverse sound were usually less than the theoretical estimates. In the case of propagation of transverse sound with q  $\|[100], e \|[110]$  the dependence of the velocity on the magnetic field was anomalously weak and in this case the experimental and theoretical values of the dispersion coefficient differed by almost an order of magnitude. Rayne and Chandrasekhar<sup>10</sup> observed an anomalously strong temperature dependence of the velocity of sound for the same direction and polarization of transverse sound. These large discrepancies in the case of the temperature and field dependences of the velocity of sound cannot be explained by the magnetohydrodynamic theory of Alpher and Rubin. A quantitative description of these dependences requires a microscopic theory allowing for the special features of the energy spectrum and elastic properties of the investigated material. The influence of a magnetic field on the velocity of transverse sound at right-angles to the field in crystals of different symmetries was considered theoretically by Kontorovich.<sup>17</sup>

However, the  $q_t \parallel H$  case investigated by us was not considered. Kontorovich showed also that in the absence of a magnetic field in that range of frequencies where the skin layer thickness  $\delta$  is comparable with the acoustic wavelength  $\lambda$  there is a characteristic resonance which increases greatly the dispersion of the velocity of sound. This increase is due to an enhancement of the contribution of transverse electric fields which appear in the course of propagation of transverse sound in a metal.

TABLE II. Experimental and theoretical values of dispersion coefficients of velocity of transverse and longitudinal sound in tin subjected to magnetic fields.

q	e	$S_0, 10^5 \text{ cm/sec}$ (Ref. 10)	$k_{exp}$ , 10 <sup>-14</sup> Oe <sup>-2</sup>	$k_{\text{theor}}, 10^{-14} \text{ Oe}^{-2}$
[100]	[100]	3.360	5.87±0.62	4,86
	[001] *	1.924	17.6±1.2	14,8
[110]	[010]	1,969	$12.7\pm0.9$	14,2
	[110]	3,684	$4.82\pm0.73$	4,03
	[001] *	1,927	$9.97\pm0.74$	14 7
[001]	[110]	1.321	5.27±0.38	31.3
	[001]	3.764	4.42±0.67	3.86

\* The dispersion coefficients were deduced from the slopes of the experimental dependences in fields up to 40 kOe.

In our study of the propagation of transverse sound along a magnetic field the conditions were such that the skin layer thickness in the plate was of the same order of magnitude as the acoustic wavelength. In this  $\delta \sim \lambda$ case the amplitude of an acoustic resonance in the plate reached its maximum and it decreased monotonically on further increase of the magnetic field.<sup>18</sup> This maximum of the field dependence of the acoustic resonance amplitude showed that the currents flowing on the opposite sides of the plate began to compensate each other effectively. The skin layer thickness then became comparable with the half-thickness of the plate. The effect was observed for two polarizations of transverse sound identified by an asterisk in Table II. The experimentally determined dependences S(H) then deviated from the Alpher-Rubin dispersion law. Figure 6 shows the experimental dependences of the velocity of transverse sound for  $q \parallel [100]$  and  $e \parallel [001]$  and also for  $q \parallel [110]$  and  $e \parallel [001]$  on the square of the magnetic field.

The magnetohydrodynamic theory gives for these two cases identical values of the dispersion coefficient in the limit  $\lambda/\lambda \ll 1$ ; the theoretical dependence is shown dashed in Fig. 6. Deviations from the quadratic dependence in the course of propagation of transverse sound along the field and saturation in strong field are of nonresonance nature and can be explained qualitatively by a strong increase in the thickness of the metal skin layer and by a corresponding increase in the correction term in the denominator of Eq. (2).

## 2. Quantum oscillations of the velocities of transverse and longitudinal sound

At the excitation frequencies of standing acoustic waves in a plate there was a strong enhancement of the amplitude of quantum oscillations of the surface impedance. The main role in this enhancement was played by oscillations of the velocity of sound.<sup>19</sup> Both measurement methods used in the present study (rf bridge and acoustic oscillator) made it possible to separate the contributions of the velocity and attenuation oscillations to the enhancement and to measure separately these quantities. A study of quantum oscillations of the velocities of transverse and longitudinal sound was made mainly using the acoustic oscillator method.

Oscillations of the velocity of acoustic waves were observed for extremal sections of all the bands of tin,



FIG. 6. Dispersion of the velocity of transverse sound in two cases  $q \parallel [100]$ ,  $e \parallel [\overline{0}01]$  (2) and  $q \parallel [110]$ ,  $e \parallel [001]$  (3) in a magnetic field. Here, line 1 represents a calculation based on the Alpher-Rubin theory.

but the fullest quantitative data were obtained for the extremal sections  $\delta_1^1$  and  $\delta_1^2$  in the third hole band and for  $\tau_2^1$  in the sixth electron band of tin. We plotted in Fig. 7 on the same scale the experimental records of the quantum oscillations of the velocity of longitudinal and of two polarizations of transverse sound in a sample whose normal coincided with the [100] axis. The difference between the amplitudes of the observed oscillations demonstrated different nature of the interactions of transverse and longitudinal sound with the conduction electrons. In the case of the shear strain which did not produce a local change in the crystal lattice volume the amplitude of the velocity oscillations of transverse sound was an order of magnitude less than the amplitude of the velocity oscillations of longitudinal sound.

