Photon emission by electrons in four-wave diffraction in a single crystal

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Some properties of the electromagnetic waves emitted by electrons in Laue four-wave scattering by two sets of mutually perpendicular reflecting crystal planes are considered. It is shown that in this case, along with electric dipole transitions there also appear mixed electric quadrupole and magnetic dipole transitions which result in the emission of waves with frequencies $\omega_\pm=\omega_1\pm\omega_2(\omega_{1,2}$ are the electric dipole radiation frequencies). The corresponding multipole moments exceed the atomic values by several orders of magnitude. The multipole structure of the matrix elements is a result of expansion of the Bloch electron functions in the parameter of deviation from the Bragg condition in the final state, and reflects the nature of the electron wave beats in the crystal. The usual approach, based on expansion in the radiation-field multipoles, is not valid in this case, since it implies a confined (e.g., by the channel boundaries or upon channeling) region of motion of the particles. As a result the radiation from a diffracted electron is found to be similar to the radiation from macroscopic multipoles moving in a refracting medium and oscillating at different frequencies. This seems to be a unique situation from the viewpoint of experimental investigation of multipole radiation of multipoles moving with "circaluminal" and "superluminal" velocities. Formulas from the radiation intensities are obtained.

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1. INTRODUCTION

It was shown earlier[1-3] that electromagnetic transitions between different dispersion surfaces or, equivalently, between transverse-motion bands,[1)] give rise to Pendellosung radiation when an electron is diffracted in a single crystal. The frequency of this radiation can range widely, from the infrared to the x-ray region. The two-wave case was considered and it was shown that the classical analog of a diffracting electron is an electric dipole oscillator moving in a refractive medium, oriented perpendicular to the velocity, and oscillating with frequency $\omega_0=2V_g/\hbar\equiv 2\pi v_\parallel/\xi_g$, where v_g is the amplitude of the corresponding harmonic of the periodic potential $V(\mathbf{r})$ of the lattice ($2V_g$ is the width of the forbidden band in the spectrum of the transverse electron motion), $v_\parallel$ is the average propagation velocity of the electron along the crystallographic planes, and ξ_g is the extinction length. This analogy has a simple physical meaning in terms of the two-wave approximation of dynamic diffraction theory. As a result of beats between electron waves having different wave vectors belonging to two branches of the dispersion surface, electrons are periodically "pumped" over the thickness of the crystal, with a period ξ_g, from the direct beam to the diffracted beam and back (the Pendellosung effect), i.e., the electron, which propagates on the average along the crystallographic planes, oscillates in the direction of the reciprocal lattice vector at a frequency $2\pi v_\parallel/\xi_g$.

The present paper deals with the properties of the electromagnetic radiation from an electron passing through a crystal when the Bragg condition is simultaneously satisfied for two systems of mutually perpendicular crystallographic planes ($\mathbf{g}_1\cdot\mathbf{g}_2=0$). In this case the incident and reflected waves are transformed in the crystal into four types of electron waves. These waves have different symmetries in the directions $\mathbf{g}_1$ and $\mathbf{g}_2$ (the x and y axes, respectively), and have different wave-vector components in a direction parallel to the planes (the z axis). The resultant picture of the beats between waves of different types is reflected in a more complicated structure of the electromagnetic transition current than in the two-beam case. It will be shown that the classical analog of such a structure is a system of charges moving in the direction of the z axis, located in a plane perpendicular to this axis, and oscillating in the directions $\mathbf{g}_1$ and $\mathbf{g}_2$ with frequencies $\omega_{01}=2V_g/\hbar$ and $\omega_{02}=2V_g/\hbar$, as indicated in Fig. 1. The result is radiation due to electric dipole oscillations of the diffracting electron in the directions $\mathbf{g}_1$ and $\mathbf{g}_2$, with frequencies ω_{01} and ω_{02}, as well as electric quadrupole and magnetic dipole oscillations with frequencies $\omega_{0\pm}=\omega_{01}\pm\omega_{02}$. The dependence of the photon frequency on the radiation direction, just as in the two-wave case, is determined either by the normal or anomalous Doppler effects, except that the photon spectrum in each direction consists of four frequencies.

We note that the quadrupole and magnetic moments produced with the radiation are of macroscopic size. In particular, the magnetic moment can amount to 10^3–10^4 Bohr magnetons.

2. BLOCH FUNCTIONS AND EQUATION OF THE DISPERSION SURFACE OF THE ELECTRON IN FOUR-WAVE DIFFRACTION

We consider a beam of electrons, having an energy E and a momentum $\hbar\mathbf{k}_0$, incident on a single crystal in a direction close to the Bragg direction for both systems of crystallographic planes in the symmetrical Laue scheme (the crystal boundary is perpendicular to the planes). We can choose, for example, a crystal with body centered cubic structure, for which the geometric structure factor of the reflection $\mathbf{g}_1+\mathbf{g}_2$ is equal to zero, i.e., $V_{g1+g2}=0$. The variables in the wave equation sep-

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arate in this case, and in the weak-coupling approximation[4] the propagation of the electron in the crystal can be described by the Bloch functions $\psi^{(\alpha,\beta)}$ corresponding to the four branches $(\alpha\beta)_E$ of the dispersion surface ($\alpha=1$ and 2; $\beta=1$ and 2):

$$\psi^{(\alpha\beta)}(\mathbf{k}^{(\alpha\beta)},\mathbf{r})=\psi^{(\alpha)}_{k_{\perp 1}}(x)\psi^{(\beta)}_{k_{\perp 2}}(y)\exp(ik_\parallel^{(\alpha\beta)}z), \quad (1)$$

where $\psi^{(\alpha)}_{k\perp j}(x_j)$ $(j=1,2)$ are the one-dimensional Bloch functions of the transverse motion in the two-wave approximation. Their form is known[3-5]:

$$\psi^{(1)}_{\perp j}(x_j)=\cos\phi_j\exp(ik_{\perp j}x_j)+\sin\phi_j\exp[i(k_{\perp j}+g_j)x_j],$$
$$\psi^{(2)}_{k_{\perp j}}(x_j)=-\sin\phi_j\exp(ik_{\perp j}x_j)+\cos\phi_j\exp[i(k_{\perp j}+g_j)x_j]. \quad (2)$$

