Collective excitations in a hot quark-gluon plasma

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A new method is proposed for finding the dispersion laws of collective excitations in systems described by non-Abelian gauge theories. The method is based on an expansion of the polarization and mass operators at high temperatures. By means of the method, the dispersion laws of collective Bose and Fermi excitations in a hot quark-gluon plasma are found explicitly. Without being inconsistent with the gauge and chiral symmetries, all the dispersion laws have an optical nature. Possible experimental consequences of the obtained results are briefly discussed.

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§1. INTRODUCTION

At the present time, the only convincing theory of the strong interactions of elementary particles is quantum chromodynamics (QCD), which has had significant successes in explaining spectroscopic data (especially for charmonium), the results of current algebra, and scaling at large momentum transfers. There is also hope that the phenomenon of quark and gluon confinement will be explained in the framework of QCD.

All the above successes of the theory are associated with the treatment at zero temperature and density. However, the development of the branches of theoretical and experimental physics associated with studying the collisions of relativistic nuclei, the early Universe, and neutron stars has posed the problem of studying QCD at nonzero temperatures and densities. It was established already in the first papers devoted to this question that because of the asymptotic freedom of QCD quarks are liberated at sufficiently high temperatures and densities, i.e., there is a phase transition from hadronic matter to a quark–gluon plasma. The estimates made in Ref. 1 show that this phase transition occurs at temperatures of several hundred MeV.

The existence of the quark–gluon plasma presents us with the task of investigating its properties. The thermodynamic properties of a quark–gluon plasma were considered in detail in Ref. 2, in which fairly good expressions were obtained for its free energy. However, to the best of our knowledge no investigation has so far been made of less trivial kinetic properties of the quark–gluon plasma and its collective excitations in particular (however, see Ref. 3).

In the present paper, we present the results of an investigation of the collective excitations in a hot quark–gluon plasma. (By this we mean a plasma in which the masses of the quarks are small compared with the temperature.)

We recall that the propagation of small excitations corresponding to some field φ is described by the equation

$$G_{\varphi^2}^{-1}(\mathbf{k};\,k_0)\,\varphi(\mathbf{k};\,k_0)=0, \qquad (1.1)$$

in which G_{φ^2} is the retarded (or advanced) Green's function of the field φ, which, as is well known,[4] is obtained by analytic continuation of the corresponding thermal Green's function.

In the case of QCD, whose Lagrangian has the form

$$L=\frac{1}{4}(\partial_\mu V_\nu^a-\partial_\nu V_\mu^a+gf^{abc}V_\mu^b V_\nu^c)^2+\bar{\psi}\gamma_\mu\left(\partial_\mu-ig\left(\frac{\lambda^s}{2}\right)V_\mu^s\right)\psi, \qquad (1.2)$$

there are both spinor and vector fields, and therefore the elementary excitations of the quark–gluon plasma will also have spinor or vector nature. In what follows, we shall, for brevity, refer to the vector and spinor excitations as Bose and Fermi excitations, respectively.

The paper is arranged as follows. In Sec. 2, we describe the general properties of quasigluon excitations, calculate the polarization operator of the gluons, and find the dispersion laws of transverse and longitudinal modes. In Sec. 3, we investigate quark-like excitations similarly. In Sec. 4, we discuss the consequences of the results obtained in Secs. 2 and 3.

§2. ELEMENTARY BOSE EXCITATIONS IN A HOT QUARK–GLUON PLASMA

As we already noted in the Introduction, the propagation of elementary Bose excitations in a quark–gluon plasma is described by the equation

$$D_{\mu\nu}^{-1}V_\nu=(D_{\mu\nu,0}^{-1}+\Pi_{\mu\nu}^{RET})\,V_\nu=0, \qquad (2.1)$$

in which $D_{\mu\nu}$ and $D_{\mu\nu,0}$ are, respectively, the exact and unrenormalized propagators of the gluons, and $\Pi_{\mu\nu}^{RET}$ are their polarization operator. It is important that all these functions must satisfy retarded (or advanced) boundary conditions.[4] To find a polarization operator satisfying this condition, we must, as is well known,[4] calculate it first in the framework of the thermal technique and then continue it analytically in the space of the real time.