Oscillations of the velocity of longitudinal sound were observed for all the investigated crystallographic directions. Measurements were made in a magnetic field parallel to the crystallographic axes [100] and [001]. In a field  $H \parallel [110]$  our method failed to detect oscillations of the velocity of sound. Moreover, oscillations of the velocity of transverse sound of either polarization were not observed for a sample whose normal to the plane coincided with the [110] axis. The



FIG. 7. Quantum oscillations of the velocity of longitudinal and two polarizations of transverse sound in a sample with  $n \parallel [100]$ . T = 4.2°K.

TABLE III. Quantum oscillations of velocities of transverse and longitudinal sound in tin (T = 4.2 K, H = 70 kOe).

q, e	e <sub>ij</sub>	C <sub>ij</sub>	Extremal section	$\begin{array}{c} (\Delta S^{\rm osc}/S_{0}) \\ \times 10^{-5} \end{array}$	$S_0 \Delta S^{\text{osc}}$ × 10 <sup>a</sup> cm <sup>2</sup> /sec <sup>2</sup>
		<b>H</b> [100]			· .
<b>q</b> [010] e [010] <b>q</b> [001] e [001] <b>q</b> [100] e [001] <b>q</b> [100] e [010]	e <sub>11</sub>	C 11	τ2 <sup>1</sup>	48.0	0.54
	823	C33	$\tau_2$ <sup>1</sup>	49.3	0.70
	8.1	C.	τ2 <sup>1</sup>	12.1	0.045
	868	C 88	$\tau_2{}^i$	1.3	0 005
	•	<b>H</b> 11 [001	]	,	
q [100]	8 <sub>11</sub>	Cii	$\delta_1^1$ $\delta_2^2$	34,6 19,9	0.38
<b>q</b> [110] <b>e</b> [110] <b>q</b> [001] <b>e</b> [100]	e <sub>xx</sub>	$\frac{1}{2}(C_{11}+C_{12})+C_{66}$	$\delta_1^1$ $\delta_1^2$	36,6 21,9	0,50 0,30
	£44	C	$\delta_1^1$ $\delta_1^2$	18.3 10.6	0.07 0.04

experimentally determined amplitudes of the quantum oscillations of the velocity of sound for the extremal cross sections mentioned above are listed in Table III. This table shows also the corresponding components of the strain and elastic moduli tensors, and also the values of the product  $S_0 \Delta S^{osc}$  governing the deformation anisotropy of the Fermi surface. The amplitudes of the quantum oscillations of the velocities of transverse and longitudinal sound given in this table were determined at T = 4.2 °K in a magnetic field H = 70 kOe. (In the case of longitudinal sound traveling along a [110] axis we used the nominal notation of the component of the strain tensor  $\varepsilon_{xx}$ .)

According to Eqs. (11) and (12) and according to the data in Table III, the deformation anisotropy of the Fermi surface for the extremal sections  $\delta_1^1$  and  $\delta_1^2$  in the third hole band and for the extremal cross section  $\tau_2^1$  in the sixth band are given by the ratios

$$\begin{split} &\delta_1^{*}(D_{11}+1):(D_{22}+1):(D_{41}-1)=&0.87:1:0.37,\\ &\delta_1^{*}(D_{11}+1):(D_{22}+1):(D_{41}-1)=&0.85:1:0.36,\\ &\tau_2^{*}(D_{11}+1):(D_{33}+1):(D_{41}-1):(D_{66}-1)=&0.88:1:0.25:0.08. \end{split}$$

The ratios of the different values of  $D_{ij}$  for each energy band govern the reaction of the Fermi surface to the crystal lattice deformation. Magnetoacoustic measurements made it possible to study not only the diagonal components of the tensor  $\Lambda_{ij}$  (as, for example, in the hydrostatic pressure experiments), but also the shear components.

The relationships obtained together with the data of Table I described, in principle, the structure of the deformation potential for the selected extremal sections of the Fermi surface. Unfortunately, the data on the diagonal components of the deformation potential tensor were contradictory, so much so that different signs of the same components of the tensor  $D_{ij}$  were obtained in different experiments. Therefore, we did not include the absolute values of the various components of the deformation potential tensor in the tabulated data.

The authors are grateful to A. P. Perov for his help in the development of an acoustic oscillator, and to V. M. Kontorovich and M. I. Kaganov for valuable discussions.

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Translated by A. Tybulewicz