Here $\tan\phi_j=-[\Delta_j+(\Delta_j^2+U_{gj}^2)^{1/2}]/U_{gj}$ and $\Delta_j=[(k_{\perp j}+g_j)^2-k_{\perp j}^2]/2$ are the parameters of the deviation from the Bragg condition for the system of reflecting planes $\mathbf{g}_j$, and $U_{gj}-2mV_{gj}/\hbar^2$. Correspondingly,

$$\cos^2\phi_j=\frac{1}{2}\left(1-\frac{\Delta_j}{(\Delta_j^2+U_{g_j}^2)^{1/2}}\right). \quad (3)$$

The values of the wave vectors $k^{(\alpha\beta)}$ $(k^{(\alpha\beta)2}=k_{\perp 1}^2+k_{\perp 2}^2+k_\parallel^{(\alpha\beta)2})$ are determined by the equation of the dispersion surface:

$$k^{(\alpha\beta)2}=K^2-(\Delta_1+\Delta_2)+(-1)^\alpha(\Delta_1^2+U_{g_1}^2)^{1/2}+(-1)^\beta(\Delta_2^2+U_{g_2}^2)^{1/2}, \quad (4)$$

where $K^2=k_0^2+U_0$, $U_0=2mV_0/\hbar^2$, and $-V_0$ is the average potential of the crystal.

3. ELECTROMAGNETIC TRANSITIONS OF DIFFRACTING ELECTRON IN THE FOUR-WAVE CASE

Let electrons with energy E_a by incident on the crystal at exactly the Bragg angle relative to both systems of crystallographic planes $(k_{\perp 1}=k_{0x}=-g_1/2,\ k_{\perp 2}=k_{0y}=-g_2/2)$. In this case the waves of all four types are excited in the crystal with equal amplitude,[5] so that the wave function ψ_a of the electron inside the crystal is of the form

$$\psi_a=\frac{1}{2}\sum_{\alpha\beta}\psi_a^{(\alpha\beta)}=\frac{1}{2}\sum_{\alpha\beta}\psi_a^{(\alpha)}(x)\psi_a^{(\beta)}(y)\exp(ik_{a\parallel}^{(\alpha\beta)}z), \quad (5)$$

where[2)]

$$\psi_a^{(1)}(x_j)=-i\sqrt{2}\sin(g_jx_j/2),\quad \psi_a^{(2)}(x_j)=\sqrt{2}\cos(g_jx_j/2), \quad (6)$$
$$k_a^{(\alpha\beta)2}=K_a^2+(-1)^\alpha U_{g_1}+(-1)^\beta U_{g_2}. \quad (7)$$

Thus, the double index $\alpha\beta$ characterizes the symmetry of the two-dimensional Bloch function if the Bragg condition is exactly satisfied with respect to the coordinates x and y. We note that different longitudinal momenta $\hbar k_{\alpha\parallel}^{(\alpha\beta)}$ correspond to different transverse energies of the electron, and the difference is due to the different symmetries of the transverse-motion functions.

As a result of emitting a photon of energy $\hbar\omega$ and momentum $\hbar\varkappa$ $(\varkappa=\omega n/c)$, the electron can go over to any of the four branches of the dispersion surface of lower energy $E_b=E_0-\hbar\omega$. The Bloch functions of the final state of the electron are determined by the quasimomentum conservation laws $\mathbf{k}_b^{(\alpha\beta)}=\mathbf{k}_a^{(\gamma\delta)}-\varkappa$:

$$\psi_b^{(\alpha\beta)}=\psi_b^{(\alpha)}(x)\psi_b^{(\beta)}(y)\exp(ik_{b\parallel}^{(\alpha\beta)}z); \quad (8)$$

where

$$\psi_b^{(1)}(x_j)=-i\sqrt{2}\left(\sin\frac{g_jx_j}{2}+i\frac{\varkappa g_j}{2U_{g_j}}\cos\frac{g_jx_j}{2}\right)\exp(-i\varkappa_jx_j),$$
$$\psi_b^{(2)}(x_j)=\sqrt{2}\left(\cos\frac{g_jx_j}{2}+i\frac{\varkappa g_j}{2U_{g_j}}\sin\frac{g_jx_j}{2}\right)\exp(-i\varkappa_jx_j). \quad (9)$$

Account is taken here of the fact that the photon emission violates weakly the Bragg condition,[3] and the terms linear in Δ_{bj}/U_{gj} of the expansion in (3) for the one-dimensional functions of the final state of the electron $(\Delta_{bj}=[(k_{bj}+g_j)^2-k_{bj}^2]/2=-\varkappa g_j)$ have been retained. We get correspondingly from (4)

$$k_b^{(\alpha\beta)2}=K_b^2+\varkappa(\mathbf{g}_1+\mathbf{g}_2)+(-1)^\alpha U_{g_1}+(-1)^\beta U_{g_2}. \quad (10)$$

The matrix element of the transition $a\to b$ from the branch $(\alpha\beta)_{E_a}$ to the branch $(\gamma\delta)_{E_b}$ with emission of a quantum with polarization $\mathbf{u}_\lambda$ is of the form

$$H_\lambda(\alpha\beta\to\gamma\delta)={}^1/_2\langle\psi_b^{(\gamma\delta)}|H_\lambda|\psi_a^{(\alpha\beta)}\rangle, \quad (11)$$

where $H_\lambda=(e/mn)(2\pi\hbar^{\varkappa r}\omega)^{1/2}e^{-i\varkappa r}(\mathbf{u}_\lambda i\hbar\nabla)$ is the operator of the interaction of the electron with the photon in the refracting medium.[6]

In the four-wave case we therefore have 16 transitions $(\alpha\beta)_{E_a}\to(\gamma\delta)_{E_b}$, which can be subdivided, in accord with the character of the change of the symmetry of the wave function of the transverse motion, into three groups:

1) four transitions of the type $(\alpha\beta)\to(\alpha\beta)$ without change of symmetry between identical branches of the dispersion surface. Just as in the two-wave case,[2] they lead to Čerenkov radiation;

