On the depolarization of negative muons in hydrogen

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The magnitude of the residual polarization of μ^- mesons stopping in hydrogen (protium or deuterium) is computed. The rate of Auger collisions between the excited mesic atoms and the target atoms is computed in the eikonal approximation; it turns out be significantly lower than the rate computed earlier in the Born approximation. The dependence of the Auger-collision rate on the target density leads to the result that the residual polarization of muons varies somewhat as the gas pressure is varied. The available experimental data on the depolarization of muons in hydrogen are discussed.

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§1. INTRODUCTION

The study of the depolarization of negative muons slowing down in hydrogen is of great interest. From the magnitude of the residual polarization of μ^- muons we can judge the relative contributions of the processes determining the mechanism underlying cascade depolarization, which in the case of hydrogen have important characteristics in view of the significant role of the collisions of the mesic atoms with the atoms of the medium. The exchange collisions of mesic atoms in the ground state induce a transition within the hyperfine doublet of the K shell,[1] so that the measurement of the magnitude of the muon polarization as a function of the time allows the determination of the rate of these exchange transitions. This information is important for the interpretation of experiments on the study of the probability for μ capture in hydrogen.[2] The investigation of μ capture by protons (nuclei) is of importance in connection with the verification of the structure of the weak hadron current; also important, besides the study of the probabilities for μ capture, is the measurement of the correlation characteristics of μ capture, among which the correlations with the muon spin, which depend on the magnitude of the residual polarization of the muon, are important.[2,3]

The data on the rate of transition between the sublevels of the K shell are important in the study of the catalysis of nuclear reaction by μ mesons,[4] which has lately aroused special interest in connection with the theoretically predicted[5] and experimentally observed[6,7] resonance mechanism of formation of the molecules $dd\mu$ and $dt\mu$. In studying the kinetics of the processes of muonic catalysis, we should take into consideration the hyperfine structure of the levels of the mesic molecules produced,[8] the population of the states of the hyperfine structure of mesic molecules being essentially dependent on the rate of transitions within the hyperfine doublet of the K shell of the mesic atoms.

The present paper is devoted to the consideration of the process of depolarization of muons in hydrogen (protium and deuterium). A similar problem was solved earlier[9-15] for arbitrary mesic atoms, but mesic hydrogen atoms possess definite characteristics connected with their electrical neutrality. This is the cause of the different—from normal—relation between the various mechanisms underlying the de-excitation of mesic atoms, which in the present case can penetrate without hindrance into other atoms. At some stage of the transitions of mesic atoms, the most important role in the de-excitation is played by the external Auger effect, i.e., the knocking out of electrons from the target atoms. The rate of this process was computed earlier,[16,18] but, as we shall see, to find the residual polarization in gaseous hydrogen, we need to refine the existing calculations. Therefore, in §3 we compute the rate of Auger de-excitation of mesic atoms with the aid of the eikonal method, which possesses a broader region of applicability than the usual Born approximation. In the final section (§4) we discuss the results obtained and compare them with the available experimental data.

§2. DEPOLARIZATION OF NEGATIVE MUONS

It is essential to the depolarization of negative muons that there occur in the de-excitation of the mesic atom a stage at which the important role is played by the spin–orbit interaction.[1)] A characteristic of the electrically neutral mesic hydrogen atom is the fact that it does not possess an electron shell, since, upon being captured into a high-lying level of the mesic atom with a principal quantum number $n \approx 14$, the muon replaces the only electron of the hydrogen atom. Therefore, only the processes that occur during the collisions of the mesic atom with the target atoms (molecules) can compete with the radiative cascade transitions occurring in the mesic atom during its de-excitation.

The decisive role in the de-excitation of the mesic atom immediately after the μ-meson capture is played by the chemical reaction involving the dissociation of the target molecules[16]:

$$p\mu + H_2 \to p\mu + H + H, \tag{1}$$

where by p and H we mean either of the hydrogen isotopes. But for $n \leqslant 10$, the most probable mechanism for the de-excitation of the mesic atoms is the target-molecule-ionization mechanism, i.e., the external Auger effect:

$$p\mu + H_2 \to p\mu + H_2^+ + e^-. \tag{2}$$

At lower levels of the mesic atom, starting from some $n = n_0$ the dominant process (besides the above-

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mentioned transitions within the hyperfine doublet of the K shell) is radiative de-excitation.

