Effect of annihilation of quasinuclear baryonium states in higher partial waves

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The cross section for the annihilation of a nucleon (N) and a antinucleon ($\bar{N}$) from continuum states, as well as the annihilation widths and shifts of the levels of a quasinuclear $N\bar{N}$ system with orbital momentum $l = 0$, is calculated within the framework of the coupled-channel model. It is shown that although the annihilation cross section is large (it is comparable with the unitary limit at nonrelativistic energies), the annihilation widths Γ_{an} of the quasinuclear states are small ($\Gamma_{an} = 1$–10 MeV at $l = 2$). It is observed that annihilation decreases the resonance width, and the physical cause of this phenomenon is explained.

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1. INTRODUCTION

It was shown[1,2] in 1969–1970 that strong nuclear attraction leads to the appearance of a rich spectrum of bound states in an $N\bar{N}$ system. It was revealed in the same papers that the annihilation widths Γ_{an} of these states do not exceed in order of magnitude, at any rate, the normal hadron widths (~100 MeV), and can be much smaller in states with nonzero orbital momentum l of the relative motion of N and $\bar{N}$. It was shown in Ref. 3 that not only bound but also resonant states should exist in an $N\bar{N}$ system, and their elastic widths were calculated. Bound and resonant states of nuclear type of the baryon-antibaryon system have been named quasinuclear baryonium. Further progress in this field is described in reviews by Shapiro *et al.*[4,5] (see also the references therein).

Special interest in the baryonium problem arose after the publication of experimental reports indicating the existence of heavy mesons (with mass in the region of 2 GeV) which are strongly coupled to the $N\bar{N}$ channel (for the present status of the experimental data see Ref. 6). The observed baryonium levels included also some that were quite narrow (width $\Gamma \lesssim 10$ MeV). In the quasinuclear approach this can be attributed to the presence of a nonzero orbital momentum (see Refs. 7 and 8). The problem is, however, that an exact calculation of the annihilation widths calls for a full account of the relativistic dynamics of the strong interactions, and is not feasible at the present time. The cited papers gave estimates of Γ_{an} based on the smallness of the annihilation radius r_a (≈ 0.1 fm) compared with the size of the quasinucleus R (≈ 1 fm). It would be desirable to be convinced of the reliability of these estimates. The question has become even more timely in connection with the planned forthcoming startup of an antiproton storage ring that offers a new possibility of finding narrow baryonium states and of investigating their properties (see Ref. 9).

The purpose of the present paper is to assess the agreement between the qualitative estimates of the influence of annihilation and quasinuclear states (see Refs. 4 and 10) with results of calculations within the framework of the exactly solvable nonrelativistic model of coupled channels (elastic annihilation). The employed model is realistic in the sense that its parameters (the radii of the annihilation and nuclear interactions, the particle masses, the binding energies of the baryonium) are in accord with the true situation that obtains for the NN system. Such a model, proposed in Ref. 11, has already been used for the theory of baryonium s-states (see Refs. 11–16). Greatest interest, however, attaches to states with $l \neq 0$, since they should have, as noted above, much lower annihilation widths. In addition, calculation of one-boson exchange with realistic potentials (see Refs. 4, 17, 18) have shown that most quasinuclear states have a nonzero orbital momentum. At the same time, the effect of annihilation on such states has not yet been investigated from a point of view of interest to us.[1)] We fill this gap in this paper, and consider d-states by way of example ($l = 2$).

The plan of the article is the following. In Sec. 2 are formulated the basics of the employed model. In Sec. 3 is clarified the character of the interaction produced between N and $\bar{N}$ on account of coupling with the annihilation channel. Section 4 is devoted to the calculation of the annihilation widths and shifts of the bound quasinuclear states. In Sec. 5 is considered the influence of annhilation on the resonant states. The $N\bar{N}$ annihilation cross section is calculated in Sec. 6 for nonrelativistic energies. Section 7 contains a brief formulation of the main results of the paper.

2. COUPLED-CHANNEL MODEL

We consider the nonrelativistic model of two coupled channels. Channel 1 consists of two particles with identical masses m_1. The interaction between them is described by a certain potential that is assumed to be strong enough to produce a bound or resonant state. Channel 2 contains two identical noninteracting particles with masses $m_2 < m_1$. All the particles are assumed to have zero spin. Channel 1 is the analog of the $N\bar{N}$ system, channel 2 corresponds to the annihilation products.