From the transversality of the polarization operator,

$$k_\mu\Pi_{\mu\nu}=0, \qquad (2.2)$$

which also holds at finite temperatures,[5] and the fact of the existence of a distinguished 4-velocity vector of the medium, it follows that $\Pi_{\mu\nu}$ has the structure

$$\Pi_{\mu\nu}=\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)A+\left(\frac{k_\mu k_\nu}{k^2}-\frac{u_\mu k_\nu+u_\nu k_\mu}{uk}+\frac{u_\mu u_\nu k^2}{(uk)^2}\right)B. \qquad (2.3)$$

In the center-of-mass system of the quark–gluon plas-

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ma ($u_4=1, \mathbf{u}=0$), in which the calculations are usually made, (2.3) takes the form

$$\Pi_{ij}=\left(\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right)A+\frac{k_ik_j}{\mathbf{k}^2}\frac{k_4^2}{\mathbf{k}^2}\Pi_{44},$$
$$\Pi_{i4}=\Pi_{4i}=-\frac{k_ik_4}{\mathbf{k}^2}\Pi_{44}\quad(i,j=1,2,3). \tag{2.4}$$

Now, knowing the structure of $\Pi_{\mu\nu}$, it is not difficult to find the general solution of Eq. (2.1), which has the form

$$V_\mu(k)=\sum_{i=1}^{3}c_i\delta(\lambda_i(k))a_\mu^{(i)}(k)+c_0k_\mu\delta(k^2). \tag{2.5}$$

In (2.5), $\lambda_i(k)$ and k^2 are eigenvalues of $D_{\mu\nu}^{-1}$ and $a_\mu^{(i)}(k)$ and k_μ are the corresponding eigenvectors. It can be seen from (2.5) that in the quark–gluon plasma elementary Bose excitations of two types can propagate (not counting, of course, the unphysical four-dimensionally longitudinal excitations). They are three-dimensionally longitudinal excitations with dispersion law determined by the equation

$$(\mathbf{k}^2-k_0^2)(1+\Pi_{44}^{RET}(\mathbf{k};k_0)/\mathbf{k}^2)=0, \tag{2.6}$$

and three-dimensionally transverse excitations with dispersion law determined by

$$\mathbf{k}^2-k_0^2+A^{RET}(\mathbf{k};k_0)=0. \tag{2.7}$$

It is here appropriate to draw attention to the fact that at small spatial momenta the polarization operator must be isotropic, which together with the transversality of $\Pi_{\mu\nu}$ leads to the important relation

$$\Pi_{44}(k_4\neq0;\ \mathbf{k}\to0)=\frac{\mathbf{k}^2}{k_4^2}A(k_4\neq0;\ \mathbf{k}\to0). \tag{2.8}$$

By means of (2.8), we readily find from (2.6) and (2.7) that in the region of small spatial momenta the spectra of the transverse and longitudinal modes are the same and thus determine a characteristic frequency of the quark–gluon plasma.

Thus, to find the dispersion laws of the longitudinal and transverse modes it is necessary to find a correct approximate expression for (2.6) and (2.7) and then solve Eqs. (2.6) and (2.7). The spectra then obtained must, of course, be independent of the gauge. Because of this and the gauge dependence of A and Π_{44} particular care is needed in the solution of the dispersion relations. Note that the investigation of the dispersion laws in QED (Ref. 4) is in principle free of such difficulties, since in that case the polarization operator does not depend on the gauge at all.

Bearing in mind these remarks, we turn to the direct finding of the dispersion laws of the transverse and longitudinal modes in the hot quark–gluon plasma.

Calculation of the functions A and Π_{44} in the single-loop approximation leads to the result (Feynman gauge[3])

$$A-A^{(vac)}=\frac{g^2T^2}{4}\left(1+\frac{N_f}{6}\right)\left(1-\frac{p_4^2}{\mathbf{p}^2}\right)-\frac{3g^2(\mathbf{p}^2+p_4^2)}{16\pi^2|\mathbf{p}|^3}$$
$$\times\int_0^\infty dkn_k^B\{(3\mathbf{p}^2-p_4^2+4k^2)\ln a+4ikp_4\ln b\}-\frac{g^2(\mathbf{p}^2+p_4^2)N_f}{16\pi^2|\mathbf{p}|^3}$$
$$\times\int_0^\infty dkn_k^F\{(\mathbf{p}^2-p_4^2+4k^2)\ln a+4ip_4k\ln b\}, \tag{2.9}$$