In the case of the cascade transitions in the mesic atoms, the μ-meson depolarization occurs at those levels whose width Γ is small compared to the magnitude of the fine level splitting Δ. For any level of the mesic atom the radiative width $\Gamma_\gamma \approx \alpha\Delta$ (α is the fine-structure constant), i.e., is always significantly smaller than the magnitude of the fine level splitting.[9,15] This means that, during the radiative transitions between the levels, the muon spin manages over a period of time equal to the lifetime at the fine-structure level to turn many times around the direction of the total angular momentum j of the mesic atom as a result of the spin–orbit interaction, i.e., depolarization should occur. But the radiative width is not the decisive factor determining the lifetime of the muon at high-lying levels of the mesic atom. For light mesic atoms the internal Auger effect is the most effective mechanism for the de-excitation of mesic atoms right down to small values of n, the conversion coefficient $\gamma = \Gamma_a/\Gamma_\gamma$ (Γ_a is the probability for the internal Auger effect) for $n > 3-4$ being large ($>1/\alpha$). Thus, the transitions resulting from the internal Auger effect occur fairly rapidly in comparison with the precession of the muon spin, and the depolarization of the μ mesons does not occur.

For mesic hydrogen atoms the external-Auger-effect-related mechanism of de-excitation is not so well-defined: its effectiveness depends on the target density. As has been shown in a number of papers,[16-18] the Auger collision mechanism is decisive down to $n_0 \approx 4-5$ for liquid and gaseous targets under normal pressure. Nevertheless, in the case of gaseous targets at pressures of up to several tens of atmospheres the Auger-collision rate Γ_e turns out to be insufficiently high in comparison with the muon-spin precession rate starting from some $n = n_1$. Thus, muon depolarization occurs also in Auger collisions of excited mesic atoms.

The magnitude of the residual polarization of muons in the K shell was found earlier[14] as a function of the value of the initial orbital angular momentum l of the depolarizing cascade (in the absence of hyperfine level splitting):

$$\lambda_l = \frac{3}{10}\frac{l+3}{2l+1} \quad (l \neq 0). \tag{3}$$

Averaging this quantity over the possible values of l under the assumption that the levels are statistically populated,[16,17] we obtain

$$\bar{\lambda} = \sum_{l=1}^{n-1} \frac{2l+1}{n^2}\lambda_l = \frac{3}{20n^2}(n+6)(n-1) \tag{4}$$

(this formula is inapplicable for $n = 1$, since it neglects the contribution of the states with $l = 0$; this contribution is proportional to n^{-2}, and is therefore insignificant when $n \geq 3$). It can be seen that for $n \geq 3$ the quantity $\bar{\lambda}$ perceptibly depends on the choice of the value of $n = n_1$.

Allowance for the hyperfine interaction in the K shell under conditions of statistical population of the levels with $F_\pm = I \pm \frac{1}{2}$ (I is the nuclear spin of the mesic atom) leads to the following values for the residual polarization at these levels (in the absence of transitions between the levels)[11]:

$$\lambda_+ = \frac{\bar{\lambda}(2I+3)(I+1)}{3(2I+1)^2}, \quad \lambda_- = \frac{\bar{\lambda}I(2I-1)}{3(2I+1)^2}. \tag{5}$$

We should also take into account the hyperfine splitting, which may affect the polarization, of the excited levels of the mesic atom.[13-16] The ratio of the hyperfine splitting is a quantity $\sim m_\mu/Zm_p$ (m_μ is the μ-meson mass; m_p, the nucleon mass; and Z, the nuclear charge), so that for hydrogen the hyperfine splitting is roughly an order of magnitude greater than the radiative width of a level of the mesic atom. Since the ratio of the width of the Auger transitions to the fine (or hyperfine) splitting very critically depends on n (see Fig. 1), but the extent of the depolarization changes insignificantly when n is changed by one, it can be assumed that, for $n \leq n_1$, both the fine and hyperfine splittings are greater than the total level width, whereas the inverse relation obtains for $n > n_1$. This means that depolarization does not occur when $n > n_1$, and that the fine and hyperfine interactions are switched on simultaneously starting from $n = n_1$.

In Table I we present the residual muon polarization values $\lambda_\pm$ obtained for the depolarizing cascade starting from $n = n_1$ with the use of the data of Ref. 14 under the assumption that the levels with different l are statistically populated.