2) eight transitions $(1\alpha)\rightleftarrows(2\alpha)$, $(\alpha 1)\leftrightarrows(2\alpha)$ with change of symmetry along one of the directions $\mathbf{g}_1$ or $\mathbf{g}_2$ lead to electric-dipole Pendellosung radiation similar to that considered earlier[1-3];

3) four transitions $(11)\rightleftarrows(22)$, $(12)\leftrightarrows(21)$ with simultaneous change of the energy along both directions lead to Pendellosung radiation of mixed electric quadrupole and magnetic dipole type. The total number of photons emitted by the diffracting electron per unit time is determined by the formula[3)]

$$N=\sum_{\substack{\alpha\beta\gamma\\ \delta\lambda}}\int\frac{2\pi}{\hbar}|H_\lambda(\alpha\beta\to\gamma\delta)|^2\delta(E_a-E_b-\hbar\omega)\frac{n^3\omega^2}{c^3}\frac{d\omega do}{(2\pi)^3}. \quad (12)$$

4. ČERENKOV RADIATION

The calculation of the matrix elements (11) for the transitions of the first group leads to the result

$$H_\lambda(\alpha\beta\to\alpha\beta)=-\frac{e}{2mn}\left(\frac{2\pi\hbar}{\omega}\right)^{1/2}\hbar\left[k_{a\parallel}^{(\alpha\beta)}(\mathbf{e}_\parallel\mathbf{u}_\lambda)+(-1)^\alpha\frac{\varkappa\mathbf{g}_1}{2U_{g_1}}\frac{\mathbf{g}_1\mathbf{u}_\lambda}{2}+(-1)^\beta\frac{\varkappa\mathbf{g}_2}{2U_{g_2}}\frac{\mathbf{g}_2\mathbf{u}_\lambda}{2}\right]. \quad (13)$$

Here $\mathbf{e}_\parallel$ is a unit vector in the direction of the z axis. Since

$$\frac{\varkappa g_j}{2U_{g_j}}=\frac{\hbar\omega}{2V_{g_j}}\beta_{\perp j}n(\mathbf{e}_j\mathbf{n}_\varkappa)\ll 1 \quad (14)$$

$(\beta_{\perp j}=v_{\perp j}/c,\ v_{\perp j}=\hbar g_j/2m$ are the transverse velocities corresponding to the momenta $\hbar g_j/2$, $\mathbf{e}_j=\mathbf{g}_j/g_j$; $\mathbf{n}_\varkappa=\varkappa/\varkappa)$, the last two terms in (13) can be neglected in this case, since they contain an additional smallness $g_j 2k_{a\parallel} \sim \beta_{\perp j}$ compared with the first terms.

From the quasimomentum conservation laws $\mathbf{k}_b^{(\alpha\beta)} = \mathbf{k}_a^{(\alpha\beta)} - \boldsymbol{\varkappa}$, with account taken of the dispersion equations for the electron and photon, we obtain

$$E_a(\mathbf{k}_a^{(\alpha\beta)}) - E_b(\mathbf{k}_b^{(\alpha\beta)}) - \hbar\omega = -\hbar\omega(1 - n\beta_\parallel^{(\alpha\beta)}\cos\theta), \qquad (15)$$

where $\beta_\parallel^{(\alpha\beta)} = \hbar k_a^{(\alpha\beta)}/mc$ and $\cos\theta = \mathbf{n}_\varkappa \cdot \mathbf{e}_\parallel$. The energy conservation law in this case leads to conditions for the onset of Čerenkov radiation.

For the number N_c of the Čerenkov photons emitted per unit time we get from (12)

$$N_c = \frac{1}{4}\alpha n \sum_{\alpha\beta} \beta_\parallel^{(\alpha\beta)2} \int \delta(1 - n\beta_\parallel^{(\alpha\beta)}\cos\theta)\sin^2\theta \frac{d\omega do}{2\pi}. \qquad (16)$$

Here $\alpha = e^2/\hbar c$. Thus, the Čerenkov radiation of the diffracting electron is described by the Tamm-Frank formula,[8] except that the radiation is in four directions because of the different propagation velocities of waves of different types in the crystal.

5. PENDELLOSUNG RADIATION OF THE ELECTRIC-DIPOLE TYPE

For the transitions $(1\alpha) \rightleftarrows (2\alpha)$ and $(\alpha 1) \leftrightarrows (\alpha 2)$ the quasimomentum conservation laws and the dispersion laws lead to the following result:

$$E_a - E_b - \hbar\omega = \pm 2V_{g_i} - \hbar\omega(1 - n\beta_\parallel\cos\theta). \qquad (17)$$

Here and below the index i takes on the following values: $i = 1$ for transitions $(1\alpha) \rightleftarrows (2\alpha)$ with change of symmetry of the wave function along the direction $\mathbf{e}_i$, $i = 2$ for transitions $(\alpha 1) \leftrightarrows (\alpha 2)$ with change of symmetry with respect to $\mathbf{e}_2$. The plus sign pertains to transitions indicated by the upper arrow $(1\alpha)_{E_a} \to (2\alpha)_{E_b}$, $(\alpha 1)_{E_a} \to (\alpha 2)_{E_b}$ with decreasing transverse energy of the electron, and the minus sign to transitions indicated by the lower arrow $(2\alpha)_{E_a} \to (1\alpha)_{E_b}$, $(\alpha 2)_E \to (\alpha 1)_{E_b}$ with increase of the transverse energy; $\beta_\parallel = v_\parallel/c$, $v_\parallel = \hbar K_{a\parallel}/m$ is the average propagation velocity of the electron along the crystallographic planes. The energy conservation laws determine the spectrum of the radiated photons:

$$\omega_i = \omega_{0i}[\pm(1 - n\beta_\parallel\cos\theta)]^{-1}, \qquad (18)$$

where $\omega_{0i} = 2V_{g_i}/\hbar$. Thus, the radiation frequencies for the given transitions are determined by beats of the electron waves in the directions $\mathbf{g}_1$ and $\mathbf{g}_2$, analogous to the beats in two-wave diffraction.[1-3] The dependence of the frequencies on the radiation direction is determined either by the normal (plus sign) or the anomalous (minus sign) Doppler effect.