$$\Pi_{44}-\Pi_{44}^{(vac)}=\frac{g^2T^2}{2}\left(1+\frac{N_f}{6}\right)-\frac{3g^2}{8\pi^2|\mathbf{p}|}\int_0^\infty dkn_k^B\{(2\mathbf{p}^2+p_4^2-4k^2)\ln a$$
$$-4ikp_4\ln b\}-\frac{g^2N_f}{8\pi^2|\mathbf{p}|}\int_0^\infty dkn_k^F\{(\mathbf{p}^2+p_4^2-4k^2)\ln a-4ip_4k\ln b\}, \tag{2.10}$$

where N_f is the number of quark species, and

$$a=\frac{(\mathbf{p}^2+p_4^2+2k|\mathbf{p}|)^2+4p_4^2k^2}{(\mathbf{p}^2+p_4^2-2k|\mathbf{p}|)^2+4p_4^2k^2}$$
$$b=\frac{(\mathbf{p}^2+p_4^2)^2-4k^2(|\mathbf{p}|-ip_4)^2}{(\mathbf{p}^2+p_4^2)^2-4k^2(|\mathbf{p}|+ip_4)^2}$$
$$n_k^B=(e^{k/T}-1)^{-1},\quad n_k^F=(e^{k/T}+1)^{-1}.$$

As in Ref. 3, we can substitute the expressions (2.9) and (2.10) in (2.6) and (2.7) and then solve the resulting equations. However, following this procedure we encounter serious difficulties of both computational and fundamental nature. The latter are associated with the fact that in the framework of such an approach there are no grounds for expecting the dispersion law obtained in such a manner to be independent of the gauge or the corrections to it obtained when the higher approximations of perturbation theory are taken into account to be small. The existence of these difficulties requires a different approach to the finding of the dispersion laws. One of the possible approaches is presented below.

The essence of this approach consists of finding the spectra by means of the high-temperature ($\sim T^2$) asymptotic behavior of the polarization operator with subsequent exact solution of the obtained dispersion relations.

An important feature of the high-temperature asymptotic behavior of the polarization operator is the fact that allowance for the diagrams of higher orders gives only small corrections to it and also that this asymptotic behavior is gauge invariant. The gauge invariance of the high-temperature asymptotic behavior, which is proportional to T^2, is readily proved by noting that in the polarization operator the terms of order T^2 arise only from the structures that diverge quadratically in the ultraviolet region and that the gauge-dependent longitudinal parts of the propagators do not lead to such structures. These features of the high-temperature asymptotic behavior make it very convenient for finding the dispersion laws. We note that a somewhat similar high-temperature expansion was used in Ref. 6 to investigate the restoration of spontaneously broken symmetry when the temperature is raised.

In the framework of our method, we find the spectrum of transverse modes for arbitrary momenta. The high-temperature asymptotic behavior necessary for this can be readily found from (2.9) and has the form

$$A(\mathbf{p};p_4)=\frac{g^2T^2}{2}\left(1+\frac{N_f}{6}\right)\left[\frac{\mathbf{p}^2-p_4^2}{2\mathbf{p}^2}\right.$$
$$\left.+\frac{(\mathbf{p}^2+p_4^2)(\mathbf{p}^2+3p_4^2)}{2\mathbf{p}^2}\int_0^1\frac{dz}{\mathbf{p}^2z^2+p_4^2}-\frac{(\mathbf{p}^2+p_4^2)p_4^2}{\mathbf{p}^2}\int_0^1\frac{dz}{(\mathbf{p}^2z^2+p_4^2)^2}\right]. \tag{2.11}$$

Making now in (2.11) an analytic continuation to the retarded Green's functions by means of the substitution $p_4\to i(\omega+i\varepsilon)$ and calculating the obtained integrals, we obtain the required gauge-invariant approximate expression for A^{RET}:

$$A^{\mathrm{RET}}(\omega;\mathbf{p})=\frac{3}{2}\omega_{pl}^2\left[\frac{\omega^2}{\mathbf{p}^2}+\frac{\mathbf{p}^2-\omega^2}{\mathbf{p}^2}F\left(\frac{\omega}{|\mathbf{p}|}\right)\right]. \qquad (2.12)$$