Of the effects neglected by us, that possibly play a definite role in the depolarization of the muon, we should mention the Stark effect occurring in the collision of a mesic atom with an atom, i.e., the effect of the electric field of the atom on the mesic atom that has penetrated into it.[2)] The estimate for the time of flight of a mesic atom through an atom, $\tau \sim a_0/v \sim 10^{-14}$ sec [a_0 is the Bohr radius of the atom and $v \sim 10^6$ cm/sec is the speed that the mesic atom acquires as a result

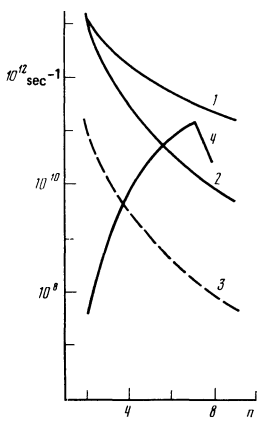

FIG. 1. Dependence on n of the magnitude of the fine splitting (curves 1 and 2), the radiative transition rate (curve 3), and the Auger-transition rate (curve 4) for the mesic atom $d\mu$ at a pressure of 10 atm. The curve 1 pertains to the p states, the curves 2 and 3 pertain to the circular ($l = n-1$) orbits, and curve 4 is the result of the averaging over the states with different l.

TABLE I. Values of the residual polarization of the muons in mesic protium ($\lambda_- = 0$) and deuterium atoms as a function of the principal quantum number n_1 starting from which the depolarizing cascade is switched on.

n_1	λ_+		λ_-
	Protium	Deuterium	
2	0.1596	0.1337	0.0100
3	0.1534	0.1251	0.0064
4	0.1392	0.1116	0.0028
5	0.1272	0.1006	0.0001
6	0.1177	0.0920	−0.0020
7	0.1103	0.0854	−0.0037
8	0.1044	0.0802	−0.0050
9	0.0996	0.0758	−0.0060
10	0.0956	0.0723	−0.0068

of the chemical dissociation of the molecules (1)] shows that this time is significantly shorter than the characteristic time of the precession of the muon spin in the excited states of the mesic atom ($\Delta^{-1} \gtrsim 10^{-13}$ sec). Therefore, it is unlikely that the Stark depolarization will be substantial, although, perhaps, the present estimate is too crude for the final conclusion to be drawn.

§3. COMPUTATION OF THE RATE OF AUGER DE-EXCITATION OF MESIC ATOMS

The rate of Auger de-excitation of mesic atoms in liquid hydrogen has been computed by Leon and Bethe[16] in the Born approximation. The use of this approximation is justified in those cases in which the rate of the process is relatively low (the cross section does not exceed the geometrical cross section). But the condition of application of the Born approximation can be strongly violated at high degrees of excitation of the mesic atoms, when the transition energies are small. Leon and Bethe noted this themselves, but they were primarily interested in the slowest of the cascade transitions (for the purpose of determining the lifetime of the π meson in a mesic atom), and as for the fast transitions, they were content with a rough estimate. Meanwhile, for our purposes, knowledge of the probabilities for the highly excited transitions is what is most important, since it is precisely these transitions that determine that value of $n = n_1$ from which the muon depolarization starts. Therefore, we shall determine these probabilities with the aid of the eikonal method,[22] which is often used in inelastic-collision calculations in atomic physics,[23,24] or, more exactly, its simplest variant: the quasistatic approximation.[3)]

In the quasistatic approximation the atoms are assumed to be moving classical straight-line trajectories and the rate $w(\mathbf{R})$ of the process under investigation as a function of the relative interatomic distance $\mathbf{R}$ is determined with the aid of perturbation theory as if the atoms at a given moment were fixed. The cross section for the process has the form

$$\sigma = \int d^2\rho (1 - e^{-I(\rho)}), \tag{6}$$

$$I(\rho) = \frac{1}{v}\int_{-\infty}^{\infty} dz w\left(\rho + z\frac{\mathbf{v}}{v}\right), \tag{7}$$

where v is the relative velocity of the atoms. The integrand in (6) has the meaning of the total transition probability for a given value of the impact parameter ρ. In the Born approximation, to which corresponds the expansion of the exponential function up to the first term, the analogous role is played by the quantity $I(\rho)$; therefore, the condition for its applicability is $I(\rho) \ll 1$. In our case the order-of-magnitude estimates yield

$$w \sim \frac{e^2}{a_0}\left(\frac{a_\mu}{a_0}\right)^2, \quad I(\rho) \sim \frac{e^2}{v}\left(\frac{a_\mu}{a_0}\right)^2$$

where e is the electron charge and a_μ is the radius of the excited mesic atom (we assume $\hbar = 1$). For the highly excited states of the mesic atom $a_\mu \sim a_0$; at the same time, for $v \approx 10^6$ cm/sec the value of $e^2/v \approx 200$. This means that in order of magnitude $I(\rho) \gg 1$. Therefore, the Born approximation should give overestimated values, although specific calculations do not lead to a discrepancy as great as might be expected on the basis of the above-presented rough estimate.