The c.m.s. momenta k_1 and k_2 in channels 1 and 2 are connected with the kinetic energy E in channel 1 by the relations[2)]

$$k_1 = (m_1 E)^{1/2}, \quad k_2 = \{m_2[E + 2(m_1 - m_2)]\}^{1/2}. \tag{1}$$

The S matrix as a function of the energy E has two square-root branch points, $E = 0$ and $E = -2(m_1 - m_2)$.

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Accordingly, the corresponding Riemann surface has four sheets. We direct the cuts on the sheets from the branch points to the right along the real axis. The joining of the sheets and their numbering are defined by the inequalities

$$\text{I. } \left.\begin{matrix}\operatorname{Im} k_1>0\\ \operatorname{Im} k_2>0\end{matrix}\right\},\quad \text{II. } \left.\begin{matrix}\operatorname{Im} k_1>0\\ \operatorname{Im} k_2<0\end{matrix}\right\},\quad \text{III. } \left.\begin{matrix}\operatorname{Im} k_1<0\\ \operatorname{Im} k_2<0\end{matrix}\right\},\quad \text{IV. } \left.\begin{matrix}\operatorname{Im} k_1<0\\ \operatorname{Im} k_2>0\end{matrix}\right\}.$$

The poles corresponding to the quasinuclear bound states, which can decay only via channel 2, will then be located on sheet II, while those corresponding to the resonant states (which decay both via channel 1 and via channel 2) will be on sheets III and IV.

We consider also the S matrix as a function of the momentum k_1, defined on a two-sheet Riemann surface. If the cuts are drawn from the branch points $k_1 = \pm i[2m_1(m_1 - m_2)]^{1/2}$ up and down along the imaginary axis respectively, then the arrangement of the poles in the second sheet will be asymmetrical (about the imaginary axis) reflection of the picture on the first sheet. On the latter, the poles corresponding to resonant states in channel 1 are located in the third and fourth quadrants above the bisectrices, while those corresponding to the bound states are above the bisectrix of the second quadrant.

The considered two-channel system is described by a Hamiltonian $\hat{H} = \hat{H}^0 + \hat{V}$, where the free-system Hamiltonian $\hat{H}^0$ and the interaction Hamiltonian $\hat{V}$ are Hermitian 2×2 matrices:

$$\hat{H}^0=\begin{pmatrix} H_1^0 & 0\\ 0 & H_2^0\end{pmatrix},\quad \hat{V}=\begin{pmatrix} U & W\\ W & 0\end{pmatrix}.$$

Here $H_1^0 = \mathbf{k}_1^2/m_1$, $H_2^0 = \mathbf{k}_2^2/m_2 - 2(m_1 - m_2)$, U is the potential of the nuclear interaction between N and $\bar{N}$ in channel 1, and W is the annihilation interaction that couples the channels. We choose U and W in separable form:

$$\langle \mathbf{k}|W|\mathbf{k}'\rangle = 5\lambda\beta_2^2\xi_2(k)\xi_2(k')P_2(\cos\theta)/(4\pi\sqrt{m_1m_2}),$$
$$\xi_2(k)=(32\beta_2/5\pi)^{1/2}k^2/(k^2+\beta_2^2)^2\quad \int_0^\infty \xi_2^2(k)k^2dk=1. \tag{2}$$

The equations for $\langle \mathbf{k}|U|\mathbf{k}'\rangle$ and $\xi_1(k)$ are obtained from (2) by making the substitutions $\lambda\to g$, $\beta_2\to\beta_1$, $\xi_2\to\xi_1$, and $m_2\to m_1$. Here g is the dimensionless constant of the diagonal (nuclear) interaction, λ is the dimensionless annihilation constant, $\beta_1 \sim 1/R$ is the reciprocal nuclear-interaction radius, $\beta_2 \sim 1/r_a$ is the reciprocal annihilation radius, $P_2(\cos\theta)$ is a Legendre polynomial, and θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$.

We use in this paper the following parameter values: $m_1 = 940$ MeV/c^2 (nucleon mass), $m_2 = 770$ MeV/c^2 (ρ-meson mass), $\beta_2 = 2m_1 = 1880$ MeV/c (corresponds to $r_a \approx 0.1$ fm), $\beta_1 = 0.1\beta_2 = 188$ MeV/c (corresponds to $R \approx 1$ fm).