The matrix elements are also analogous to the two-wave elements[3] and have an electric-dipole structure:

$$H_\lambda \begin{pmatrix} 1\alpha \rightleftarrows 2\alpha \\ \alpha 1 \rightleftarrows \alpha 2 \end{pmatrix} = A_i \left[\mathbf{e}_i \mathbf{u}_\lambda \pm (\mathbf{e}_i \mathbf{n}_\varkappa)(\mathbf{e}_\parallel \mathbf{u}_\lambda) \frac{\hbar\omega}{2V_{g_i}} \beta_\parallel n \right], \qquad (19)$$

where

$$A_i = \frac{1}{2}\frac{e}{n} v_{\perp i} \left(\frac{2\pi\hbar}{\omega}\right)^{1/2}. \qquad (20)$$

In expression (19) it is important to take into account the terms linear in the parameter of the deviation from the Bragg condition in the final state of the electron, since their smallness is offset by the large longitudinal current of the electron.

The radiation intensity at the frequencies ω_1 and ω_2 is given by

$$\frac{dN_p^{\omega_1}}{do} = \frac{\alpha n}{4\pi}\beta_{\perp 1}\omega_{01} \frac{[(1-\beta_\parallel n\cos\theta) - (1-\beta_\parallel^2 n^2)\cos^2\varphi\sin^2\theta]}{(1-\beta_\parallel n\cos\theta)^4}, \qquad (21)$$

$$\frac{dN_p^{\omega_2}}{do} = \frac{\alpha n}{4\pi}\beta_{\perp 2}\omega_{02} \frac{[(1-\beta_\parallel n\cos\theta)^2 - (1-\beta_\parallel^2 n^2)\sin^2\varphi\sin^2\theta]}{(1-\beta_\parallel n\cos\theta)^4}, \qquad (22)$$

where $dN_p^{\omega_i}/do$ is the number of photons having a frequency ω_i and emitted in a unit time into a unit solid angle. The polar axis is directed along $\mathbf{e}_\parallel$ and the azimuthal angle φ is measured from the $\mathbf{e}_1$ direction.

For the total number of photons emitted per unit time we have (at $n = \text{const}$, $n\beta_\parallel < 1$)

$$N_p^{\omega_i} = \frac{2}{3}\alpha n \beta_{\perp i}^2 \frac{\omega_{0i}}{1 - \beta_\parallel^2 n^2}. \qquad (23)$$

At $n = 1$ and high electron energies ($\beta_\parallel \approx \beta$)

$$N_p^{\omega_i} = {}^2/_3 \alpha \beta_{\perp i}^2 \gamma^2 \omega_{0i},$$

where $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic factor, which represents the total mass of the electron in rest-mass units (we recall that m is taken to be the total mass[3]). Since the quantities $\beta_{\perp i}^2$ contain the relativistic factor γ^{-2}, at high electron energies the total number of radiated photons does not depend on the electron energy, while the energy losses increase with increasing energy like γ^2, owing to the Doppler shift of the photon frequency into the harder region.

Possible classical analogs of the transitions $(1\alpha) \rightleftarrows (2\alpha)$ and $(\alpha 1) \leftrightarrows (\alpha 2)$ are dipole oscillators that move in the direction of $\mathbf{e}_\parallel$ [Fig. 1(a)] and oscillate in the directions of $\mathbf{e}_i$ with frequencies ω_{0i} $(i = 1, 2)$, so that

$$\mathbf{d}_1(t) = 2ex_m\mathbf{e}_1\cos\omega_{01}t, \qquad \mathbf{d}_2(t) = 2ey_m\mathbf{e}_2\cos\omega_{02}t, \qquad (24)$$

where $\mathbf{d}_1$ and $\mathbf{d}_2$ are the corresponding electric dipole moments; x_m and y_m are the oscillation amplitudes of the charges shown in Fig. 1(a). The expressions for the radiation intensity of such oscillators moving in a refracting region were obtained by Frank.[9] Equations (21)–(23) coincide with them if the oscillation ampli-

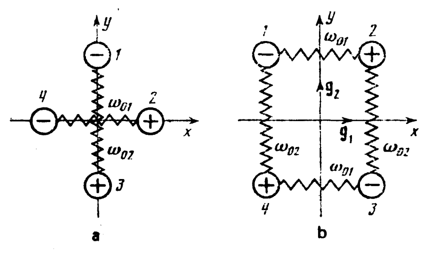

FIG. 1. Classical analogs that can be set in correspondence with electromagnetic transitions of a diffracting electron: a) system of two electric dipole oscillators oscillating in the directions $\mathbf{g}_1$ and $\mathbf{g}_2$ at frequencies ω_{01} and ω_{02}. It corresponds to transitions with a change in the symmetry of the wave function relative to one of the directions $\mathbf{g}_1$ or $\mathbf{g}_2$; b) system having quadrupole and magnetic moments with frequencies ω_{0+} and ω_{0-}. It corresponds to transitions with simultaneous change of the symmetry with respect to directions $\mathbf{g}_1$ and $\mathbf{g}_2$. Both systems are shown at the instant of time $t = 0$.

tudes x_m and y_m are set equal to $x_m = v_{\perp 1}/\omega_{01}$ and $y_m = v_{\perp 2}/\omega_{02}$, and if account is taken of the population of the states from which the transitions take place.

We note that in the relativistic case ($\gamma \gg 1$) at $n=1$ the radiation turns out to be concentrated near zero angles θ. Equations (21) and (22), integrated with respect to the angle φ, are then reduced to the form

$$dN_p^{\omega_i}/d\theta^2 = \alpha\beta_{\perp i}^2\omega_{0i}(\theta^4+\gamma^{-4})/(\theta^2+\gamma^{-2})^4. \tag{25}$$

Allowance is made here for the fact that at $\gamma \gg 1$ we have $\beta_\parallel \approx \beta \approx 1 - 1/2\gamma^2$.

