Magnetic channeling of neutrons in nonmagnetic crystals

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Features of the motion of neutrons through nonmagnetic crystals at resonant and higher velocities are discussed. It is found that a magnetic potential well arises in a reference system at rest with respect to a neutron moving through a crystal as a result of the relativistic transformation of the electrostatic field of the lattice, and its structure is investigated. The appearance of a set of plane potential wells substantially alters the initially linear trajectory and leads to controlled localization of the particles either near the barrier, where the lattice nuclei are located, or in the region between barriers. This effect tends to enhance or suppress nuclear reactions involving channeling neutrons or other neutral particles having a magnetic moment. The characteristic range of angles of the paraxial motion within which such channeling takes place is an order of magnitude smaller than in the case of channeling of charged particles.

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We have previously discussed the possibility of the channeling of neutrons and other electrically neutral particles having a magnetic moment in magnetic structures,[1] a possibility that is realized in the presence of dynamic or static regularly nonuniform changes in the magnetization. In this case the forces responsible for the channeling are due to the dipole-dipole interaction of the magnetic moment of the neutron with the magnetic moments of the lattice atoms.

We propose below an essentially different magnetic-channeling mechanism, which provides a much stronger interaction and consequently a larger acceptance angle for channeling. When we speak of neutrons in what follows, we mean to imply that the effect is applicable to any neutral particle having a magnetic moment.

The interaction energy between the neutron and the nonmagnetic lattice involves not only the purely nuclear potential V_n, but is also connected with features of the electrodynamics of moving media. In the coordinate system fixed to a neutron that is moving through the crystal in the electrostatic field E of the crystal atoms there is a magnetic field $\mathbf{H}=\mathbf{E}\times\mathbf{v}/c$. To estimate this field we note that the motion of the lattice nuclei and electrons in the rest system of the neutron is equivalent to a current I. The maximum strength of the field H produced by such a current, averaged over the plane, can be estimated with the aid of the integral theorem

$$\oint \mathbf{H}dl=4\pi I/c.$$

The integration contour runs in the immediate vicinity of the static crystal plane. Noting that we have $I=ZevL/S_0$ for a plane of width L on which the nuclear charges Ze are disposed at the lattice points of elementary cells of area S_0, we find $H^{\max}=2\pi Zev/S_0c$. As follows from the definition of H and the usual behavior of E in a crystal, the field H sharply changes its direction on crossing the plane and slowly decreases in magnitude on moving away from the plane because of the screening action of the electrons, which is equivalent to a decrease in the total current I. The field changes sign at the center of the space between planes and then increases in absolute magnitude.

The presence of such a field of strength $H\sim10^5$ Oe for resonance neutrons and $H\sim10^8$ Oe for fast ones that alternates in direction and magnitude may give rise to a comparatively deep potential well, which will substantially affect the motion of the neutrons in the crystal. The motion in the presence of the effective field H is described by the Pauli equation

$$\left(-\frac{\hbar^2}{2m}\Delta+V_n+V_\mu\right)\Psi=E_0\Psi,\quad V_\mu=\mu\sigma\mathbf{H}(r). \tag{1}$$

We shall use the following considerations to simplify Eq. (1). The potential V_n determines the s scattering of neutrons and is characterized by the isotropic scattering amplitude $f_n(\theta)=R_0$ (R_0 is the nuclear radius). The second part of the potential, the potential V_μ for magnetic scattering of a particle of magnetic moment μ by nuclei and atomic electrons, reduces essentially to the Schwinger interaction.[2,3] The amplitude for such scattering is highly anisotropic, and for $\theta\ll\pi$ we have $f_\mu(\theta)=iZ\mu e/\hbar c\theta$. It is known (see, e.g., Ref. 4) that the particle-channeling process and the use of the average crystal potential on the plane to describe it are possible only in the presence of coherent small-angle forward scattering from a portion of the plane whose dimensions considerably exceed the interatomic distance, with subsequent interference of the scattering amplitudes. The second part of the potential in (1) will clearly play the principal part in such a process (estimates show that $|f_\mu/f_n|\gtrsim10$–100 for incidence angles $\theta\lesssim10^{-3}$ onto the plane). As a result, a zeroth approximation to the solution of (1) can be found without allowing for the nuclear interaction potential V_n. Taking the spatial distribution of the particles in the zeroth approximation as given, one can use the known wave function to calculate the change in the yield of a nuclear reaction in which the neutrons take part.