The separable form of $\hat{V}$ enables us to obtain the $N\bar{N}$ elastic-scattering amplitude f in algebraic form. An explicit expression for f is given in the Appendix.

3. ANNIHILATION AND GENERALIZED OPTICAL POTENTIAL

To ascertain the character of the interaction that is produced between N and $\bar{N}$ as a result of the coupling with the annihilation channel, we consider the generalized optical potential

$$V(E)=\langle \mathbf{k}|WG_2(E)W|\mathbf{k}'\rangle,$$

where G_2 is the Green operator of the channel 2 (in the absence of coupling with channel 1). We assume in the present section that channel contains n identical noninteracting particles with mass m_2. We consider the l-th partial wave. For a separable interaction W of the form (2) we then obtain

$$V(E)=\beta_2^2B\int_0^\infty \xi_2^2(p)p^{3n-5}d\varepsilon/(E+Q-\varepsilon+i0). \tag{3}$$

Here $Q = 2m_1 - nm_2$ is the energy released in the annihilation channel, $p = (m_2\varepsilon)^{1/2}$, and

$$B=\lambda^2(\beta_2^2/m_1)\xi_2(k)\xi_2(k')P_l(\cos\theta).$$

From (3) we easily obtain

$$\operatorname{Im} V(E)=-\pi\beta_2^2B\xi_2^2(p_0)p_0^{3n-5},\quad p_0=[m_2(E+Q)]^{1/2}, \tag{4}$$

$$\operatorname{Re} V(E)=\beta_2^2B\,⨍_0^\infty \xi_2^2(p)p^{3n-5}d\varepsilon/(E+Q-\varepsilon). \tag{5}$$

At large p (starting with $p\sim\beta_2$) the form factor $\xi_2(p)$ decreases rapidly enough, ensuring convergence of the integral in Eq. (5) at values $\varepsilon \sim \beta_2^2/m_2$. Since we are interested in energies $E \ll m_1$ and in an energy release $Q \ll m_1$, it follows that $E + Q \ll \beta_2^2/m_2$ (we recall that $\beta_2 \approx 2m_1$). It follows from the foregoing that the integral in (5) is negative. This means that annihilation leads to an additional attraction between N and $\bar{N}$ at nonrelativistic energies. This conclusion is not a consequence of the specifics of the employed model, but is general in character. Consider, e.g., the simplest canonical diagram of the annihilation scattering of N and $\bar{N}$ (two nucleons are transferred through the t channel, and two mesons through the s channel). We assume for simplicity the nucleons and mesons to have zero spin. In the Mandelstam representation, this diagram corresponds to a generalized optical potential of the form

$$V(r,s)\sim-(1/r)\int_{4m_1^2}^\infty ds'(s-s'-i0)^{-1}\int_{t_0(s)}^\infty \exp(-rt^{1/2})y(s',t)dt. \tag{6}$$

Here s and t are known invariant variables, and $y(s,t)$ is the double spectral density

$$y(s,t)=g^4s^{1/2}/\{t(s-4m_1^2)[(s-4m_2^2)(s-4m_1^2)(s+t)-s(s-2m_2^2)^2]\}^{1/2}, \tag{7}$$

where g is the meson–nucleon coupling constant. The lower integration limit in (6) is given by

$$t_0(s)=\begin{cases}4m_1^2, & 4m_2^2\leqslant s\leqslant 4m_1^2,\\ 4m_1^2s(s-4m_2^2+m_2^4/m_1^2)/[(s-4m_1^2)(s-4m_2^2)], & s>4m_1^2.\end{cases}$$

The spectral density (7) leads to a negative real part of the potential (6) at $s\approx 4m_1^2$ (near the $N\bar{N}$ threshold). In other words, we obtain additional attraction between N and $\bar{N}$.[3)]