The angle region from 0 to $1/2\gamma$ contains ~20% $N_p^{\omega_i}$, while 50% $N_p^{\omega_i}$, are contained in the region from 0 to $1/\gamma$. At the angle $\theta = 1/2\gamma$ the photons emitted have frequencies $\omega_i = 0.8\omega_{mi}$, where $\omega_{mi} \approx 2\gamma^2\omega_{0i}$ are the maximum frequencies of the photons emitted at zero angle. Thus, at $\gamma \gg 1$ in the range of angles θ from 0 to $1/2\gamma$ it is possible to observe two intense quasimonochromatic lines (the degree of monochromaticity is ~20%) with frequencies ~$2\gamma^2\omega_{0i}$, due to electric dipole transitions of the diffracting electron. Their intensities at crystal thicknesses ~$5\xi_g$ and at an electron current ~1 μA amount to ~10^8 photons/sec, and the frequencies can lie in the x-ray region.

6. PENDELLOSUNG RADIATION OF HIGHER MULTIPOLARITY

We consider the transitions of the third group with simultaneous change of the symmetry of the electron wave function in both directions

$$(11)_{E_a}\to(22)_{E_b},\quad (22)_{E_a}\to(11)_{E_b}\text{and}(12)_{E_a}\to(21)_{E_b},\quad (21)_{E_a}\to(12)_{E_b}.$$

The quasimomentum conservation laws, when allowance is made for the dispersion equations, lead in this case to the following result:

$$E_a - E_b - \hbar\omega = \pm 2V_s - \hbar\omega(1 - n\beta_\parallel\cos\theta). \tag{26}$$

The index s takes on here plus values for the transitions (11) ⇆ (22) and minus for (12) ⇆ (21), with $V_\pm = V_{g_1} \pm V_{g_2}$. The energy conservation laws determine the dependence of the photon frequency on the radiation direction:

$$\omega_s = \pm\omega_{0s}/(1 - n\beta_\parallel\cos\theta). \tag{27}$$

Here $\omega_{0\pm} = 2V_\pm/\hbar = \omega_{01} \pm \omega_{02}$. Just as in the preceding case, the plus sign in (27) gives the normal Doppler effect, and the transverse energy of the electron decreases (the transitions 11 → 22 and 12 → 21), while the minus sign gives the anomalous effect, at which the transverse energy increases (transitions 22 → 11 and 21 → 12). Thus, the transitions of the third group lead to radiation at the combined frequencies $\omega_\pm = \omega_1 \pm \omega_2$.

For the matrix elements H_λ (11 ⇆ 22) and H_λ (12 ⇆ 21) we obtain the following expressions:

$$H_\lambda(11\rightleftarrows 22) = \mp\frac{1}{2}\frac{e}{mn}\left(\frac{2\pi\hbar}{\omega}\right)^{1/2}\hbar\left[\frac{(\mathbf{g}_1\mathbf{u}_\lambda)}{2}\frac{(\varkappa\mathbf{g}_2)}{2U_{g_2}} + \frac{(\mathbf{g}_2\mathbf{u}_\lambda)}{2}\frac{(\varkappa\mathbf{g}_1)}{2U_{g_1}} \pm \frac{(\varkappa\mathbf{g}_1)}{2U_{g_1}}\frac{(\varkappa\mathbf{g}_2)}{2U_{g_2}}k_{a\parallel}(\mathbf{e}_\parallel\mathbf{u}_\lambda)\right],$$

$$H_\lambda(12\rightleftarrows 21) = \pm\frac{1}{2}\frac{e}{mn}\left(\frac{2\pi\hbar}{\omega}\right)^{1/2}\hbar\left[\frac{(\mathbf{g}_1\mathbf{u}_\lambda)}{2}\frac{(\varkappa\mathbf{g}_2)}{2U_{g_2}} - \frac{(\mathbf{g}_2\mathbf{u}_\lambda)}{2}\frac{(\varkappa\mathbf{g}_1)}{2U_{g_1}} \pm \frac{(\varkappa\mathbf{g}_1)}{2U_{g_1}}\frac{(\varkappa\mathbf{g}_2)}{2U_{g_2}}k_{a\parallel}(\mathbf{e}_\parallel\mathbf{u}_\lambda)\right]. \tag{28}$$

Contributing to these equations are terms starting with those linear in the parameter of the deviation from the Bragg condition in the final state of the electron. An important role is played also by allowance for the quadratic terms [the third terms in (28)], owing to the compensation, as in the preceding case, of the smallness by the large longitudinal momentum of the electron.

Separating the symmetrical and antisymmetrical parts of the tensor $g_{1\mu}g_{2\nu}$ in (28), we can represent the latter in the form

$$H_\lambda(11\rightleftarrows 22) = \mp A(\omega)\left[V_+(\mathbf{D}\mathbf{u}_\lambda) + V_-([\mathbf{n}_\varkappa\times\mathbf{m}]\mathbf{u}_\lambda) \pm \frac{1}{2}(\mathbf{D}\mathbf{n}_\varkappa)(\mathbf{e}_\parallel\mathbf{u}_\lambda)\hbar\omega\beta_\parallel n\right],$$

$$H_\lambda(12\rightleftarrows 21) = \pm A(\omega)\left[V_-(\mathbf{D}\mathbf{u}_\lambda) + V_+([\mathbf{n}_\varkappa\times\mathbf{m}]\mathbf{u}_\lambda) \pm \frac{1}{2}(\mathbf{D}\mathbf{n}_\varkappa)(\mathbf{e}_\parallel\mathbf{u}_\lambda)\hbar\omega\beta_\parallel n\right], \tag{29}$$

where

$$A(\omega) = \frac{e}{2}v_{\perp 1}\beta_{\perp 2}\left(\frac{2\pi\hbar}{\omega}\right)^{1/2}\frac{\hbar\omega}{V_+^2 - V_-^2} = \frac{ev_{\perp 1}\beta_{\perp 2}\hbar\omega}{8V_{g_1}V_{g_2}}\left(\frac{2\pi\hbar}{\omega}\right)^{1/2},$$

$$\mathbf{D} = \mathbf{e}_1(\mathbf{n}_\varkappa\mathbf{e}_2) + \mathbf{e}_2(\mathbf{n}_\varkappa\mathbf{e}_1),\quad [\mathbf{n}_\varkappa\times\mathbf{m}] = \mathbf{e}_1(\mathbf{n}_\varkappa\mathbf{e}_2) - \mathbf{e}_2(\mathbf{n}_\varkappa\mathbf{e}_1). \tag{30}$$