In (2.12), we have used the notation

$$F(x)=\frac{x}{2}\left[\ln\left|\frac{x+1}{x-1}\right|-i\pi\theta(1-|x|)\right],$$
$$\omega_{pl}^2=\frac{g^2T^2}{3}\left(1+\frac{N_f}{6}\right). \qquad (2.13)$$

With allowance for (2.12), the dispersion relation determining the dispersion law of the transverse modes takes the form

$$\mathbf{p}^2-\omega^2+\frac{3}{2}\omega_{pl}^2\left[\frac{\omega^2}{\mathbf{p}^2}+\frac{\mathbf{p}^2-\omega^2}{\mathbf{p}^2}F\left(\frac{\omega}{|\mathbf{p}|}\right)\right]=0 \qquad (2.14)$$

and has an exact undamped solution which is described by the parametric equations

$$\omega_\perp^2(\xi)=\xi^2\mathbf{p}_\perp^2(\xi)\quad(1<\xi<\infty),$$
$$\mathbf{p}_\perp^2(\xi)=\frac{3}{2}\omega_{pl}^2\left[\frac{\xi^2}{\xi^2-1}-\frac{\xi}{2}\ln\frac{\xi+1}{\xi-1}\right]. \qquad (2.15)$$

We emphasize that the spectrum (2.15) does not depend on the choice of the gauge by virtue of the gauge invariance of (2.12).

It is readily found from the expression (2.15) that at small momenta ($|\mathbf{p}|\ll\omega_{\mathrm{pl}}$) the dispersion law of the transverse modes is described by the expression

$$\omega_\perp^2=\omega_{pl}^2+{}^6/_5\mathbf{p}^2 \qquad (2.16)$$

and at large momenta ($|\mathbf{p}|\gg\omega_{\mathrm{pl}}$) by the expression

$$\omega_\perp^2=\mathbf{p}^2+{}^3/_2\omega_{pl}^2. \qquad (2.17)$$

We note that the asymptotic behaviors (2.16) and (2.17) were obtained earlier in Ref. 3 by semi-intuitive (and very complicated) manipulations.

For arbitrary momenta, the spectrum of transverse modes described by Eq. (2.15) is shown in Fig. 1.

The case of the longitudinal modes can be investigated similarly. Using (2.9) to find the high-temperature asymptotic behavior of Π_{44},

$$\Pi_{44}(\mathbf{p};p_4)=\frac{g^2T^2}{2}\left(1+\frac{N_f}{6}\right)\left[1-(3p_4^2+\mathbf{p}^2)\int_0^1\frac{dz}{\mathbf{p}^2z^2+p_4^2}\right.$$
$$\left.+2p_4^2(\mathbf{p}^2+p_4^2)\int_0^1\frac{dz}{(\mathbf{p}^2z^2+p_4^2)^2}\right], \qquad (2.18)$$

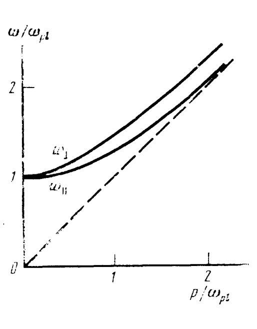


FIG. 1. Spectrum of elementary Bose excitations in quark–gluon plasma.

making the analytic continuation in it to the retarded functions $[p_4\to i(\omega+i\varepsilon)]$, and calculating the obtained integrals, we find the required gauge-invariant approximate expression for Π_{44}^{RET}:

$$\Pi_{44}^{\mathrm{RET}}(\omega;\mathbf{p})=3\omega_{pl}^2\left[1-F\left(\frac{\omega}{|\mathbf{p}|}\right)\right] \qquad (2.19)$$

[F and ω_{pl} are defined above by means of (2.13)].