In order to judge the degree of accuracy of the quasistatic approximation used by us, we shall determine somewhat more accurately the connection between the formula (6) and the eikonal method. The expression (6) corresponds entirely with the eikonal approximation in the case in which the initial states of the atoms are not degenerate. As a rule, however, there exist an entire set of initial stages with the same energy; in our case these are the states of the mesic atom with a given n and different l and m. The inelastic cross section in the eikonal approximation then has a somewhat different form (the subscript i numbers the degenerate initial states):

$$\sigma_i = \int d^2\rho (1 - |S_{ii}(\rho)|^2), \tag{8}$$

$$S(\rho) = P\exp\left[-\frac{i}{v}\int_{-\infty}^{\infty} dz U\left(\rho + z\frac{\mathbf{v}}{v}\right)\right]. \tag{9}$$

The difference between (6) and (7) is that now the "eikonal phase" $S_{ij}(\rho)$ is a matrix; its determination from the formula (9), where P exp denotes a z-ordered exponential function, is equivalent to the solution of the system of differential equations

$$iv\partial S_{ij}/\partial z = U_{ii'}S_{i'j} \tag{10}$$

with the initial condition $S_{ij}(z = -\infty) = \delta_{ij}$ and the subsequent passage to the limit as $z \to +\infty$. The matrix $U_{ii'}$, which plays the role of a non-Hermitian Hamiltonian in Eq. (10), is determined by the perturbation potential V according to the relation

$$U = V - \frac{i\Gamma}{2}, \quad \Gamma_{ii'} = 2\pi\sum_f V_{if}V_{fi'}\delta(E_i - E_f), \tag{11}$$

where the subscript f numbers the final states (which contains an electron in the continuous spectrum). The diagram elements of the matrix Γ, as can be seen from (11), coincide with the rates w for the transitions from the corresponding states in first-order perturbation theory.[28] Therefore, if the matrix U is diagonal for all z, so that can drop the ordering symbol P in the formula (9), then the expressions (8) and (9) with allowance for the Hermitian character of the operator V go over into (6) and (7). Below we shall discuss the

question: In what sense can we consider U to be diagonal? (Strictly speaking, this matrix is never diagonal.)

Generally speaking, the application of the eikonal approximation requires the fulfillment of two conditions: first, the wavelength of the colliding particles should be small compared to the interaction range, and, secondly, the interaction potential V should be small compared to the kinetic energy E_k. The first condition is characterized by the parameter $\zeta = 1/m_p v a_0$, which, for the velocity $v = 10^6$ cm/sec, has a value of the order of 1/10. As to the second condition, it is in our case formally not fulfilled, but it can be significantly weakened. The point is that, as follows from (6), it is not necessary to require a high degree of accuracy in the region where $I(\rho) \gg 1$, since here the integrand becomes equal to one, and the error in the determination of the small exponential function does not play any role. Therefore, the condition $V \ll E_k$ should be fulfilled only for $I(\rho) \lesssim 1$.

The perturbation operator V in our case is the mesic-atom-atom interaction potential: (12)

$$V = \frac{e^2}{R} - \frac{e^2}{|\mathbf{R}-\mathbf{r}_\mu|} - \frac{e^2}{|\mathbf{R}+\mathbf{r}_e|} + \frac{e^2}{|\mathbf{R}+\mathbf{r}_e-\mathbf{r}_\mu|} \approx -\frac{e^2(\mathbf{r}_\mu\mathbf{R})}{R^3} + \frac{e^2\mathbf{r}_\mu(\mathbf{R}+\mathbf{r}_e)}{|\mathbf{R}+\mathbf{r}_e|^3},$$

where r_μ and r_e are the coordinates of the muon and the electron as measured from "their" nuclei and R is the distance between the nuclei. In the last expression we have used the approximation $r_\mu \ll R, r_e$, which is justified even for fairly highly excited states of the mesic atom.[16] According to (11), the transition rate is given by the expression