The components of the vector $\mathbf{D}$ are of the form $\mathbf{D}_\mu = q_{\mu\nu}n_{\varkappa\nu}$, where $q_{\mu\nu} = e_{1\mu}e_{2\nu} + e_{2\mu}e_{1\nu}$ has an electric quadrupole moment structure. The quantity $\mathbf{m} = \mathbf{e}_1 \times \mathbf{e}_2 \equiv \mathbf{e}_\parallel$ has a magnetic dipole moment structure.

Thus, it follows from the form of the matrix elements (29) that the radiation produced in transitions of the third group is similar to radiation of a system having alternating magnetic dipole and electric quadrupole moments.

Denoting by $\mathbf{u}_1$ the polarization vector in the plane of the vectors $\mathbf{n}_\varkappa$ and $\mathbf{e}_\parallel$ ($\mathbf{u}_2$ will lie in a plane perpendicular to the direction of $\mathbf{e}_\parallel$), we have for the matrix elements that correspond to different polarizations, in a spherical coordinate system,

$$\left|H_1\begin{pmatrix}11\rightleftarrows 22\\12\rightleftarrows 21\end{pmatrix}\right| = A(\omega_s)\left|\frac{V_s\sin\theta\sin 2\varphi(\cos\theta - \beta_\parallel n)}{1 - \beta_\parallel n\cos\theta}\right|,$$

$$\left|H_2\begin{pmatrix}11\rightleftarrows 22\\12\rightleftarrows 21\end{pmatrix}\right| = A(\omega_s)|\sin\theta(V_s\cos 2\varphi - V_{-s})|. \tag{31}$$

Here ω_s is defined by Eq. (27). As a result we obtain for the numbers of the photons with frequencies ω_+ and ω_-, emitted by a diffracting electron per unit time into a unit solid angle,

$$\frac{\partial N_p^{\omega_\pm}}{do} = \frac{\alpha n^3\beta_{\perp 1}^2\beta_{\perp 2}^2}{32\pi\omega_{01}^2\omega_{02}^2}\omega_{0\pm}^3\sin^2\theta\; F_\pm(\theta,\varphi), \tag{32}$$

where

$$F_\pm(\theta,\varphi) = \frac{(\omega_{0+}^2 - 2\omega_{0+}\omega_{0-}\cos 2\varphi + \omega_{0-}^2)(1 - \beta_\parallel n\cos\theta)^2 - \omega_{0\pm}^2(1 - \beta_\parallel^2 n^2)\sin^2 2\varphi\sin^2\theta}{(1 - \beta_\parallel n\cos\theta)^6}. \tag{33}$$

The term containing the quantity $2\omega_{0+}\omega_{0-}\cos 2\varphi$ in (33) describes the interference between the electric quadrupole and the magnetic dipole radiations. Upon averaging over the angle φ, the interference term vanishes, and (32) can be represented in the form

$$dN_\gamma^{\omega_\pm}/do = dN_Q^{\omega_\pm}/do + dN_M^{\omega_\pm}/do, \tag{34}$$

where $dN_Q^\pm/do$, $dN_m^\pm/do$ are the intensities of the quadrupole and magnetic dipole radiations, respectively:

$$\frac{dN_Q^{\omega_\pm}}{do} = \frac{\alpha n^3}{32\pi}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\omega_{0\pm}^5 \sin^2\theta \frac{\left[(1-\beta_\parallel n\cos\theta)^2 - \frac{1}{2}(1-\beta_\parallel^2 n^2)\sin^2\theta\right]}{(1-\beta_\parallel n\cos\theta)^6}$$
$$\frac{dN_M^{\omega_\pm}}{do} = \frac{\alpha n^3}{32\pi}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\omega_{0\mp}^2\omega_{0\pm}^3 \frac{\sin^2\theta}{(1-\beta_\parallel n\cos\theta)^4}. \tag{35}$$

We note that the angular distribution of the radiation at the frequencies ω_+ and ω_- differ substantially from the distribution of the electric dipole radiation at the frequencies ω_1 and ω_2 [Eqs. (21) and (22)]. In particular, at zero angle we have zero intensity for the radiation at the frequencies ω_+ and ω_-, whereas for ω_1 and ω_2 the intensity is a maximum.

The expressions presented are valid both at $\theta > \theta_0$ [$\theta_0 = \arccos(1/\beta_\parallel n)$] in the region of the normal Doppler effect. The difficulties connected with the infinitely large Doppler frequency and intensity at $\theta \approx \theta_0$ for the case $n = \text{const}$ are eliminated if account is taken of the dispersion of the medium, i.e., of the $n(\omega)$ dependence. A complicated Doppler effect appears in this case,[9] and the equations for the angular distributions acquire an additional factor[4)] (ref. 10), equal to $(1-\beta n\cos\theta)/[1-\beta(n+\omega dn/d\omega)\cos\theta]$ (due to differentiation of the arguments of the δ-functions).