With allowance for (2.19), the dispersion relation for the longitudinal modes becomes

$$1+\frac{3\omega_{pl}^2}{\mathbf{p}^2}\left[1-F\left(\frac{\omega}{|\mathbf{p}|}\right)\right]=0 \qquad (2.20)$$

and has an undamped solution described by the paraetric equations

$$\omega_\parallel^2(\xi)=\xi^2\mathbf{p}_\parallel^2(\xi)\quad(1<\xi<\infty),$$
$$\mathbf{p}_\parallel^2(\xi)=3\omega_{pl}^2\left(\frac{\xi}{2}\ln\frac{\xi+1}{\xi-1}-1\right). \qquad (2.21)$$

By virtue of the gauge invariance of the high-temperature asymptotic behavior (2.19) the dispersion law (2.21) is also independent of the gauge.

At small momenta ($|\mathbf{p}|\ll\omega_{\mathrm{pl}}$), we readily find from (2.21) an explicit expression for the dispersion law of the longitudinal modes:

$$\omega_\parallel^2=\omega_{pl}^2+{}^3/_5\mathbf{p}^2. \qquad (2.22)$$

Similarly, at large momenta

$$\omega_\parallel^2=\mathbf{p}^2\left[1+4\exp\left(-\frac{2\mathbf{p}^2}{3\omega_{pl}^2}-2\right)\right]. \qquad (2.23)$$

The spectrum of longitudinal modes described by Eqs. (2.21) for arbitrary momenta is shown in Fig. 1.

All the above calculations were made in the framework of the single-loop approximation. In the considered case of high temperatures, it is not difficult to improve the obtained dispersion laws by means of the renormalization group. The anomalous dimensions can be ignored, so that the result of applying the renormalization group reduces to replacing the coupling constant in (2.15) and (2.21) by an effective constant, i.e., it reduces to the substitution

$$g^2\to g_{eff}^2(T)=4\pi\left[1+\left(\frac{11}{2\pi}-\frac{N_f}{3\pi}\right)\ln\left(\frac{T}{T_0}\right)\right]^{-1} \qquad (2.24)$$

or to the substitution

$$\omega_{pl}^2\to\omega_{pl,eff}^2=\frac{2\pi T^2(N_f+6)}{9[1+(11/2\pi-N_f/3\pi)\ln(T/T_0)]}. \qquad (2.25)$$

The smallness of the effective charge in the region of high temperatures (and we consider the case of a hot plasma!) guarantees that the expressions (2.15) and (2.21) after the substitution (2.25) correctly represent the exact dispersion laws of the transverse and longitudinal modes of the hot quark–gluon plasma.

To conclude this section, we point out that the dispersion law of the fictitious particles that arise in the case of correct quantization of Yang–Mills theory in relativistic gauges[7] is not changed when allowance is made for the presence of the medium, i.e., in the case of the quark–gluon plasma as well the dispersion law of the fictitious particles has the form $\omega^2=\mathbf{p}^2$. This fact is readily verified by direct calculations, although its validity is already clear from general considerations.

§3. ELEMENTARY FERMI EXCITATIONS IN A HOT QUARK–GLUON PLASMA

Fermi excitations in a hot quark–gluon plasma can be investigated in complete analogy with the case of Bose excitations investigated in Sec. 2. Therefore, the propagation of small-amplitude Fermi excitations in a hot quark–gluon plasma is described by the equation

$$G^{-1}\psi=(G_0^{-1}+\Sigma^{RET})\psi=0, \tag{3.1}$$

which is completely analogous to Eq. (2.1). In (3.1), G and G_0 are the exact and unrenormalized propagators of the quarks, and Σ^{RET} is their mass operator. It is important that all these functions must satisfy retarded (or advanced) boundary conditions.

Using now the fact that in the considered massless case the mass operator can, by virtue of γ_5 invariance, be represented in the form

$$\Sigma(p)=-i\gamma_\mu\Sigma_\mu(p), \tag{3.2}$$

we can invert the expression (3.1):

$$G=i\gamma_\mu(p_\mu+\Sigma_\mu)/(p_\mu+\Sigma_\mu)^2. \tag{3.3}$$

Now, setting the denominator of (3.3) equal to zero, we find that the dispersion relation determining the spectrum of the Fermi excitations has, with allowance for the subsequent analytic continuation, the form

$$(p_\mu+\Sigma_\mu(p))^2=0. \tag{3.4}$$

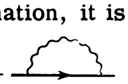
We consider the general structure of $\Sigma_\mu(p)$. If a medium is absent ($T=0$), it follows from relativistic invariance that $\Sigma_\mu(p)$ must be proportional to p_μ, and, therefore, the dispersion relation (3.4) has the unique solution $p_\mu^2=0$ or $\omega^2=\mathbf{p}^2$.