$$w_i(\mathbf{R}) = \Gamma_{ii}(\mathbf{R}) = 2\pi \int \frac{d^3k}{(2\pi)^3} |V_{fi}(\mathbf{R})|^2 \delta\left(\frac{k^2}{2m_e} - \Delta E\right) = \frac{km_e}{(2\pi)^2} \int d\Omega_k |V_{fi}(\mathbf{R})|^2, \quad (13)$$

where k is the momentum of the conversion electron, m_e is the electron mass, ΔE is the energy yield, equal to the difference between the mesoatomic-transition energy $(E_n - E_{n'})$ and the ionization potential δ_H of the hydrogen atom (molecule).

The matrix element $V_{fi}(\mathbf{R})$ can be written in the form

$$V_{fi}(\mathbf{R}) = e^2 \mathbf{M} \mathbf{N}(\mathbf{R}), \quad (14)$$

$$\mathbf{M} = \int d^3r_\mu \psi_f^* \mathbf{r}_\mu \psi_i, \quad \mathbf{N}(\mathbf{R}) = \int d^3r_e \varphi_f^* \frac{\mathbf{R}+\mathbf{r}_e}{|\mathbf{R}+\mathbf{r}_e|^3} \varphi_i, \quad (15)$$

where $\psi_{i,f}$ and $\varphi_{i,f}$ are the wave functions of the muon and the electron in the initial (i) and final (f) states [the first term on the right-hand side of (12) drops out upon being integrated over the electron coordinates]. The wave function φ_f of the continuous spectrum is assumed to be normalized to one particle in a unit volume. As the wave function φ_i of the electron in the initial state, we choose the wave function of the ground state of the hydrogen atom:

$$\varphi_i = (\pi a_0^3)^{-1/2} \exp(-r_e/a_0). \quad (16)$$

As in (16), allowance for the molecular state will amount to our taking for δ_H the magnitude of the ionization potential of the hydrogen molecule (15.2 eV), and not that of the atom (13.6 eV).

In accordance with the foregoing, large distances for which $I(\rho) \lesssim 1$ will be of greatest interest to us. Therefore, let us consider the limit $R \to \infty$ in the formulas (13)–(15):

$$\mathbf{N}(\mathbf{R}) \approx N' \frac{k\mathbf{R}^2 - 3\mathbf{R}(\mathbf{kR})}{kR^5}, \quad w_i(\mathbf{R}) \approx \frac{kme^4}{\pi} N'^2 \frac{|\mathbf{M}|^2R^2 - 3|\mathbf{MR}|^2}{3R^8}, \quad (17)$$

$$N' = \frac{1}{k} \int d^3r_e \varphi_f^* \mathbf{k}\mathbf{r}_e \varphi_i. \quad (18)$$

Assuming that $a_\mu \sim a_0$, $k \sim a_0^{-1}$ at high degrees of excitations, we find that in order of magnitude

$$N' \sim a_0^{5/2}, \quad M \sim a_0, \quad V \sim \frac{e^2a_0^2}{R^3}, \quad w(\mathbf{R}) \sim \frac{e^2a_0^5}{R^6}, \quad I(\rho) \sim \frac{e^2}{v}\left(\frac{a_0}{\rho}\right)^5. \quad (19)$$

For $I(\rho) \sim 1$, i.e., for $R \sim \rho \sim a_0(e^2/v)^{1/5}$, we shall have

$$\frac{V}{E_k} \sim \frac{e^2}{a_0}\left(\frac{e^2}{v}\right)^{-2/5} \frac{1}{m_p v^2} = \zeta\left(\frac{e^2}{v}\right)^{2/5}, \quad \zeta = \frac{1}{m_p v a_0}. \quad (20)$$

It therefore follows that for the eikonal approximation to be applicable, it is sufficient that the condition $\zeta(e^2/v)^{2/5} \ll 1$ be fulfilled besides the condition $\zeta \ll 1$, whereas the requirement that $V \ll E_k$ for any R would imply that $\zeta e^2/v \ll 1$, which in our case is clearly incorrect. To be sure, for $\zeta = 0.1$ and $e^2/v \approx 200$ the parameter $\zeta(e^2/v)^{2/5} \approx 1$, but since we are not striving for a high computational accuracy, we can assume that this will be sufficient for our purposes.