Starting from the expression for a screened Coulomb potential,

$$V_e=Zee^{-r/R}/r,\quad R=\hbar^2/m_e e^2Z^{1/3},$$

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we obtain the following expressions for the local electric and magnetic fields $\mathbf{E}_r$ and $\mathbf{H}_r$:

$$\mathbf{H}_r=[\mathbf{E}_r\times\mathbf{v}]/c,\quad \mathbf{E}_r=-\nabla_r V_e=Zec^{-1}(1/r^2+1/rR)e^{-r/R}\mathbf{e}_r. \tag{2}$$

To determine the field on the plane we introduce a laboratory coordinate system with the y axis coinciding with the channel axis, the x axis perpendicular to the crystal planes, and the z axis parallel to the planes. It is traditional in the case of channeling to assume that the average potential $\langle V_\mu\rangle$ may be used in place of the actual potential V_μ, which contains the variable components $\mu\sigma_x H_x$, $\mu\sigma_y H_y$, and $\mu\sigma_z H_z$, of which the first two change sign periodically and average out ($\langle H_x\rangle=\langle H_y\rangle=0$), and the last is associated with E_x. In addition to the quantum treatment presented above, there is a classical basis for the use of the averaged potential, associated with the insensitivity of the paraxial motion of massive particles (protons, ions, neutrons) or fast particles (electrons and positrons) to the low-amplitude high-frequency spatial variations of the local field. Because of the weakness of the interaction of the neutron with each individual atom, the averaged parameters alone shape the trajectory.

Employing the usual method of averaging over the plane[5] and introducing the cylindrical coordinate $\rho=(r^2-x^2)^{1/2}$, which is perpendicular to the x axis, we obtain the following formula for the average magnetic potential of the static plane of the crystal:

$$\langle V_\mu\rangle=\mu\sigma_z(\boldsymbol{\xi}\mathbf{e}_x)\int_0^\infty\{H_z(x^2+\rho^2)^{1/2}2\pi\rho x/S_0(x^2+\rho^2)^{1/2}\}d\rho,$$

where $\boldsymbol{\xi}$ is a vector normal to the yz plane. Performing the integration with the aid of (2), we obtain

$$\langle V_\mu\rangle=(\boldsymbol{\xi}\mathbf{e}_x)2\pi\mu Zev\sigma_z\exp(-x/R)/S_0c.$$

This equation does not take the thermal motion of the lattice into account. On averaging $\langle V_\mu\rangle$ with the aid of the function

$$f(x)=(2\pi u^2)^{-1/2}\exp(-x^2/2u^2)$$

for the fluctuational deviations of the atoms from their equilibrium positions and taking into account the effect of the two planes closest to the potential well, we obtain

$$\langle\langle V_\mu\rangle\rangle=\sigma_z\{V_0(x)-V_0(a-x)\}v/c, \tag{3}$$

where

$$V_0(x)=\frac{\pi\mu Ze}{S_0}\exp\left(\frac{u^2}{2R^2}\right)\left\{e^{-x/R}\left[1-\Phi\left(\frac{u}{R\sqrt{2}}-\frac{x}{u\sqrt{2}}\right)\right]-e^{x/R}\left[1-\Phi\left(\frac{u}{R\sqrt{2}}+\frac{x}{u\sqrt{2}}\right)\right]\right\},$$

u is the rms deviation in the direction perpendicular to the plane of an atom from its equilibrium position, $\Phi(\alpha)$ is the probability integral, and a is the distance between planes.