In the case of nonzero temperature, there is a preferred 4-velocity vector u_μ of the center of mass of the medium, and therefore $\Sigma_\mu(p)$, like the polarization operator $\Pi_{\mu\nu}$ (see Sec. 2), is determined by two scalar functions:

$$\Sigma_\mu(p)=p_\mu\mathcal{A}(p;\,u)+u_\mu\mathcal{B}(p;\,u). \tag{3.5}$$

The last circumstance has the consequence that the dispersion relation (3.4) in the case $T\neq0$ has two different solutions, and the spectrum of elementary quark-like excitations has two branches, which may have an optical nature.

Bearing in mind these general comments, we turn to the direct finding of the dispersion law of the Fermi excitations. We first find the mass operator of the quarks in the thermal technique. In the single-loop approximation, it is described by the diagram

$\Sigma = -$  (3.6)

whose calculation by means of the ordinary expressions for the propagators and vertices leads to an expression of the form

$$\Sigma(p)=\frac{8g^2}{3}T\sum_{k_4}\int\frac{d^3k}{(2\pi)^3}\frac{-ik_\mu\gamma_\mu}{k^2(p-k)^2}. \tag{3.7}$$

In (3.7), k and p have odd frequencies in units of πT and we have used the Feynman gauge (we shall discuss the dependence of the obtained results on the gauge below). Summing over k_4 in (3.7), we obtain the expression

$$\Sigma(p)=\Sigma^{(vac)}(p)+\frac{8g^2}{3}\int\frac{d^3k}{(2\pi)^3}\frac{n_k^F}{2|\mathbf{k}|}\left[\frac{|\mathbf{k}|\gamma_4-i\gamma\mathbf{k}}{\mathbf{p}^2+p_4^2+2\mathbf{kp}+2i|\mathbf{k}|p_4}-\text{h.c.}\right]$$
$$+\frac{8g^2}{3}\int\frac{d^3k}{(2\pi)^3}\frac{n_k^B}{2|\mathbf{k}|}\left[\frac{(|\mathbf{k}|-ip_4)\gamma_4-i\gamma(\mathbf{k}+\mathbf{p})}{\mathbf{p}^2+p_4^2+2\mathbf{kp}+2i|\mathbf{k}|p_4}-\text{h.c.}\right], \tag{3.8}$$

in which $\Sigma^{(vac)}$ is the value of the mass operator Σ for $T=0$ and, as usual, $n_k^B=[\exp(k/T)-1]^{-1}, n_k^F=[\exp(k/T)+1]^{-1}$.

To separate from (3.8) the part that is affected little by allowance for the higher orders of perturbation theory, we proceed as in §2, i.e., we restrict ourselves to the first term in the high-temperature expansion:

$$\Sigma(p)=\frac{g^2T^2}{6}\left[-\frac{i\gamma\mathbf{p}}{\mathbf{p}^2}\left(1-p_4^2\int_0^1\frac{dz}{\mathbf{p}^2z^2+p_4^2}\right)-i\gamma_4p_4\int_0^1\frac{dz}{\mathbf{p}^2z^2+p_4^2}\right]. \tag{3.9}$$

In the expression (3.9), for convenience in the analytic continuation to the retarded Green's functions, we have not calculated the integrals over z.

Performing now in (3.9) the analytic continuation to the retarded Green's functions, $p_4\to i(\omega+i\varepsilon)$, and calculating the obtained integrals, we find that the required high-temperature asymptotic behavior of the mass operator which satisfies retarded boundary conditions has the form

$$\Sigma^{RET}(\omega;\mathbf{p})=-i\gamma\mathbf{p}\frac{\omega_0^2}{\mathbf{p}^2}\left(1-F\left(\frac{\omega}{|\mathbf{p}|}\right)\right)-\gamma_4\frac{\omega_0^2}{\omega}F\left(\frac{\omega}{|\mathbf{p}|}\right). \tag{3.10}$$

In (3.10), we have used the notation

$$\omega_0^2=\frac{g^2T^2}{6},\quad F(x)=\frac{x}{2}\left[\ln\left|\frac{x+1}{x-1}\right|-i\pi\theta(1-|x|)\right]. \tag{3.11}$$

It is important that the expression (3.10) does not depend on the gauge, which is readily seen, since the terms that depend on the gauge-fixing parameter are proportional at high temperatures to T and not T^2.