For the total numbers of photons with frequencies ω_+ and ω_- radiated in a unit time, integration of (34) and (35) at $n = \text{const}$ and $n\beta_\parallel < 1$ yields

$$N_\gamma^{\omega_\pm} = N_Q^{\omega_\pm} + N_M^{\omega_\pm}, \tag{36}$$

where

$$N_Q^{\omega_\pm} = \frac{\alpha n^3}{20}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\frac{\omega_{0\pm}^5}{(1-n^2\beta_\parallel^2)^2}, \quad N_M^{\omega_\pm} = \frac{\alpha n^3}{12}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\frac{\omega_{0\mp}^2\omega_{0\pm}^3}{(1-n^2\beta_\parallel^2)^2}. \tag{37}$$

At $n=1$ and at high electron energies ($\beta_\parallel \approx \beta$, $\gamma \gg 1$) we have $N_{Qm}^{\omega\pm} \sim \beta_{\perp 1}^2\beta_{\perp 2}^2\gamma^4$; consequently the total number of the emitted photons does not depend on the electron energy, just as in the case of electric dipole radiation. The angular distributions (35) take then the form

$$\frac{dN_Q^{\omega_\pm}}{d\theta^2} = \frac{\alpha}{2}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\omega_{0\pm}^5\frac{\theta^2(\theta^4+\gamma^{-4})}{(\theta^2+\gamma^{-2})^6},$$
$$\frac{dN_M^{\omega_\pm}}{d\theta^2} = \frac{\alpha}{2}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\omega_{0\mp}^2\omega_{0\pm}^3\frac{\theta^2}{(\theta^2+\gamma^{-2})^4}. \tag{38}$$

We note that at equal structure factors of the systems of the reflecting planes, i.e., at $V_{g_1} = V_{g_2}$, $\omega_{0-} = 0$, and $\omega_{0+} = 2\omega_{01}$, the magnetic dipole radiation vanishes. The classical analog that can be compared with the transitions 11 ⟷ 22 and 12 ⟷ 21 is a system of oscillating charges moving in the $e_\parallel$ direction, as shown in Fig. 1(b). The oscillation laws are the following:

$$x_{1,4} = -x_m\cos\omega_{01}t, \quad x_{2,3} = x_m\cos\omega_{01}t,$$
$$y_{1,2} = y_m\cos\omega_{02}t, \quad y_{3,4} = -y_m\cos\omega_{02}t. \tag{39}$$

The system in Fig. 1(b) is shown as the instant of time $t=0$. It has electric quadrupole and magnetic dipole moments. The electric quadrupole moment $Q_{\mu\nu}$ has two nonzero components:

$$Q_{xy} = Q_{yx} \equiv Q = 12ex_my_m\cos\omega_{01}t\cos\omega_{02}t,$$

which oscillate at two frequencies ω_{0+} and ω_{0-}. Recognizing that $x_m = v_{\perp 1}/\omega_{01}$ and $y_m = v_{\perp 2}/\omega_{02}$, we have

$$Q = Q_{\omega+}\cos\omega_{0+}t + Q_{\omega-}\cos\omega_{0-}t,$$

where

$$Q_{\omega\pm} = 6ev_{\perp 1}v_{\perp 2}/\omega_{01}\omega_{02}.$$

The magnetic moment μ is directed along the z axis, i.e., $\mu = \mu e_\parallel$, where

$$\mu = \frac{1}{2c}\sum_{i=1}^4 e_i(x_i\dot{y}_i - y_i\dot{x}_i)$$
$$= -\frac{2ex_my_m}{c}(\omega_{02}\cos\omega_{01}t\sin\omega_{02}t - \omega_{01}\cos\omega_{02}t\sin\omega_{01}t).$$

Similarly

$$\mu = \mu_{\omega+}\sin\omega_{0+}t + \mu_{\omega-}\sin\omega_{0-}t,$$

where

$$\mu_{\omega\pm} = ev_{\perp 1}v_{\perp 2}\omega_{0\mp}/c\omega_{01}\omega_{02}$$

are the amplitudes of the oscillations of the magnetic moment at the frequencies $\omega_{0\pm}$.

Using the classical formula for the radiation intensity of the electric quadrupole and of the magnetic dipole[11] ($\beta_\parallel = 0$, $n=1$):

$$I = \frac{1}{180c^5}\dddot{Q}_{\mu\nu}^2 + \frac{2}{3c^3}\ddot{\mu}^2 \tag{40}$$

and dividing it by $\hbar\omega_{0\pm}$, we obtain for the numbers of photons with frequencies ω_{0+} and ω_{0-} emitted per unit time

$$N_Q^{\omega_\pm} = \frac{\omega_{0\pm}^5}{180\hbar c^5}Q_{\omega_\pm}^2 = \frac{\alpha}{5}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\omega_{0\pm}^5,$$
$$N_M^{\omega_\pm} = \frac{\omega_{0\pm}^3}{3\hbar c^3}\mu_{\omega_\pm}^2 = \frac{\alpha}{3}\frac{\beta_{\perp 1}^2\beta_{\perp 2}^2}{\omega_{01}^2\omega_{02}^2}\omega_{0\mp}^2\omega_{0\pm}^3. \tag{41}$$

The equations in (41) coincide with those in (37) at $n=1$ and $\beta_\parallel = 0$, if it is recognized that the expressions in (37) contain an additional factor 1/4 that determines the population of the states $(11)_{E_a}$, $(12)_{E_a}$, $(22)_{E_a}$, and $(21)_{E_a}$ from which the transitions take place.

This analogy has the following physical meaning. In transitions of an electron with change of the symmetry of the wave functions in both directions e_1 and e_2, the electron density becomes periodically distributed, over the thickness of the crystal, between the direct and diffracted beams. This redistribution is due to beats between the electron waves simultaneously in two directions. In the case of electric dipole transitions, the beats take place in each of the directions independently.

To compare the intensities of the different radiation types, we consider the case $n=1$, $\beta_\parallel \approx \beta$ and $g_1 = g_2 = 2\pi/d$, where d is the distance between the planes; in this case $\beta_{\perp 1} = \beta_{\perp 2} = \beta_\perp$. Let $\omega_{01}/\omega_{02} = \eta$ ($\eta \geq 1$) and $\omega_{02} = \omega_0$; then $\omega_{01} = \eta\omega_0$ and $\omega_{0\pm} = (\eta \pm 1)\omega_0$. Putting

$$N_\gamma^{\omega_2} \equiv N = {}^2/_3\alpha\beta_{\perp 0}^2\omega_0, \tag{42}$$

where $\beta_{\perp 0} = \pi\hbar/dm_0c$ is the transverse velocity corresponding to the rest mass of the electron m_0, we obtain from (23) and (37)

$$N_\gamma^{\omega_1} = \eta N, \quad N_Q^{\omega_\pm} = \frac{3}{40}\frac{(\eta\pm1)^5}{\eta^2}\beta_{\perp 0}^2 N, \quad N_M^{\omega_\pm} = \frac{1}{8}\frac{(\eta\mp1)^2(\eta\pm1)^3}{\eta^2}\beta_{\perp 0}^2 N. \tag{43}$$