Returning to our model, we estimate the ratio of the imaginary and real parts (4) and (5) of the generalized optical potential. At $p \ll \beta_2$ the form factor $\xi_2(p)$ is given by $\xi_2(p) \sim (p/\beta_2)^l$. Since it is normalized by the condition

$$\int_0^\infty \xi_2^2(p) p^{3n-4} dp = 1$$

and depends on only one dimensional parameter β_2, it can be represented in the form

$$\xi_2(p) = C\beta_2^{-(3n-3)/2} h(x),$$

where $x = p/\beta_2$, $h(x)$ is a dimensionless function, and the dimensionless constant C is chosen such that $h(x) \approx x^l$ at $x \ll 1$. Since $E + Q \ll \beta_2^2/m_2$, Eqs. (4) and (5) take the form

$$\operatorname{Im} V(E) \approx -\pi BC^2 (p_0/\beta_2)^{2l+3n-5},$$
$$\operatorname{Re} V(E) \approx -2BC^2 \int_0^\infty h^2(x) x^{3n-6} dx. \qquad (8)$$

We recall that the function $h(x)$ ensures convergence of the integral in (8) at the values $x \sim 1$.

Thus, we ultimately obtain

$$A = \operatorname{Im} V(-E_B^0)/\operatorname{Re} V(-E_B^0) \sim [m_2(Q-E_B^0)/\beta_2^2]^{(2l+3n-5)/2}, \qquad (9)$$

where E_B^0 is the initial (at $\lambda = 0$) binding energy of the considered quasinuclear state. In the case $n = 2$ (two-particle annihilation channel) and $l \neq 0$, the exponent in (9) is $\geq 3/2$. Consequently $A \ll 1$ independently of E_B^0 i.e., the attraction due to the annihilation is much stronger than the absorption. The baryonium coupling energy is therefore expected to increase monotonically with increasing annihilation constant λ, and the annihilation width is expected to be much less than the shift. The number of particles in the annihilation channel enters in Eq. (9) in the same manner as the orbital momentum l. This means that if $l = 0$ but $n \geq 3$ the character of the motion of the quasinuclear pull will be the same as at $l \neq 0$ and $n = 2$ (this was in fact obtained earlier in Ref. 14 by numerical calculation for $n = 3$). When $l = 0$ but $n = 2$, the exponent in (9) is $1/2 < 1$. Therefore, depending on the value of E_B^0, the pole can behave in two ways. If the initial binding energy is high enough ($E_B^0 \sim Q$), then $A \ll 1$ and the pole moves in the same manner as in the cases described above (see Ref. 12). When the binding energy is not too large ($E_B^0 \ll Q$), we have $A \sim 1$ and the pole motion is finite about the initial position (see Ref. 11).[4)]

The character of the motion of the quasinuclear poles with increasing annihilation constant λ is thus determined mainly by the fact that the annihilation produces between N and $\bar{N}$, an attraction which in the sense indicated above is much stronger than the absorption.

4. ANNIHILATION WIDTHS AND SHIFTS OF BOUND QUASINUCLEAR STATES

We consider now the pole trajectories (Fig. 1) corresponding to a quasinuclear bound state and obtained by numerical calculation from the equations of the Appendix. Curve 1 of Fig. 1 corresponds to $E_B^0 = 0$ (the bound state is located at $\lambda = 0$ at the threshold of the NN channel), and curve 2 corresponds to $E_B^0 = 70$ MeV.

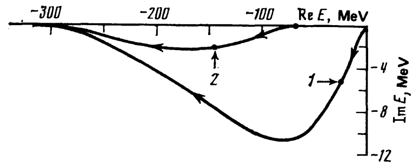

FIG. 1. Motion of poles corresponding to bound quasi-nuclear states with increasing annihilation constant λ.

As expected, annihilation increases the binding energy of the level, which acquires a small annihilation width. The maximum width Γ_{an}^{max} is smaller the larger the initial binding energy E_B^0 [in accordance with Eq. (9)]. Thus, at $E_B^0 = 0$ the width is $\Gamma_{an}^{max} = 21$ MeV, while at $E_B^0 = 70$ MeV we have $\Gamma_{an}^{max} = 5$ MeV.

The annihilation widths of the d levels thus turn out to be in our realistic model smaller by one or two orders of magnitude than the widths of the s levels, and amount to several MeV. The physical cause of the decrease of the annihilation width on going to higher orbital momenta is the action of the centrifugal barrier which prevents the distance between N and $\bar{N}$ to decrease to the annihilation value.