With allowance for (3.4) and (3.10), the dispersion relation that determines the spectrum of elementary quark-like excitations takes the form

$$\left[\omega-\frac{\omega_0^2}{\omega}F\left(\frac{\omega}{|\mathbf{p}|}\right)\right]^2=\left\{|\mathbf{p}|+\frac{\omega_0^2}{|\mathbf{p}|}\left[1-F\left(\frac{\omega}{|\mathbf{p}|}\right)\right]\right\}^2 \tag{3.12}$$

and decomposes into the two equations

$$\omega-\frac{\omega_0^2}{\omega}F\left(\frac{\omega}{|\mathbf{p}|}\right)=|\mathbf{p}|+\frac{\omega_0^2}{|\mathbf{p}|}\left[1-F\left(\frac{\omega}{|\mathbf{p}|}\right)\right],$$
$$\omega-\frac{\omega_0^2}{\omega}F\left(\frac{\omega}{|\mathbf{p}|}\right)=-|\mathbf{p}|-\frac{\omega_0^2}{|\mathbf{p}|}\left[1-F\left(\frac{\omega}{|\mathbf{p}|}\right)\right]. \tag{3.13}$$

In the region $\omega>|\mathbf{p}|$, the dispersion relation (3.13) does not have imaginary parts in accordance with (3.11) and can be solved exactly. It is then found that the spectrum of quark-like excitations has in accordance with the remark made in connection with (3.5) two (undamped) branches described by the parametric equations

$$\omega_\pm^2(\xi)=\xi^2\mathbf{p}_\pm^2(\xi)\quad(1<\xi<\infty),$$
$$\mathbf{p}_\pm^2(\xi)=\omega_0^2\left[\frac{\pm1}{\xi\mp1}\mp\frac{1}{2}\ln\frac{\xi+1}{\xi-1}\right], \tag{3.14}$$

in which the upper sign corresponds to one branch, and the lower to the other. In accordance with the gauge invariance of (3.10), the expression (3.14) is independent of the gauge, which is necessary for an observable quantity. Note that in the case of quantum electrodynamics we would obtain exactly the same (up to an isotopic factor in ω_0^2) dispersion laws, and therefore our results can also be applied in the case of an electron–photon plasma.

It follows from (3.14) that in the region of small momenta both branches have an optical nature:

$$\omega_\pm^2(\mathbf{p})=\omega_0^2\left[1\pm\frac{2}{3}\frac{|\mathbf{p}|}{\omega_0}+\frac{7}{9}\frac{\mathbf{p}^2}{\omega_0^2}+O(|\mathbf{p}|^3)\right]. \quad (3.15)$$

The optical nature of the spectrum of the quark-like excitations is very nontrivial, since the γ_5 invariance of the considered case of a massless quark–gluon plasma prohibits the occurrence in the mass operator of the quarks of a part proportional to the unit matrix with respect to the spinor indices, i.e., the ordinary mass term of the quarks. An entirely similar situation obtains in the case of a cold quark–gluon plasma, in which the spectrum of quark-like excitations also has a gap: $\omega^2(0)=g^2\mu^2/6\pi^2$ (μ is the chemical potential).