At $d=1$ Å we have $\beta_{\perp 0}$ $1.2\cdot 10^{-2}$. Thus, the intensity of the radiation of the higher multipoles is suppressed by a factor ~10^4 compared with the electric dipole radiation, on account of the factor $\beta_{\perp 0}^2$, and depends substantially on the frequency ratio η. For example, at $\eta=3$ (in this case the spectrum of the photons radiated in the specified direction is equidistant) we have $N_p^{\omega_1}=3N$; $N_Q^{\omega_+}\approx 1.2\cdot 10^{-3}N$; $N_M^{\omega_+}\approx 5.2\cdot 10^{-4}N$; $N_Q^{\omega_-}\approx 0.4\cdot 10^{-4}N$; $N_M^{\omega_-}\approx 2.6\times 10^{-4}N$. After traversing in the crystal a distance equal to one extinction length $\xi_{g_2}=2\pi v_\parallel/\omega_0$, the diffracting electron radiates $N_\xi^{\omega_2}=2\pi N/\omega_0$ photons with frequencies ω_2:

$$N_\xi^{\omega_2}=4\pi\alpha\beta_{\perp 0}^2/3.$$

The corresponding number of photons with other frequencies ω_1 and $\omega_\pm$, radiated over the same distance, is determined by Eqs. (43). In the example considered above $N_\xi^{\omega_2}=4.4\cdot 10^{-6}$, so that passage through a crystal of thickness ~$5\xi_{g_2}$ of an electron current of 1 μA ($6\cdot 10^{12}$ electrons/sec) will cause the emission of $\approx 10^8$ sec^{-1} photons with frequencies ω_1 and ω_2 on account of electric dipole transitions, and ~10^4–10^5 sec^{-1} photons with frequencies ω_+ and ω_- on account of electric quadrupole and magnetic dipole transitions. We note that in the region of optical frequencies the intensity of the latter can be relatively increased on account of the additional factor $n^2/(1-n^2\beta_\parallel^2)$.

7. CONCLUSION

The equations derived above for the Pendellosung radiation intensity are based in fact on a two-wave approximation, relative to each of the directions $\mathbf{e}_1$ and $\mathbf{e}_2$, of the dynamic theory of diffraction. At sufficiently high electron energies this approximation does not hold. Indeed, with increasing energy, the effective crystal reflectivity $U_{g_j}\sim\gamma$ increases, and at sufficiently high energies there are excited in the crystal, besides the electron waves with wave vectors $\mathbf{k}$ and $\mathbf{k}+\mathbf{g}_j$, also waves with vectors $\mathbf{k}-\mathbf{g}_j$ and $\mathbf{k}+2\mathbf{g}_j$ (systematic reflections[5]). Their amplitude is low if the corresponding parameters of the deviation from the Bragg condition are large compared with U_{g_j}, i.e., $g_j^2\gg U_{g_j}$ ($j=1,2$) or (we omit the indices 1 and 2 from now on):

$$\hbar\omega_\perp/V_g\gamma\gg 1,$$

where $\omega_\perp=2\pi v_{\perp 0}/d$. This condition means that the transverse energy of the diffracting electron should be much higher than the amplitude of the periodic potential of the crystal lattice or, in other words, the diffraction angle width should be less than the Bragg angle. Consequently, the equations that describe the Pendellosung radiation are valid at electron energies for which

$$\gamma\ll\gamma_c=\hbar\omega_\perp/V_g. \tag{44}$$

At $d=1$ Å we have $\hbar\omega_\perp\approx 150$ eV. At low values $V_g\approx 1$ eV we have $\gamma_c\approx 150$, so that the equations can be valid up to electron energies of several dozen MeV. At higher energies the systematic reflections become substantial and account must be taken of the higher harmonics of the periodic crystal-lattice potential.

At electron energies $\gamma>\gamma_c$ the diffraction effects give way to channeling effects,[12] inasmuch as the Bragg angles become smaller than the critical channeling angles (the transverse energy becomes less than the height of the periodic potential).

Radiation produced by channeling of fast electrons and positrons ($\gamma\gg\gamma_c$) was considered by a number of workers.[13-17] It results from transitions between levels[13-16] or, more accurately, bands[17] of transverse motion of the particle in the averaged potential of the atomic planes or chains that make up the channels. The band structure of the levels of the transverse motion in channeling is due to tunneling of the particles through a barrier into neighboring channels. In the case of diffraction, on the other hand, the energy bands are produced as a result of above-barrier reflection of the particle from the periodic potential of a system of planes. The last circumstance determines essentially the quantum character of the Pendellosung radiation, which, in contrast to channeling radiation, cannot be described in the classical limit. Indeed, as $\hbar\to 0$, as follows from (44), the region of applicability of the equations for the Pendellosung radiation becomes zero.

We note that to describe the radiation of ultrarelativistic channelled particles ($\gamma\gg\lambda_c$), above-barrier states can be considered in a quasiclassical approximation, and below-barrier states in the strong-coupling approximation.[7,10] In the intermediate energy region ($\gamma\sim\gamma_c$) the problem can be solved analytically only in certain models of the Kronig-Penney type.[17] Therefore a more constructive approach in this region may be the one based on a consistent allowance for the systematic reflections in the dynamic theory of diffraction.

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1) In the direction of the reciprocal-lattice vector $\mathbf{g}$ perpendicular to the reflecting planes. Transitions between identical branches (intraband) lead to Čerenkov radiation.[2]

2) We assume that $U_{g1}\geqslant U_{g2}>0$, with the origin located on the intersection of the planes.

3) Effect connected with interference between different transition are not considered. For crystals of finite size they can lead to oscillations of the intensity with the crystal thickness.[2,7]

4) The author thanks V. G. Baryshevskiĭ for calling his attention to this circumstance.

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