The mesoatomic matrix element $\mathbf{M}$ in (15) has the well-known form[16]

$$M_\alpha = a_0 \frac{m_e}{m_\mu} C_{l\,m_1\,1\alpha}^{l_1 m_1} C_{l_1 0\,1 0}^{l_1 0} \left(\frac{2l+1}{2l_1+1}\right)^{1/2} R_{nl}^{n_1 l_1}, \quad (21)$$

where the subscript α numbers the cyclic vector components ($\alpha = 0, \pm 1$); R::: is the radial integral, the explicit form of which is given in Ref. 29 (the numbers n, l, and m pertain to the initial state of the mesic atom; the numbers n_1, l_1, and m_1, to the final state). Analysis of the expression (21) allows us to follow the transition from the eikonal approximation (8), (9) to the quasistatic one (6), (7).

Using the expressions (21) and (17), we can find with the aid of (14) and (11) the matrix $\Gamma_{ii'}$ (by i is meant the set of quantum numbers n, l, m), and verify that it contains, besides the diagonal elements, elements corresponding to transitions in which l changes by 2. The latter elements arise as a result of the fact that there remains in the product $N_\alpha N_\beta^*$ after integration over k the tensorial term $R_\alpha R_\beta$, which leads to the dependence of Γ on the direction of the vector R; this dependence should disappear only after the cross section (8) has been averaged over the initial states. We shall, however, not introduce a large error if we carry out the averaging over the directions of R directly in the expression for Γ by replacing $R_\alpha R_\beta$ by $R^2\delta_{\alpha\beta}/3$. The matrix Γ, on account of the orthogonality relations for the Clebsh-Gordon coefficients, then becomes diagonal; exact calculations carried out in Ref. 27 for a particular case indeed confirm the smallness of its off-diago-

nal elements. Furthermore, let us note that for large R we can discard the Hermitian part of V in the expression (11) for U, since $V_{ii'}$ is proportional to the integral

$$\int d^3r_e \left(\frac{\mathbf{R}+\mathbf{r}_e}{|\mathbf{R}+\mathbf{r}_e|^3}-\frac{\mathbf{R}}{R^3}\right)|\varphi_i|^2,$$

which falls off exponentially with increasing R. After this, the matrix $U=-i\Gamma/2$ also becomes diagonal, and the formulas (8) and (9) go over into (6) and (7).

The Coulomb wave function φ_f of the electron in the final state has the form[28]

$$\varphi_f=\exp(\pi/2ka_0)\Gamma\left(1+\frac{i}{ka_0}\right)\exp(i\mathbf{kr}_e)F\left(-\frac{i}{ka_0},1,-i(kr_e+\mathbf{kr}_e)\right). \quad (22)$$

The substitution of this expression into (15) gives rise to an integral, the exact computation of which is complicated and inexpedient, bearing in mind the degree of accuracy of our approximation. At the same time, it would not have been sufficient for us to have limited ourselves to the consideration of the limit $R\to\infty$ if we wanted our calculations to remain valid in the region of applicability of the Born approximation, since for large k small distances are important. Therefore, we shall consider separately the two cases $ka_0\sim 1$ and $ka_0\gg 1$, each of which can be simplified.

For $ka_0\sim 1$, as we shall see, large distances ($R\sim a_0(e^2/v)^{1.5}\gg a_0$) are important, so that we can use the formulas (17) and (18).

After carrying out a partial-wave expansion[28] of the wave function (22), we can notice that only the term with $L=1$ makes a contribution to the integral (18); therefore, (22) can be replaced by the following expression

$$\varphi_f^{(1)}=\exp\left(\frac{\pi}{2ka_0}\right)\Gamma\left(2-\frac{i}{ka_0}\right)i\mathbf{kr}_e\exp(-i\mathbf{kr}_e)F\left(2+\frac{i}{ka_0},4,2i\mathbf{kr}_e\right). \quad (23)$$

The integral (18) can then be computed explicitly[28]:

$$N'=\frac{16ka_0^2(\pi a_0^3)^{1/2}}{(1+k^2a_0^2)^3}\exp\left(-\frac{2}{ka_0}\operatorname{arctg}ka_0+\frac{\pi}{2ka_0}\right)\Gamma\left(2-\frac{i}{ka_0}\right). \quad (24)$$