At large momenta, the branches of the spectra are described by the expressions

$$\omega_+^2(\mathbf{p})=\mathbf{p}^2+2\omega_0^2-\left(\frac{\omega_0^4}{\mathbf{p}^2}\right)\ln\frac{\mathbf{p}^2}{\omega_0^2}+O\left(\frac{1}{\mathbf{p}^2}\right), \quad (3.16)$$

$$\omega_-^2(\mathbf{p})=\mathbf{p}^2+4\mathbf{p}^2\exp\left(-\frac{2\mathbf{p}^2}{\omega_0^2}-1\right)+O\left(\exp\left(-\frac{4\mathbf{p}^2}{\omega_0^2}\right)\right). \quad (3.17)$$

As in the case of the spectrum of the Bose excitations, the expression (3.14) must be improved by means of the renormalization group, since it is only after this has been done that it will, by virtue of the smallness of the effective coupling constant, correctly represent the exact dispersion law of the quark-like excitations in the hot quark–gluon plasma. The anomalous dimensions can here be ignored, so that the result of applying the renormalization group to the expression (3.14) reduces to the substitution (2.24) in it, i.e., to the substitution

$$\omega_0^2\to\omega_{0,eff}^2=\frac{2\pi T^2}{3}\left(1+\left(\frac{11}{2\pi}-\frac{N_f}{3\pi}\right)\ln\left(\frac{T}{T_0}\right)\right)^{-1}. \quad (3.18)$$

The behavior of the spectrum of quark-like excitations at arbitrary momenta [see (3.14)] is shown in Fig. 2. The spectrum shown in this figure is remarkable not only for its optical nature but also on account of the fact that the branch ω_- has a minimum at nonzero momenta.

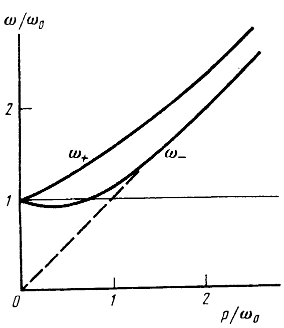


FIG. 2. Spectrum of elementary Fermi excitations in quark–gluon plasma.

§4. CONCLUSIONS

In Secs. 2 and 3, we have investigated the collective excitations of a hot quark–gluon plasma and determined their dispersion laws. These dispersion laws are good in that they do not depend on the choice of the gauge and in that the corrections to them when the higher orders of perturbation theory are taken into account are small.

The interest in the collective excitations of the hot quark–gluon plasma is not purely academic, since they are very important for describing real processes in quark stars and in collisions of relativistic nuclei. The obtained dispersion laws can also be readily generalized to the case of the early Universe, filled with the plasma of all elementary particles.

We now turn to the direct analysis of the obtained dispersion laws of the collective excitations of the hot quark–gluon plasma. Here, in the first place, it is noteworthy that both the vector and spinor collective excitations have a mass gap in their spectra. The apparent contradiction between this fact and the gauge and chiral symmetries disappears when we recall that in the case of quantum statistics there is no manifest relativistic invariance. It is important that the optical nature of the obtained dispersion laws will be verified already in the very near future in the experimental analysis of collisions of relativistic nuclei, since in this case the energies of the secondary particles will evidently be multiples of the energies of the corresponding quasiparticles, i.e., multiples of either ω_{pl} or ω_0.

A second very important feature of the spectra found above is the fact that one of the branches of the Fermi spectrum has a minimum at nonzero momenta (see Fig. 2). We do not completely understand the significance of this minimum, but it is not impossible that its existence could lead to a fundamental consequence such as the formation of vortices in the quark–gluon plasma. It is also possible that this minimum could be an indication of a spontaneous breaking of the translational invariance of the theory, this being expressed as a motion of the quark–gluon plasma as a whole. For the final solution of the question of the significance of this minimum, it is necessary to solve the kinetic equations for the system with the Hamiltonian $H=\sum_{\mathbf{p}}\omega_-(\mathbf{p})\times a_{\mathbf{p}}^+a_{\mathbf{p}}$ and equilibrium initial conditions ($a_{\mathbf{p}}$ and $a_{\mathbf{p}}^+$ are the ordinary Fermi operators of creation and annihilation).

A third important feature of all our spectra is the fact that the refractive indices obtained by means of them for the corresponding modes are always less than unity, and therefore elementary particles passing through the hot quark–gluon plasma will not radiate coherently. The finding of the connection between this fact and the results of Ref. 8 undoubtedly warrants very careful study.

Finally, the dispersion laws we have found for the vector excitations are also remarkable in that their functional form is identical to the functional form of the dispersion laws in a plasma of scalar particles, in an ultrarelativistic electron plasma, and in a degenerate plasma. It is clear that this identity is a consequence of some general principle which we have not as yet recognized.

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