At larger values of the constant λ, the quasinuclear pole goes off far from its initial position. At the same time another ("annihilation") pole (not shown in Fig. 1) approaches the physical region. The behavior of such poles, which appear near the physical region in the case of intense annihilation depends strongly on the dynamics at short distances, $r \lesssim r_a$. They have therefore no physical meaning within the framework of the nonrelativistic approach (which is used by us to describe near-threshold phenomena that are determined mainly by the nuclear interaction between N and $\bar{N}$).

5. EFFECT OF ANNIHILATION ON RESONANT QUASINUCLEAR STATES

Two-channel resonance is known to correspond to two pairs of poles located on two sheets (III and IV) of the Riemann surface E. It is therefore more convenient to track the motion of the resonant poles on the Riemann surface of the variable k_1.

It was shown in Sec. 3 that coupling with the annihilation channel produces additional attraction between N and $\bar{N}$. One can therefore expect the resonance energy and its elastic width to decrease with increasing annihilation constant. The resonant state can become bound at a sufficiently large constant λ.

These expectations are confirmed by numerical calculations (Fig. 2). Figure 2(a) corresponds to a ratio of the annihilation and nuclear-interaction radii $r_a/R = 0.1$, while Fig. 2(b) corresponds to $r_a/R = 0.2$ ($\beta_1 = 0.2\,\beta_2 = 376$ MeV/c). In Fig. 2(a) the total width of the resonance decreases from $\Gamma_0 = 15$ MeV (at $\lambda = 0$) to $\Gamma_{min} = 5$ MeV (at $\lambda^2 = 21.3$), while in the case b we have respectively $\Gamma_0 = 19$ MeV and $\Gamma_{min} = 3$ MeV (at $\lambda^2 = 13$). We note that the decrease of the total width of a quasinuclear s resonance on account of annihilation was observed earlier in Ref. 16. Here this phenomenon is

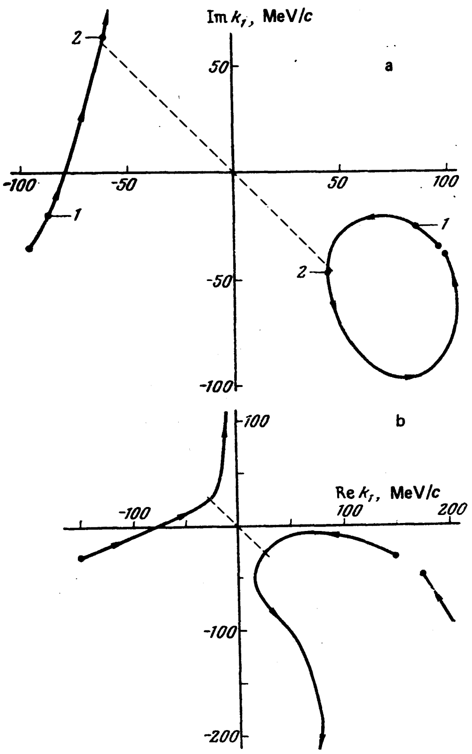

FIG. 2. Motion of poles corresponding to quasinuclear resonance: a) $r_a/R = 0.1$; b) $r_a/R = 0.2$ ($\beta_1 = 0.2$ $\beta_2 = 376$ MeV/c).

even more pronounced, for in the case of the d states the effect of attraction due to annihilation is much stronger than the absorption effect (see Sec. 3).

At $\lambda^2 = 21.9$ in case a and $\lambda^2 = 13.6$ in case b, the resonant state turns into a bound state at the $N\bar{N}$ threshold (the poles cross in this case the bisector of the second and fourth quadrants). With further increase of the constant λ, the binding energy of this state increases.

On Fig. 2(b) (at $r_a/R = 0.2$) the right-hand pole goes off at large λ far from its initial position, and an "annihilation" pole arrives in its place. In Fig. 2(a) (at $r_a/R = 0.1$) no such restructuring takes place (the "annihilation" pole passes far from the physical region, and the quasinuclear pole executes finite motion near the initial position). The difference is due to the fact that in the case $r_a/R = 0.2$ a larger part of the quasinuclear-state wave function lands in the annihilation region then at $r_a/R = 0.1$.