For simplicity, in what follows we shall write V for $\langle\langle V_\mu\rangle\rangle$. Figure 1 shows the structure of the magnetic potential V in the crystal. One feature of the potential well that arises from the motion is the sharp dependence of the position of its minimum on the ratio of the parameters R, u, and a, the last two of which are adjustable since they depend on the temperature and the specific choice of the channel. The possible values of u lie in the range from $u\sim3\times10^{-9}$ cm for light and weakly bound atoms when $T>\Theta_D$ to $u\sim2\times10^{-10}$ cm for crystals having the maximum value of $M\Theta_D$ when $T<\Theta_D$ (M is the mass of an atom and Θ_D is the Debye temperature). When $u\ll R$ and $u\ll a$, the coordinate x_0 of the bottom of the well is

$$x_0\approx u\sqrt{2}\{\ln[R\sqrt{2}/u\sqrt{\pi}(1+\exp(-a/R))]\}^{1/2}. \tag{4}$$

In the opposite case, when $R<u<a$, we have

$$x_0\approx u\{1+(R^2/2u^2)+(u/2R)\exp(-a^2/2u^2)\}. \tag{5}$$

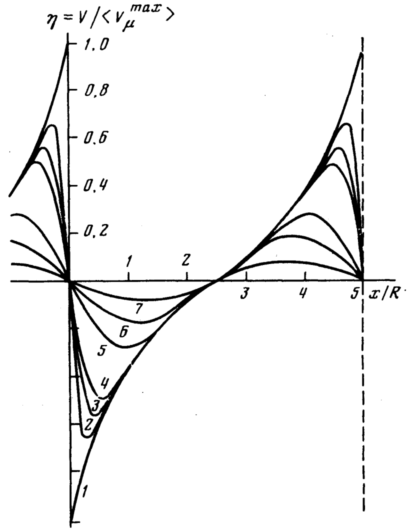

FIG. 1. Potential energy of a neutron in a planar channel reduced to a static lattice for $R=a/5$ and various values of u: 1—$u=0$, 2—$u=R/6$, 3—$u=R/4$, 4—$u=R/3$, 5—$u=R\sqrt{2}$, 6—$u=R$, 7—$u=R\sqrt{2}$.

It follows from (4) and (5), as well as from the numerical calculations shown in Fig. 1, which were based on formula (3), that as u increases the potential well becomes shallower and the bottom of the well shifts toward the center of the channel. Increasing the channel width a results in the same shift, but with a certain increase in the depth of the well.

In view of the expression obtained for V and the diagonal form of the Pauli matrix $\sigma_z=-(-1)^n\delta_{nm}$, we see that the initial equation (1) breaks up into two independent equations for neutrons having two opposite polarizations (with their spins oriented parallel or antiparallel to the z axis):

$$\Delta\Psi_{1,2}+(2m/\hbar^2)(E_0\pm V)\Psi_{1,2}=0. \tag{6}$$

When the neutron polarization is reversed the potential well transforms to its mirror image in the line $\eta=0$ on Fig. 1, and this leads to a symmetric disposition of the well minima with respect to the plane.

Because of the complicated form of the potential $V(x)$, an accurate solution of (6) can be obtained only by numerical methods. Nevertheless, the principal qualitative features of the motion of neutrons in that field can be studied using a simple quasiclassical analysis. In that approximation the standard solution of Eq. (6) has the form

$$\Psi_{1,2}=C_{1,2}(p_{0x}/p_x)^{1/2}\exp\left\{i\left[p_{0y}y\pm\int_{x_0}p_x dx\right]/\hbar\right\}, \quad (7)$$

where $p_x=(p_{0x}^2\pm 2mV)^{1/2}$ is the effective local transverse momentum on the classical trajectory between the turning points x_1 and x_2, which are defined by the condition $p_{0x}^2=2mV(x_{1,2})$.