Averaging the expression (17) for w over the R directions in accordance with the foregoing, we obtain

$$w(R)=\gamma\left(\frac{a_0}{R}\right)^6,\quad I(\rho)=\frac{3\pi}{8}\frac{\gamma a_0}{v}\left(\frac{a_0}{\rho}\right)^5, \quad (25)$$

$$\gamma=\frac{2^{10}\pi e^2}{3a_0}\left(\frac{m_e}{m_\mu}\right)^2\cdot\frac{\exp[(-4/ka_0)\operatorname{arc\,tg}ka_0)]}{(1+k^2a_0^2)(1-\exp(-2\pi/ka_0))}(C_{l_0l_0}^{l_0})^2(R_{nl}^{n_il_i})^2. \quad (26)$$

If $ka_0\gg 1$, then the distances $R\lesssim a_0$ are important. In this case we can approximately compute N after replacing the function φ_f given by (22) by a plane wave, and assuming φ_i to be a slowly varying function at distances $\sim 1/k$:

$$\mathbf{N}\approx\int d^3r_e e^{-i\mathbf{kr}_e}\frac{\mathbf{R}+\mathbf{r}_e}{|\mathbf{R}+\mathbf{r}_e|^3}\varphi_i(\mathbf{r}_e)\approx\varphi_i(-\mathbf{R})\nabla_R\int d^3r_e\frac{e^{-i\mathbf{kr}_e}}{|\mathbf{R}+\mathbf{r}_e|}$$
$$=-\frac{4\pi i\mathbf{k}}{k^2}e^{i\mathbf{kR}}\varphi_i(-\mathbf{R}). \quad (27)$$

Using (16), (14), (13), and (7), we obtain

$$w(R)=\gamma_1\exp\left(-\frac{2R}{a_0}\right),\quad I(\rho)=\frac{2\gamma_1\rho}{v}K_1\left(\frac{2\rho}{a_0}\right), \quad (28)$$

$$\gamma_1=\frac{16e^2}{3ka_0^2}\left(\frac{m_e}{m_\mu}\right)^2(C_{l_0l_0}^{l_0})^2(R_{nl}^{n_il_i})^2. \quad (29)$$

Here $K_1(x)$ is a MacDonald function.

It is not difficult to combine the expressions (25) and (28) in order to obtain a function $w(R)$ that allows us to find the cross section with a high accuracy from the formula (6) for any k. We need only to first modify these expressions slightly, smoothing out the meaningless singularities [R^{-6} in (25) and k^{-1} in (29)]. One of the simplest ways of doing it is the following:

$$w(R)=\gamma\left(\frac{a_0^2}{R^2+b^2}\right)^3+\gamma_1\frac{(ka_0)^2}{1+(ka_0)^2}\exp\left(-\frac{2R}{a_0}\right). \quad (30)$$

The true behavior of $w(R)$ for $R\to 0$ and $k\to 0$ is not important here. Indeed, after the substitution into (7) and (6) only those ρ for which one of the terms in (30) is small compared to the other will turn out to be important. For $ka_0\sim 1$, the first term is the dominant term, and only its asymptotic form in the region $R\gg a_0$ is important; if, on the other hand, $ka_0\gg 1$, then the second term, which decreases with increasing k much more slowly than the first, is the dominant term. By varying the parameter $b\sim a_0$ in the formula (30), we can graphically verify that the magnitude of the cross section changes little in the process. In the computations we set $b=1.5a_0$ for the reason that for such a choice the expansion of the exponential function in (6) up to the first term yields roughly the same result as the exact computation in the Born approximation.[16] This agreement of course becomes strongly violated as the parameter b is varied, but the eikonal expression (6) is insensitive to this variation.

Let us say a few words about the dependence of the cross section computed by us on the velocity v of the mesic atom. In the Born approximation the de-excitation cross section is inversely proportional to v, and the transition frequency Γ_e does not depend on v. In our case, as the formulas (6) and (25) show, for $v\to 0$ (if we remain within the framework of the eikonal approximation), the cross section increases more slowly than $1/v$, specifically, in proportion to $v^{-2/5}$, while Γ_e decreases with decreasing v like $v^{3/5}$. Therefore, the lowering of the velocity of the mesic atom in comparison with the value 10^6 cm/sec adopted by us will decrease the probability for external Auger de-excitation and increase the deviation of our results from those obtained in the Born approximation.