We note that trajectories similar to those in Figs. 1 and 2(a) were obtained recently in the case of an s wave in a single-channel model with complex separable potential in Ref. 22 (see also Ref. 23). This result could be expected beforehand, since the authors of these papers used an attractive potential with sufficiently small ratio of the imaginary and real parts.

6. DEPENDENCE OF THE ANNIHILATION CROSS SECTION ON THE MOMENTUM AT LOW ENERGIES

We consider the dependence of the annihilation probability $v\sigma_{an}$ (v is the relative velocity of N and $\bar{N}$, and σ_{an} is the annihilation cross section) on the momentum k_1 of the antinucleon in the c.m.s. The curves on Figs. 3(a), (b), (c) were calculated from Eq. (A.3) of the Appendix at $\lambda^2 = 21.96$, while those in Fig. 3(c) were calculated at $\lambda^2 = 19.52$. Figure 3(a) corresponds to the existence of a quasinuclear state with binding energy $E_B = 150$ MeV and an annihilation width $\Gamma_{an} = 4$ MeV. Despite the small width of the level, the annihilation cross section reaches 65% of the unitary level for the d wave at a momentum $k_1 = 200$ MeV/c. In the case of Fig. 3(b), the level binding energy is $E_B = 25$ MeV, and its width is $\Gamma_{an} = 10$ MeV. Since this bound state is closer to the $N\bar{N}$ threshold, a noticeable increase of the annihilation cross section takes place at low temperatures. Thus, at a momentum $k_1 = 200$ MeV/c it amounts already to 82% of the unitary limit. Figure 3(c) corresponds to the presence of quasinuclear resonance with energy $E_R = 7$ MeV and total width $\Gamma = 8.5$ MeV. As expected, the annihilation cross section has in this case a clearly pronounced resonant character. At resonance, at a momentum $k_1 = 100$ MeV/c, it amounts to 30% of the unitary limit. Finally, Fig. 3(d) corresponds to the conversion (on account of annihilation) of the resonant state into a bound state that is very close to the NN threshold ($E_B = 1$ MeV and $\Gamma_{an} = 15$ MeV). The presence of such a near-threshold level leads to a considerable increase of the cross section for annihilation of slow $\bar{N}$. At a momentum $k_1 = 100$ MeV/c it amounts to 68% of the unitary limit.

7. CONCLUSION

The principal results of this paper are the following:

1. Annihilation increases the binding energies of quasinuclear d states by several dozen MeV. Their annihilation widths, at any rate, do not exceed 10 MeV.

2. The total width of the quasinuclear resonances

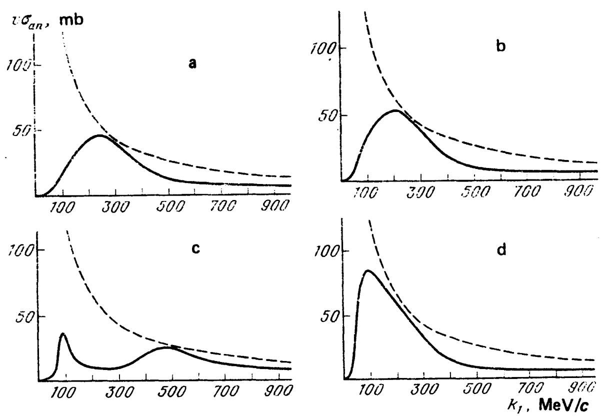

FIG. 3. Dependence of the annihilation probability $v\sigma_{an}$ on the antinucleon c.m.s. momentum k_1 in the following cases: a) in the presence of a quasinuclear bound state with binding energy $E_B = 150$ MeV and an annihilation width Γ_{an} (Fig. 4 MeV (point 2 of Fig. 1); b) the same at $E_B = 25$ MeV and $\Gamma_{an} = 10$ MeV (point 1 on Fig. 1); c) in the case of existence of a quasinuclear resonance with energy $E_R = 7$ MeV and total width $\Gamma = 8.5$ MeV (point 1 on Fig. 2a); d) the same as a) at $E_B = 1$ MeV and $\Gamma_{an} = 15$ MeV (point 2 on Fig. 2a). The dashed curves shows the unitary limit for the d wave $\sigma_{an}^{vL} = 10\pi/(m_1 k_1)$.

may decrease as a result of annihilation, while the resonances can turn into bound states close enough to the $N\bar{N}$ threshold.