Solution (7) is valid for resonance neutrons and faster ones except within negligibly small regions near the points x_1 and x_2 where $p_x\approx 0$. The low barrier height $V^{max}\sim 10^{-3}-10^{-8}$ eV leads to high transparency of the interchannel barriers and to the absence of bound levels of purely channeling motion, for which the following well-known condition must be satisfied:

$$|p_x^{max}|a/\hbar=(2mV^{max})^{1/2}a/\hbar>1.$$

In connection with this the transverse motion of the neutrons is delocalized and corresponds to the geometric optics of a plane laminated medium having a variable refractive index, the necessary criterion for the admissible rate of change of the refractive index being determined together with other variable parameters of the longitudinal motion.[6,7]

Because of the periodicity of the function $V(x)$, the solution (7) must satisfy Floquet's theorem:

$$\Psi_{1,2}(x)=g\Psi_{1,2}(x\pm a), \quad |g|=1.$$

In particular, if the neutrons are incident at the Bragg angle we have a case of higher-order diffraction. In the general case, from the continuity condition we find the dispersion equation

$$|\cos\varphi_1\,\mathrm{ch}\,\varphi_2|\leq 1, \quad \varphi_1=\int_{x_1}^{x_2}p_x dx/\hbar, \quad \varphi_2=\int_{x_2}^{x_1+a}|p_x|dx/\hbar,$$

which is satisfied for the only possible small values of the arguments $\varphi_{1,2}\ll 1$ provided $\varphi_1\geqslant\varphi_2$. Bearing in mind the symmetric form of the potential curve, we find that this condition will be satisfied only if $|x_2-x_1|\geqslant a/2$, i.e., if the region of classically allowed motion exceeds the halfwidth of the channel. This result determines the region of allowed energies for the transverse motion, which lie above the middle of the potential barrier, i.e., at $\eta\geqslant 0$. Motion with $|x_2-x_1|<a/2$ and $\eta<0$ will be damped. Motion above the barrier is characterized by the absence of forbidden energy bands.

The motion of neutrons discussed here differs substantially from ordinary channeling, but many properties of the latter are, nevertheless, characteristic of it. Actually, it is not so important for the nuclear interaction of a particle with the lattice whether the particle always remains within a single channel or, remaining preferentially in the space between planes, jumps rapidly back and forth through the barrier.

Another sort of motion is also possible, in which the neutron rapidly crosses the space between planes and "hovers" for a long time near the barrier where there is a high density of nuclei. In view of the quasiclassical nature of the motion and the transparency of the barrier, such features follow immediately from the relation $|\Psi_{1,2}|^2\sim 1/|p_x(x)|$. It is obvious that the last two modes of motion, which are qualitatively similar to the channeling of positively or negatively charged particles, will tend to enhance or suppress nuclear reactions involving neutrons. Above-barrier particles will have the same sort of motion. The reaction will consequently be enhanced for neutrons, the energy of whose transverse motion is close to the top of the barrier. If the crystal consists of alternating planes having different potentials, the motion near the barrier with enhancement of the reaction yield at one plane will be accompanied motion above or below the barrier at the other plane and the reaction will be suppressed on that plane.

With the aid of Eq. (3), we see that the optimal angle for such quasichanneling is

$$\theta_0\approx[4V^{max}/(p^2/2m)]^{1/2}=4[\pi\mu Ze\eta_0/S_0mcv]^{1/2}, \quad (8)$$

where $\eta_0\sim 0.3-0.7$ is the relative depth of the potential well (see Fig. 1), which varies within that interval for all realistic relations between u and R. The slow decrease in the characteristic angle $\theta_0\sim 1/\sqrt{v}$ as compared with the relation $\theta_0\sim 1/v$ for the channeling of heavy charged particles is connected with the fact that the well depth and barrier height increase linearly with v. For crystals having the parameter values $Z\sim 10-50$ and $S_0\sim 10^{-16}$ cm^2 we have $\theta_0\sim 2'-5'$ for $v\sim 4\times 10^6$ cm/sec ($E_0\sim 10$ eV) and $\theta_0\sim 10''-20''$ for $v\sim 10^9$ cm/sec ($E_0\sim 1$ MeV) in the case of neutrons, and $\theta_0\sim 0.7-1.4°$ and $\theta_0\sim 5'-7'$ for the same respective velocities in the case of light atoms having magnetic moments.

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