§4. DISCUSSION OF THE RESULTS

The external Auger de-excitation rates for mesic atoms, computed in the eikonal approximation, and referred to the density $N_0=4.25\times 10^{22}$ cm^{-3} of liquid hydrogen,

$$\Gamma_e=\sigma v N_0\ [\text{sec}^{-1}], \quad (31)$$

are compared in Table II with the values obtained in Refs. 16 and 17 in the Born approximation (the results given are for mesic deuterium atoms; in the case of mesic protium atoms the Γ_e values are roughly 10–15% higher). As can be seen, the values for the principal

TABLE II. Values, referred to the liquid-hydrogen density, of the rate Γ_e (10^{10} sec^{-1}) of Auger collisions of mesic deuterium atoms during $n \to n'$ transitions when the mesic-atom velocity $v = 10^6$ cm/sec. The row a) corresponds to transitions from the states with the largest possible l (circular orbits); the row b), to the result of the averaging over the states with different l.

	n	2	3	4	5	6	7	8
	n'	1	2	3	4	5	6	6
Ref. 16	a)	0.51	19.2	160	722	2310	5910	–
	b)	0.38	13.1	105	460	1440	3640	220
Present paper	a)	0.51	18.7	144	516	1120	2090	–
	b)	0.38	12.8	96	355	827	1620	179

quantum number values $n \approx 6$, i.e., in the region that is important for the choice of the value of n_1, differ (by a factor of more than two). The values given in the rows b) were obtained by averaging the corresponding quantities for different l ($0 \leq l \leq n-1$) with weights equal to the statistical weights.[17] In Fig. 1 these values, as recalculated for gaseous deuterium at a pressure of 10 atm (the curve 4), are compared with the magnitudes of the fine splittings of the levels of the mesic atom (the curves 1 and 2), as well as with the radiative widths, which become dominant at small n. In comparing the level width and the fine level splitting, we can assume that the splitting is approximately given by the curve 2, which corresponds to large l, since the contribution of the small l values is statistically suppressed. The curve 1 is presented for the purpose of illustration; it shows that the value of n_1 for the cascades with small l values is higher than the value for typical cascades with large l values.

As the gas pressure is varied, the curve 4 moves vertically since, according to (31), Γ_e is proportional to the density, whereas the curves 1, 2, and 3 do not depend on pressure at all. Therefore, the value of n_1, which is roughly determined by the point of intersection of the curves 2 and 4, decreases with increasing pressure, while the magnitude of the residual polarization, as can be seen from Table I, increases in the process. Figure 2 shows the resulting pressure dependence of the residual polarization (no particular importance should be attached to the linearity of the graphs: the calculations are not so exact as to warrant a discussion of the detailed form of the dependence). The data pertain to polarization in the state with $F = I + \frac{1}{2}$; for deuterium in the state with $F = \frac{1}{2}$ the polarization turns

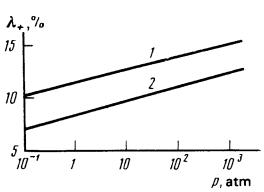

FIG. 2. Dependence of the residual polarization of μ mesons on the gas pressure in the case of total initial polarization and in the absence of transitions between the sublevels of the K shell. The curve 1 is for the protium; the curve 2, for deuterium.

out to be too small for it to be reliably estimated.

The results of measurements of the residual polarization of negative muons in gaseous hydrogen at a pressure of 40 atm are reported in Ref. 30. The small magnitude of the experimentally detected residual muon polarization leads the authors of that paper to the conclusion that the rate of transition within the hyperfine doublet of the K shell of the mesic atom is high, or that there are additional mechanisms for muon depolarization at the excited levels.

Our analysis of the cascade mechanism for muon depolarization does not predict for the muons that reach the K shell of the mesic atom a polarization significantly smaller than the estimated values, used in Ref. 30 (the difference is ~20%). Experiments on the measurement of negative-muon polarization in gaseous hydrogen at low pressures should, however, be performed in order to obtain reliable information about the various mechanisms underlying the depolarization of muons.

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[1)]We shall not touch upon here the questions of kinematic muon depolarization, which are discussed in Refs. 19–21.

[2)]Following the results of Refs. 16 and 17, we take into account the reestablishment of the statistical population of the levels as a result of the Stark mixing of the states with different l.

[3)]There are quite a large number of variants of the eikonal method, differing both in their degrees of accuracy and in their regions of applicability. Some variants applicable in atomic physics are listed in Ref. 25. The quasistatic approximation is discussed in Refs. 24, 26, and 27 in connection with the investigation of the ionization de-excitation of ordinary atoms (i.e., of the Penning process).

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