3. The cross section for NN annihilation at momenta $k_1 \gtrsim 200$ MeV/c is comparable with the unitary limit for the d wave. The presence of quasinuclear states close to the $N\bar{N}$ threshold increases considerably the cross section for annihilation at low energies (up to $\gtrsim 50\%$ of the unitary limit at a momentum $k_1 = 100$ MeV/c). Since the numerical results of the present paper are physically lucid and agree with qualitative estimates (see Refs. 4 and 10), the main results are valid also for other orbital momenta $l \neq 0$.

In conclusion, the author is sincerely grateful to V.E. Markushin and I.S. Shapiro for numerous helpful discussions of the results of the work and for reading the manuscript.

APPENDIX

We present below equations for the partial amplitude f for elastic scattering of an antinucleon $\bar{N}$ of a nucleon N and for the annihilation probability $v\sigma_{an}$ in the considered model.

We introduce the notation

$$\bar{g}=g\beta_1^2/m_1,\quad \bar{\lambda}=\lambda\beta_2^2/(m_1m_2)^{1/2},$$

$$G_{inj}(E)=\int_0 \xi_i(p)\xi_j(p)p^2dp/[E+2(m_1-m_n)-p^2/m_n+i0]. \quad \text{(A.1)}$$

Equation (A.1) yields the matrix element of the free Green operator of the channel n (in the absence of coupling between the channels). The indices k, j, and n can take on values 1 and 2. Substituting in (A.1) expression (2) for ξ_i and integrating, we obtain

$$G_{inj}(E)=8m_n(\beta_i\beta_j)^{1/2}\{\beta_i\beta_jk_n^2+[(\beta_i+\beta_j)k_n +i\beta_i\beta_j]^2\}[5(\beta_i+\beta_j)^3(k_n+i\beta_i)^2(k_n+i\beta_j)^2]^{-1}, \quad \text{(A.2)}$$

where k_n is the momentum in channel n and is connected with the energy E by Eqs. (1).

The amplitude f is expressed in terms of the quantities in (A.2) with the aid of the Lippman-Schwinger equation in the following manner

$$f(E)=-\pi m_1N(E)/[2D(E)],$$

where

$$D(E)=(1-\bar{g}G_{111})(1-\bar{\lambda}^2G_{222}G_{212})-\bar{g}\bar{\lambda}^2G_{222}G_{112}^2,$$

$$N(E)=\bar{g}\xi_1^2(1-\bar{\lambda}^2G_{212}G_{222})+\bar{\lambda}^2\xi_2^2G_{222}(1-\bar{g}G_{111}) +2\bar{g}\bar{\lambda}^2\xi_1\xi_2G_{222}G_{112}.$$

The poles of this amplitude are determined by the solution of the algebraic equation $D(E) = 0$. The annihilation probability $v\sigma_{an}$ is connected with the amplitude by the formula

$$v\sigma_{an}=10\pi(1-|1+2ik_1f|^2)/(m_1k_1). \quad \text{(A.3)}$$

1) The question was considered,[19] using the coupled-channel scheme, on the basis of the N/D method, using the discontinuity of the Born amplitude on the left-hand potential) cut in the complex energy plane. As shown by a special investigation[15] for the s states, the results of such calculations differ greatly from the exact ones. The baryonium-level spectrum obtained by this method does not agree with the result of the exact solution of the Schrödinger eigenvalue problem even in the absence of annihilation (see Ref. 10). The "Born N/D method" yields annihilation widths of the order of 100 MeV independently of the orbital momentum of the states.

2) Here and elsewhere we use a system of units in which $\hbar = c = 1$.

3) It is concluded in Ref. 20 on the basis of a similar analysis that repulsion takes place between N and $\bar{N}$. This erroneous conclusion is due to the fact that the authors used for the spectral density an incorrect expression corresponding to massless initial and final particles (in analogy with photon-photon scattering in electrodynamics, see Ref. 21). It is clear, however, that such a formula can be used only if $s, t \gg 4m_1^2$, i.e., in the ultrarelativistic region, whereas we are interested in nonrelativistic energies.

4) The change of the character of the pole trajectories when the energy-independent ratio of the imaginary and real parts of the optical potential is varied was investigated in the single-channel model in the case of $l = 0$ in Ref. 22 (see also Ref. 23 and the literature therein).

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